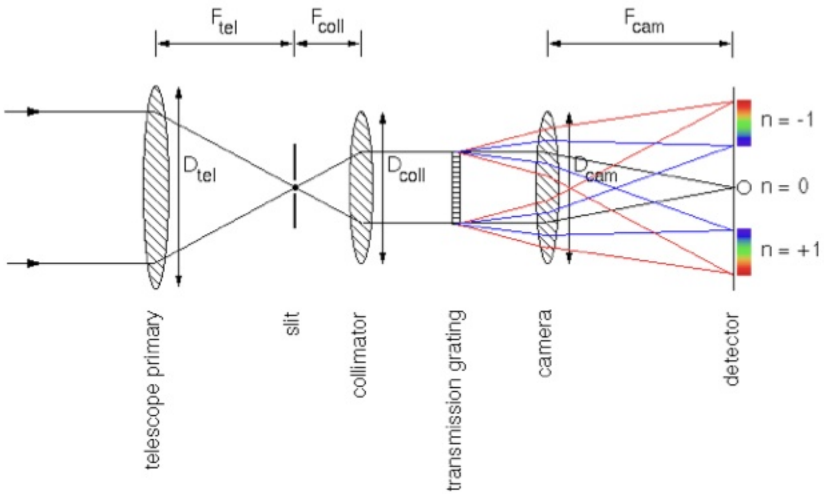
Optical Spectroscopy

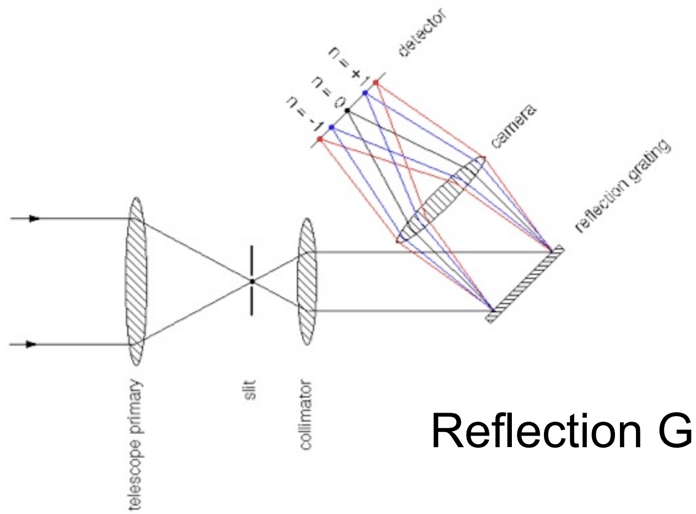
Overview

- Why spectroscopy?
- Basic configuration & dispersive elements
- Diffraction optics
  - Interference, dispersion, resolution
- Blazed gratings
- Slit-less spectroscopy, slits, slit-losses
- Long-slit, multi-slit, multi-fibre observations

Basic Configuration

Transmission Grating





Reflection Grating

- Slit: isolates portion of sky that is imaged (not required)
- Collimator: makes the beam parallel
- Dispersive element: disperses light in wavelength
- Camera: focus light on detector, where the spectrum is recorded

Why spectroscopy?

