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Project Title

CHSH SIMULATION VIOLATING THE BELL INEQUALITY

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1. Abstract

The CHSH game was proposed by Clauser, Horne, Shimony, and Holt demonstrates the violation of Bell's inequality, which fundamentally distinguishes quantum mechanics from classical physics.

The CHSH game is a thought experiment that is often used to illustrate the concept of quantum entanglement
The CHSH game is a game of strategy between two players, Alice and Bob. Alice and Bob are given a set of possible inputs $x,y \in \{0,1\}$,
where Alice produces an output
$a \in \{0,1\}$ based on her input x and Bob produces an output b{0,1}
without communicating with each other.

Win Condition: Alice and Bob win if:
$a \bigoplus b = x \wedge y$
where $\bigoplus$ is the XOR operation and $\wedge$ is the AND operation, and they must produce a set of possible outputs. The goal of the game is to maximize the probability of winning

2. Introduction

a. Background :

In quantum Mechanics a Bell inequality is an inequality that is bound between certain quantities in a given system whose violation cannot be explained by the classical theory whereas Quantum strategy predicts its violation.

b. Problem Statement :

Efficient transmission of classical information over quantum channels is crucial for quantum networks. This project addresses the challenge of achieving optimal data transmission efficiency using quantum protocols.
Bell-type inequality is the Clauser-Horne-Shimony-Holt (CHSH) inequality [2]. The CHSH inequality can also be illustrated using a simple game with 2 players and binary inputs and outputs. This game is known as the CHSH game
The **CHSH (Clauser-Horne-Shimony-Holt) game** is a thought experiment in quantum mechanics designed to test the principles of quantum entanglement and non-locality. It provides a mathematical framework to demonstrate the violation of **Bell's inequality**, which distinguishes between classical and quantum correlations.

The CHSH inequality defines a classical upper limit on the strength of correlations between the inputs and outputs:

$S=E(0,0) + E(0,1) + E(1,0) - E(1,1)$

where $E(x,y)$ is the expectation value of $(-1)^{a \oplus b}$ for each combination of x and y.

- **Classical limit**: $|S| \leq 2|$
- **Quantum limit**: $|S| \leq 2\sqrt{2}$ (maximum achievable via quantum mechanics)

c. Scope:

This project focuses on simulation, analysis and understanding of Quantum entanglement through CHSH Game in order to verify the violation of bell inequality demonstrating the non-local correlations inherent in quantum mechanics.

d. Goals :

- Prove the classical strategy and the Quantum Strategy of CHSH Game
- Implement CHSH game to show that the probability of winning is 85.4%
- Prove the CHSH game for Bell inequality Violation by using the bell inequality theorem calculating Expectation Value and validate its Simulation.

3. Literature Review:

Bell inequality or bell theorem is named after John Stewart Bell and he stated that "There is no physical theory for local hidden variables which can reproduce the quantum mechanics predictions", this inequality was proposed to distinguish between predictions of classical local hidden Variable theories and Quantum Mechanics.

The inequality serves as a test for non-local correlations, in which entangled particles exhibit correlations that cannot be explained by any classical model. Clauser, Horne, Shimony, and Holt (CHSH) extended Bell's inequality in 1969 to create a more experimentally feasible test, now known as the CHSH inequality.

This inequality was used experimentally to prove the bell theorem where this theorem asserts that local hidden variable theories cannot account for some consequences of entanglement in quantum mechanics. The violation of the CHSH inequality is used to show that quantum mechanics is incompatible with local hidden-variable theories.

4. Methodology

A. Tools and Frameworks :

- **Quantum Computing Platform**: Qiskit (Python library for quantum programming).
- **IDE**: VS CODE
- **Hardware**: IBM Quantum ,AerSimulator and Quantum Circuit for simulation.

B. Theoretical Foundations :

1. **Quantum Entanglement:**
   - This State demonstrated the non classical correlations between two Qubits .

2. **Bell theorem:**
   - Bell's theorem establishes that no classical local hidden variable theory can reproduce all predictions of quantum mechanics.
   - The CHSH inequality is a mathematical expression derived from Bell's theorem, providing a testable criterion to distinguish between classical and quantum correlations.

3. **CHSH inequality:**
   - The CHSH inequality is expressed as $|S| \leq 2$ where S is calculated using expectation values from four specific measurement settings on two qubits.
   - In quantum mechanics, entangled states can violate this inequality, achieving values up to $S \leq \mathbf{2\sqrt{2}} \approx 2.828$,confirming the presence of non-classical correlations.

4. Quantum Measurements:

- Measurement settings are chosen based on specific angles that maximize the violation of the CHSH inequality.

