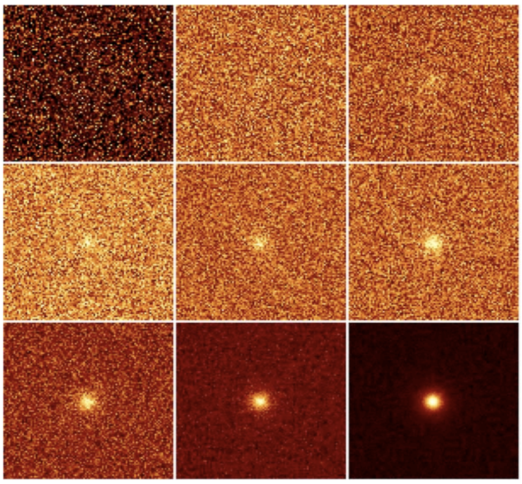
Signal, noise, and photometry

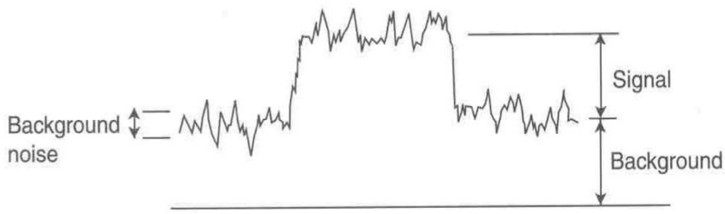
Signal and Poisson Noise



Courtesy: Ben Maughan

Signal and Background

- In general: Signal = (Signal + Background) – Background
- Backgrounds:
  - Sky – especially bright in the red / infrared
  - Dark current
  - [Residual cosmic rays; Flat-fielding errors; Residual fringing]
- Backgrounds need to be measured and subtracted
- Noise is introduced by the precision to which background is measured



Noise in CCD Observations

- Photon counting
  - The Poisson error on a detection of N photons
- Dark current
  - Error on the subtraction of the dark current
- Sky background
  - Error on the subtraction of the sky background
- Read noise
  - Error introduced by the digitization of the measured signal

$$\sigma^2 = \sigma^2_{\text{Poisson}} + \sigma^2_{\text{Dark}} + \sigma^2_{\text{Sky}} + \sigma^2_{\text{RN}}$$

CCD Equation

- Consider a CCD for which the dark current is **D** electrons/pixel/sec and the read noise is **R** electrons
- This CCD is used to observe a star for a total of **t** seconds
- A total of **F** photons/sec are detected from the star in a measurement aperture of **N** pixels
- The sky background is measured to be **B** photons/sec/pixel

$$\text{Signal} = S = Ft$$
$$\sigma_{\text{Poisson}} = \sqrt{Ft}$$
$$\sigma_{\text{Dark}} = \sqrt{DNt}$$
$$\sigma_{\text{Sky}} = \sqrt{BNt}$$
$$\sigma_{\text{RN}} = R\sqrt{N}$$

$$\frac{S}{N} = \frac{Ft}{\sqrt{\sigma_{\text{Poisson}}^2 + \sigma_{\text{Dark}}^2 + \sigma_{\text{Sky}}^2 + \sigma_{\text{RN}}^2}}$$
$$\frac{S}{N} = \frac{Ft}{\sqrt{(F + DN + BN)t + R^2 N}}$$

Worked Example

$$\frac{S}{N} = \frac{Ft}{\sqrt{\sigma_{\text{Poisson}}^2 + \sigma_{\text{Dark}}^2 + \sigma_{\text{Sky}}^2 + \sigma_{\text{RN}}^2}}$$
$$\frac{S}{N} = \frac{Ft}{\sqrt{(F + DN + BN)t + R^2 N}}$$