Stellar types / Stellar composition of galaxies

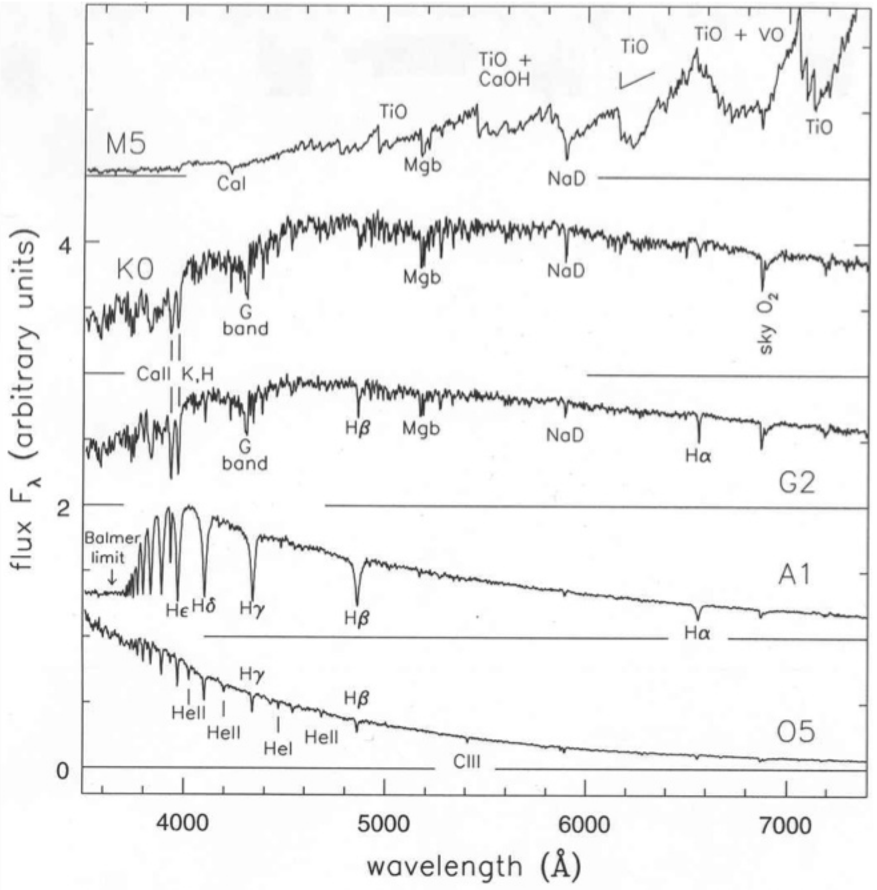
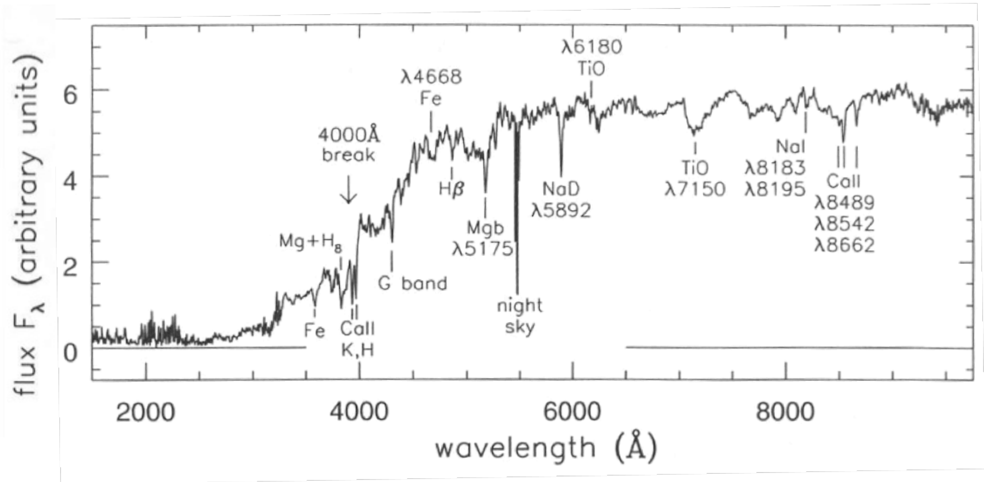


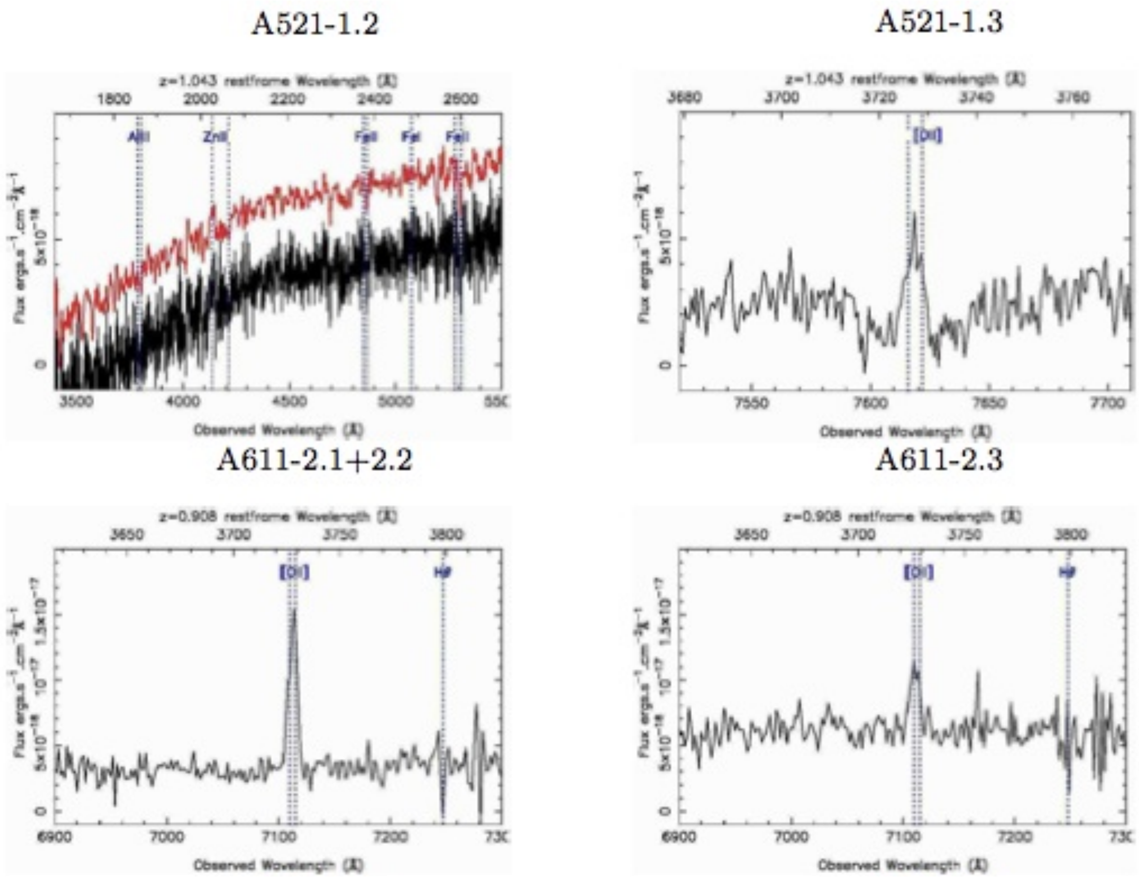
Figure 1.1 Optical spectra of main-sequence stars with roughly the solar chemical composition. From the top in order of increasing surface temperature, the stars have spectral classes M5, K0, G2, A1, and O5 – G. Jacoby *et al.*, spectral library.

