Assessment 1.2: Portfolio optimisation using Genetic Algorithm based on 3 crossover methods

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Word Count: 3206

Abstract

Stocks are one of the things that keep a country's economy thriving. Stock exchanges enables people to invest their own money for capital gain. Using this report people with even a minimal understanding of the stock market will be able to understand just how to invest in a balanced portfolio and how to improve upon an investor's portfolio. The method used in this report is a genetic algorithm. A genetic algorithm can be defined a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution coined by Charles Darwin. The crossover operators used are single point,arithmetic and blend. A crossover operator is a genetic operator used to combine the genetic information of two parents to generate new offspring. This algorithm will feature real values so as a result the crossover operator type will be real valued.

1. Introduction

The aim of this report is to examine historical data of 8 stocks in order to achieve the optimal weights.Different crossover methods will be applied to the genetic algorithm in order to achieve a better result. The weights will then be used to get the rolling portfolio returns and the rolling cumulative returns.This is a very important aspect in investment as it can be used as an indicator as to how a portfolio will do in the market. Using a genetic algorithm the best weights will be allocated to each stock.Further analysis helps us understand how this portfolio will do in the date period of the historical data.

A big limitation of portfolio optimisation is the idea that stocks are a frictionless market meaning that there no transaction costs or constraints to trading when in reality this is not the case and there is friction in the market (Srivastav, 2022). The results achieved for this genetic algorithm fared a lot better than the benchmark used which in this case was a paper on portfolio optimisation using genetic algorithm and Particle Swarm optimisation (Venkiah,2020).

The first part of the report will feature the importance of portfolio optimisation followed by a comparison of the Markowitz model against Sharpe Ratio. The details of the dataset used will be introduced after as well as a description of the 3 crossovers.The experimental setup will then be explained including the choice of parameters and the reasons for it. The next section will feature the results. The results include the best fitness values,weights,historical returns and rolling performance. This will be followed by the discussion of the results and conclusion.

2. Background: Problem Domain

Optimisation models play an important role in making important financial decisions. Many computational financial problems such as asset allocation and risk management can be done using optimisation methods (Cornuejols&Tütüncü, 2006).

The word stock means ownership or equity in a corporation. The stock market provides a venue in which you can trade shares in exchange for money. Financial markets such as the stock market are important for companies as it gives you an opportunity to invest money in shares to make money in the future (Chen, 2023). It also provides finance for a company so that they can hire,invest and grow(2022). A common misconception about the stock market is that when you sell a stock it goes back to the company. This is not the case but rather you are selling to another investor on the exchange.

An index is a way to check the performance of a group of assets (Chen, 2023) A market index is a popular measure of stock index performance. The S&P 500 is a market-cap-weighted index of the 500 largest companies in the U.S and is what will be used in this paper to compare the weighted stocks to in order to view the performance compared to the other stocks.(Chen, 2023)

Investment decisions are an important area in the finance

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Modern Optimisation Report, CMP 7213 - Modern Optimisation.

industry. A portfolio is a collection of financial investments like stocks, bonds, commodities, cash, and cash equivalents.(Tardi,2023). Portfolio optimization is a formal mathematical approach to making investment decisions across a collection of financial instruments or assets.

3. Problem Instance (Markowitz Model vs Sharpe Ratio)

The focus of this paper will be Stock Portfolio optimisation as it is an important topic in the field of finance.The Modern Portfolio Theory was introduced in 1952 by Harry Markowitz in an article in the Journal Of Finance. It is widely known as the foundation of MPT and has been widely referenced. Modern Portfolio theory states that given a desired level of risk an investor can optimise the expected returns through portfolio diversification. He went on to introduce a notion of an efficient portfolio using mean-variance optimisation(Sharpe & Markowitz, 1987). MVO provides the minimum possible variance whilst maximising the return for that variance.The mathematical formula for the mean-variance model is as follows(Chang et al., 2009):

$$Minimize \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$Subject\ to\ \Sigma_{i=1}^{N} w_i u_i = R^* \tag{2}$$

$$\Sigma_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \leqslant w_i \leqslant 1, i = 1, ..., N \tag{4}$$

In this series of equations N is the number of assets available,w_i is the proportion of the assets and can also be referred to as the weight of each asset,u_i is the expected return of asset i and σ_{ij} is the covariance between price of assets i and j.

