Antisensationalism and Alternate Thought Patterns in Mathematics

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Achilles and the Tortoise

There are few, if any, significant movements in the history of philosophy whose roots are not to be found in the thought of some ancient Greek philosopher. During the height of Greek philosophy, the idea of "antisensationalism" was born; simply described as "the protest against the truth of the knowledge acquired through our sensory organs". While in the modern age we might think this is foolish, the practice does live on in one form or another today, since many mathematicians do all of their work in thought up scenarios or conditions that will never occur in real life (eg. the idea of an n-sphere, or a sphere in more than 3 dimensions).

One of the most famous advocates of antisensationalism was Zeno of Elea (490-430 BC). We don't know much about Zeno, since most of his works were never recovered, but one of his famous paradoxes was published in Aristotle's "Physics":

> In a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.

A bold statement; following Zeno's reputation, all of our senses immediately tell us that this is false. But what about if we examine this mathematically?

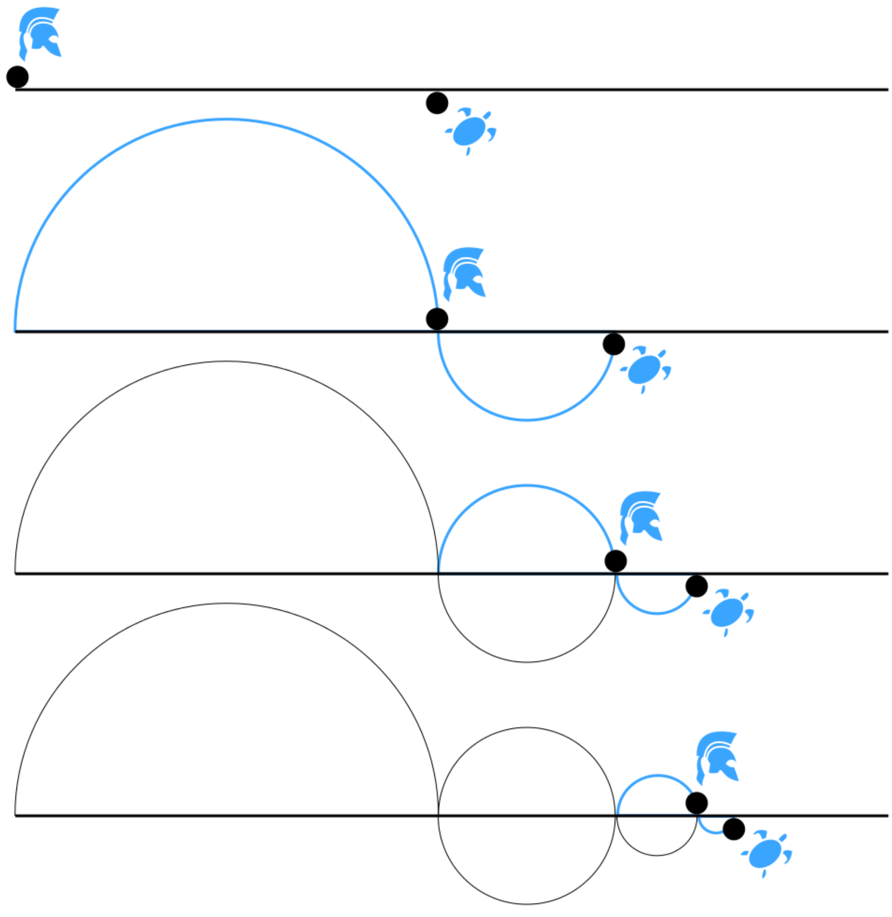

Figure 1: Achilles and the Tortoise

So then, we see that every time Achilles travels to the point that the tortoise was before, the tortoise has moved on. But this is still very abstract (mathematicians love thinking in the abstract). Let's instead put some numbers to this:

Achilles and a tortoise are in a race against one another. Achilles is so confident in his own ability to win, the tortoise is given a 10m head start in the race. If Achilles travels at a constant velocity of 10 m/s and the tortoise travels at a constant velocity of 1 m/s, will Achilles ever catch up to the tortoise? If so, where?