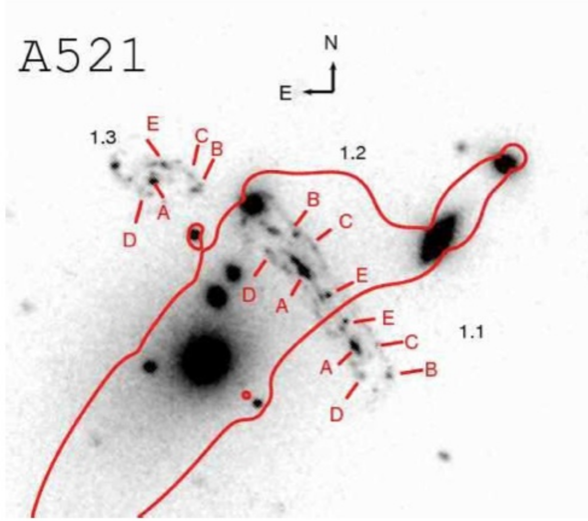
Elliptical galaxy

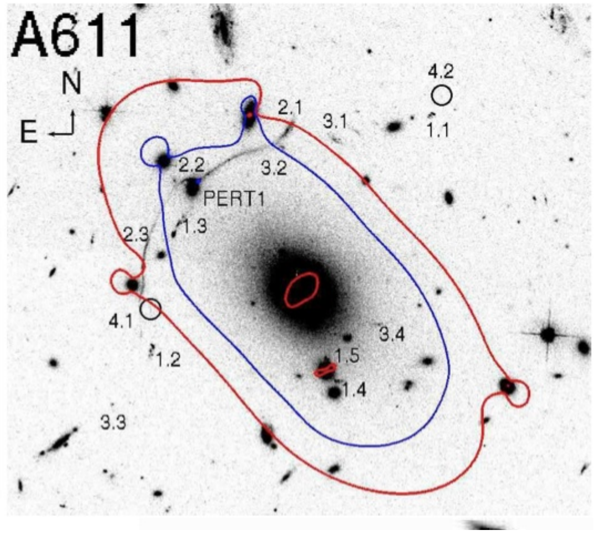


Why spectroscopy?

