

1 Modeling transient mortality shocks in  
2 low-mortality populations

3 Joshua R. Goldstein  
4 Ronald D. Lee  
Proposal for Presentation at PAA 2021  
Extended Abstract  
5 September 23, 2020

6 **Abstract**

7 The standard Lee-Carter model is useful for looking at the long-  
8 term evolution of progress in longevity but does not include the effect  
9 of short-term, transient shocks such as the life expectancy declines  
10 seen in recent years in the United States or the coronavirus epidemic.  
11 In this work, we extend the Lee-Carter approach by adding an an-  
12 nual transitory component, meant to represent the effects of weather  
13 and infectious diseases. Our preliminary findings show that the model  
14 works well for some, but not all, countries. The transient shocks we  
15 detect are correlated across countries, suggesting that they are picking  
16 up real external shocks to mortality, rather than measurement error.  
17 Our hope is that these models will be of use to understanding the mag-  
18 nitude and nature of modern mortality shocks and the implications of  
19 these shocks for future mortality trends.

20 **1 Overview**

21 The canonical time-series model used for studying modern mortality trends is  
22 the Lee-Carter’s model of the random walk with drift. The logic behind this  
23 is that there is a steady march of progress toward lower mortality which varies  
24 in its pace. The recent three-year run of declining mortality in the United

States, the French heatwave of 2003, the annual flu, and – of course - the current coronavirus pandemic are all examples of transient shocks, highlighting that factors other than technological progress and changes in population health may be important for mortality in any given year.

Modeling these short-term transient shocks is interesting in its own right, because of what it reveals about the nature of population mortality levels and changes. It may also be useful for understanding the long-run implications of mortality reversals like that from the U.S. opioid crisis or the worldwide coronavirus pandemic.

Here we present a model and preliminary results for the modeling of transient mortality shocks based on Lee-Carter estimates of the evolution of mortality over time. The model we fit includes two random terms: the first is the standard term for the evolution of the underlying trend in the Lee-Carter model, a random walk with deterministic drift; the second is a new term for transient shocks, which we interpret as single-year events due to weather or annual infectious diseases like the flu.

These preliminary models only work well for some countries. Nonetheless it is still possible to see that the transient shocks of neighbors are correlated, suggesting that they are picking up real shocks to mortality, perhaps caused by the effects of severe weather and contagious diseases.

## 2 Modeling

The approach we use to model mortality shocks are based on structural time series approaches for distinguishing between long-lasting changes from transient shocks. These models include two kinds of random terms, one which influences the underlying state of the system and an additional term that translates the state of the system into what is observed at a given time. In our application, we think of mortality rates as consisting of an underlying level of technology and population health that evolves slowly over time, combining with short-term fluctuations in conditions such as the weather and contagious disease that can vary greatly from one year to the next. Each year’s observed mortality is the result of both factors. The model tries to separate them.

We begin by using the standard Lee-Carter approach. This model reduces the logarithm of the full set of age-period mortality rates to an average age-schedule  $a_x$  and a time index  $k_t$  that drives age-specific changes  $b_x$ . The

model has the form

$$\log M_{x,t} = a_x + b_x k_t.$$

The usual time series model for forecasting used by Lee and Carter is the random walk with drift

$$k_t = k_{t-1} + d + \epsilon_t.$$

To this, we add another layer to the estimation of the time series, decomposing the observed  $k_t$  into a latent  $k_t$  that still evolves as a random walk with drift as well as an annual transitory component  $n_t$ . In state-space modeling  $n_t$  is sometimes called “observation error” or “noise”. We are conceiving of it not so much as error but as a transitory perturbation – for example due to weather or the severity of the annual flu or to another kind of contagious disease such as COVID-19.

The model has the form

$$k_t^{\text{observed}} = k_t^{\text{latent}} + n_t \tag{1}$$

$$k_t^{\text{latent}} = k_{t-1}^{\text{latent}} + d + \epsilon_t \tag{2}$$

The model has two features.

- The observed value is the latent value plus “noise”  $n_t$ , which is assumed to be normally distributed with constant variance and independent over  $t$ .
- The latent value evolves as a random walk with drift, with a fixed (deterministic) value of  $d$ .

(Note it is also possible to add an additional random layer to this model by making  $d$  itself a stochastic evolving term. The standard form of this “random trend” model is to let  $d$  evolve as a random walk,  $d_t = d_{t-1} + \eta_{t-1}$ . Other extensions include considering non-normal distributions for  $n_t$  as well as adding time-series dependence to the evolution of  $n_t$  over time. We don’t consider these extensions here.)

### 3 Data and methods

Because we are interested in short-term fluctuations that are often small in magnitude, we limit our application to high-quality data available in the

Human Mortality Database. It would also be of interest to include a broader range of countries, but this would require a careful evaluation of data quality.

We estimate the Lee-Carter model using the **demography** package written by Hyndman et al. We model both sexes together and use the option of calibrating the estimates of observed  $k_t$  to life expectancy at birth.

We estimate the structural time series models using the MARSS (Multivariate Autoregressive State-Space Modeling) package written by Holmes et al.

We note that it is also possible to estimate short term shocks by smoothing the observed time series and looking at differences between the observed and smoothed rates. This produces similar results when we look at the correlations between countries. An advantage of the time series approach is that it makes explicit the underlying assumptions and nature of the model. A further advantage is that the explicit time series model used to fit the past can also be used for forecasting the future.

## 4 Results

We begin by showing the estimates of the latent  $k_t$  for France in Figure . The time trend is generally downward, corresponding to improvements in mortality rates over time. As expected the latent  $k_t$  is considerably smoother than the observed term. The differences between observed and latent mortality correspond in some cases to known shocks, such as the heat-wave of 2003 and the global influenza (H3N2) of 1968.

Figure 2 shows the observed and latent  $k_t$  for a larger set of 19 countries. Some cases resemble France in that one can clearly see that the latent  $k_t$  is a smoothed version of what is observed. However, in other cases, such as the United States and Russia, the estimates of the latent  $k_t$  are so similar to what is observed as to make the two lines essentially indistinguishable.

Figure 3 shows the same information as figure 2 in the form of the differences between the observed and latent mortality time trends ( $k_t$ ). These are plotted on the same scale to make it apparent that the magnitude of the estimated transient shocks are similar in many countries, including France, Sweden, Japan, Italy, Great Britain and Spain, but smaller in West Germany, and even smaller in the United States and Russia.

One reason for the small magnitudes of mortality shocks estimates for the United States and Russia is that the variations that we see in these

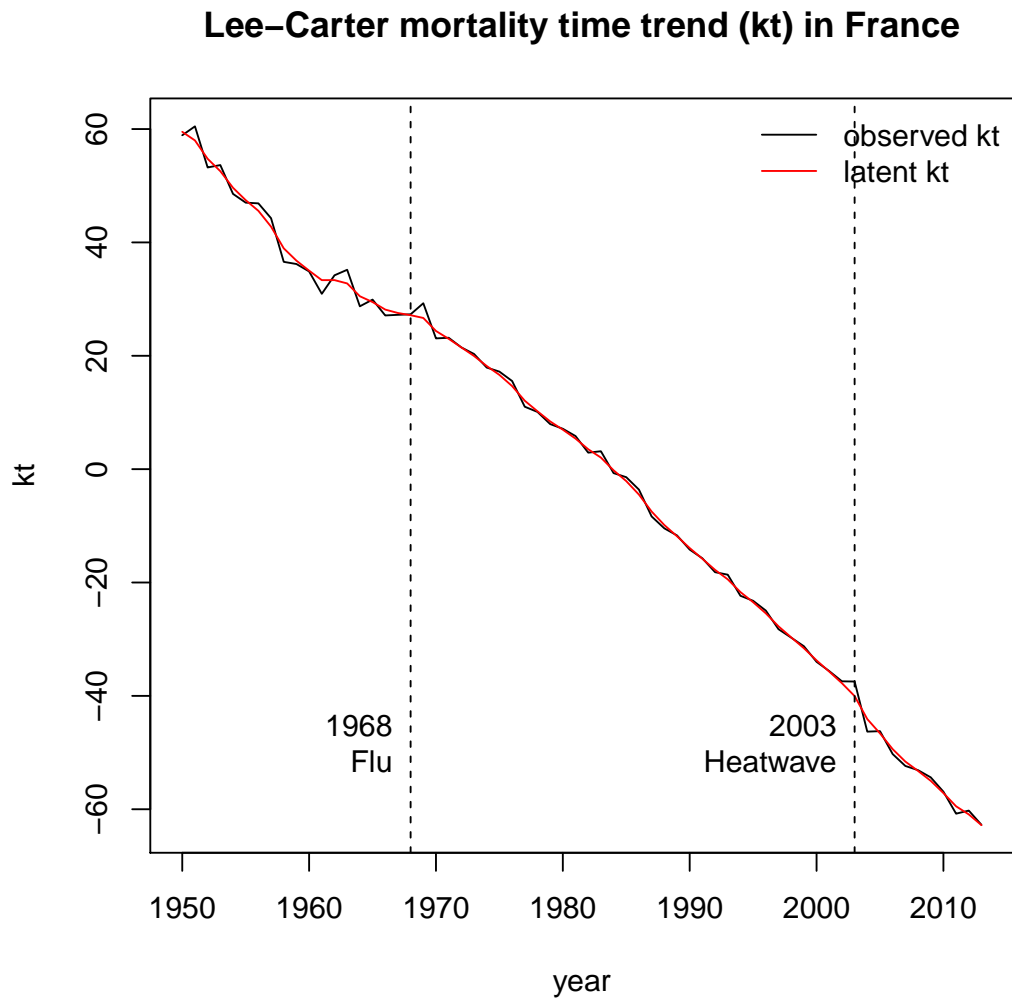


Figure 1: Estimates of observed and latent values of  $k_t$  in Lee-Carter model using MARSS package based on HMD data. Mortality shocks for 1968 Flu (which reportedly had more severe death rates in Europe in 1969) and the 2003 heatwave are indicated.

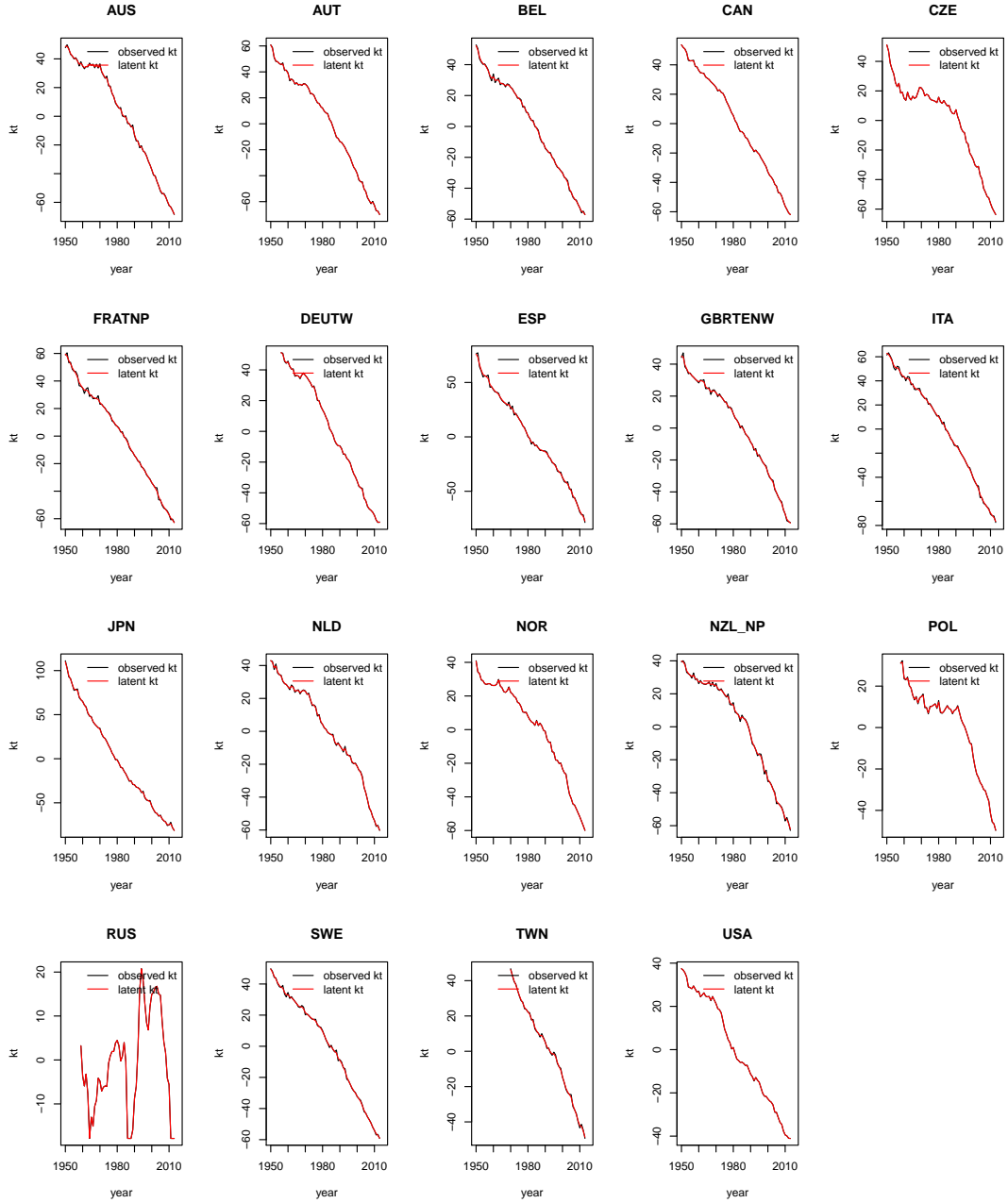


Figure 2: Estimates of observed and latent values of  $k_t$  in Lee-Carter model using MARSS package based on HMD data. Note: in some countries (e.g., Russian and the United States) the estimated shocks are so small that the latent and observed  $k_t$  values are overlapping and indistinguishable to the eye.

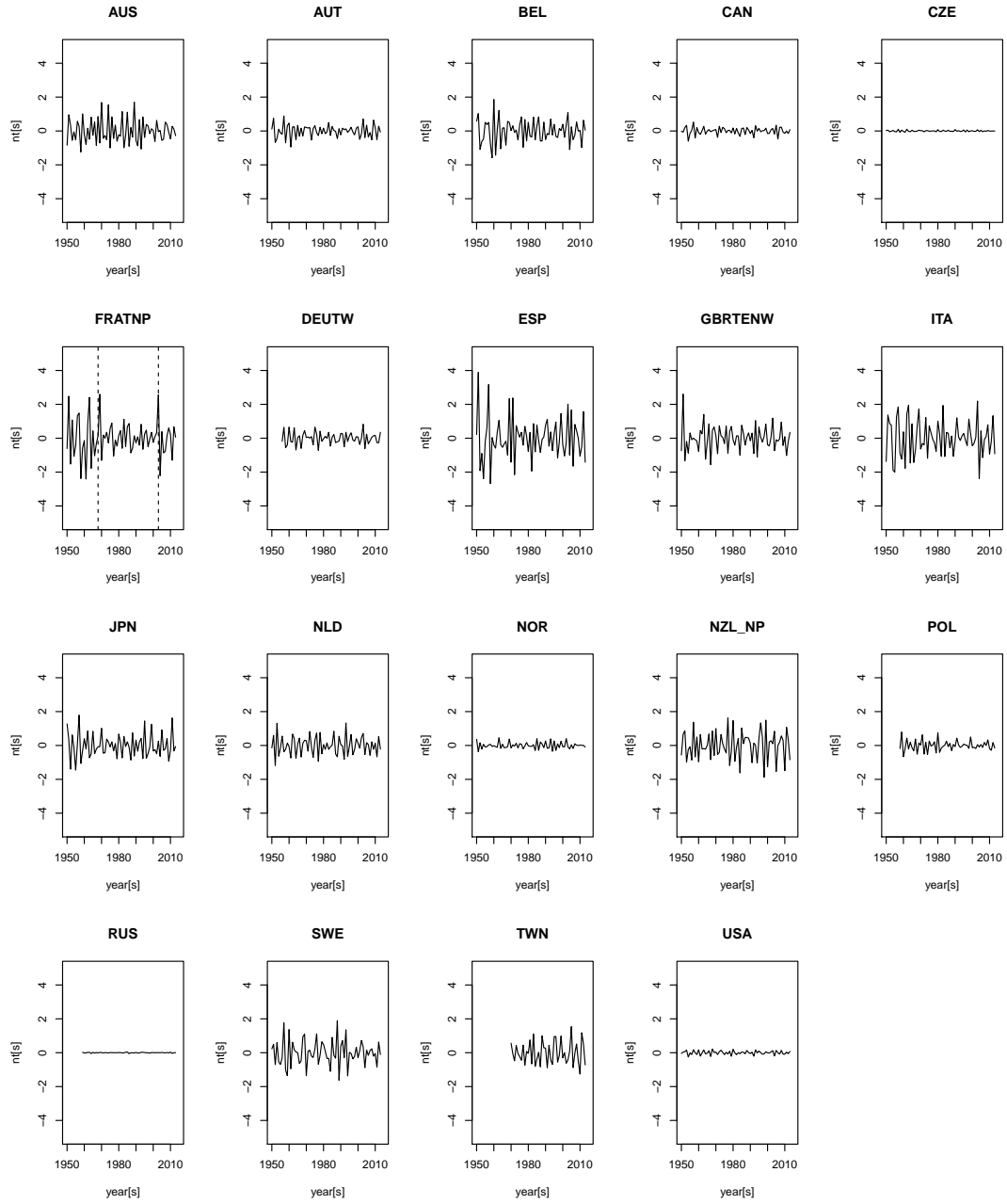


Figure 3: Estimated shocks  $n_t = k_t^{latent} - k_t^{observed}$ , shown on a common scale. Note: the vertical lines in the “FRATNP” panel correspond to the 1968 Flu and 2003 Heatwave shown in Figure ??

114 countries in the observed  $k_t$  tend to consist less of high frequency, year-to-  
 115 year fluctuations and more of what appears to be multi-year departures from  
 116 trend. The estimation approach we are using then assigns these departures  
 117 from trend to persistent changes in the latent state ( $\epsilon_t$ ), rather than to a  
 118 high-frequency transient effect.

119 This may be an undesirable idiosyncrasy of the model we are using. On  
 120 the other hand, it may reflect something fundamentally different about the  
 121 evolution of mortality in some countries. The United States is a large country  
 122 consisting of many sub-populations and that may be part of the reason it  
 123 appears to evolve differently over time. The evolution of mortality in Russia  
 124 of course has been one of repeated crises in recent decades and it is not  
 125 surprising that the same time series model that works well for France and  
 126 Sweden behaves differently for Russia.

127 Setting the magnitudes of the shocks aside, we can see if the timing co-  
 128 incides across some countries, and if so, which countries experience the same  
 129 shocks at the same time. In Figure 4, we plot the shocks for all pairs of  
 130 countries and give the correlation, ordering the using the 1st principle com-  
 131 ponent. We see that the Continental European countries (France, Germany,  
 132 Blegium, Italy, Spain and Austria) appear to be highly correlated. The En-  
 133 glish speaking countries (U.S, U.K, and Canada) also have similar temporal  
 134 patterns of mortality shocks.

135 The two countries with the highest correlation (0.8) in short-term shocks  
 136 are Italy and France. Other notable correlations include Taiwan and Japan  
 137 (0.6) and Austria and Czechoslovakia (0.6). Australia has shocks that are  
 138 largely uncorrelated with the rest of the world, except New Zealand (0.3),  
 139 and New Zealand is most correlated wtih Taiwan (0.4).

140 There is some suggestive evidence that there is more than geographic  
 141 distance linking the short-term fluctuations in mortality. For example, New  
 142 Zealand is more correlated with Canada, the United States, and the U.K. that  
 143 it is with continental European countries. Some surprising correlations, e.g.,  
 144 between Japan and Czechoslakia suggest that there may be some random  
 145 element, too, in the correlations we find.

## 146 5 Discussion

147 Our estimation of transitory mortality is not complete. But a few preliminary  
 148 conclusions can be reached.



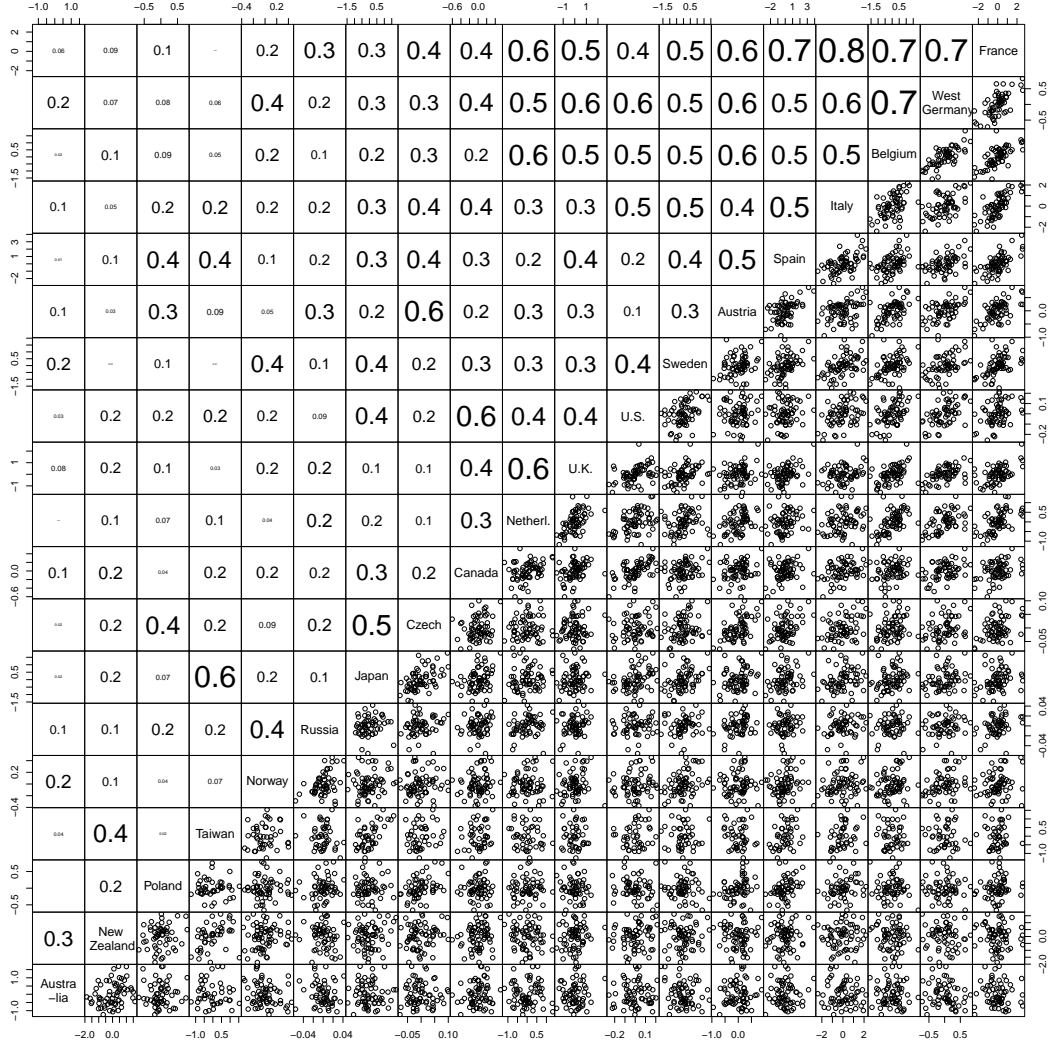


Figure 4: Bivariate scatter plots in annual short-term mortality shocks ( $n_t$ ) among countries along with the corresponding correlation. The countries are ordered by the strength of correlation, as measured by the 1st principle component. For example, the correlation in between shocks measured in Canada and the United States is 0.6

- 149 1. The structured time series model of annual transient shocks appears to  
150 be a good description of what is happening in countries with otherwise  
151 steadily declining mortality such as many countries in Europe as well  
152 as Japan.
- 153 2. These shocks are correlated across countries suggesting the common  
154 influence of weather and contagious diseases.
- 155 3. In other cases, notably the United States, the simple model of annual  
156 transient shocks does not appear to fully describe the time series. Ad-  
157 ditional explanation, involving longer shocks that last multiple years,  
158 would seem required. The influence of population heterogeneity and  
159 crises that unfold more slowly (e.g., HIV, opioids) are likely factors.

## 160 **6 Future Plans**

161 Our future plans for this work include

- 162 1. Including more countries in the analysis and describing the pattern of  
163 covariation across space and cultures.
- 164 2. Trying to incorporate the effect of transitory shocks that last longer  
165 than a single year.
- 166 3. Including known weather and influenza epidemics to see if the shocks  
167 we are detecting correspond.