The Lamp Switch

Now, for some reason even though mathematically it can be seen that Achilles will never actually catch the tortoise, there is a point where we consider him "sufficiently close enough". Well what if the problem is rephrased in a way where there is no "close enough"?

There's a few different number systems that we have been introduced to in life. The obvious one is the decimal or "base 10" system, where each additional digit in a number expresses a factor of 10 increase. The other one that we know intuitively is the binary system. This the base 2 system where there are only two states to a problem—on and off, true and false, dead and alive, these systems are ones we run into every day.

Just planting this seed, in case I want to use it later. But the discussion of binary vs. spectrum solutions in quite fascinating.

So then let's consider an additional problem:

Consider a lamp with an on-off switch. Now, the lamp is initially off and I switch it on. After 1 minute I switch it off. After half a minute I switch it back on. After a quarter of a minute I switch it off. After one eighth of a minute I switch is back on and so on, each time halving the length of time I wait before I switch the lamp on or off as appropriate (I have very quick reflexes).

1. How long before I "finish" this sequence of turning the lamp on and off?
2. Is the lamp on or off at the end?
3. If I begin the problem with the lamp initially on, is the lamp on or off at the end?

Instructions

The above problems are usually less intensive mathematically than you'd expect on a normal day in class, but logically tougher. So the difficulty lies in ensuring that you know your logic is sound and you can **explain to your reader** your reasoning. Since much of the mathematical work will be done in class collaborating with your peers, the majority of your marks will be obtained through fully outlining the problem, and explaining your own personal solution to it.

This could lead easily into a lecture on the Bisection method I was discussing using Python for root-finding, which might be more time consuming so would require another class. The other thing I realized is that bisection method could be covered while you're still in limits and then Newton's method would be more comfortable to cover once you're in derivatives since they'll have done something extremely similar.
Another thing this could lead into if time permits (in the same class) would be a discussion of the Dirichlet function, which would refresh their memory on piece-wise notation from PC 11, as well as lead into the mindset of "close enough" when you're dealing with neighborhoods in Epsilon-Delta proofs.

References

[1] K. Simonyi "A Cultural History of Physics" ISBN 978-1-56881-3295

[2] MartinGrandjean, "Zeno Achilles Paradox" Licensed under CC BY-SA 4.0 via Commons - `https://commons.wikimedia.org/wiki/File:Zeno_Achilles_Paradox.png#/media/File:Zeno_Achilles_Paradox.png`

Marking Rubric

Criteria	1	2	3	4	Value
Problem Solving: Specifically dealing with the logic necessary to solve problems, not numerical skills	Little or no understanding of the problem is evidenced	Numerous errors when solving problem	Few errors when solving problem	No errors when solving problem	
Math Content: Purely numerical skills, including applying the correct formulas and executing calculations.	Demonstrates little or no knowledge or application of math skills	Demonstrates a limited knowledge and application of math skills	Demonstrates a general knowledge and application of math skills	Demonstrates a clear knowledge and application of math skills	
Math Communication:	Inaccurately communicates solution to problem and concepts	Limited communication of solution to problems and concepts	Satisfactorily communicates solution to problems and concepts	Accurately communicates solution to problem and concepts. Makes effort to discuss alternate viewpoints to controversial problems	
Presentation: The successful student will provide a solution that will only require basic mathematical knowledge to follow and understand; all steps should be outlined fully before being executed.	The reader is unable to follow the steps taken in the solution	Solution is difficulty to follow at times. Reader has to make assumptions about steps taken.	Solution is presented in a logical manner with no guesswork necessary by the reader	Solution is presented in an easy to follow step-by-step model	
Use of Mathematical Terminology: Where applicable, references to sources including the names of formulae applied and their relevance to the problem.	No mathematical terminology is used or attempted	Some mathematical terminology is presented, but not correctly used	Mathematical terminology is correctly used	Mathematical terminology is prevalent and used correctly	
				Total:	