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Composite Materials
HW 2

Problem 1. A carbon/epoxy (AS/3501) unidirectional laminate with a fiber orientation $\theta = 30°$ is exposed to the following stresses, $\sigma_x = 40$ ksi, $\sigma_y =$ -20 ksi, $\sigma_{xy} = 20$ ksi. Determine the engineering strains in the fiber coordinate system. Use the following typical properties for carbon/epoxy $E_1 = 18.4$ Msi, $E_2 = 1.6$ Msi, $G_{12} = 0.95$ Msi, $\nu_{12} = 0.28$

$$|\sigma_{12}| = [T]|\sigma_{xy}|$$
$$|\sigma_{12}| = [Q]|\gamma_{12}|$$

Tensor stress in coordinates of xy can be related to engineering strain in fiber coordinates by applying a transformation matrix to transform from xy to 12, then inverse Q to transform from stress to strain, and finally an inverse Reuter's matrix to transform from tensor to engineering units.

$$|\gamma_{12}| = [R^{-1}][Q^{-1}][T]|\sigma_{xy}|$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} E_1/\Delta & \nu_{12}E_1/\Delta & 0 \\ \nu_{12}E_1/\Delta & E_2/\Delta & 0 \\ 0 & 0 & G_{12} \end{bmatrix}^{-1} \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.06264 \\ -0.2156 \\ -0.008411 \end{bmatrix}$$

Problem 2. An off-axis tensile specimen of an orthotropic lamina with fiber orientation $\theta = 45°$ is subjected to a known uniaxial stress, σ_x, and the resulting normal strains, ϵ_x, and ϵ_y are measured with strain gages. Show how the in-plane shear modulus, G_{12}, can be determined from the measured stress and strain data. Assume that no other data are available.

Following from the transformation matrix T, I may solve for γ_{12} in terms of longitudinal strain and transverse strain:

$$\frac{\gamma_{12}}{2} = -\cos(45°)\sin(45°)\epsilon_x + \cos(45°)\sin(45°)\epsilon_y + (\cos^2(45°) - \sin^2(45°))\frac{\gamma_{xy}}{2}$$

$$\gamma_{12} = -\epsilon_x + \epsilon_y$$

Knowing that σ_x is the only non-zero stress, my equation for τ_{12} simplifies a lot.

$$\tau_{12} = -\cos(45°)\sin(45°)\sigma_x = -\frac{\sigma_x}{2}$$

Finally, I may plug in the computed values into my expression for G_{12}.

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{\sigma_x}{2(\epsilon_x - \epsilon_y)}$$

Problem 3. Derive the rule of mixtures equations for series and parallel material combinations.

Longitudinal Modulus:
$\sigma_{c1} A_c = \sigma_{F1} A_F + \sigma_{m1} A_m$
Divide both sides by A_c to get the packing density term.
$\sigma_{c1} = \sigma_{F1} V_F + \sigma_{m1} V_m$
Substitute in modulus of elasticity.
$E_1 \epsilon_1 = E_{F1} \epsilon_{F1} V_F + E_m \epsilon_{m1} V_m$
All strain terms are equal. $E_1 = E_{F1} V_F + E_m V_m$
Parallel combination.

Poisson's Ration:
$\nu_{12} = -\epsilon_{c2}/\epsilon_{c1}$
Substitute in the longitudinal modulus equation.
$\nu_{12} = (\epsilon_{F1} V_F + \epsilon_{m1} V_m)/\epsilon_{c1}$
Strains are equal.
$\nu_{12} = (\nu_{F12} V_F + \nu_m V_m)$
Parallel combination.

Transverse Modulus:
$\epsilon_{c2} L_2 = \epsilon_{F2} L_F + \epsilon_{m2} L_m$
Divide by length to get packing density.
$\epsilon_{c2} = \epsilon_{F2} V_F + \epsilon_{m2} V_m$
Substitute in modulus of elasticity relation.
$\frac{\sigma_{c2}}{E_{c2}} = \frac{\sigma_{F2}}{E_F} V_F + \frac{\sigma_{m2}}{E_m} V_m$
Assume that stress terms are equal (bad assumption!!)
$\frac{1}{E_{c2}} = \frac{V_F}{E_F} + \frac{V_m}{E_m}$
Series combination.

Shear modulus:
$\gamma_c L_2 = \Delta F + \Delta m$
$\gamma_c L_2 = L_f \delta_F + L_m \delta_m$
$\frac{\tau_{12}}{G_{12}} = \frac{\tau_F}{G_F} V_F + \frac{\tau_m}{G_m} V_m$
Assume that shear stress terms are equal (bad assumption!!)
$\frac{1}{G_{12}} = \frac{V_F}{G_F} + \frac{V_m}{G_m}$
Series combination.

