Now, in this video, we're going to go over integration—that is, how to find the antiderivative of certain functions.

If you recall, according to the power rule, the derivative of x^n (a variable raised to a constant) is

$$\frac{d}{dx}(x^n) \; = \; n\,x^{n-1}.$$

When dealing with antiderivatives (integration), the power rule is reversed: instead of subtracting 1 from the exponent, you add 1; instead of multiplying by the exponent, you divide by the new exponent; and you always add the constant of integration, C. Let us work through some examples.

1. **Derivative example.** Find the derivative of x^3.

$$\frac{d}{dx}(x^3) \; = \; 3x^2.$$

2. **Basic antiderivative.** Find the antiderivative of $3x^2$.

$$\int 3x^2\,dx = x^3 + C.$$

3. Find the antiderivative of x^4.

$$\int x^4\,dx = \frac{x^5}{5} + C \quad \text{or} \quad \frac{1}{5}x^5 + C.$$

4. Practice:

$$\int x^2\,dx = \frac{x^3}{3} + C, \qquad \int x^7\,dx = \frac{x^8}{8} + C.$$

5. Antiderivative of x.

$$\int x\,dx = \frac{x^2}{2} + C.$$

6. $\displaystyle\int \frac{x}{4}\,dx$. Rewrite as $\frac{1}{4}\int x\,dx$:

$$\frac{1}{4}\cdot\frac{x^2}{2} + C = \frac{x^2}{8} + C.$$

7. $\displaystyle\int 8x^3\,dx$. Factor out 8:

$$8\int x^3\,dx = 8\cdot\frac{x^4}{4} + C = 2x^4 + C.$$

8. $\displaystyle\int 4\,dx$. Integrating a constant gives

$$\int 4\,dx = 4x + C.$$

More generally, $\int a\,dy = ay + C$. To see the power rule in action, write $4 = 4x^0$:

$$\int 4x^0\,dx = 4\cdot\frac{x^{0+1}}{0+1} + C = 4x + C.$$

9. Antiderivative of a binomial:

$$\int (7x-6)\,dx = \int 7x\,dx - \int 6\,dx = \frac{7x^2}{2} - 6x + C.$$

10.

$$\int (6x^2 + 4x - 7)\,dx = 6 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^2}{2} - 7x + C = 2x^3 + 2x^2 - 7x + C.$$

Radical functions can be handled by rewriting with rational exponents.

11. $\int \sqrt{x}\,dx$. Rewrite $\sqrt{x} = x^{1/2}$:

$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C.$$

12. $\int \sqrt[3]{x^4}\,dx$. Rewrite $x^{4/3}$:

$$\int x^{4/3}\,dx = \frac{x^{7/3}}{7/3} + C = \frac{3}{7}x^{7/3} + C.$$

13. $\int \sqrt[4]{x^7}\,dx$. Rewrite $x^{7/4}$:

$$\int x^{7/4}\,dx = \frac{x^{11/4}}{11/4} + C = \frac{4}{11}x^{11/4} + C.$$

Next, trigonometric integrals. Memorize these basic antiderivatives:

$$\begin{aligned}
\int \cos x\,dx &= \sin x + C,\\
\int \sin x\,dx &= -\cos x + C,\\
\int \sec^2 x\,dx &= \tan x + C,\\
\int \csc^2 x\,dx &= -\cot x + C,\\
\int \sec x\,\tan x\,dx &= \sec x + C,\\
\int \csc x\,\cot x\,dx &= -\csc x + C.
\end{aligned}$$

Hence,

$$\int (4\sin x - 5\cos x + 3\sec^2 x)\,dx = -4\cos x - 5\sin x + 3\tan x + C.$$

Indefinite vs. definite integrals:

An *indefinite* integral $\int f(x)\,dx$ yields an antiderivative plus C. A *definite* integral $\int_a^b f(x)\,dx$ evaluates to a number via the Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is any antiderivative of f.

15. Indefinite example:

$$\int 6x^2\,dx = 2x^3 + C.$$

16. Definite example:

$$\int_1^2 6x^2\,dx = \left[2x^3\right]_1^2 = 2 \cdot 2^3 - 2 \cdot 1^3 = 16 - 2 = 14.$$

17. Another definite integral:

$$\int_2^3 (8x-3)\,dx = \left[4x^2-3x\right]_2^3 = (36-9)-(16-6) = 27-10 = 17.$$

Exponential functions with linear exponent:

$$\int e^u\,dx = \frac{e^u}{u'} + C, \quad u = u(x).$$

18. $\displaystyle\int e^x\,dx = e^x + C.$

19. $\displaystyle\int e^{5x}\,dx = \frac{e^{5x}}{5} + C.$

20. $\displaystyle\int e^{-7x}\,dx = -\frac{e^{-7x}}{7} + C.$

21. $\displaystyle\int e^{3x-5}\,dx = \frac{e^{3x-5}}{3} + C.$

22. U-substitution example: $\displaystyle\int e^{8x}\,dx$. Let $u = 8x$, so $du = 8\,dx$.

$$\int e^{8x}\,dx = \frac{1}{8}\int e^u\,du = \frac{1}{8}e^u + C = \frac{1}{8}e^{8x} + C.$$

23. Another U-substitution: $\displaystyle\int 4x\,e^{x^2}\,dx$. Let $u = x^2$, $du = 2x\,dx$.

$$\int 4x\,e^{x^2}\,dx = 4\int x\,e^u\frac{du}{2x} = 2\int e^u\,du = 2e^u + C = 2e^{x^2} + C.$$

Rational functions and negative exponents:

21. $\displaystyle\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + C = -x^{-1} + C = -\frac{1}{x} + C.$

22. $\displaystyle\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C.$

23. $\displaystyle\int \frac{8}{x^4}\,dx = 8\int x^{-4}\,dx = 8\cdot\frac{x^{-3}}{-3} + C = -\frac{8}{3x^3} + C.$

24. $\displaystyle\int \frac{1}{(4x-3)^2}\,dx$. Let $u = 4x-3$, $du = 4\,dx$.

$$\int \frac{1}{u^2}\frac{du}{4} = \frac{1}{4}\int u^{-2}\,du = -\frac{1}{4u} + C = -\frac{1}{4(4x-3)} + C.$$

25. $\displaystyle\int \frac{7}{(5x-3)^4}\,dx$. Let $u = 5x-3$, $du = 5\,dx$.

$$\int \frac{7}{u^4}\frac{du}{5} = \frac{7}{5}\int u^{-4}\,du = -\frac{7}{15u^3} + C = -\frac{7}{15(5x-3)^3} + C.$$

26. Special case: $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C.$

27. $\int \frac{7}{x}\, dx = 7 \ln |x| + C.$

28. $\int \frac{1}{x+5}\, dx = \ln |x+5| + C.$

Throughout integration, practice each of these techniques–power rule, term-by-term integration, rational exponents, u-substitution, and known antiderivatives for trigonometric, exponential, and logarithmic functions–to build fluency.