Monday, August 7, 2017

MAT 21A - Lecture 1

Set theory

Intro: Set theory is one of the most fundamental aspects of maths.

Definition: A well defined mental collection of objects.

Example: A set of all the students in a room

In a set an object can be one of two things:

1. An element of the set
2. **Not** an element of the set

Set notation:

Example: $\{1,3,7\}$

- Use of curly brackets
- Elements separated via commas
- The order of elements doesn't matter

Set properties:

- Sets can be finite or infinite
- Sets are defined by rules, for example, "*if a person is a student*, they are part of the set"

Real numbers

In MAT 21A, we mostly deal with only real numbers.

Definition: A real number is a number that can be represented on a number line. This includes rational numbers.

A set of real numbers can be written like so:

Example: $\{ \ x : where \ x \ is \ a \ real \ number \ \} \ = \ \mathbb{R}$

Real numbers can also be defined through set notation:

Example: $\{ \ -\infty : \infty \ \} \ = \ \mathbb{R}$

Sets can also make use of interval notation. Interval notation represents an interval of real numbers through the use of a single pair of numbers.

Example: $\{ \ x \ : \ 5 < x < 7 \ \} = (5, \ 7)$

It's important to note that when one of the pairs of numbers in the interval notation is included in the set, a square bracket is used instead of a normal bracket. For example:

Example: $\{ \ x \ : \ 5 \leq x < 7 \ \} = [5, \ 7)$

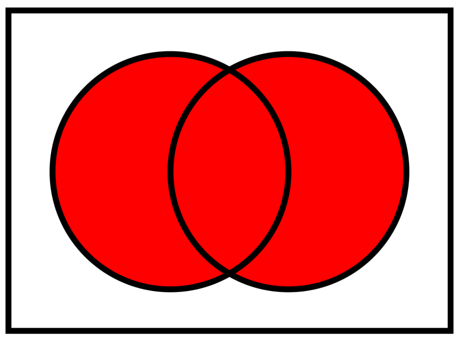
Set operations

Basic yet important set operations include:

- Union
- Intersection

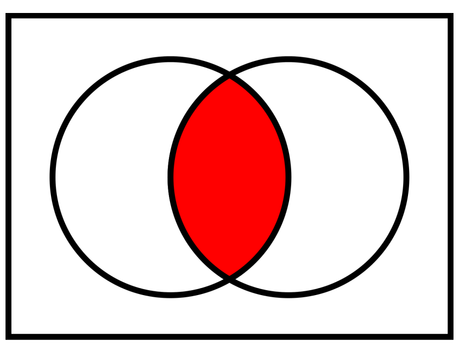
Union

A union between two or more sets is also known as the “or” operation. For example, a union between set A and B is just like saying, “A or B”. Venn diagrams can be used to visualize a union between two sets:



Intersection

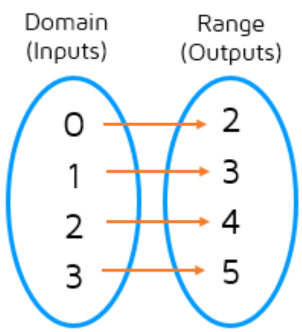
An intersection between two or more sets is also known as the “and” operation. For example, a intersection between A and B is like saying “show only elements that are apart of both set A **and** B”. The Venn diagram for an intersection looks like so:



Functions

Functions are essentially rules that map every object in one set, the domain, to a unique object in a second set, called the co-domain. Take a look at the following example:

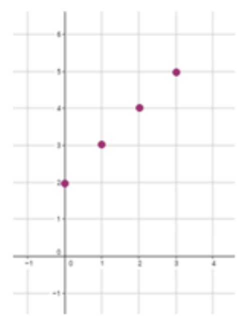
Mapping



Table

x	y
0	2
1	3
2	4
3	5

Graph



Ordered Pairs

$\{(0,2),(1,3),(2,4),(3,5)\}$

To the left we can see two sets, where each value from the domain is mapped to the range.

Here there a few things to note:

- Co-domain is the set of values that could possibly come out the function, while the range is the set of values that do come out the function. Thus, co domain is sometimes interchanged with range and vice versa.
- The ordered pairs to the right contain ordered pair notation inside the set, **not interval notation.**

Functions can also be defined via set notation:

$$f(x) = x + 7 = \{(x,\ x + 7) : x\ is\ a\ real\ number\}$$

An equation can only be considered a function if it's all of the elements in the domain set map to a maximum of 1 value in the co-domain. If this condition is not satisfied, the equation can not be considered a function. For example:

$$y^2 = x$$

In the above equation, if the domain (the x value) was 1, y could be either -1 or 1, thus it shows how the domain is mapping to both -1 and 1, and the definition of a function does not allow for an element from the domain set to map to more than one element of the co-domain set.