Problem 4. Show that the Halpin-Tsai relation reduces to the series and parallel rule of mixtures expressions when $\xi = 0$ and $\xi = \infty$, respectively.

$$\frac{M}{M_m} = \frac{1+\xi\eta V_F}{1-\eta V_F}, \quad \text{where } \eta = \frac{M_F/M_m - 1}{M_F/M_m + \xi}$$

Evaluated at $\xi = 0$.

$$\frac{M}{M_m} = \frac{1}{1 - V_F[\frac{M_m}{M_F}(M_F/M_m - 1)]}$$

$$\frac{M}{M_m} = \frac{1}{1 - V_F + V_F M_m/M_F}$$

$$\frac{M_m}{M} = 1 - V_F + V_F M_m/M_F$$

$$\frac{1}{M} = \frac{1-V_F}{M_m} + \frac{V_F}{M_F}$$

$$\frac{1}{M} = \frac{V_m}{M_m} + \frac{V_F}{M_F}$$

Series combination.

Evaluated at $\xi = \infty$.

$$\eta = \frac{M_F/M_m - 1}{M_F/M_m + \xi} = \frac{M_F/M_m - 1}{\infty} = 0$$

$$\xi\eta = \xi\frac{M_F/M_m - 1}{M_F/M_m + \xi} = (\frac{\xi}{\xi})\frac{M_F/M_m - 1}{M_F/(M_m\xi) + 1} = \frac{M_F/M_m - 1}{M_F/\infty + 1} = M_F/M_m - 1$$

$\eta = 0$
$\xi\eta = M_F/M_m - 1$

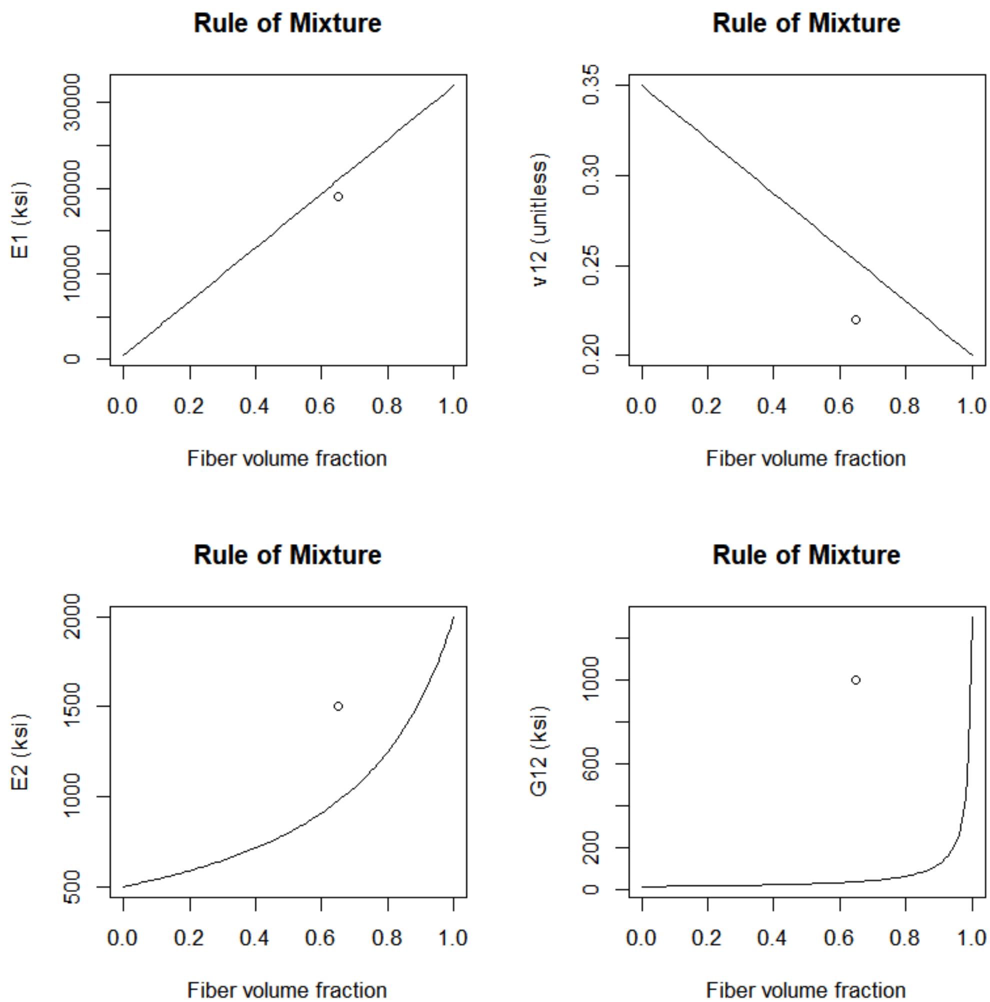
$$\frac{M}{M_m} = 1 + V_F(M_F/M_m - 1)$$