Measure the redshift of distant galaxies



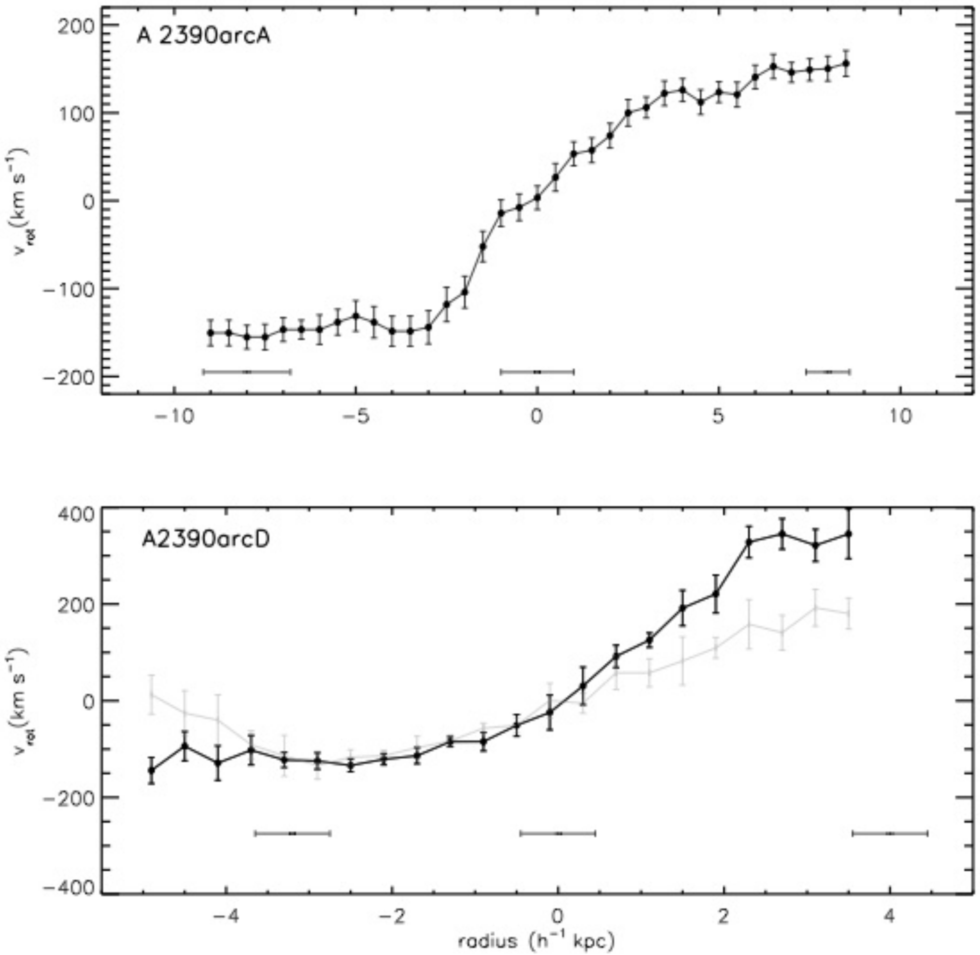


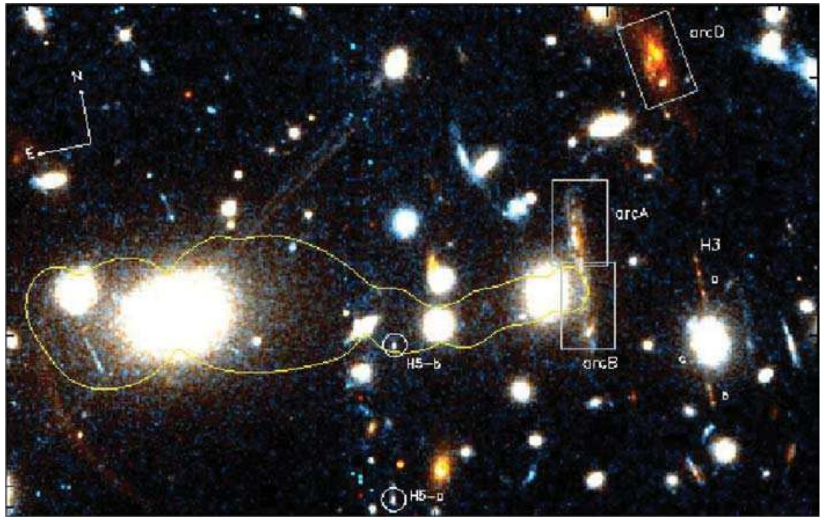


From: Richard, Smith, et al., 2010, MNRAS, 404, 325

Why spectroscopy?

