Two Way ANOVA

Evaluation Methods & Statistics- Lecture 11

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Research Example (Last Time)

- Consequences of a secondary task on driving
  - Texting
  - Talking on phone
  - Control (no secondary task)

Research Example (This week)

Consequences of a secondary task on driving

- Texting
- Talking on phone
- Control (no secondary task)

Gender effect

- Male
- Female

What are our DV and IVs?

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IV 1- Secondary Driving Task

- Level 1- Control Group (No secondary task)
- Level 2- Texting
- Level 3- Talking

IV 2- Participant Gender

- Male
- Female

DV-Driving score

How would we analyse the data?

- We could do lots of t-tests
  - Control to Texting for Males
  - Control to Talking for Males
  - Talking to Texting for Males
  - Control to Texting for Females
  - Control to Talking for Females
  - Talking to Texting for Females
  - Control (Males) to Control (Females) etc…..
- This would inflate our *Type I error rate*

Factorial ANOVA- The Idea

- One Way ANOVA cannot deal with two factors (i.e. two Independent variables)

- We need to use a different type of ANOVA if we have two independent variables

Types of Factorial ANOVA

- Independent factorial design
  - All independent variables are between subjects
- Repeated Measures
  - All independent variables are within subjects
  - E.g. Measuring satisfaction pre and post (IV 1) 3 different interfaces (IV 2) all experienced one after another
- Mixed Design
  - Some independent variables are within subjects and some are between subjects
  - Gender (IV1) on pre and post scores (IV2)

Types of Factorial ANOVA

- What type of ANOVA are we going to use in our example?

Types of Factorial ANOVA

What type of ANOVA are we going to use in our example?

- Independent factorial design
  - All independent variables are between subjects

ANOVA Names

- Number of IV's
- Number of levels in IV's
- Experiment design used to gather data
  - Independent (Between subjects)
  - Repeated Measures (Within subjects)

Example

If we had:

- IV 1- Gender (2 levels- Between)
- IV2- Secondary Task (2 levels- Between)

2x2 Independent ANOVA

– or-

Two Way Independent ANOVA

Example

If we had:

- IV 1- Gender (2 levels- Between Subjects)
- IV2- Secondary Task (3 levels- Between Subjects)

2x3 Independent ANOVA

– or-

Two Way Independent ANOVA

Example

If we had:

- IV 1- Gender (2 levels) (Between Subject)
- IV2- Secondary Task (3 levels) (Within Subjects)