The limitations of MVO are that risk and return are weighed equivalently which does not apply in the real world as investors are more worried about risk than reward. The underlying assumption is that all investors are risk averse which is not the case(Marakbi, 2016). Another risk is that covariance is used in this set of equations and the problem with this is that it only measures assets with each other and cannot show relationship with other assets. A better way to check relationships with other assets would be too check it's correlation coefficient (Investopedia, 2022).

To simplify the Markowitz model, economist William F. Sharpe proposed the Sharpe ratio in 1966. Similarly to the Markowitz model, the Sharpe ratio compares the return of an investment with it's risk but simplifies it by taking covariance out of the equation.

$$S_r = \frac{[E(r_p) - r_f]}{\sigma_p} \tag{5}$$

The Sharpe ratio can be denoted by Equation 5 where $E(r_p)$ is the expected return on investment of the portfolio,r_f is the risk free rate of interest andσ_p is the standard deviation of the portfolio(Liao et al., 2015).The statistical significance for the Sharpe Ratio can be done using one sample and 2 sample hypothesis tests (Liao et al., 2015)

3.1. Dataset Description

The data set used in this paper will be tech stocks' data as this is an industry that has thrived in recent times especially during the lockdown period.

The dataset that will be used is taken from Yahoo Finance and will provide historical data for each stock and index for the last 5 years for the training data and data from the last 3 years for the testing data.The time period that will be used for the training data is 19/04/2018 to 19/04/2023.The dataset that will be used for testing data will be from 19/04/2020 to 05/05/2023. The relevant fields will be the date and the closing price.The stocks chosen will be the 7 biggest tech stocks which are Oracle,Intel,IBM,Alphabet,Amazon, Apple and Microsoft(Kiplinger,2021).The investment benchmark that will be used in this paper will be the NASDAQ index as these are American stocks.The testing dataset will not include the benchmark index.

4. Candidate Optimisation Methods

4.1. Literature Review

The three candidate optimisation for this paper are single point crossover, blend crossover and arithmetic crossover specifically whole arithmetic combination.Mous, Van den Berg and Dallagnol compared Particle Swarm optimisation and Genetic algorithms in portfolio management. The crossover operators used here were the basic basic crossover and arithmetic crossover (Dallagnol et al., 2009). Benbouziane and Sefiane wrote a great paper that was very relatable to the one that will be conducted for this paper as it focused on the same 3 crossover operators. The only difference being that this paper featured 5 stocks rather than 7(Sefiane, S. & Benbouziane,2012).

4.2. Fitness Function Formulation

The purpose of the objective function in this paper is to assign a weight to each of the assets so that they will maximise returns and minimise risk.The objective function in this case is the Sharpe ratio. The Sharpe ratio should be maximised in order to obtain the best weights for each asset.The constraint for this genetic algorithm is shown in equation 3 and 4 for the Markowitz model.

To begin setting up this genetic algorithm data will be imported in the form of a csv and the columns that will be used are date and closing price.A time series of price will be created and the previous price will be calculated by minusing the value from the previous closing price. Returns will then be calculated using the formula (input formula). This will then be inputted into a dataframe called returns and will contain the date and each asset's returns. The date and the first column with NAs will be removed creating a dataframe with asset returns. The constraints will then be introduced as well as the penalty function.

$$X_i \leqslant 1 \rightarrow [max(0, X_i)]^2 \tag{6}$$

$$X_i \geq 1 \rightarrow [max(0, X_i)]^2 \tag{7}$$

The penalty functions can be denoted by equation 6 and 7 where X_i is the weight and if the weight is less than 1 and greater than 0 the constraint will be returned as 0 whereas if it is any thing outside these intervals it will square the value thus increasing the value of the fitness value exponentially and punish the optimisation.The fitness function can be denoted as

$$(-(-sharpe(x) + constraint(x))) \tag{8}$$

Equation 8 includes 2 minus signs.The minus sign before the Sharpe variable refers to the penalty function.Penalty functions can only compute minimisation so the addition of the minus sign changes it to a maximisation function.The genetic algorithm can only compute maximisation so to change it to minimisation a minus sign must be added to the whole function.