- Rotational gates (like Ry) are applied to orient the qubits before performing standard measurements in the computational basis.

5. Quantum Circuits and Simulation and Probability and Expectation Values:

- Quantum gates: The Hadamard (HHH) and CNOT gates are used to create entanglement.

- Simulation: The Aer-Simulator mimics the behaviour of a quantum system, allowing testing of the CHSH inequality without a physical quantum computer.

- Parity-based measurements are used to compute probabilities P(0) and P(1).

- Expectation values for each measurement configuration are derived, forming the basis for calculating S.

C. Implementation Steps :

- Implementation of CHSH game using a simple classical python function and NumPy to prove the CHSH algorithm

- Finding the Classical and Quantum strategy to prove the winning probability

- USING Qiskit CHSH game satisfying the criteria of the game violating the bell inequality

Steps involved :

- Create a bell state by creating a Quantum Circuit of 2 qubits and 2 classical bits
- Applying the measurement by defining the angles of Alice and Bob
    - and then followed by the rotations in order orient the qubits before performing standard measurements in the computational basis.
- Simulate the CHSH game for 10000 and calculate the winning probabaility

- PROOF OF BELL INEQUALITY violation by defining the angles of Alice and bob and also simulating the measurements of the state

5. Results and Discussion

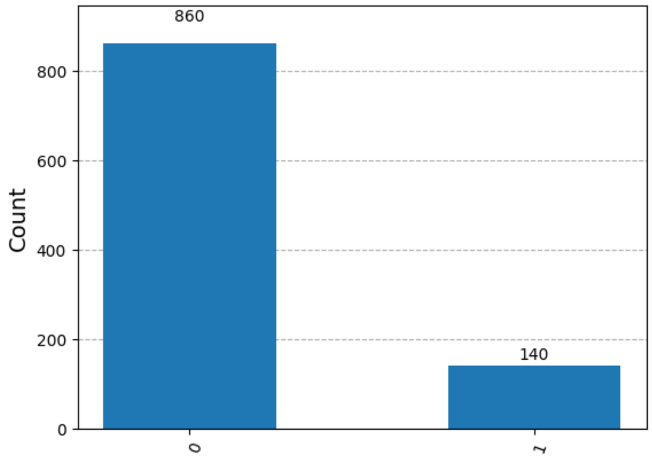
Simulation Results :

The main aim was to achieve the Quantum winning probability of 85.4%
And also prove the bell inequality violation where these were the results achieved

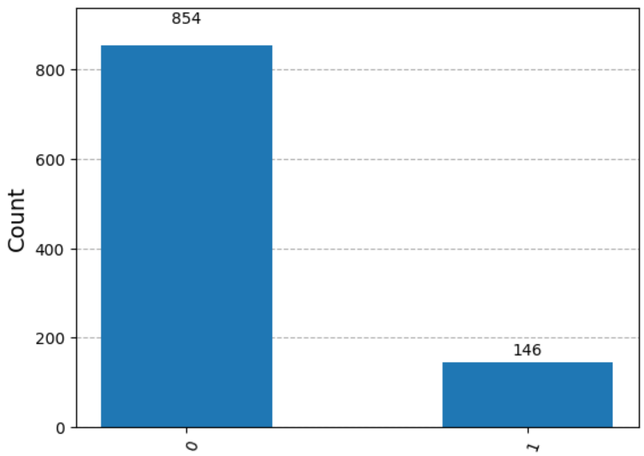
Expectation Values:
E(0, 0) = 0.7200
E(0, 1) = 0.7080
E(1, 0) = 0.6920
E(1, 1) = -0.7240

CHSH Inequality Value S = 2.8440
The CHSH inequality is violated, demonstrating quantum entanglement.

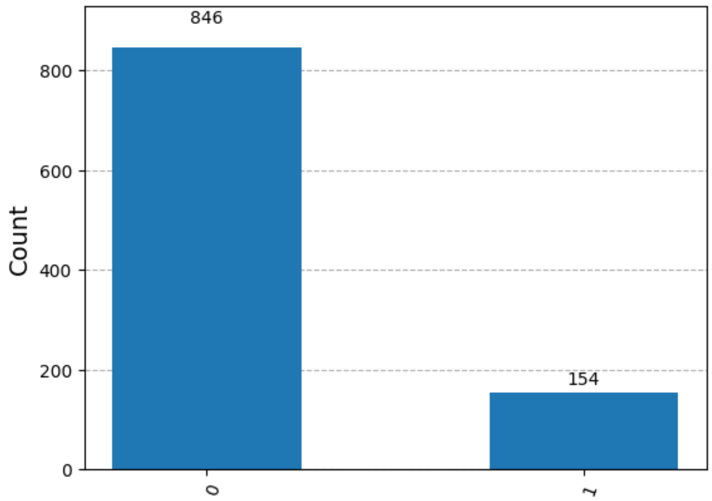
Graphs and Visualizations :
Measurement Results:
For inputs x=0, y=0:
P(0) = 0.8600, P(1) = 0.1400
Counts: {0: 860, 1: 140}



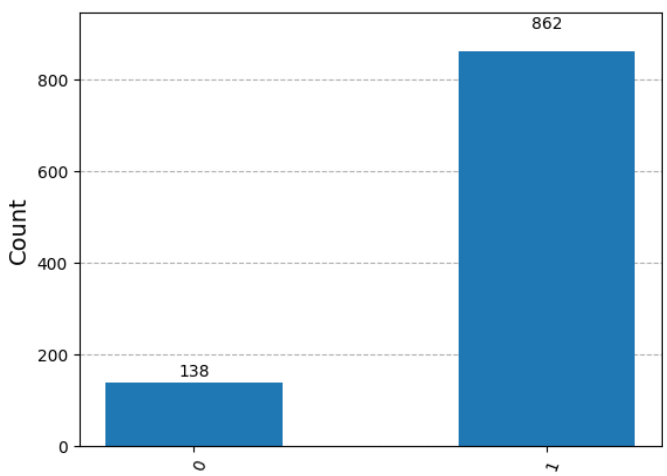
For inputs x=0, y=1:
P(0) = 0.8540, P(1) = 0.1460
Counts: {0: 854, 1: 146}



For inputs x=1, y=0:
P(0) = 0.8460, P(1) = 0.1540
Counts: {0: 846, 1: 154}



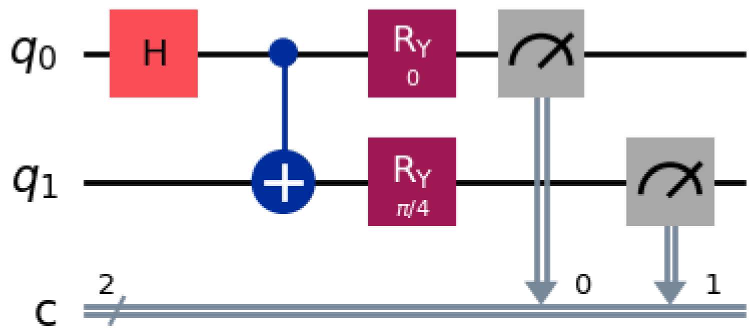
For inputs x=1, y=1:
P(0) = 0.1380, P(1) = 0.8620
Counts: {0: 138, 1: 862}



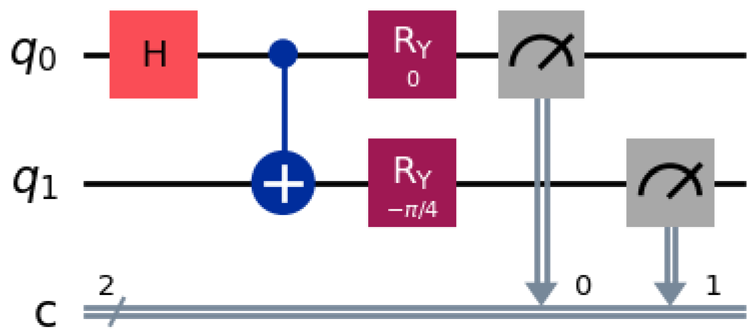
Quantum Circuit Diagrams for each states :

Circuit Diagram

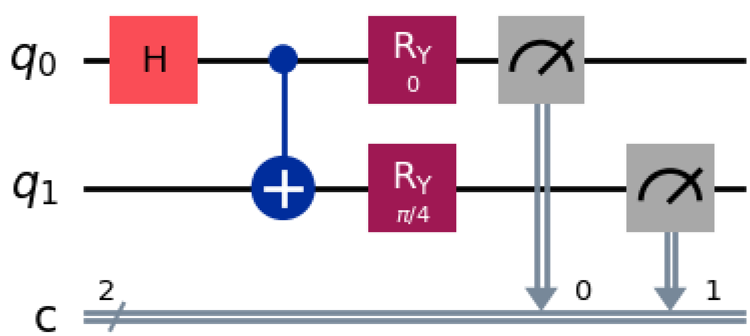
Quantum Circuit for CHSH game :



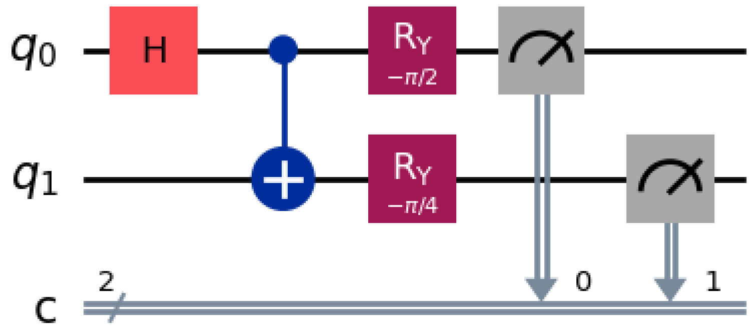
Quantum Circuit for E(0,0):



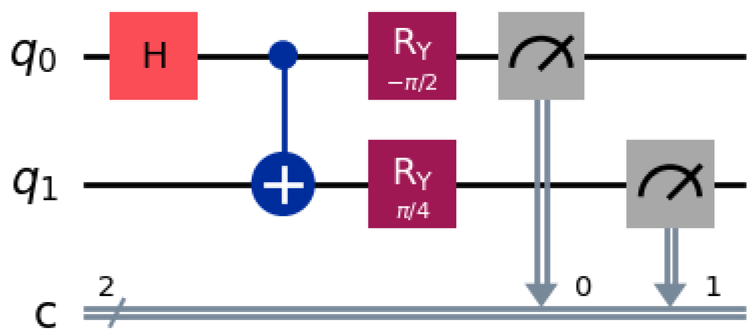
Quantum Circuit for E(0,1):



Quantum Circuit for E(1,0):



Quantum Circuit for E(1,1):



Analysis :

In the above analysis we see the

- Look into the difference between classical and quantum strategies.
- Show cases wherein quantum violates the Bell inequality.
- Visualize and analyze outcomes of measurements for justification of advantage in quantum relativity.

The CHSH, or Clauser-Horne-Shimony-Holt, game is a quantum mechanics thought experiment demonstrating the principles of quantum entanglement and violating classical Bell inequalities.

This is important because entanglement is the basis behind the superiority of quantum over classical strategies.

Entanglement leads to correlations in the measurement of Alice and Bob, which cannot be classically accounted for.

- **Measurement Settings:**

  Alice and Bob must pick one of the two possible measurement settings corresponding to their input bits (x for Alice, y for Bob).

  The choice of rotation angles are very carefully chosen

  - Alice: $\theta_{AO} = 0\,, \theta_{A1} = \boldsymbol{\pi/4}$
  - Bob : : $\theta_{BO} = \frac{\pi}{8}, \theta_{B1} = -\pi/8$

    Where ,these angles are taken from Quantum mechanics to optimize the bell inequality, matching the quantum correlation

  - The CHSH game is won when the outputs $a \oplus b = x \wedge y$ (logical AND of inputs).
  - Classical strategies can only have a maximum winning probability of 75%, limited by the Bell inequality whereas Quantum strategy Exploits the entanglement with the winning probability of 85.4%

Comparison with Classical Strategy vs Quantum Strategy :

Key observation:

We see that in classical strategy we get 75% whereas Quantum strategy outperforms more than classical and also when theoretically exploring it satisfies the whole theorem of CHSH game

Also another point to be note down where there is a some role in entanglement classical system.

Challenges :

a. Quantum decoherence in real-world hardware.
b. Alignment and calibration of quantum gates for accurate results.

6. Conclusion and Future Work:

This project successfully demonstrates the power of quantum strategies in the CHSH game. Combining theoretical foundations with practical quantum circuit simulations highlights the quantum advantage and lays a solid foundation for understanding the implications of entanglement in quantum mechanics.

Future Work :

- Compare Classical and Quantum Strategies:
  a. Implement and compare classical strategies to make the contrast more obvious.
- Scalability:
  a. Extend the game to higher-dimensional systems or more players to explore generalized Bell inequalities.
- Noise Analysis:
  a. Add noise and analyze how the noise affects the winning probability in noisy quantum system

7. References

a. Qiskit Documentation: https://qiskit.org/documentation/

b. Solis-Labastida, A.F., Gastelum, M. and Hirsch, J.G. (2021) ‘The violation of Bell-CHSH inequalities leads to different conclusions depending on the description used’, *Entropy*, 23(7), p. 872. doi:10.3390/e23070872.

c. Clauser, F.; Horne, M. A.; Shimony, A.; Holt, R. A. Proposed Experiment to Test Local Hidden-Variable Theories, Physical Review Letters, volume 23, pages 880-884, 1969.

8. Appendices:

GitHub :

https://github.com/joshIsac/Quantum-Computing-project/blob/main/CHSH%20GAME%20SIMULATION/CHSH_AN_BELL_INEQUALITY.ipynb