2x3 Mixed Design ANOVA

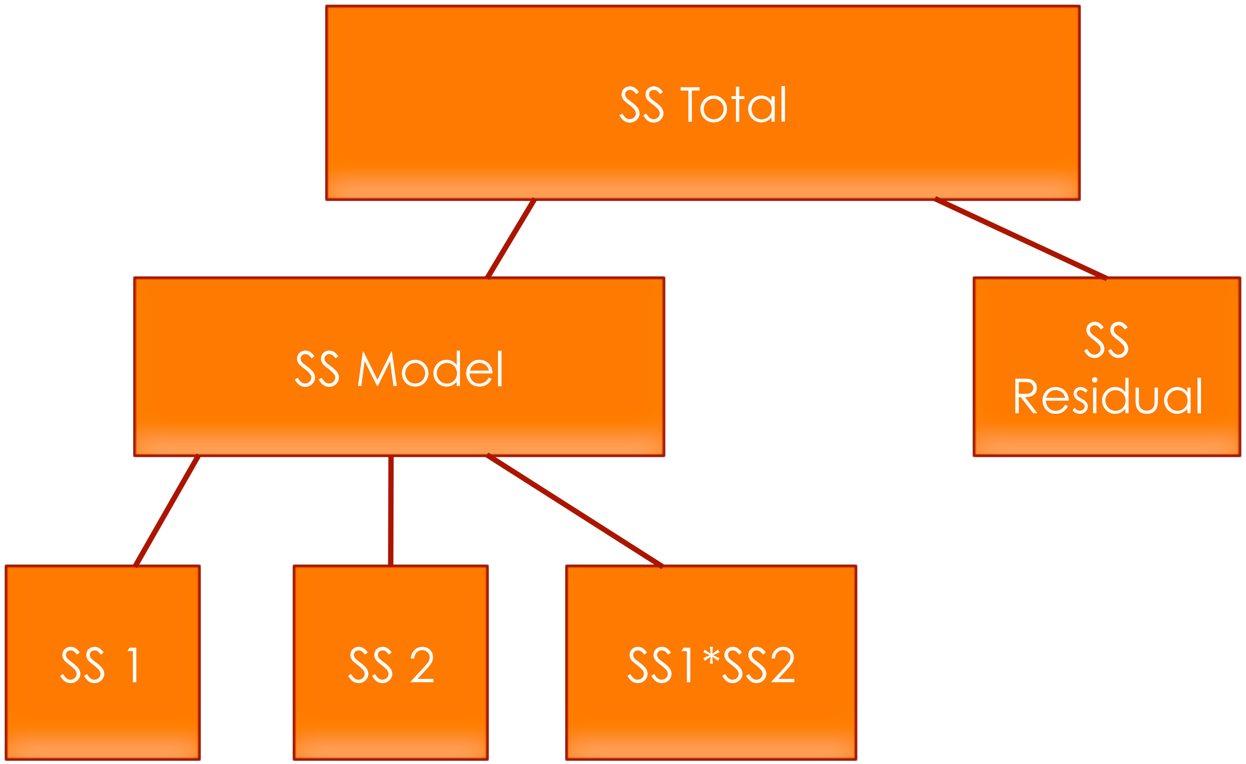
-or-

Two Way Mixed Design ANOVA

The Key (again): ANOVA & F Ratio

- F ratio is the ratio of **explained (that accounted for by the model we are proposing)** to **unexplained** variation