Measure the rotation speed of disk galaxies

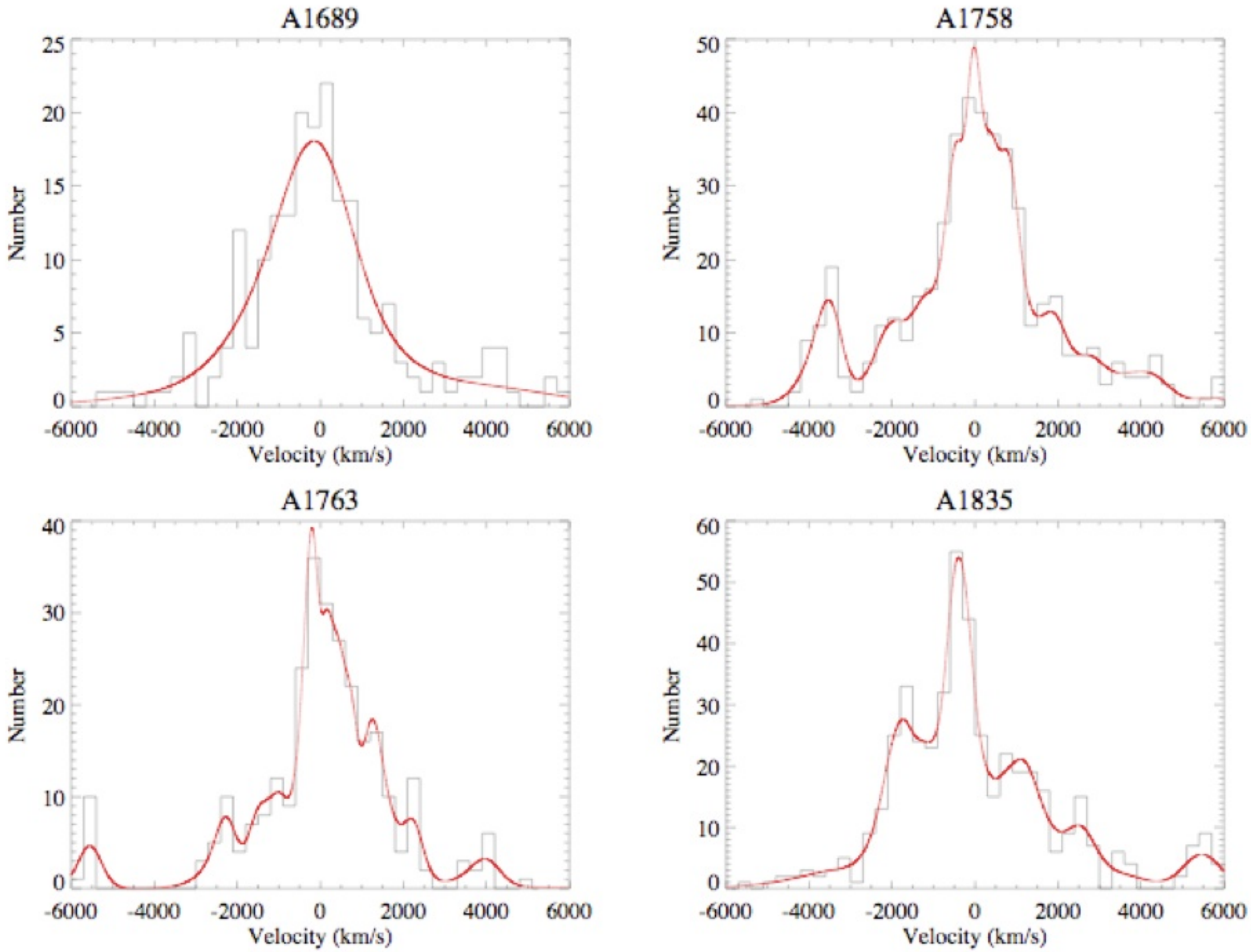




From: Swinbank, Bower, Smith, et al., 2006, MNRAS, 368, 1631

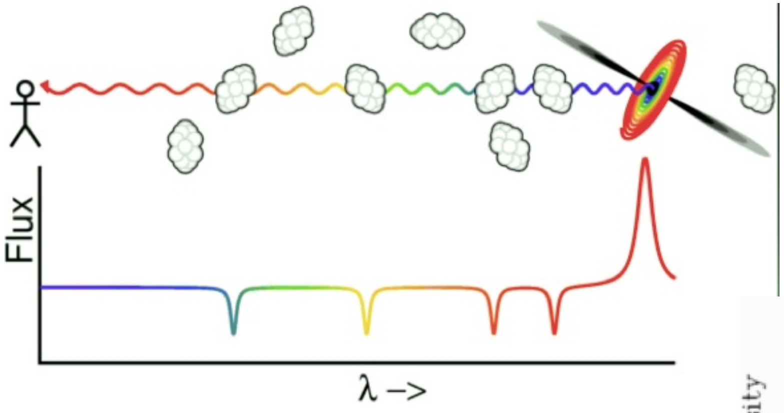
Why spectroscopy?

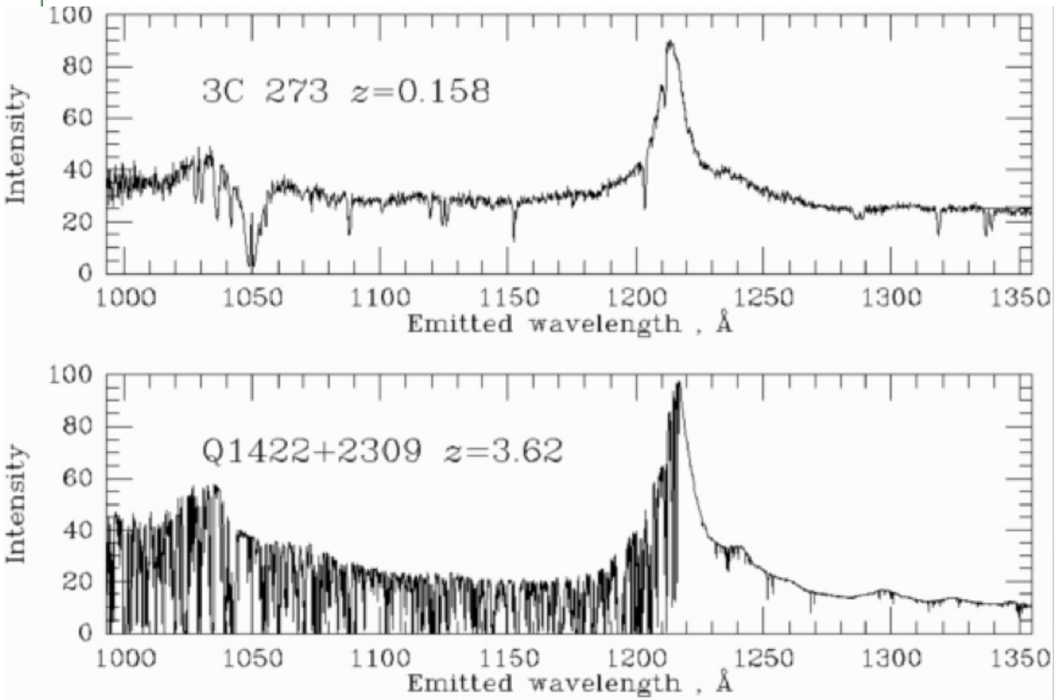
Measure the mass and internal structure of galaxy clusters



Why spectroscopy?

