The "Sinc" Function

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Introduction

When you were a kid in a restaurant, one of the most annoying things you could do would be to run your finger around the rim of a water glass. Of course this produced a loud ringing sound, which changed based on the amount of water in your glass. Similarly, people who wear glasses are eternally grateful for the anti-reflective coating on the back of their lenses that prevents light reflecting straight back into their eyes.

Both of these phenomenon rely on the exact some principle of "interference". The connotation of the word today has become almost entirely negative, talking about something being "stopped", or "interrupted". But in fact, there are two types of interference: constructive, and destructive. Today, we'll be talking about a phenomenon that relies on both of them.

Diffraction

A common example of interference patterns is using light. Something interesting happens when light passes through a very small hole, or in this case a very thin slit. As waves of light coming completely parallel to one another encounter the

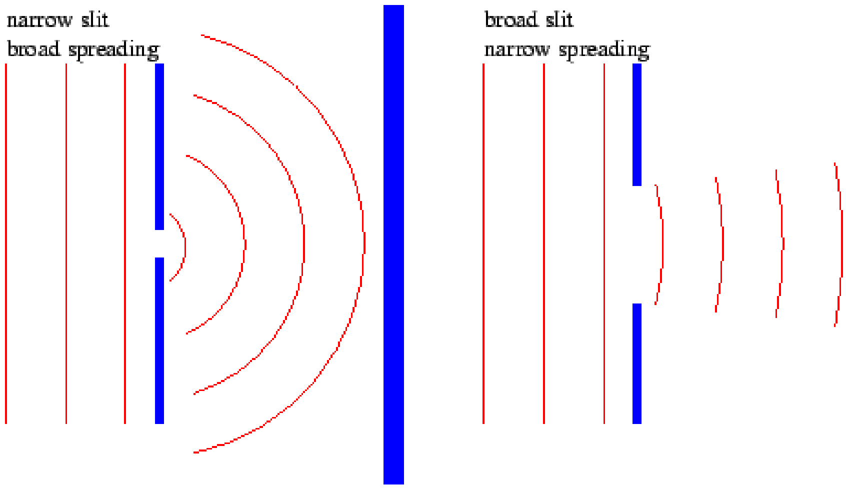

Figure 1: Plane Waves Transforming to Spherical Waves

slit, the waves shift into a circular pattern radiating from the slit. If it helps, you can think of it similar to water waves.

Now we're less concerned about the radiating itself than with what happens when the new waves hit a wall. The original waves would have been uniform against the surface, but the new waves will hit the surface at different times since they're now curved.

When the dust settles after all of the ray tracing, the pattern of light intensity on the screen follows the equation:

$$I(\theta) = I_0 \left(\frac{Sin(x)}{x} \right)^2 \tag{1}$$

Which can be seen in Figure 2. What we're investigating today is the behavior of this very unique function. From Pre-Calculus 11, we know what the standard $\frac{1}{x}$ function looks like and from Pre-Calculus 12, we know what $Sin(x)$ looks like. Composing the two functions together would normally not be a problem, but because of the hyperbolic nature of the $\frac{1}{x}$ function, we would assume that our new function would have issues at $x = 0$, which by the looks of it, it doesn't. What changes?

What is $\lim_{x \to 0} \frac{Sin(x)}{x}$?

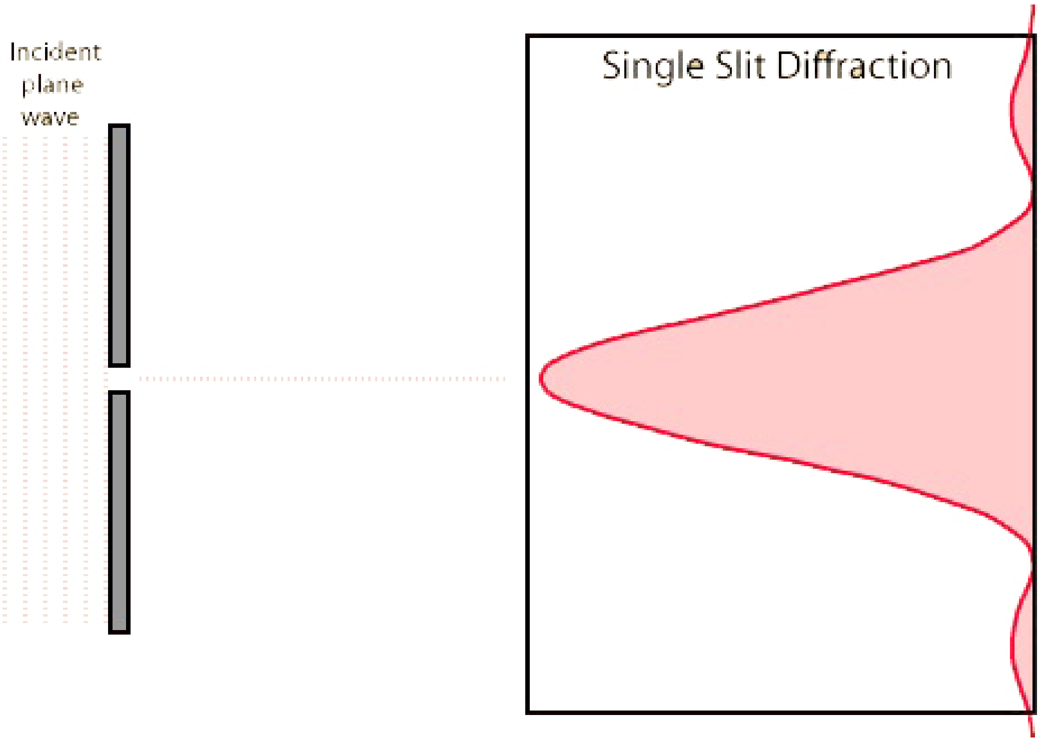

Figure 2: Intensity Pattern from Single Slit Diffraction

We can guess what the limit is graphically, but how would we prove it mathematically?

Hint: There's a famous theorem that might help you. It goes like this:

Two police officers are escorting a drunk man into a jail cell. If the drunk man is always between the two officers, and the officers make it to the jail cell, then no matter how much the drunk staggers, he too will make it to the jail cell.