- A 20th magnitude star (in the R-band) is observed for 20 seconds with LRIS on the Keck-I telescope in seeing of FWHM=0.7arcsec
- Count rate from the star:
  F = 1890 electrons/sec
- Signal (F) measured within circle of diameter 4arcsec (~6 x seeing)
- LRIS pixels subtend 0.14arcsec:
  N = π(2/0.14)² = 640pixels
- Dark current is negligible:
  D ~ 0.001electrons/pix/sec
- Read noise = R = 5 electrons
- Sky background:
  B = 39 electrons/sec/pix

$$\sigma_{\text{Poisson}} = \sqrt{1890 \times 20} = 194\text{electrons}$$

$$\sigma_{\text{Dark}} \approx \sqrt{0.001 \times 640 \times 20} = 3.6\text{electrons}$$

$$\sigma_{\text{RN}} = 5 \times \sqrt{640} = 126\text{electrons}$$

$$\sigma_{\text{Sky}} = \sqrt{39 \times 640 \times 20} = 706\text{electrons}$$

$$\frac{S}{N} = \frac{1890 \times 20}{\sqrt{194^2 + 3.6^2 + 126^2 + 706^2}} = 51$$

Limiting Cases

$$\frac{S}{N} = \frac{Ft}{\sqrt{(F + DN + BN)t + R^2 N}}$$

- Source photon noise limited
  - Source is much brighter than the background
  - E.g. bright stars observed in the optical

$$\frac{S}{N} = \sqrt{Ft} \propto \sqrt{t}$$

- Sky/background limited
  - Sky is much brighter than the source
  - E.g. faint galaxies observed in the near-infrared

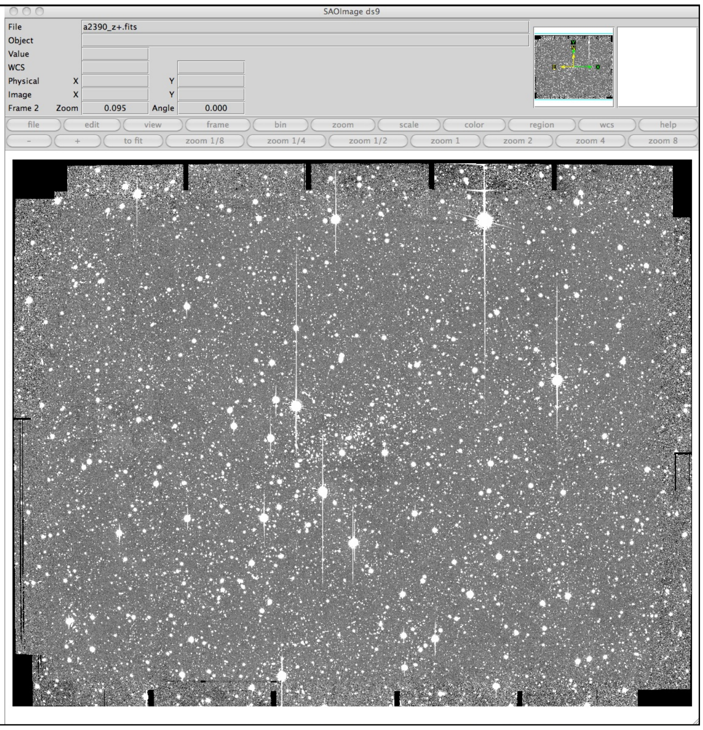
$$\frac{S}{N} = F\sqrt{\frac{t}{BN}} \propto \sqrt{t}$$

Photometry

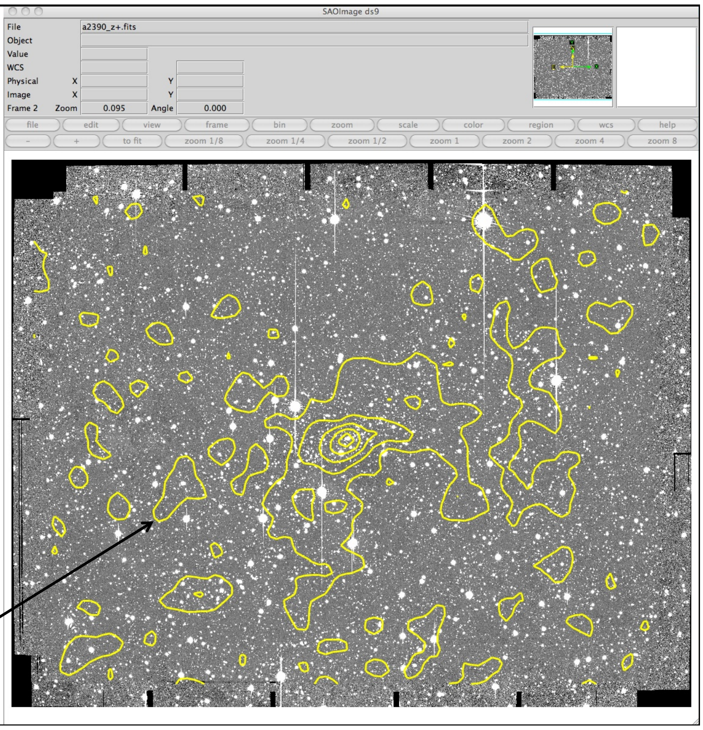
Abell 2390
- Galaxy cluster
- Gravitational lens
- z=0.2