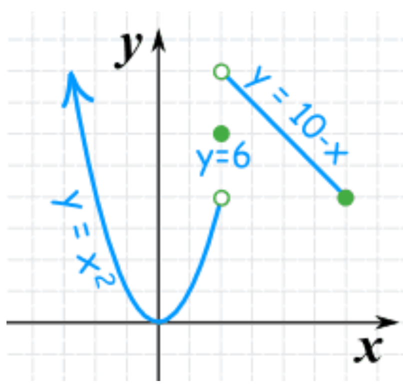
Piecewise functions

Piecewise functions are essentially functions that behave differently based different x input values (MathIsFun https://www.mathsisfun.com/sets/functions-piecewise.html).

For example, a function could be written with the following three rules.

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ 10 - x & \text{if } x > 2 \text{ and } x \leq 6 \end{cases}$$

When graphed, it will look like the following, segmented into different pieces (hence the name):



Properties:

- Uses multiple rules
- No obligation for the graph to be continuous
- Discontinuous gaps are allowed

Properties of functions

How to tell when a function is increasing or decreasing?

Through mathematical notation, the formal way to tell whether a function is increasing or decreasing at a point can be done like so (where x represents the domain and y the range):

Increasing: $f(x) < f(y)$

Decreasing: $f(y) < f(x)$

Even or Odd functions

Even functions can be flipped around the y axis and remain symmetrical. A parabola is a fantastic example of an even function. Mathematically, a function is even if the following condition is satisfied:

Even: $f(-x) = f(x)$

Odd functions exist when the function can be symmetrical via "rotating the origin", or in other words flipping both the x and y axis:

Odd: $f(-x) = f(x)$

Cubic functions are a great example of odd functions.

Common types of functions

The three most common type of functions (not in any particular order) are:

1. Polynomials -> Follow the binomial theorem
2. Root functions -> Where a function is inside of a root
3. Rational functions -> A function divided by a function

Function composition

Function composition can be described as using the result from one function as the input to another function, ultimately resulting in a third, composed, function. For example, the notation is as follows:

$$f \circ g(x) = f(g(x))$$

In addition, composed functions can be defined via set notation:

$$f \circ g = \{x : g(x)\ is\ in\ the\ domain\ of\ f\}$$

Transformation of Graphs

Vertical shifts: $f(x) + a = g(x)$

Horizontal shifts: $f(x - a) = g(x)$

Reflections:

- Y axis: $f(-x) = g(x)$
- X axis: $-f(x) = g(x)$

Scaling:

- Horizontal: $f(x/a) = g(x)$
- Vertical: $a \cdot f(x) = g(x)$

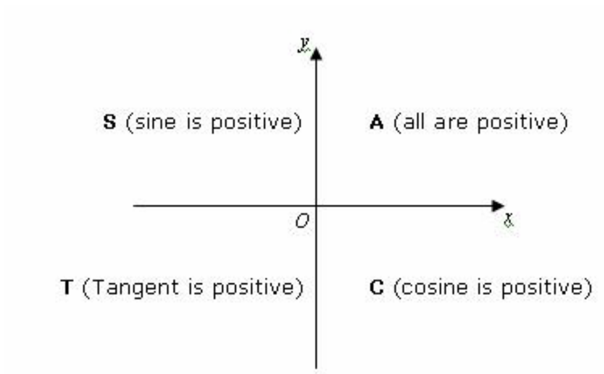
Trigonometric functions

Trigonometry can be broadly defined as the relative study of angles.

The unit circle

The unit circle has a radius of one, centered at the origin. It is used to understand sin and cosine angles in right angled triangles.

The unit circle can be divided in to four quadrants using the letters, CAST, as explained in the diagram below:



Sin is represented as the y value of each point in the unit circle, and cosine is represented as the x value. Thus, tan can be derived from any point on the unit circle by dividing the sine of the y value by the sine of the x value:

$$tan(x) = \frac{sin(x)}{cos(x)}$$

Radians

Both degrees and radians can be used as measurement units for angles. 2 pi is equal to 360 degrees. Below are some common values of radians:

$$2\pi = 360^\circ$$

$$\pi = 180^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{6} = 30^\circ$$

Easy angles

The below triangles can be used to simply obtain many useful angles in an intuitive way through making use of Pythagoras's theorem: