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Homework 6

[All LP programming done in LINDO]

1. Shortest Paths using LP:

a) Find the distance of the shortest path from G to C in the graph below.

The shortest path from G to C is 16, from G to H to B to C.

Lindo Code:

```
max dc
ST
        dg = 0
        dh - dg <= 3
        da - dh <= 4
        db - dh <= 9
        db - da <= 8
        df - da <= 10
        da - df <= 5
        db - df <= 7
        dc - df <= 3
        de - df <= 2
        dc - db <= 4
        dd - dc <= 3
        de - dd <= 25
        dd - de <= 9
        dd - dg <= 2
        dg - de <= 7
        de - db <= 10
        df - dd <= 18
END
```

Output:

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)       16.00000

 VARIABLE        VALUE          REDUCED COST
       DC        16.000000          0.000000
       DG         0.000000          0.000000
       DH         3.000000          0.000000
       DA         4.000000          0.000000
       DB        12.000000          0.000000
       DF        13.000000          0.000000
       DE         0.000000          0.000000
       DD         0.000000          0.000000
```

b) Find the distances of the shortest paths from G to all other vertices.

The shortest distances from G are:

A: 7
B: 12
C: 16
D: 2
E: 19
F: 17
G: 0
H: 3

2. Product Mix: Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal numbers of ties of each type to maximize profit? Include a copy of the code and output.

To reach the maximum profit of $120,196.00, the company should produce these quantities of ties:

Silk:	7,000
Polyester:	13,625
Blend 1:	13,100
Blend 2:	8,500

Before simplifying, the objective function looks like this:

6.7s + 3.55p + 4.31b + 4.81c - 2.5s - 0.48p - 0.75b - 0.81c - 0.75(s + p + b + c)
(selling price x each product) – (material cost x each product) – (labor cost x each product)

After simplifying:

3.45s + 2.32p + 2.81b + 3.25c

And operates under these constraints:

The number of silk ties must be between 6,000 and 7000 (this max is below the available silk yardage)
The number of polyester ties must be between 10,000 and 14,000
The number of blend 1 ties must be between 13,000 and 16,000
The number of blend 2 ties must be between 6,000 and 8,500
The polyester, blend 1, and blend 2 ties must not use more than 2,000 yards of polyester
The blend 1 and blend 2 ties must not use more than 1,250 yards of cotton

```
max 3.45s + 2.32p + 2.81b + 3.25c
ST
        s >= 6000
        s <= 7000
        p >= 10000
        p <= 14000
        b >= 13000
        b <= 16000
        c >= 6000
        c <= 8500
        0.08p + 0.05b + 0.03c <= 2000
        0.05b + 0.07c <= 1250
END
```

```
LP OPTIMUM FOUND AT STEP      1

      OBJECTIVE FUNCTION VALUE

       1)      120196.0

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000          0.000000
        P     13625.000000          0.000000
        B     13100.000000          0.000000
        C      8500.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      1000.000000          0.000000
       3)         0.000000          3.450000
       4)      3625.000000          0.000000
       5)       375.000000          0.000000
       6)       100.000000          0.000000
       7)      2900.000000          0.000000
       8)      2500.000000          0.000000
       9)         0.000000          0.476000
      10)         0.000000         29.000000
      11)         0.000000         27.200001

NO. ITERATIONS=       1
```

3. Transshipment Model: Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost. Include a copy of the code and output.

The minimum cost is 17,100 and it involves using the routes in this manner:

Route	Cost	Route	Cost
A	150	K	150
C	200	L	100
D	250	O	200
F	150	P	200
G	100	T	150
I	150	U	100
J	100		

Plant-to-Warehouse Variable Key:

Cost	W1	W2	W3
P1	A	B	
P2	C	D	
P3	E	F	G
P4		H	I

Warehouse-to-Retailer Variable Key:

Cost	R1	R2	R3	R4	R5	R6	R7
W1	J	K	L	M			
W2			N	O	P	Q	
W3				R	S	T	U

```
LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)        17100.00

 VARIABLE        VALUE          REDUCED COST
        A      150.000000          0.000000
        B        0.000000          8.000000
        C      200.000000          0.000000
        D      250.000000          0.000000
        E        0.000000          2.000000
        F      150.000000          0.000000
        G      100.000000          0.000000
        H        0.000000          7.000000
        I      150.000000          0.000000
        J      100.000000          0.000000
        K      150.000000          0.000000
        L      100.000000          0.000000
        M        0.000000          5.000000
        N        0.000000          2.000000
        O      200.000000          0.000000
        P      200.000000          0.000000
        Q        0.000000          1.000000
        R        0.000000          7.000000
        S        0.000000          3.000000
        T      150.000000          0.000000
        U      100.000000          0.000000

      ROW   SLACK OR SURPLUS      DUAL PRICES
        2)        0.000000          1.000000
        3)        0.000000          0.000000
        4)        0.000000          0.000000
        5)        0.000000          1.000000
        6)        0.000000        -16.000000
        7)        0.000000        -17.000000
        8)        0.000000        -18.000000
        9)        0.000000        -16.000000
       10)        0.000000        -18.000000
       11)        0.000000        -21.000000
       12)        0.000000        -15.000000
       13)        0.000000        -11.000000
       14)        0.000000         -8.000000
       15)        0.000000         -9.000000
       16)      150.000000          0.000000
       17)        0.000000          0.000000
       18)      200.000000          0.000000
       19)      250.000000          0.000000
       20)        0.000000          0.000000
       21)      150.000000          0.000000
       22)      100.000000          0.000000
       23)        0.000000          0.000000
       24)      150.000000          0.000000
       25)      100.000000          0.000000
       26)      150.000000          0.000000
       27)      100.000000          0.000000
       28)        0.000000          0.000000
       29)        0.000000          0.000000
       30)      200.000000          0.000000
       31)      200.000000          0.000000
       32)        0.000000          0.000000
       33)        0.000000          0.000000
       34)        0.000000          0.000000
       35)      150.000000          0.000000
       36)      100.000000          0.000000

 NO. ITERATIONS=       2
```

```
min     10A + 15B + 11C +  8D + 13E +  8F  +  9G  + 14H +  8I +  5J +
         6K +  7L + 10M + 12N +  8O + 10P  + 14Q  + 14R + 12S + 12T + 6U

ST
        A + B <= 150
        C + D <= 450
        E + F + G <= 250
        H + I <+ 150

        J >= 100
        K >= 150
        L + N >= 100
        M + O + R >= 200
        P + S >= 200
        Q + T >= 150
        U >= 100

        A + C + E - J - K - L - M >= 0
        B + D + F + H - N - O - P - Q >= 0
        G + I - R - S - T - U >= 0
END
```

4. A Mixture Problem:

(a) Determine the combination of ingredients that minimizes calories but meets all nutritional requirements. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What is the cost of the low calorie salad?

The lowest calorie count for a salad that meets all nutritional requirements is 114.7541 and it contains:

Lettuce: ~58.5 grams
Smoked Tofu: ~87.8 grams

The cost of this low calorie salad is ~$2.33

Ingredient	Name
Tomato	TO
Lettuce	LT
Spinach	SP
Carrot	CT
Sunflower Seeds	SF
Smoked Tofu	ST
Chickpeas	CP
Oil	OL

```
LP OPTIMUM FOUND AT STEP     12

        OBJECTIVE FUNCTION VALUE

        1)      114.7541

 VARIABLE        VALUE          REDUCED COST
       TO         0.000000         16.901640
       LT         0.585480          0.000000
       SP         0.000000         14.513662
       CT         0.000000         36.289616
       SF         0.000000        408.387970
       ST         0.878220          0.000000
       CP         0.000000         97.551910
       OL         0.000000        886.404358


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.650273
        3)         2.508197          0.000000
        4)         3.491803          0.000000
        5)         0.022248          0.000000
        6)        78.220139          0.000000
        7)         2.327283          0.000000
        8)         0.000000         -6.010929

NO. ITERATIONS=      12
```

```
min 21TO + 16LT + 40SP + 41CT + 585SF + 120ST + 164CP + 884OL
ST
        0.85TO + 1.62LT + 2.86SP + 0.93CT + 23.4SF +   16ST +    9CP +   0OL >= 15
        0.33TO + 0.20LT + 0.39SP + 0.24CT + 48.7SF +    5ST +  2.6CP + 100OL >= 2
        0.33TO + 0.20LT + 0.39SP + 0.24CT + 48.7SF +    5ST +  2.6CP + 100OL <= 8
        4.64TO + 2.37LT + 3.63SP + 9.58CT +   15SF +    3ST +   27CP +   0OL >= 4
           9TO +   28LT +   65SP +   69CT +  3.8SF +  120ST +   78CP +   0OL <= 200
           1TO + 0.75LT +  0.5SP +  0.5CT + 0.45SF + 2.15ST + 0.95CP +   2OL >= 0
            LT + SP - 0.4TO - 0.4LT - 0.4SP - 0.4CT - 0.4SF - 0.4ST - 0.4CP - 0.4OL >= 0
END
```

(b) Determine the combination of ingredients that minimizes cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. How many calories are in the low cost salad? Include a copy of the code/file with the HW.

The lowest-costing salad has a price of $1.55 and contains these ingredients in these proportions:

Spinach:	~83.2 grams
Sunflower Seeds:	~9.6 grams
Chickpeas:	~115.2 grams

There are 278 calories in this salad.

Ingredient	Name
Tomato	TO
Lettuce	LT
Spinach	SP
Carrot	CT
Sunflower Seeds	SF
Smoked Tofu	ST
Chickpeas	CP
Oil	OL

```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      1.554133

 VARIABLE        VALUE          REDUCED COST
       TO         0.000000          1.002081
       LT         0.000000          0.402912
       SP         0.832298          0.000000
       CT         0.000000          0.486914
       SF         0.096083          0.000000
       ST         0.000000          0.405609
       CP         1.152364          0.000000
       OL         0.000000          7.281258

      ROW    SLACK OR SURPLUS      DUAL PRICES
        2)       278.488403          0.000000
        3)         0.000000         -0.131261
        4)         6.000000          0.000000
        5)         0.000000          0.051847
        6)        31.576324          0.000000
        7)        55.651089          0.000000
        8)         0.000000         -0.241358

 NO. ITERATIONS=       4
```

```
min        1TO + 0.75LT +  0.5SP +  0.5CT + 0.45SF + 2.15ST + 0.95CP +   2OL
ST
          21TO +   16LT +   40SP +   41CT +  585SF +  120ST +  164CP + 884OL >= 0
       0.85TO + 1.62LT + 2.86SP + 0.93CT + 23.4SF +   16ST +    9CP +   0OL >= 15
       0.33TO + 0.20LT + 0.39SP + 0.24CT + 48.7SF +    5ST +  2.6CP + 100OL >= 2
       0.33TO + 0.20LT + 0.39SP + 0.24CT + 48.7SF +    5ST +  2.6CP + 100OL <= 8
       4.64TO + 2.37LT + 3.63SP + 9.58CT +   15SF +    3ST +   27CP +   0OL >= 4
          9TO +   28LT +   65SP +   69CT +  3.8SF +  120ST +   78CP +   0OL <= 200
           LT + SP - 0.4TO - 0.4LT - 0.4SP - 0.4CT - 0.4SF - 0.4ST - 0.4CP - 0.4OL >= 0
END
```