- This is calculated using the **Mean Squares**

What ANOVA is doing



Step 1- Total Sum of Squares

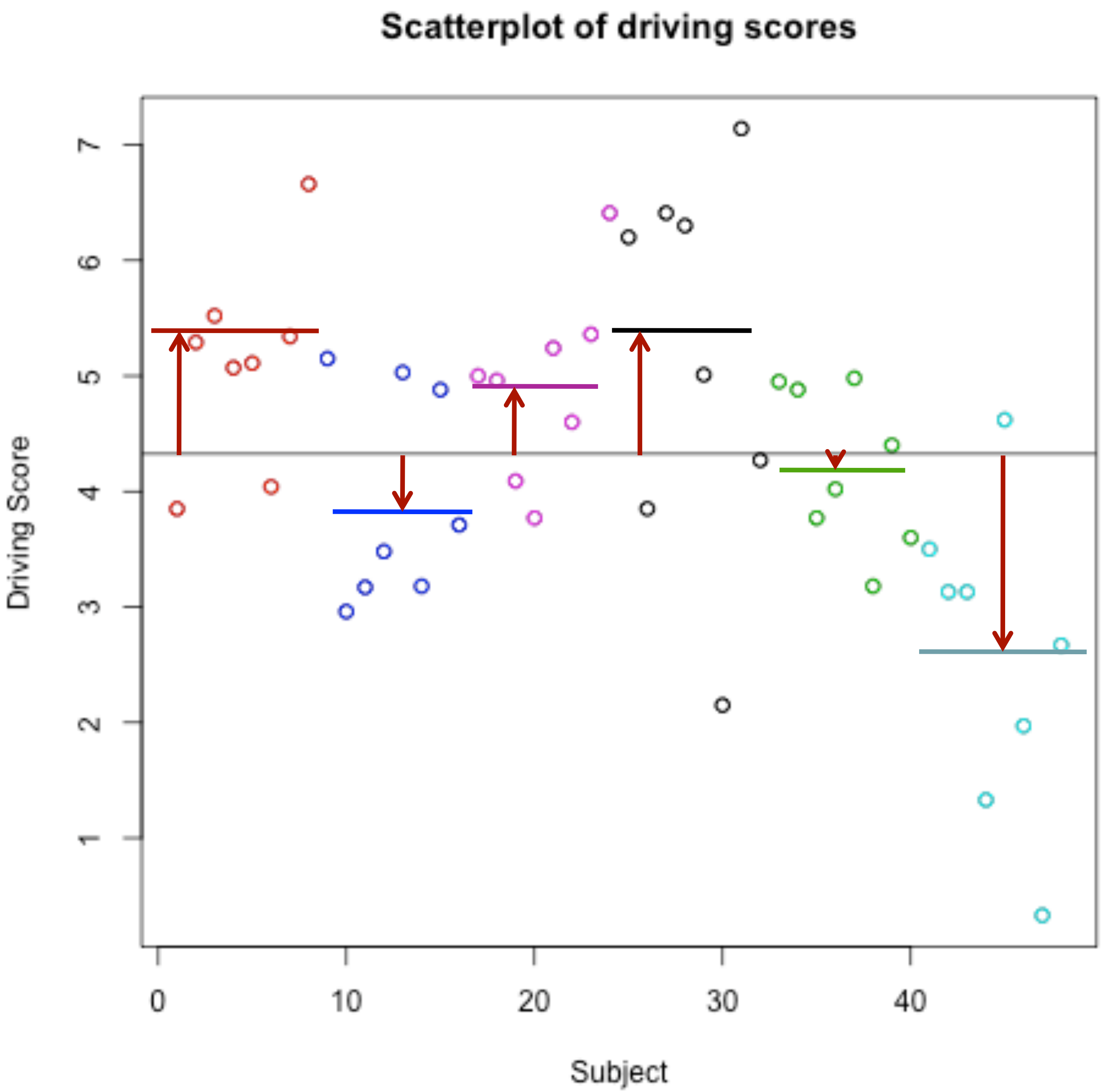
- The total amount of variation in our data
- This should look familiar (see Lecture 6)

