

1 Intro

What is the goal? Intuitive understanding of the Fourier/Lagrange transforms.

What is the Fourier/Lagrange transform used for? It transforms differential equations into algebraic equations. Give example

Two main questions arise

- Why is this the case?
- How can we interpret this?

2 Vector explanation

We are going to use an analogy between ordinary vector and functions. Here it is used in a heuristic way, but it can be made rigorous.

We take the simplest equation

$$D\mathbf{x} = \mathbf{b}.$$

In this case since we are thinking about \mathbf{x} and \mathbf{b} as ordinary vector the operator D can be a linear transformation, represent by some matrix.

The effect of some matrix is this, with this basis vectors and this with these basis vector.

In order to solve our problem in the simplest way possible what would be the ideal case scenario? How could we choose the basis of we are using to render the problem as simple as possible?

Ideally applying the operator D on a vector \mathbf{x} would be multiplying it a constant. Thus,

$$D\mathbf{x} = \lambda\mathbf{x}.$$

For most matrices this problem will render a set of eigen vectors and eigen values, which can be interpreted as direction which only suffer extension or compression.

To change to this new simpler basic style we do

$$\mathbf{x}'_S = \sum_i \langle \mathbf{x}, \mathbf{v}_i \rangle \mathbf{v}_i.$$

This, is the same as multiplying \mathbf{x} a matrix L

So now we have

$$LDL^{-1}L\mathbf{x} = \mathbf{b},$$

or

$$D'\mathbf{x}' = \mathbf{b}',$$

In this new coordinate system, for most linear transformations, matrix D' will be diagonal.

3 Moving on to functions

Intuition for functions as vectors.

Same problem but now D .

$$Df = g,$$

The simplest effect the operator could have is to multiply by a number.

$$-iDf = \lambda f.$$

So

$$f(x) = \frac{e^{-i\lambda x}}{\sqrt{\pi}}.$$

$$f(x) = \sum_i \langle f, \varphi_i \rangle \varphi_i,$$

Substituting we get

Now if the function is not periodic

$$f^*(x) = \int_{-\infty}^{\infty} f(y) e^{-ixy} dy$$

$$\mathcal{L}(Df) = s\mathcal{L}(f) - f(0).$$