# Fundamentals/ICY: Databases 2013/14

## ***WEEK 8 –Monday***

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# Reminder

# Some More Examples

- **{JAB, “JAB”}** has 2 members: me, and a 3-char string.
- **{3, {4,5}, 4, 6}**has 4 members, one of which is a set.
- **{3, {5,4}, 4, 6}** is that same set.
- **{ {4,5} }** has 1 member, which is a set.
- **{4,5}** has 2 members, both numbers.
- **{∅}** is a singleton set. Its only member is the empty set.
- **{{∅}}** is a different singleton set.

# New

# Membership Relationship

- **a** $\in$ **A** means that **a is a member** of **A**.
  - **5 $\in$ {4,5}**
  - **{5,4} $\in$ {3, {4,5}, 4, 6}**
- **a** $\notin$ **A** means that **a** is **not** a member of **A**.
  - **5 $\notin$ {3, {4,5}, 4, 6}**
  - **{5} $\notin$ {3, {4,5}, 4, 6}**
  - **{4,6} $\notin$ {3, {4,5}, 4, 6}**
  - **{3,4,5} $\notin$ {3, {4,5}, 4, 6}**

# Subsets and Supersets

- **A $\subseteq$ B** means that A is a "**subset**" of B (and that B is a "**superset**" of A). I.e., every member of A is also a member of B.
  - Carefully distinguish between ***subset-of*** and ***member-of*** !!!
  - The symbol $\subset$ means the same as $\subseteq$
  - $\subset$ does NOT mean that there cannot be equality.
- Examples:
  - **$\emptyset \subseteq \{4,5\}$**
  - **$\{5\} \subseteq \{4,5,6\}$, $\{6,4\} \subseteq \{4,5,6,7\}$, $\{6,4,7,5\} \subseteq \{4,5,6,7\}$**
  - **$\{n \mid n$ is an EVEN whole number$\} \subseteq \{n \mid n$ is a whole number$\}$**

# Subsets and Supersets

- $\emptyset \subseteq A$ for any set A.
- $A \subseteq A$ for any set A. ***(Reflexivity)***
- If $A \subseteq B$ and $B \subseteq A$ then $A = B$. ***(Antisymmetry)***
- If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. ***(Transitivity)***

# A Note about Antisymmetry

- In arithmetic, both $<$ and $\leq$ (and similarly $>$ and $\geq$) are antisymmetric under the definition:

  ***A (mathematical) relationship is antiymmetric iff: <u>IF</u> it applies in both directions between items A and B then A and B are the same item.***

- It’s just that in the case of $<$ and $>$ it can never apply in both directions between any A, B.

# A Note about Antisymmetry
## (with extra explanation added after lecture)

- In arithmetic, both $<$ and $\leq$ (and similarly $>$ and $\geq$) are antisymmetric under the definition:

  ***A (mathematical) relationship is antiymmetric iff the following holds: IF it applies in both directions between items A and B then A and B are the same item.***

- Another way of putting that definition:

  ***A (mathematical) relationship is antiymmetric iff the following holds: IF it applies in one direction between two DIFFERENT items A and B then it cannot apply in the other direction.***

- $<$ (similarly $>$) can never apply in both directions between any A, B —so the first purple rule is vacuously satisfied. But $\leq$ (similarly $\geq$) can be in both directions—though only when A=B.

# Some Operations on Sets

- ***Union*** of sets A and B:
- A $\cup$ B = the set of things that are in A **or** B (or both).
- NB: ***no repetitions created***.
- ***Intersection*** of sets A and B:
- A $\cap$ B = the set of things that are in **both** A **and** B.
- ***Difference*** of sets A and B:
- A $-$ B = the set of things that are in A but not B.
- Note: also notated by a backslash instead of a minus sign.
- ***The minus sign is also more standardly used as in*** A $-$ **a** to mean remove member **a** from A (if it’s a member of A at all).

# Some Properties of those Operations

- Union and intersection are ***commutative* *("can switch")***:
  - $A \cup B = B \cup A$
  - $A \cap B = B \cap A$
- Union and intersection are ***associative***
  ***("can group differently"):***
  - $A \cup (B \cup C) = (A \cup B) \cup C$
  - $A \cap (B \cap C) = (A \cap B) \cap C$
- Because of associativity, we can omit parentheses for union-only or intersection-only cases:
  - $A \cup B \cup C \cup D$ $\qquad$ $A \cap B \cap C \cap D$

# Bad Associations …

- ***Caution: if an operation is not associative, the position of parentheses is normally important.***
  - In arithmetic, division is non-associative.

    ***(x/y)/z*** is usually a different value from ***x/(y/z).***

# Two Other Properties

- Union ***distributes over*** intersection:
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- Intersection ***distributes over*** union:
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$