$$SS_T = \sum \left(x_i - \bar{x}_{grand} \right)^2$$

Step 2- Model Sum of Squares

- We now need to know how much variation our model can explain
- How much the total variation can be explained due to data points coming from different groups in "the perfect model"
- n_k is the number of people in that condition
- Sum all levels of IVs together

$$SS_M = \sum n_k (\bar{x}_k - \bar{x}_{grand})^2$$

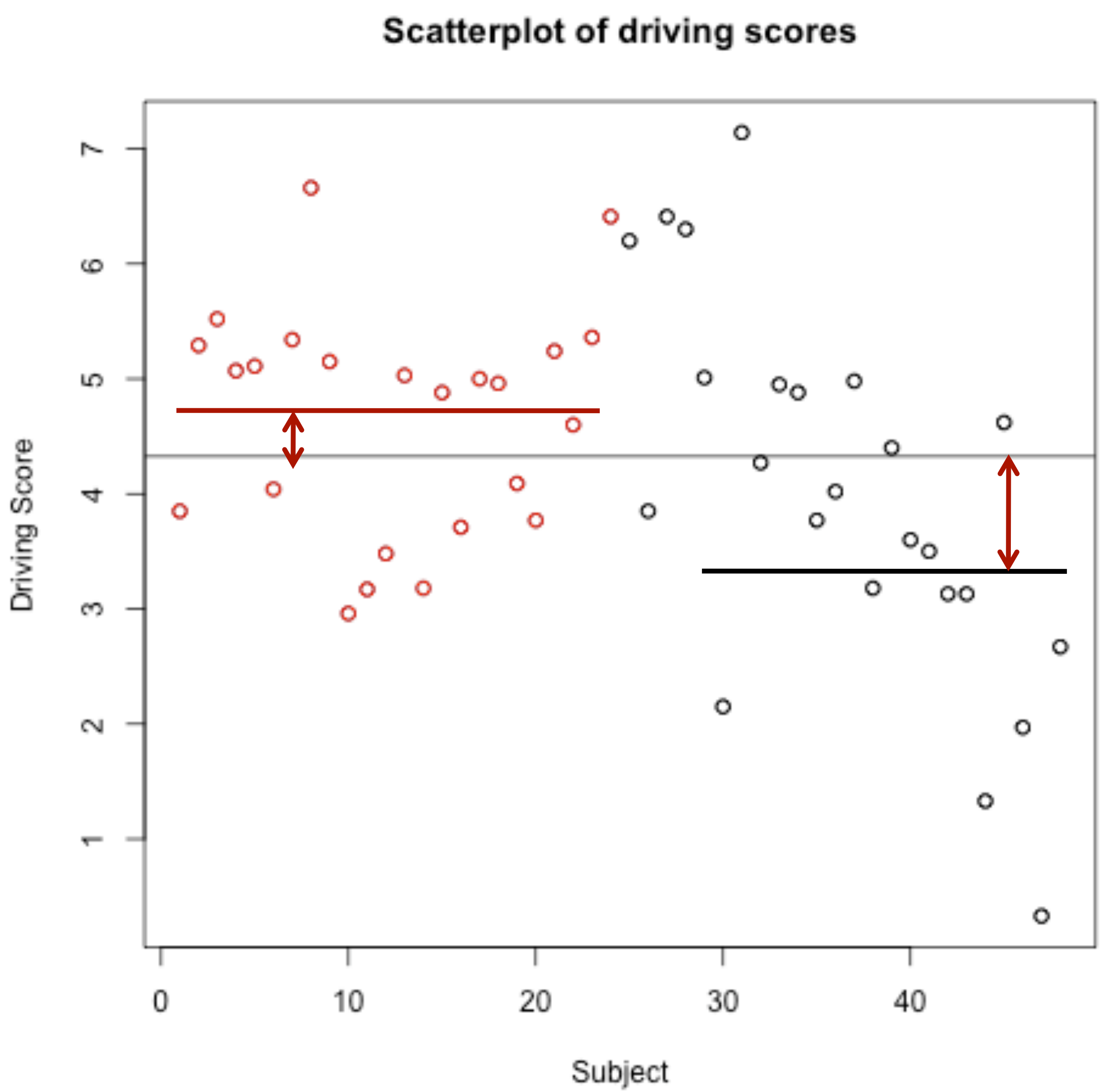


Scatterplot of driving scores
Driving Score
Subject
1
2
3
4
5
6
7
0
10
20
30
40

Step 3- Main Effect of Gender

- We group data by levels of Gender alone
- How much the total variation can be explained due to data points coming from different gender groups only in "the perfect model"
- n_k is the number of people in that condition
- Sum both levels of IV together