4.3. Single Point Crossover

Single point crossover is one of the most approved crossovers in use(Kora & Yadlapalli,2017).The first step is to choose a point on the parent organism and string. This is called the crossover point. All the data between those two points is swapped to create a new offspring referred to as children.Figure 1 shows chromosome 1 in yellow and chromosome 2 in green. Offspring 1 inherits all the data past the crossover point in chromosome 2 and Offspring 2 inherits all the data past the crossover point in chromosome 1.(Geeksforgeeks)

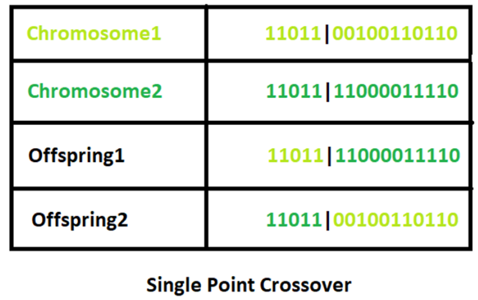

Figure 1. Single Point Crossover

4.4. Arithmetic Crossover

The arithmetic crossover operator linearly combines the 2 parent chromosomes.Two chromosomes are randomly chosen and are combined linearly to create 2 offsprings.The focus of this paper will be whole arithmetic combination which takes a percentage of each parent and adds them to produce new solutions. The formulae to do this are

$$Child1 = \alpha.x + (1 - \alpha).y \tag{9}$$

$$Child2 = \alpha.x + (1 - \alpha).y \tag{10}$$

If alpha is equal to 0.5 then both children will be the same as shown in figure 2.

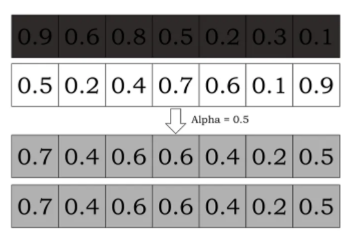

Figure 2. Arithmetic Crossover

4.5. Blend Crossover

The blend crossover operator is determined where alpha determines the exploration and exploitation level of the offspring.Exploitation uses the intervals between the values of the 2 parents and exploration uses an interval outside of these limits. Similarly to whole arithmetic combination to ensure balance the alpha must be set to 0.5. Figure 3 shows what the blend crossover looks like.

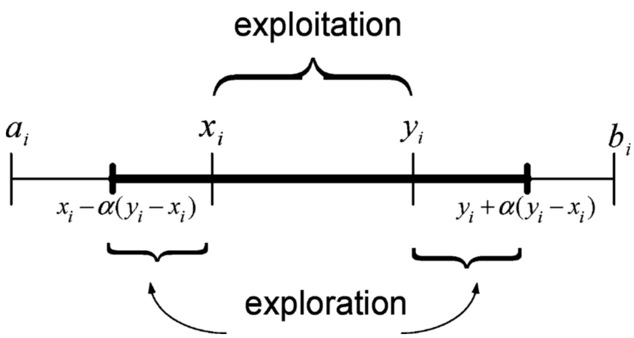

Figure 3. Fitness Values and error bar for Daily stock prices of training data

5. Experiments

Table 1. Configuration parameters for Genetic Algorithm.

Population Size	100.
Maximum Generations	300(max)
Type	Real Valued; Point mutation: 0.1.
Crossover Probability	0.6.
Mutation Probability	0.4
Fitness Function	$(-(-sharpe(x)$ $+constraint(x)))$

The GA parameters chosen by VD Vasiani(2019) were population size 100, a mutation probability of 0.4, a crossover probability of 0.6 and a generation of 1000.These crossover and mutation probabilities have also been used by Sasongko(2019) but with a generation of 250 and a population of 170.

This paper will use the parameters shown in Figure 1.It will use a population size of 100,a generation of 300 generations depending on whether the same fitness value repeats 50 times along with a mutation probability of 0.4,a crossover probability of 0.6 and the type will be Real Valued.The fitness value is $(-(-sharpe(x) + constraint(x)))$. The reason for a generation of 300 is that it was evident that the 3 algorithms were converging at around the 50th generation according to both Sasongko (2019) and Vasiani (2019).

6. Results

The best fitness values and portfolio weights for training data of daily stock prices using single point,arithmetic and blend crossover respectively. This helps to understand which crossover method performs the best.This is shown by Figure 4.

Figures 5,6 and 7 show the weighted portfolio vs historical data. This helps us in identifying how the weighted portfolio against other stocks.Table 5,6 and 7 show the best fitness values and portfolio weights for training data of monthly stock prices using single point,arithmetic and blend crossover respectively.This will be compared to the results from the daily stock prices data.