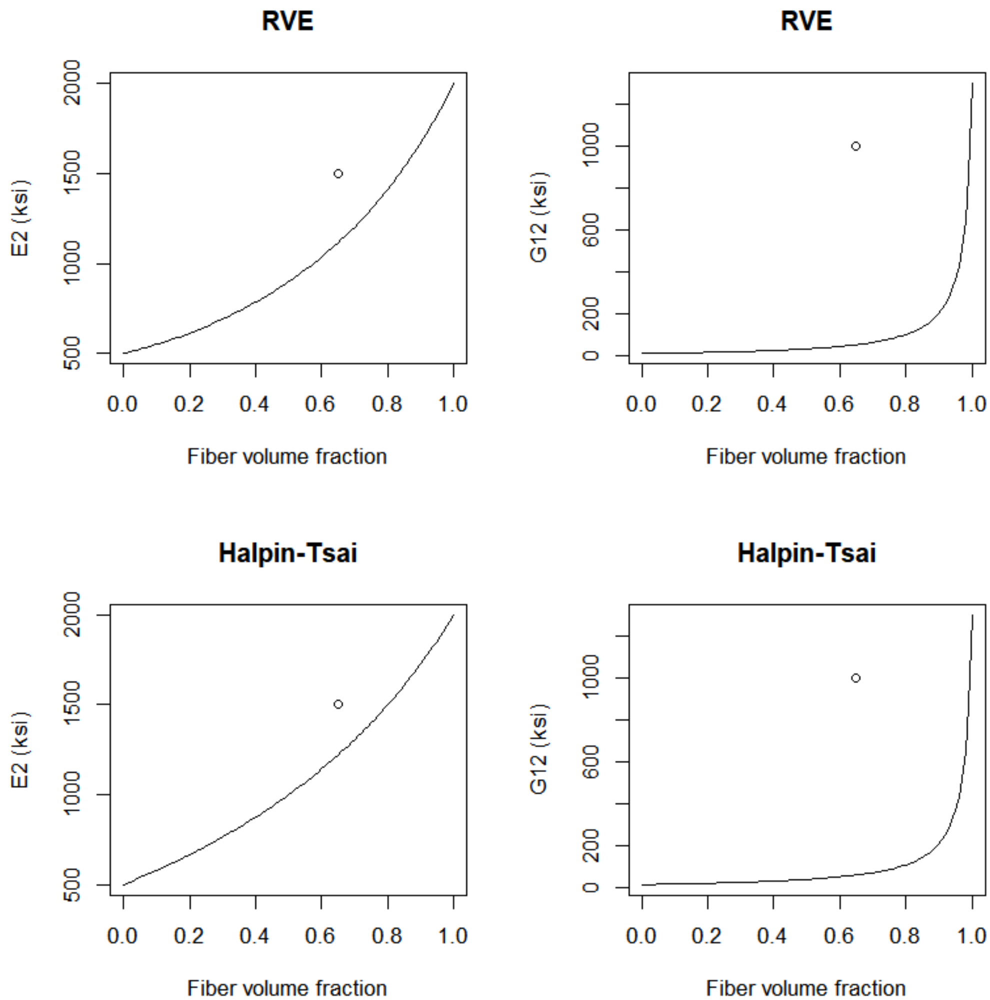
$$M = (1-V_F)M_m + V_F M_F$$

$$M = V_m M_m + V_F M_F$$

Parallel combination.

Problem 5. In the following, use the data from Table 3.1 and 3.2 for T300 carbon fiber, and intermediate modulus high strength epoxy. Plot E_1, E_2, G_{12}, and ν_{12} vs. V_F, using the rule of mixtures, RVE, and Halpsin-Tsai relations. Compare these plots with data found in Table 2.2





Values from Table 2.2 are plotted as a circular point. Each relation is plotted across values for fiber volume fraction from 0 to 1. No relation is able to get a good estimate for shear modulus (G12). Halpin-Tsai gets closest to predicting E2 correclty, although it is still ¿250 ksi off. The Rule of Mixtures predicts E1 well, although it fails to predict ν_{12} accurately.