CS365
Homework #2

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6. (a) There exists a student in my school who has visited North Dakota.
   (c) There does not exist a student in my school who has visited North Dakota.
   (f) No students in my school have visited North Dakota.

12. (d) $\exists x Q(x)$ says that for some x, $x + 1 > 2x$.
    This statement is **True**. (for example when $x = 0$)
    (e) $\forall x Q(x)$ says that for every x, $x + 1 > 2x$.
    This statement is **False**. (for example when $x = 1$)
    (g) $\forall x \neg Q(x)$ says that for every x, $x + 1 \leq 2x$.
    This statement is **False**. (for example when $x = 0$)

24. $F(x) : x$ is in your class.
    (a) $G(x) : x$ has a cellular phone.
        i. $\forall x G(x)$
        ii. $\forall x (F(x) \to G(x))$
    (c) $H(x) : x$ cannot swim.
        i. $\exists x H(x)$
        ii. $\exists x (F(x) \wedge H(x))$
    (e) $I(x) : x$ does not want to be rich.
        i. $\exists x I(x)$
        ii. $\exists x (F(x) \wedge I(x))$

32. (a) $F(x) : x$ has fleas.
    Domain of x is all dogs
    i. $\forall x F(x)$
    ii. $\exists x \neg F(x)$
    iii. There is a dog that does not have fleas.
(b) $G(x) : x$ can add.
    Domain of x is all horses
    i. $\exists x G(x)$
    ii. $\forall x \neg G(x)$
    iii. There are no horses that can add.
(e) $H(x) : x$ can swim.
    $I(x) : x$ can catch fish.
    Domain of x is all pigs.
    i. $\exists x (H(x) \wedge I(x))$
    ii. $\forall x (\neg H(x) \vee \neg I(x))$
    iii. There are no pigs that can catch fish and swim.

60. (a) $\forall x (P(x) \rightarrow Q(x))$
(b) $\exists x (R(x) \wedge \neg Q(x))$
(c) $\exists x (R(x) \wedge \neg P(x)$
(d) Just because all clear explanations are satisfactory and some excuses are not satisfactory then there is no guarantee that an unclear excuse exist. **No** (c) does not follow from (a) and (b).

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10. (c) $\forall x \exists y F(x, y)$
(e) $\forall y \exists x F(x, y)$
(j) $\exists x \exists y ((x \neq y) \wedge F(x, y) \wedge \forall z ((F(x, z) \wedge x \neq z) \rightarrow (z = y)))$

20. (a) $\forall x \forall y (((x < 0) \wedge (y < 0)) \rightarrow (x \times y > 0))$
(b) $\forall x \forall y (((x > 0) \wedge (y > 0)) \rightarrow (\frac{x + y}{2} > 0))$
(c) $\neg \forall x \forall y (((x < 0) \wedge (y < 0)) \rightarrow (x - y > 0)))$
(d) $\forall x \forall y (|x + y| \leq |x| + |y|)$

28. (e) This reads... For every real number x that is not zero, there exists another real number y such that $x \times y = 1$. This is **True** because if $x \times y = 1$ then $y = \dfrac{1}{x}$ which means y exists as long as $x \neq 0$.

(f) This reads... There exists some real number x that for every non-zero real number y $x \times y = 1$. This is **False** because there is no such number.

(i) This reads... For every real number x there exists a real number y such that $x + y = 2$ and $2x - y = 1$. This is **False** because there are numbers that exists where no real number can satisfy both of these equations. For example if $x = 2$ then for $x + y = 2$, $y = 0$, but for $2x - y = 1$, $y = 3$.

(j) This reads... For all real numbers x and y there exists another real number z where $z = \dfrac{x+y}{2}$. This is **True** because the expression $\dfrac{x+y}{2}$ always evaluates to a real number given that x and y are also real numbers.

40. (a) If $x = 2$ then y must be equal to $\dfrac{1}{2}$, which is not an integer value.

(b) If $x = -100$ then $y^2 - (-100) < 100$ evaluates to $y^2 < 0$ which is not true for any integer values.

(c) If $x = 1000$ and $y = 100$ then $x^2 = y^3$, which means that for every x and y it is not true that $x^2 \neq y^3$.