Table 8,9 and 10 show the annual means and volatility.This shows us how risky it is to invest in the portfolio.This will be compared to the benchmark's results in table 11.

Table 12 shows the statistical significance of the returns for all the historical training data.Figure 8,9 and 10 show the rolling portfolio returns visualising the volatility of the portfolio.

Figure 11,12 and 13 gives a better look into the portfolio returns using rolling cumulative returns so it is visible exactly how well the portfolio is doing.

Table 2. Best Fitness Values and Weights for training data of daily stock prices using Single Point Crossover.

Best Fitness Value	0.06228711
Weight1	0.7446935
Weight2	0.1571194
Weight3	0.002223555
Weight4	0.002053284
Weight5	0.001701968
Weight6	0.013832
Weight7	0.074012381
Weight8	0.005350376

Table 3. Best Fitness Values and Weights for training data of daily stock prices using Arithmetic Crossover.

Best Fitness Value	0.05674194
Weight 1	0.3248925
Weight 2	0.3486532
Weight 3	0.03296642
Weight 4	0.01652007
Weight 5	0.00188008
Weight 6	0.09436359
Weight 7	0.09224465
Weight 8	0.07065816

Table 4. Best Fitness Values and Weights for training data of daily stock prices using Blend Crossover.

Best Fitness Value	0.06221189
Weight 1	0.6429242
Weight 2	0.2005517
Weight 3	0.001109108
Weight 4	0.001915082
Weight 5	0.0009572059
Weight 6	0.02938665
Weight 7	0.1146674
Weight 8	0.00801748

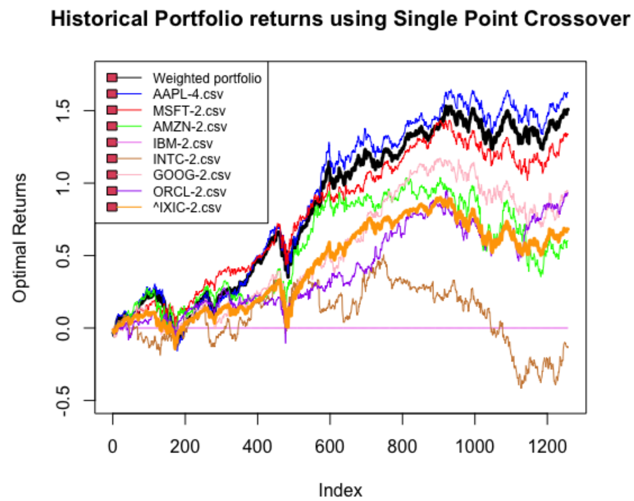

Figure 5. Graph to show the historical returns vs the weighted using single point crossover

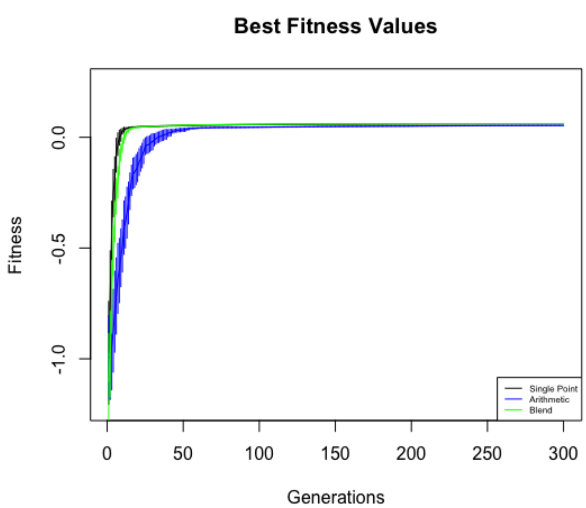

Figure 4. Fitness Values and error bar for Daily stock prices of training data

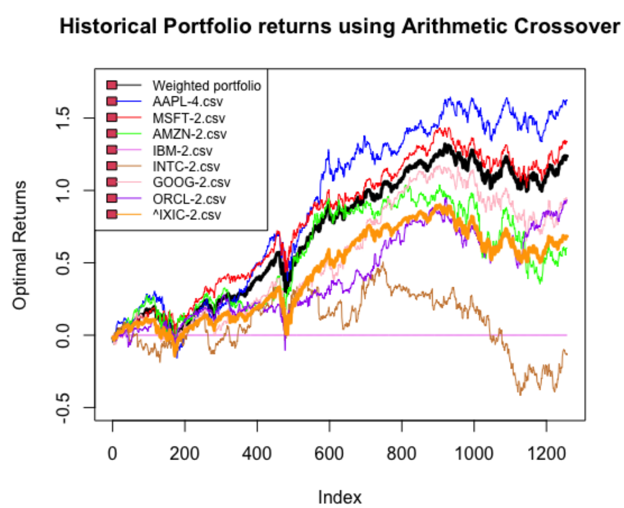

Figure 6. Graph to show the historical returns vs the weighted using Arithmetic crossover