Subaru Telescope
- 8.2m primary mirror
- f/2 prime focus

Suprime-CAM
- ~30arcmin fov
- z-band observation
- 9x4minutes
- FWHM~0.6arcsec



Photometry



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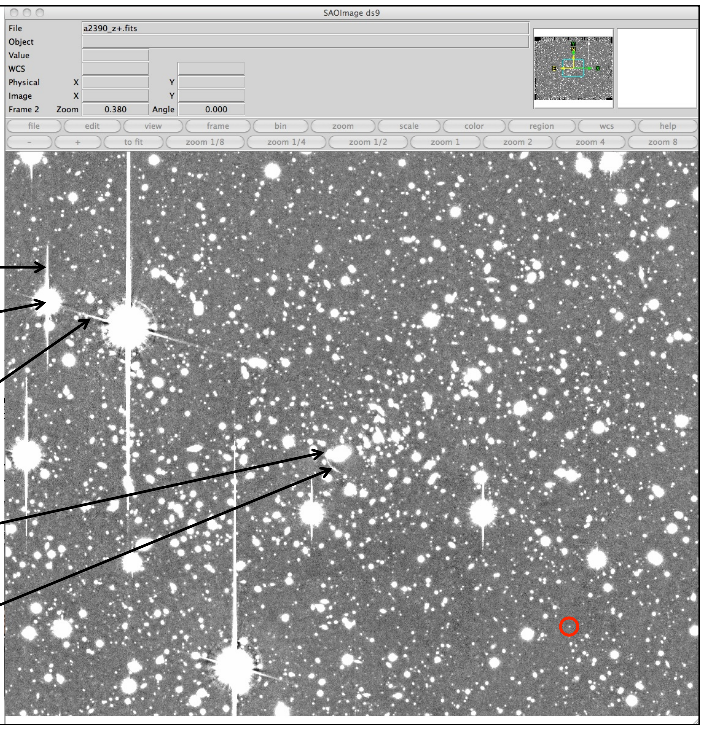
Contours show density of dark matter

Photometry



Charge bleed

Saturated star

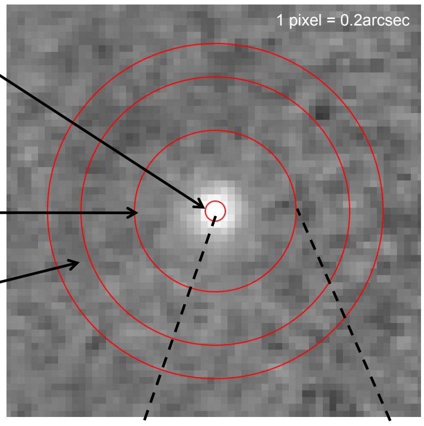
Diffraction spike

Center of cluster

Gravitational arc

Photometry

FWHM=0.6arcsec



Measure flux:
$F = F_{object} + F_{sky}$
inside aperture of
radius = 4xFWHM

Calculate $B=\langle B_i \rangle$ from
sky background annulus

Number of pixels in flux
measurement aperture:
$N = \pi(4x0.6/0.2)^2 = 452$pixels

$F_{sky} = BN$

Ignoring dark current and read noise:

$$\frac{S}{N} = \frac{(F - BN)t}{\sqrt{(F + BN)t}} = \frac{F_{\text{object}}}{\sqrt{F_{\text{object}} + 2F_{\text{sky}}}}\sqrt{t}$$