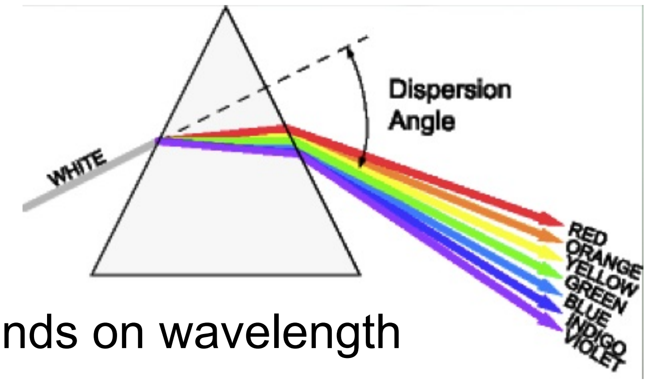
Probe the physics of the inter-galactic medium



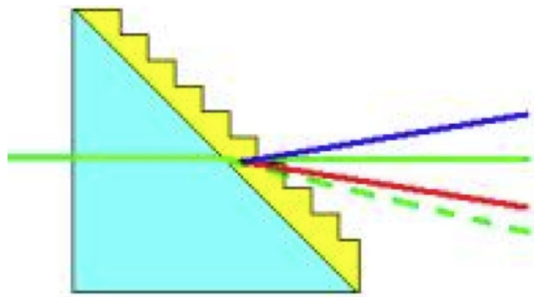


Dispersive Elements

- Prisms
  - Disperse light by refraction (Snell's law)
  - First surface disperses light
  - Second surface disperses light further
  - Angles of incidence at second surface depends on wavelength

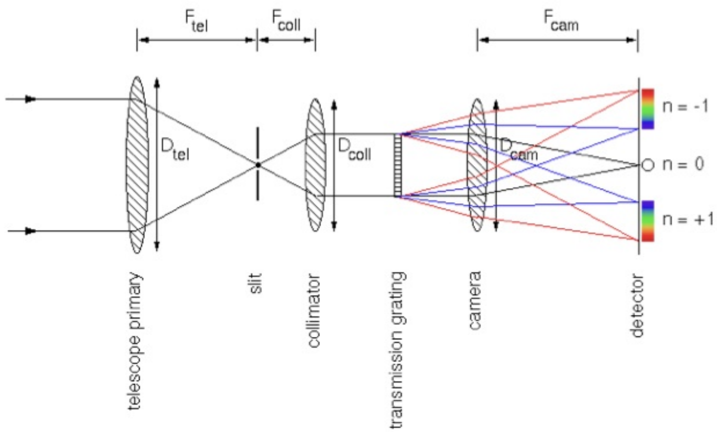


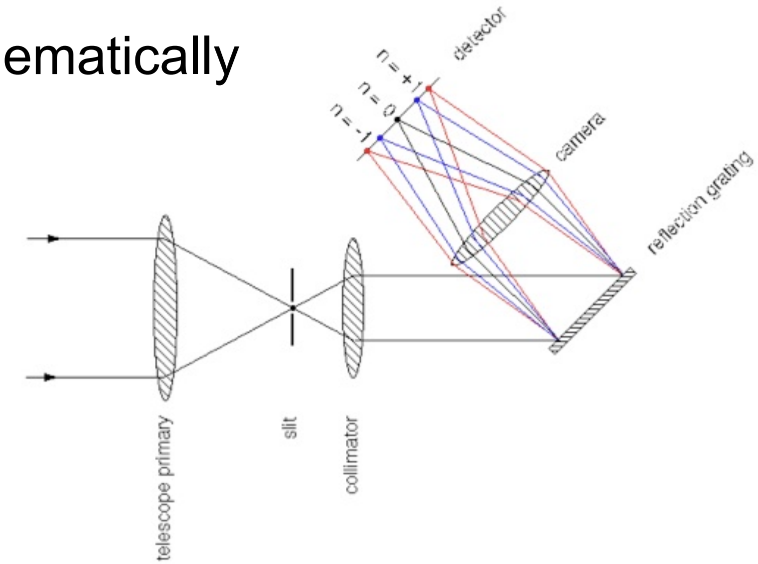
- Gratings
  - Disperse light by diffraction
  - Transmission gratings similar to Young's slits with N>>2
  - Reflection gratings have many finely spaced grooves
- Grisms
  - Cross between a prism and a transmission grating
  - Light at chosen central wavelength is transmitted un-deviated