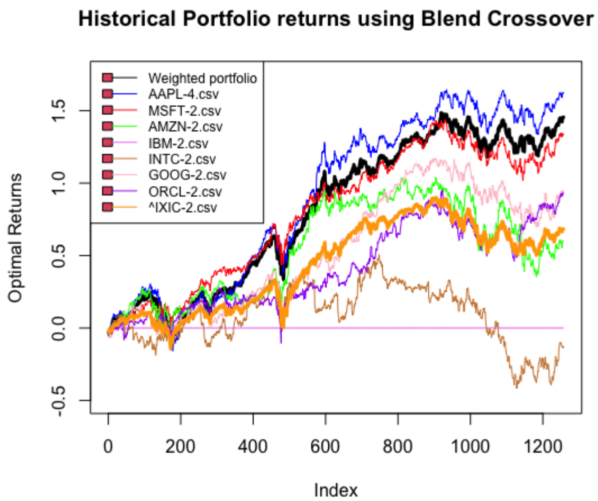

Figure 7. Graph to show the historical returns vs the weighted using blend crossover

Table 5. Best Fitness Values and weights for training data of monthly stock prices using single point crossover.

Best Fitness Value	0.33874
Weight 1	0.1613942
Weight 2	0.7930933
Weight 3	0.0003249943
Weight 4	0.005455315
Weight 5	0.0004471838
Weight 6	0.003338188
Weight 7	0.03263214
Weight 8	0.001073956

Table 6. Best Fitness Values and weights for training data of monthly stock prices using Arithmetic crossover.

Best Fitness Value	0.3161311
Weight 1	0.2534956
Weight 2	0.5001556
Weight 3	0.008877213
Weight 4	0.0465967
Weight 5	0.01193405
Weight 6	00.0207361
Weight 7	0.1341032
Weight 8	0.0265124

Table 7. Best Fitness Values and weights for training data of monthly stock prices using blend crossover.

Best Fitness Value	0.3383299
Weight 1	0.2352597
Weight 2	0.7295554
Weight 3	0.001309663
Weight 4	0.00246872
Weight 5	0.0001445115
Weight 6	0.001873508
Weight 7	0.02012935
Weight 8	0.006831557

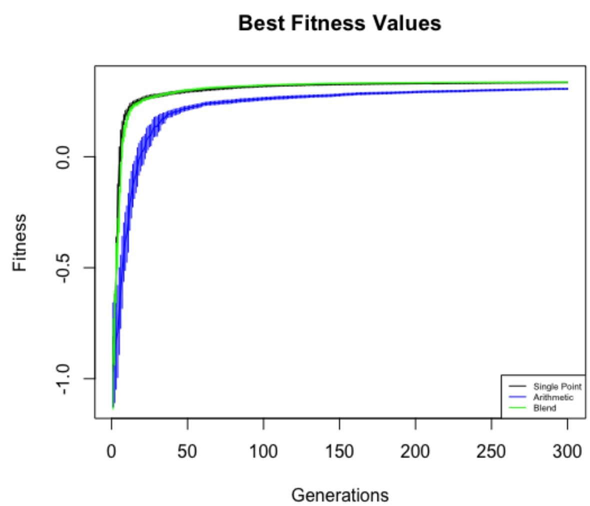

Figure 8. Graph to show the fitness values and error bars of 3 crossover methods for monthly training data

Table 8. Annualised Mean and Volatility for single point crossover

Annualised Mean Return	0.2978
Annualised Standard Deviation	0.2671
Annualised Sharpe Ratio	0.9508

Table 9. Annualised Mean and Volatility for Arithmetic crossover.