$$SS_1 = \sum n_k(\overline{x}_k - \overline{x}_{grand})^2$$

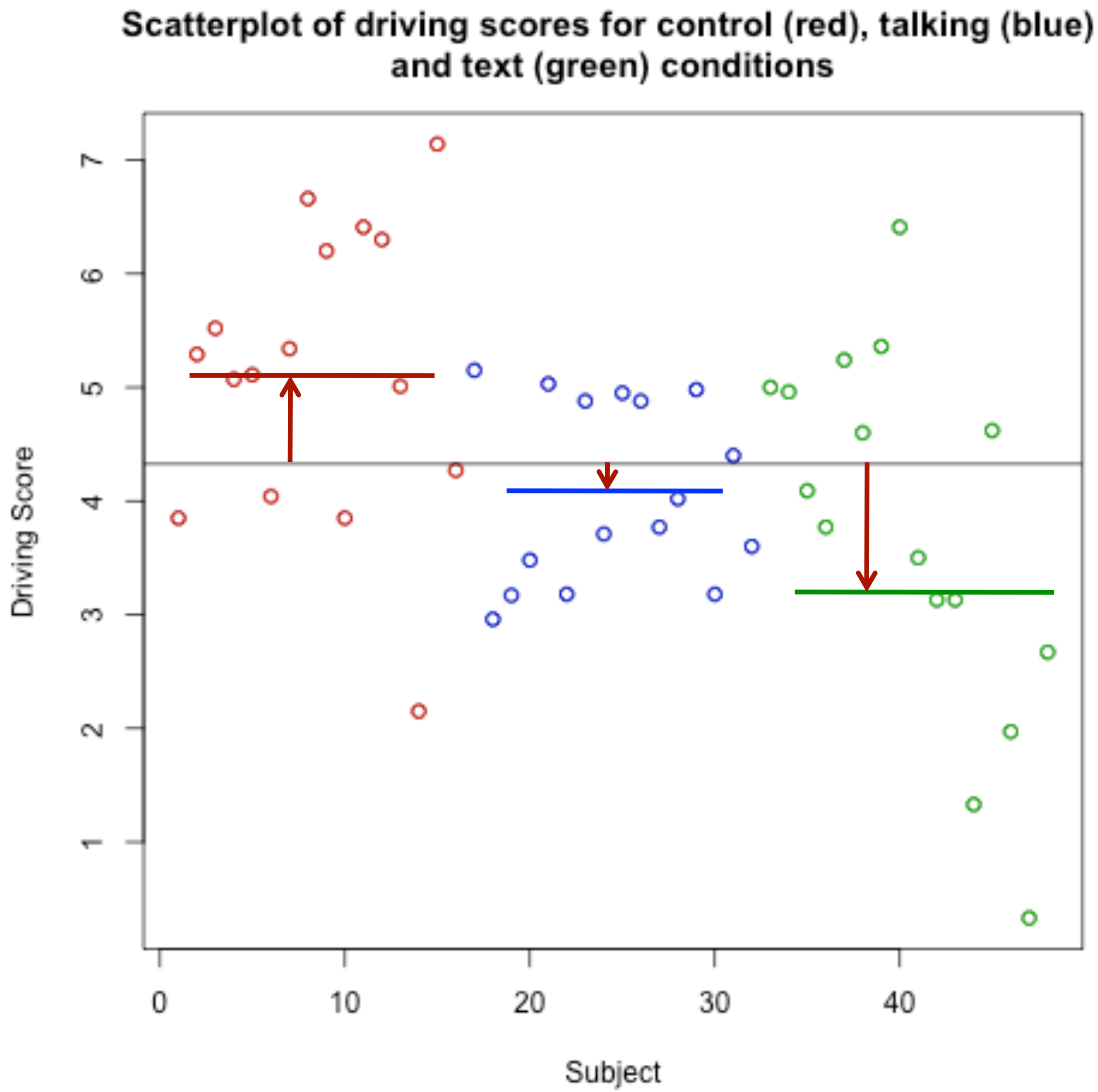


Scatterplot of driving scores
Driving Score
Subject
1
2
3
4
5
6
7
0
10
20
30
40

Step 4- Main Effect of Task

- We group data by levels of Task alone
- How much the total variation can be explained due to data points coming from different Task conditions alone in "the perfect model"
- n_k is the number of people in that condition
- Sum levels of IV together

$$SS_2 = \sum n_k(\overline{x}_k - \overline{x}_{grand})^2$$


Scatterplot of driving scores for control (red), talking (blue) and text (green) conditions
Driving Score
Subject
0
10
20
30
40
1
2
3
4
5
6
7

Step 5- Interaction (Gender *Task)

- SSm made up of 3 components:
  - SS1, SS2, SS1*2
  - Therefore easiest way is to take away SS1 and SS2 from SSM

$$SS_{1*2} = SS_M - SS_1 - SS_2$$

Step 6- Residual Sum of Squares

- How much of the variation cannot be explained by the model i.e. what error is there in the model prediction?

- Easy way to calculate: $SS_R = SS_T - SS_M$

Degrees of Freedom for each SS

- Degrees of Freedom for SS_T (dfT):
  - N-1 = 48-1 = 47
- Degrees of Freedom for SS_M(dfM):
  - Gender: Number of Conditions (k) -1 = 1
  - Task: Number of Conditions (k) -1 = 2
  - Gender*Task= df Gender*df Task = 2
- Degrees of Freedom for SS_R(dfR):
  - (N-1)*Number of groups = (8-1)*6= 42

F Ratio

- F Ratio gained for each effect (IV) and the interaction

- Mean Squares model (MS_M):
  - SS_M/df_M
  - This is done for main effects SS and interaction SS

- Mean Squares residual (error) (MS_R):
  - SS_R/df_R

F Ratio

$$\frac{\text{Mean Square Model } (MS_M)}{\text{Mean Square Residual } (MS_R)}$$

F Ratio

$$\frac{\text{Variation explained by our model}}{\text{Variation unexplained by our model}}$$

