Tuesday, August 8, 2017

MAT 21A - Lecture 2

Trig Identities

Trig identities are useful equations used in trigonometry that are always true. The most fundamental identity is as follows:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

From the above identity, several more identities can be derived. For example:

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

Note: the position of the plus and minus operators. Many of the other identities can be obtained via solving / using algebra on the above, especially the first, equation.

A few more valuable identities are as follows:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$$

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

Exponential functions

Exponential functions involve a function where x is the power of a base. For example:

$$f(x) = a^x$$

A few notes on this:

- a is assumed to be positive and not 0. This can be explained via the following example: $a^{\frac{1}{2}} = \sqrt{a}$
- Thus if a is a number less than zero we will be attempting to take the square root of a negative number which is of course, not possible.

On a side note, a valuable formula to recognize when dealing with exponential functions is the following:

$$x = \frac{p}{q}$$

Then:

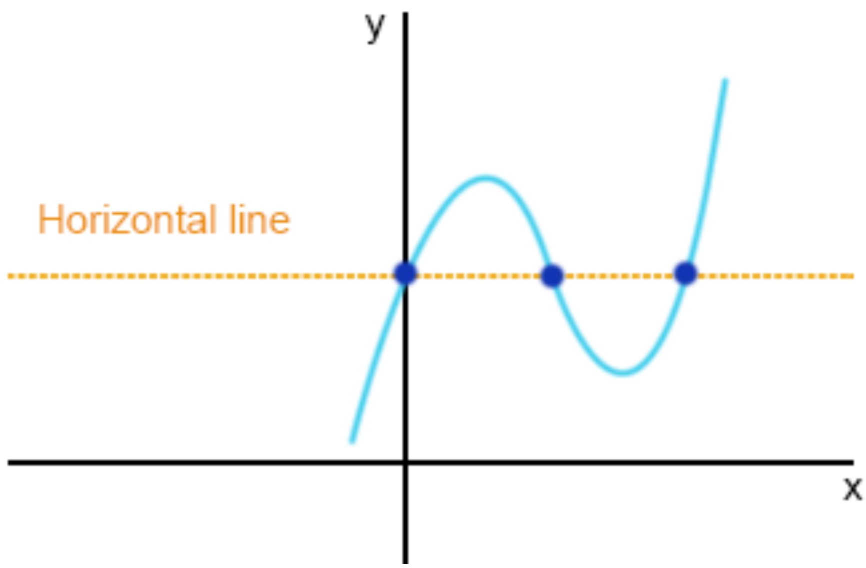
$$a^x = \sqrt[q]{a}^p$$

Assuming $q > 0$

One to one functions

A one to one function always gives a unique value for a unique input, e.g $f(x_1) \neq f(x_2)$ unless $x_1 = x_2$.

A horizontal line test can be done to test if a function is one to one. Similar to the vertical line test, in the horizontal line crosses more than two points of a graph at once, it is **not** a one to one function. For example, the below function is not a one to one:

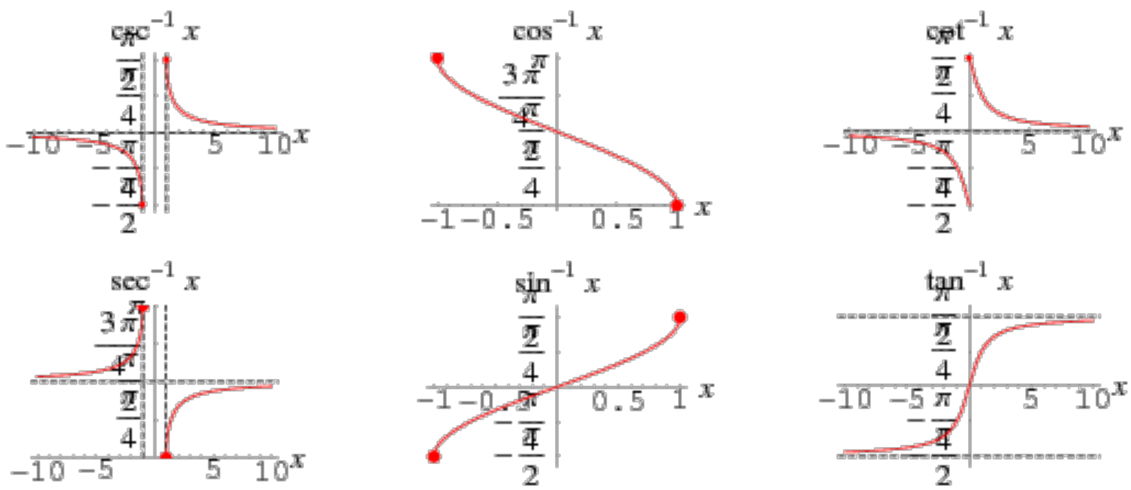


Inverse trig functions

Just like getting an inverse of a function, trig functions (sin and cos etc) must be restricted or else the result will **not** be a function.

Below is an example of how the different graphs have been restricted in order to show the inverse. Note, the superscript of -1 does not indicate the reciprocal, but instead the inverse, just like inverse function notation.

Another term is $arcsin$ where "arc" is in front of the trig function.



Logarithms

- Logarithms are inverses of exponential functions.
- The domains are the same between functions and inverses.

It is important to be careful or generic logs, where the subscript is implied. In can either be base e or base 10, so it is important to pay attention to the equation and context.

There are also a common set of logarithmic rules that are important to memorize:

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where: $b > 1$, and M, N and k can be any real numbers

but M and N must be positive!