Annualised Mean Return	0.2547
Annualised Standard Deviation	0.2483
Annualised Sharpe Ratio	0.8551

Table 10. Annualised Mean and Volatility for blend crossover

Annualised Mean Return	0.2979
Annualised Standard Deviation	0.2686
Annualised Sharpe Ratio	0.9455

Table 11. Mean and Volatity of Benchmark

Mean Return	0.114
Standard Deviation	1.740
Sharpe Ratio	0.0654

Table 12. Statistical Significance test of training data

[H] Stock Returns	Test Statistic	p-value	skewness
AAPL	0.94953	2×10^{-16}	-0.04227109
MSFT	0.93888	2×10^{-16}	-0.0205069
AMZN	0.95342	2×10^{-16}	0.07711387
IBM	0.89588	2×10^{-16}	-0.4576691
INTC	0.8976	2×10^{-16}	-0.2571215
GOOG	0.95191	2×10^{-16}	0.02392842
ORCL	0.85091	2×10^{-16}	1.364081
IXIC	0.93612	2×10^{-16}	-0.3971105

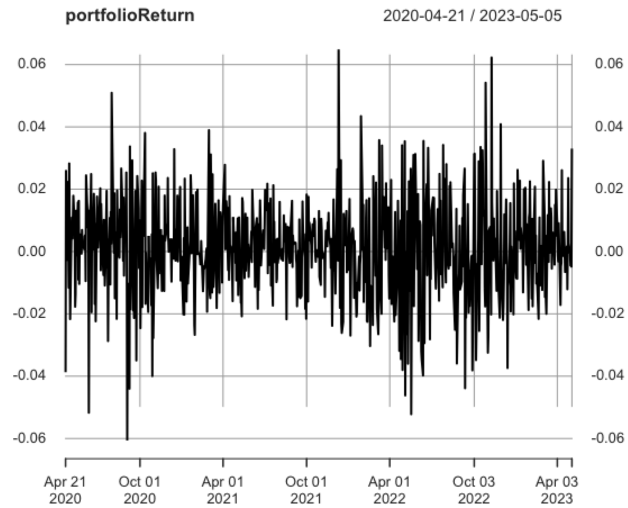

Figure 10. Rolling portfolio returns for Arithmetic Crossover

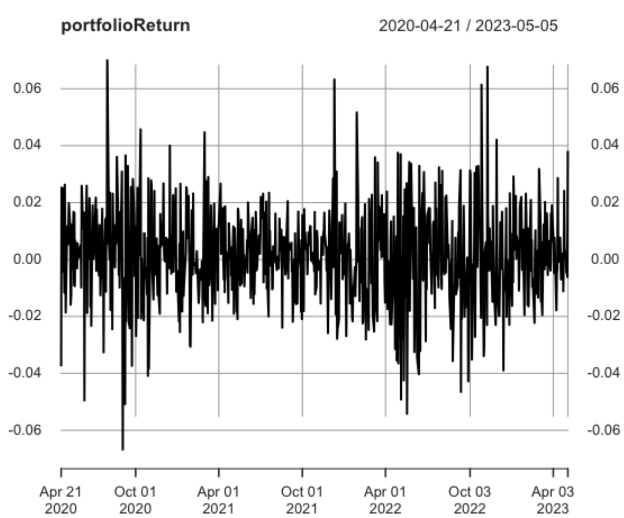

Figure 9. Rolling Portfolio returns using single point crossover

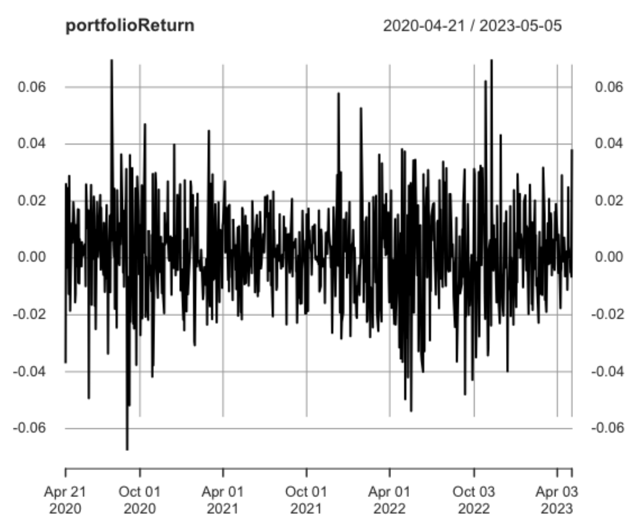

Figure 11. Rolling portfolio returns for blend crossover

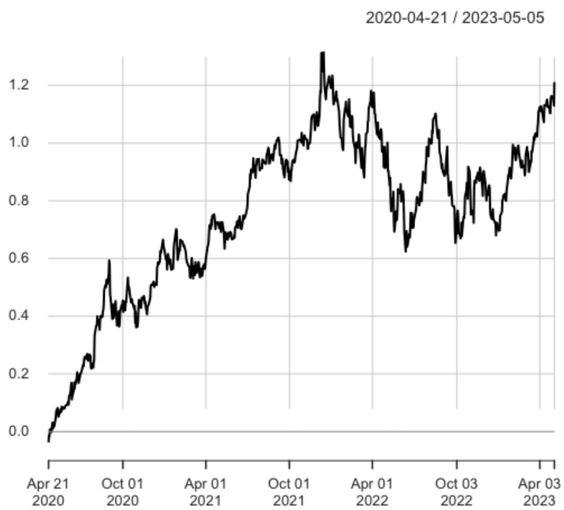

Figure 12. Rolling Cumulative returns using single point crossover

Figure 13. Rolling Cumulative returns using arithmetic crossover

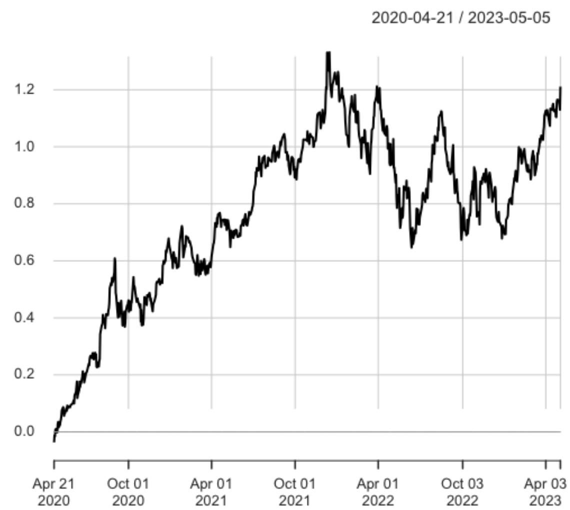

Figure 14. Rolling Cumulative returns using blend crossover

7. Discussion

As seen in the results the genetic algorithm has been implemented using three crossovers. The three crossovers were single point crossover, whole arithmetic recombination and blend crossover. The best fitness values were computed and the weight of each asset. The weights were used to compute a weighted portfolio which has been graphed along with historical returns in order to compare how well it compared to the other assets' historical returns. Each genetic algorithm has been run 30 times and the crossover operator has changed for each one.The best fitness for the 3 different crossovers have been graphed.

