1 Derivation of Thermal Background

Number of incident photons, N_γ, is

$$N_\gamma = \int_{\nu_1}^{\nu_2} \frac{\epsilon B_\nu(T)}{h_n u}\,\mathrm{d}\nu \tag{1.1}$$

This integral spans the filter from the red to the blue end.

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\mathrm{e}^{\frac{h\nu}{kT}} - 1} \tag{1.2}$$

where $\mathrm{e}^{\frac{h\nu}{kT}} \ll 1$. Using the substitution

$$x = \frac{h\nu}{kT}, \qquad \nu = \frac{kTx}{h} \tag{1.3}$$

$$\Rightarrow \mathrm{d}\nu = \frac{kT}{h}\,\mathrm{d}x \tag{1.4}$$

$B_\nu(T)$, then, is

$$B_\nu(T) = \frac{2k^3T^3x^3}{h^2c^2}\frac{1}{\mathrm{e}^x} \tag{1.5}$$

Thus, the number of photons is

$$N_\gamma = \frac{2k^3T^3\epsilon}{h^2c^2}\int_{x_1}^{x_2} x^2\mathrm{e}^{-x}\,\mathrm{d}x \tag{1.6}$$

$$= \frac{2k^3T^3\epsilon}{h^2c^2}\left[-x^2\mathrm{e}^{-x} + \int \mathrm{e}^{-x}\cdot 2x\,\mathrm{d}x\right]_{x_1}^{x_2} \tag{1.7}$$

$$= \frac{2k^3T^3\epsilon}{h^2c^2}\left[-x^2\mathrm{e}^{-x} - 2x\mathrm{e}^{-x} + \int 2\mathrm{e}^{-x}\,\mathrm{d}x\right]_{x_1}^{x_2} \tag{1.8}$$

$$= \frac{2k^3T^3\epsilon}{h^2c^2}\Big[-\mathrm{e}^{-x}\left(x^2 + 2x + 2\right)\Big]_{x_1}^{x_2} \tag{1.9}$$

By integrating from the blue end of the filter to the red, we can remove the negative sign to give

$$N_\gamma = \frac{2k^3T^3\epsilon}{h^2c^2}\Big[\mathrm{e}^{-x}\left(x^2 + 2x + 2\right)\Big]_{x_1}^{x_2} \tag{1.10}$$