F Distribution

- F Distribution for specific pair of degrees of freedom

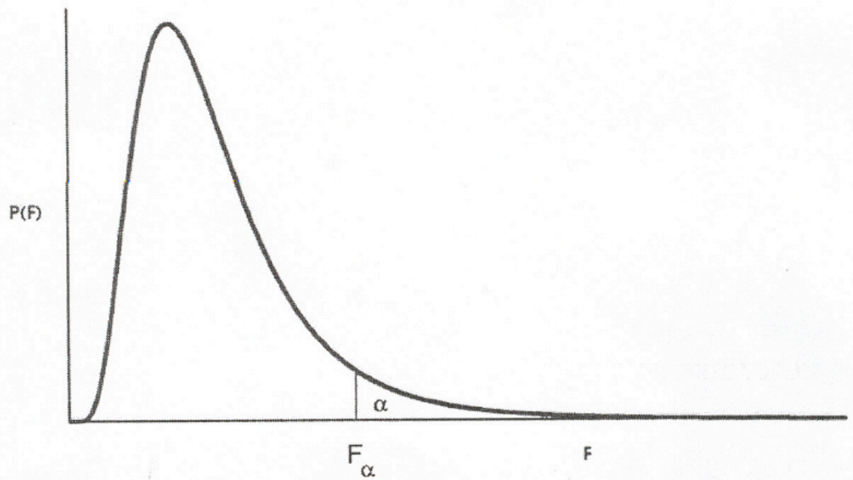


Figure K.1: The F distribution

- Table of Critical Values

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91

Output in R

```
> summary(analysis)
                 Df Sum Sq Mean Sq F value  Pr(>F)
gender            1   5.39   5.387   4.409 0.04180 *
condition         2  16.67   8.337   6.823 0.00272 **
gender:condition  2  16.91   8.453   6.918 0.00253 **
Residuals        42  51.32   1.222
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

Main Effect

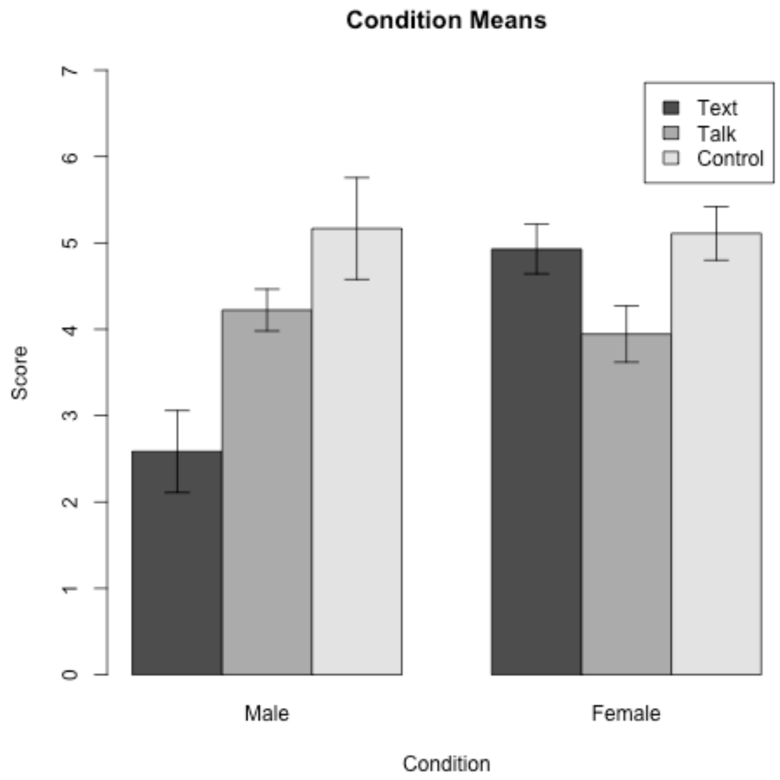
- Significant effect of an IV irrespective of the other IV

- Significant effect of gender on driving score irrespective of the task completed

Output in R

```
> summary(analysis)
                 Df Sum Sq Mean Sq F value  Pr(>F)   
gender            1   5.39   5.387   4.409 0.04180 * 
condition         2  16.67   8.337   6.823 0.00272 **
gender:condition  2  16.91   8.453   6.918 0.00253 **
Residuals        42  51.32   1.222                   
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

Interactions



- The effect of secondary task on score is not that same for male and females
- interaction effect
- Females have a higher score in the texting condition compared to males
- Interactions supersedes main effects

Omnibus test & Post Hoc

- ANOVA is omnibus test
- How do they break down?
  - Male vs Female ? (Main Effect)
  - Control vs Text, Control vs Talk, Talk vs Text ? (Main effect of Task)
  - Male:Control vs Female Control etc (interaction)
- Again we need ***post hoc tests (See last lecture)***

Reporting ANOVA

- F ratio
- Degrees of Freedom (dof_M, dof_R)
- P value
  - **$F(2, 42) = 5.097$, $p<0.05$**

Output in R

```
> summary(analysis)
                 Df Sum Sq Mean Sq F value  Pr(>F)
gender            1   5.39   5.387   4.409 0.04180 *
condition         2  16.67   8.337   6.823 0.00272 **
gender:condition  2  16.91   8.453   6.918 0.00253 **
Residuals        42  51.32   1.222
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

Reporting ANOVA

e.g. Main effect of Gender

- **$F(1, 42) = 4.409$, $p<0.05$**

Reading

- Field (2012) Discovering Statistics Using R, Chapters 12 -14