Table 2 shows that the best fitness value for the genetic algorithm on the daily stock prices is (0.06228711) .Table 3 shows that the best fitness value for the genetic algorithm is (0.05674194).Table 4 shows that the best fitness value for the genetic algorithm is (0.06221189).The weights have also been computed for all tables for the optimal portfolio

This shows us that the best fitness value for single crossover is better than the arithmetic crossover and blend crossover in terms of the average solution and best solution.

Out of the 3 experiments the single point crossover had the highest fitness value meaning that it had the most optimal weights.There were many similarities between the 3 like the fact that Weight 1 was similar in both the Single point operator and the blend operator.It was also revealed that weight 3 had the worst weightage out of the 8 weights.

The experiment was repeated for the monthly training data as shown in tables 5,6 and 7 using single point, Arithmetic and blend crossover respectively.The single point crossover performed best just as it did with the daily training data.

Figure 4,5 and 6 show the optimal returns of the weighted portfolio and the historical data of each asset.The thick black line is the weighted portfolio. The weighted portfolio is calculated by multiplying the portfolio returns by the weights that were calculated using the genetic algorithm. The black line is being compared to the other lines which is the historical data mainly the NASDAQ index denoted by the orange line . As visible on the 3 figures all 3 of the weighted portfolios have performed better than the NASDAQ index. The single point crossover's weighted portfolio has outperformed the other 2 weighted portfolios but the difference between the blend crossover's weighted portfolio and single point crossover's weighted portfolio is very minimal.

