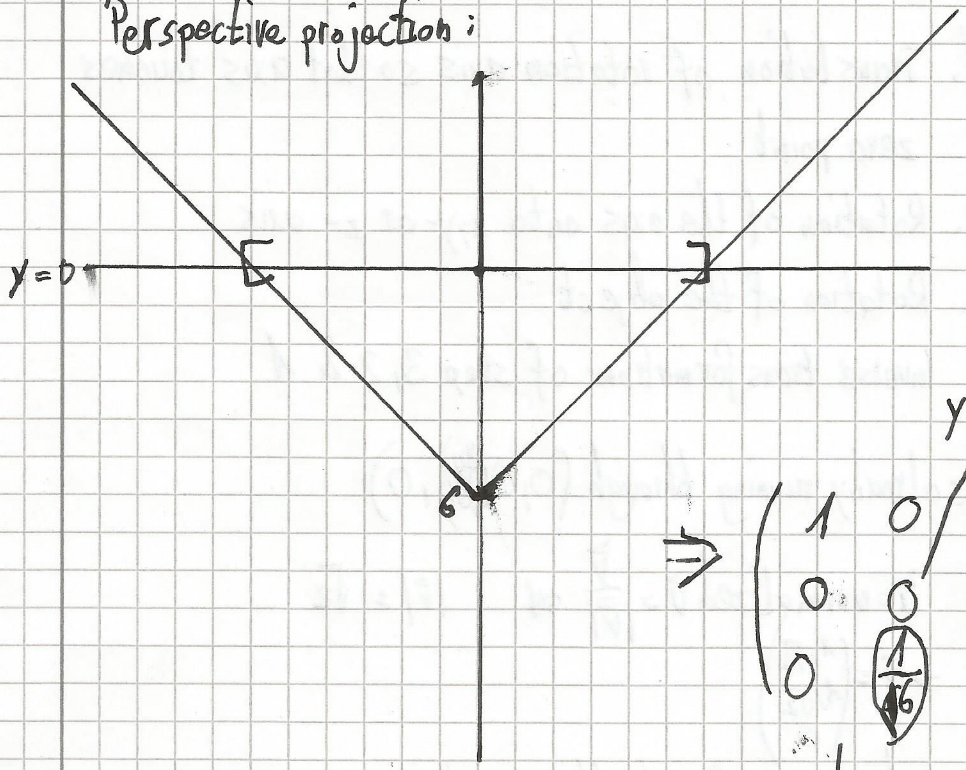
GED Assignment 6 Questions:

Perspective projection:



$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{6} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

y=0

because of distance between $y=0$ & center of projection point $(0,-6)$

(unnecessary ~~unnecessary~~: just for understanding $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{6} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ \frac{1}{6}y+1 \end{pmatrix} : W \Rightarrow \begin{pmatrix} x/(\frac{1}{6}y+1) \\ 0 \end{pmatrix}$ = projected point)

Rotation matrix:

steps:
1. Translation of rotation axis so that axis touches zero point
2. Rotation of the axis onto x-, y- or z- axis
3. Rotation of the object
4. Inverse transformations of step ~~3~~, 2 & 1.

step 1: vector is already running through (0, 0, 0)

Step 2: $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\vec{u} = \text{normalized } \vec{v} = \frac{\vec{v}}{|\vec{v}|}$ $|\vec{v}| = \sqrt{2}$

$\rightarrow \vec{u} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$

$\Rightarrow$ rotation around z-axis onto the y-axis

necessary: $\cos\alpha$, $\sin\alpha$; α is the degree between the rotation axis and the y-axis.

$$\Delta x = 1/\sqrt{2} \quad \Delta y = 1/\sqrt{2}$$

$$r_{xy} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{\left(\tfrac{1}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2} = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}} = 1$$

$$\sin\alpha = \Delta x / r_{xy} = 1/\sqrt{2}$$

$$\cos\alpha = \Delta y / r_{xy} = 1/\sqrt{2}$$

$$\Rightarrow \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Step 3: rotation of the object around y-axis with $\varphi = 90°$

Point P:

$$p' = \begin{pmatrix} \cos 90° & 0 & \sin 90° \\ 0 & 1 & 0 \\ -\sin 90° & 0 & \cos 90° \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} p_3 \\ p_2 \\ -p_1 \end{pmatrix}$$

step 4: Rotation around z-axis with $-\alpha$:

$\cos -\alpha = \cos\alpha$, $\sin -\alpha = -\sin\alpha$

$$\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p3 \\ p2 \\ -p1 \end{pmatrix} = \begin{pmatrix} \cos\alpha \cdot p3 + \sin\alpha \cdot p2 \\ -\sin\alpha \cdot p_3 + \cos\alpha \cdot p2 \\ -p1 \end{pmatrix}$$

this is the 3×3 rotation matrix to rotate a point $P = \begin{pmatrix} p1 \\ p2 \\ p3 \end{pmatrix}$ around the axis $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ with $90°$