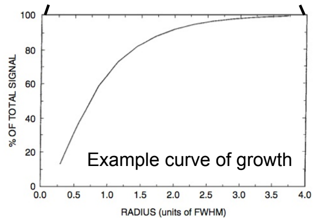
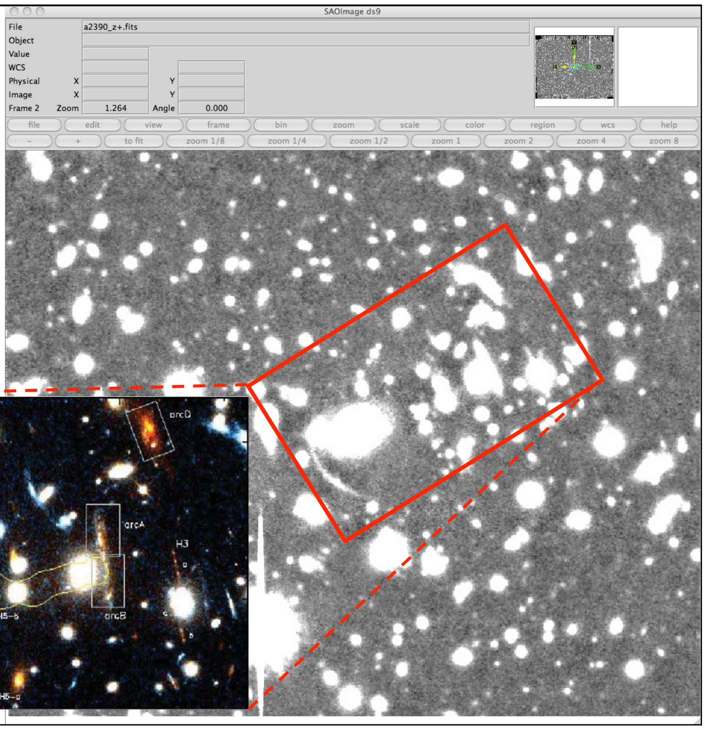


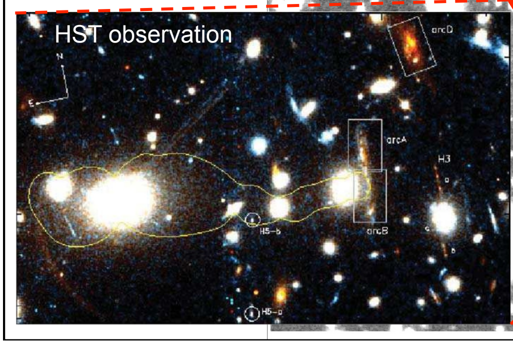
Fig. 5.6. For any reasonable PSF approximation, the figure above shows the run of the total encircled signal with radius of the PSF in FWHM units. Note that within a radius of 3·FWHM essentially 100% of the signal is included.

Photometry

The center of A2390 contains many gravitational arcs.

Photometry of irregular structures and overlapping galaxies is very challenging, even with HST data!





Relating S/N to Magnitudes

$$
\begin{aligned}
m \pm \delta(m) &= m_0 - 2.5\log_{10}(S \pm N) \\
&= m_0 - 2.5\log_{10}\left[S\left(1 \pm \frac{N}{S}\right)\right] \\
&= m_0 - 2.5\log_{10}(S) - 2.5\log_{10}\left(1 \pm \frac{N}{S}\right) \\
&= m - 2.5\log_{10}\left(1 \pm \frac{N}{S}\right) \\
\delta(m) &\approx \pm 2.5\log_{10}\left(1 + \frac{N}{S}\right) \\
&\approx \pm 2.5\log_{10}(e)\log_e\left(1 + \frac{N}{S}\right) \\
&\approx \pm 1.087\left[\frac{N}{S} - \frac{1}{2}\left(\frac{N}{S}\right)^2 + \frac{1}{3}\left(\frac{N}{S}\right)^3 - \ldots\right] \\
&\approx \pm 1.087\left(\frac{N}{S}\right)
\end{aligned}
$$

Errors are not symmetric when logged

Taylor expansion

Errors on magnitude scale are (within 10%) the same as fractional flux errors

S/N, Magnitudes, Statistical significance

Statistical Significance	Fractional flux error	δm	
100	1%	0.011	
50	2%	0.022	
10	10%	0.11	
5	20%	0.22	Threshold for scientific discovery
3	33%	0.36	Marginal detection
2	50%	0.54	Not publishable

At low significance the factor 1.087 is important