Figure 7 shows the best fitness values for all 3 crossovers. It shows that all 3 crossovers converge towards the best fitness value by the 75th generation. The single point crossover gets to the convergence point the fastest with the blend crossover coming closely after. It is also revealed that the single point crossover and the blend crossover achieve a better fitness value than the arithmetic crossover by the 300th generation. The error bars on the arithmetic crossover are also much bigger than the other crossovers.

Table 8,9 and 10 show the annualised mean volatility for the 3 crossover methods for the testing data. Single point crossover had the highest sharpe ratio whereas arithmetic had the lowest.However when comparing to the benchmark in table 11 it is evident that it performed considerably better than the benchmark.The risk free asset used for this was 3.44 as this is the current risk free rate.The standard deviation helps determine market volatility or the spread of asset prices from their average price. When prices move wildly, standard deviation is high, meaning an investment will be risky(investopedia,2022).

Table 12 shows the results of the statistical tests conducted on the testing data. The Shapiro-Wilks test was used and it shows that the data is statistically significant as the p-value is considerably lower than 0.05.The skewness was also measured to see just how the data was doing.

Figure 8,9 and 10 show the rolling portfolio returns visualising the volatility of the portfolio.It can be seen that there are many similarities when looking at all 3 portfolio returns so it is not a good measure to see the difference between the 3.

Figure 11,12 and 13 gives a better look into the portfolio returns using rolling cumulative returns so it is visible exactly how well the portfolio is doing. This is a better indicator of how well the portfolio would do in the same time period.

8. Conclusions and Future Work

To conclude this report implements a genetic algorithm to deal with portfolio optimisation . The objective function for this is Sharpe's ratio.Using Sharpe's ratio a decision can be made about the optimal portfolio.The crossover methods that have been used are single point,arithmetic and blend. The results obtained showed that the single point crossover performed better than the others but the blend crossover method came close to the performance of the single point crossover.Overall the genetic algorithm performed better for the monthly data than the daily data as it had a better fitness value.

The 3 experiments were applied to a portfolio of stocks with 8 assets, 1 of which is the NASDAQ index which serves as a benchmark index for an optimal portfolio. A benchmark index is a standard against which the performance of a security, investment strategy, or investment manager can be measured.(Investopedia,2022).This report compares weighted portfolios against the NASDAQ index and shows it has outperformed this through each of the three crossover methods as it has a higher sharpe ratio in each case. The testing data also showed us that the Sharpe ratio was also the highest when it came to the single point crossover. The genetic algorithm outperformed the benchmark in every statistic.

It is also show that the data is statistically significant. The skewness also helps us understand what was happening with the stocks. Negative skewness usually indicates frequent small gains and a few large losses whereas positive skewness usually indicate small losses and a few large gains(Venkiah,2020). This tells us that if you are a risk averse investor looking not to make risky trades you should invest in the negative skewed stocks but if you are looking to be making risky transactions you should look at the positive skewed stocks.

Further research can be conducted to obtain a more optimal portfolio. This can be done by further narrowing down the parameters. This can be done through hyperparameter optimisation which can be defined as tuning the parameters to help find either a minimum or a maximum depending on the objective function.We can do this by increasing the number of assets so that investors can aim to not only invest in these 7 stocks but other ones that may have done better in this genetic algorithm.

Acknowledgements

I want to thank Dr Atif Azad for all the help and consistent feeback.

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A. Code&fitness value/weights for testing data

https://github.com/josh236916/Portfolio-Optimisation

Table 13. Fitness Values and optimal weights for single point crossover for testing data

Best Fitness Value	0.0724431
Weight 1	0.5960315256
Weight 2	0.01062591
Weight 3	0.001187965
Weight 4	0.009006962
Weight 5	0.0002122074
Weight 6	0.01454383
Weight 7	0.3683916

Table 14. Fitness Values and optimal weights for arithmetic crossover for testing data

Best Fitness Value	0.06700488
Weight 1	0.41510186
Weight 2	0.04489304
Weight 3	0.02068862
Weight 4	0.05994131
Weight 5	0.02149226
Weight 6	0.05723101
Weight 7	0.3806519

Table 15. Fitness Values and optimal weights for blend crossover for testing data

Best Fitness Value	0.07212468
Weight 1	0.6058907387
Weight 2	0.005832082
Weight 3	0.001580685
Weight 4	0.01235124
Weight 5	0.0002819143
Weight 6	0.04804814
Weight 7	0.3260152