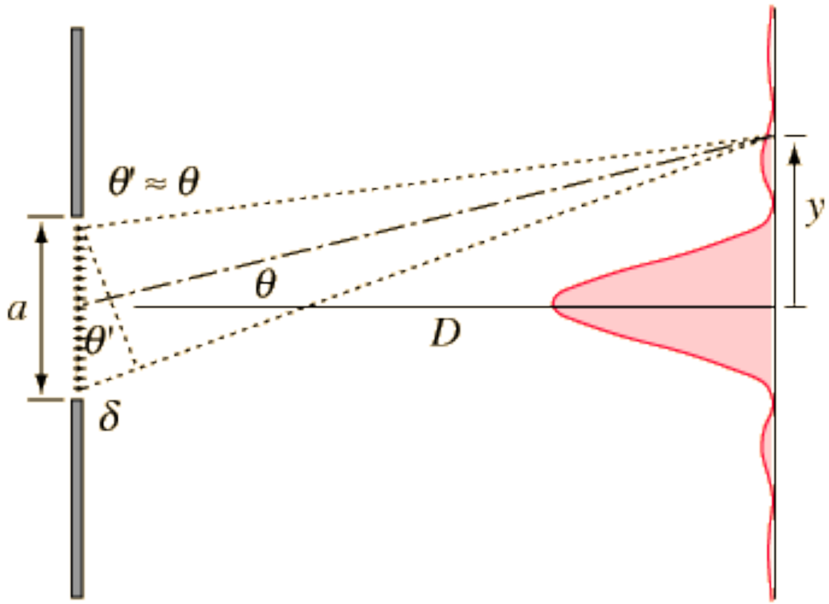
Gratings

- Transmission gratings
  - Equivalent to Young's double slit experiment, with large number of slits
- Reflection gratings
  - Closely spaced grooves act as parallel mirrors
  - Grooves are typically cut with a diamond
  - Mirrors (grooves) are typically inclined – known as blazing (more later)
  - Light reflected from adjacent mirrors interferes
- Both can be treated the same mathematically





Fraunhofer Diffraction by a Single Slit

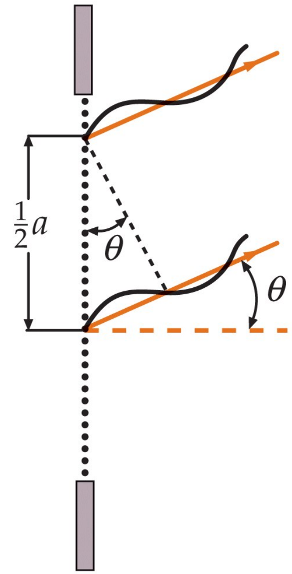


Minima occur when path difference across the width of the slit is an integer number of wavelengths:

$$p = m\lambda = a\sin\theta \approx a\tan\theta = \frac{ay_{\min}}{D}$$

Minima located at: $y_{\min} = \dfrac{m\lambda D}{a}$

Each pair of wavefronts originating from points separated by a/2 interfere destructively if p=mλ



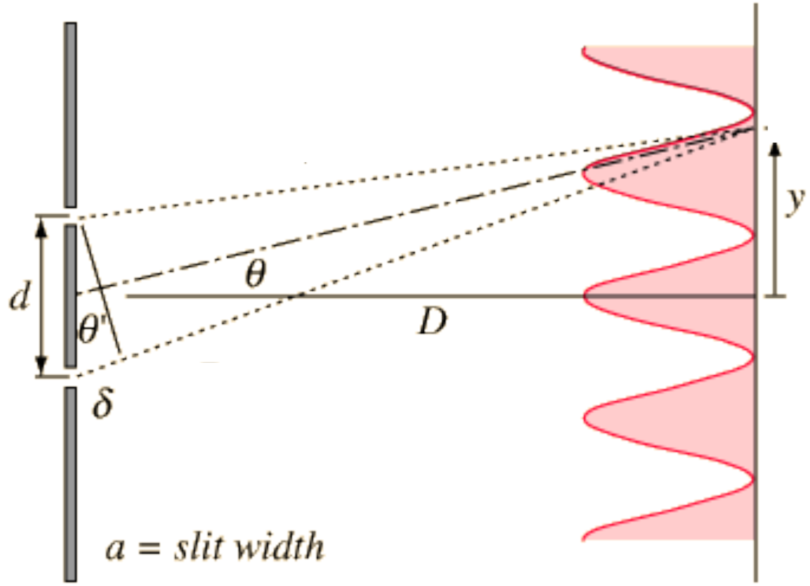
Path difference:

$$p = a\sin\theta$$

Phase difference:

$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi a\sin\theta}{\lambda}$$

Young's Double Slit Experiment



Maxima occur when path difference between the wavefronts emerging from each slit is an integer number of wavelengths:

$$p = m\lambda = d\sin\theta \approx d\tan\theta = \frac{dy_{\max}}{D}$$

Maxima located at: $y_{\max} = \dfrac{m\lambda D}{d}$

[Assumptions:
- D>>a, therefore ϑ'=ϑ and sinϑ =tanϑ =ϑ
- Incident wave is a plane wave, therefore all amplitude elements are in phase]

Path difference:

$$p = d\sin\theta$$

Phase difference:

$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi d\sin\theta}{\lambda}$$

Grating Equation

Path difference between adjacent reflections:

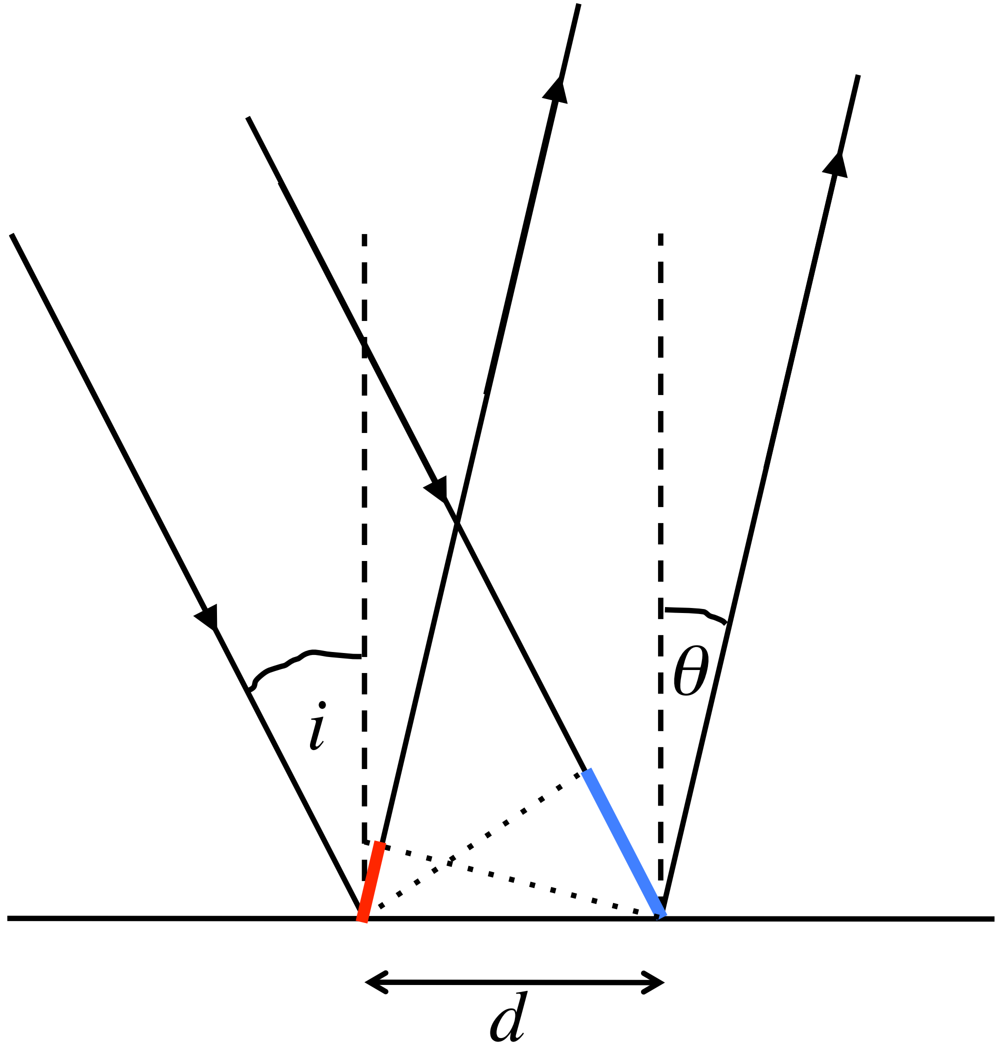
$$p = d(\underline{\sin\theta} + \underline{\sin i})$$

Phase difference between adjacent reflections:

$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi d(\sin i + \sin\theta)}{\lambda}$$

Condition for constructive interference:

$$m\lambda = d(\sin i + \sin\theta)$$



Interference in Diffracted Beam

Amplitude and phase of the diffracted beam is the sum over all N slits/grooves in the grating:

$$Ae^{i\phi} = \sum_{n=0}^{N-1} \alpha e^{in\delta} = \alpha \frac{(1-e^{iN\delta})}{(1-e^{i\delta})}$$

Phase difference between adjacent slits/grooves:

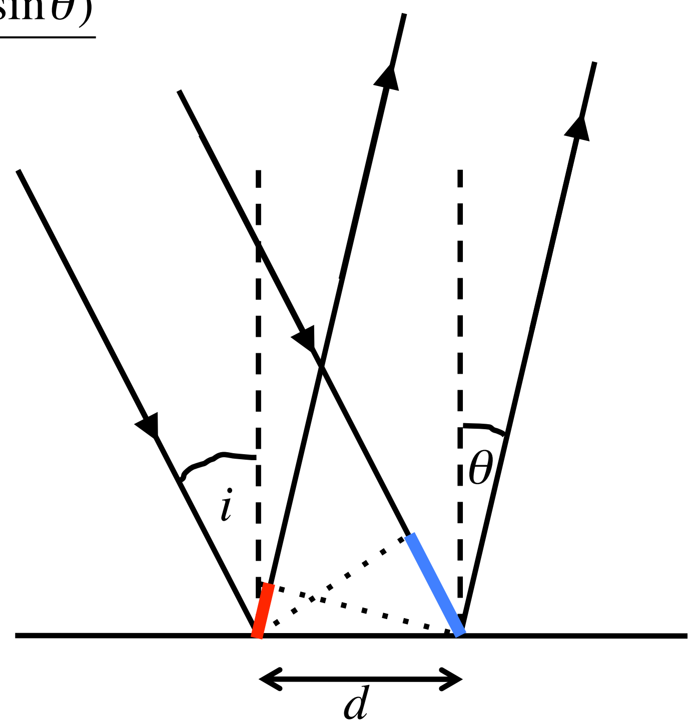
$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi d(\sin i + \sin\theta)}{\lambda}$$

Amplitude and phase of the diffraction pattern of an individual slit/groove:

$$\alpha \propto \frac{\sin\beta}{\beta}$$

Phase difference across an individual slit/groove (a = width of each slit/groove):

$$\beta = \frac{2\pi a}{\lambda}\sin\theta$$



Intensity of Diffracted Beam

Amplitude and phase of diffracted beam:

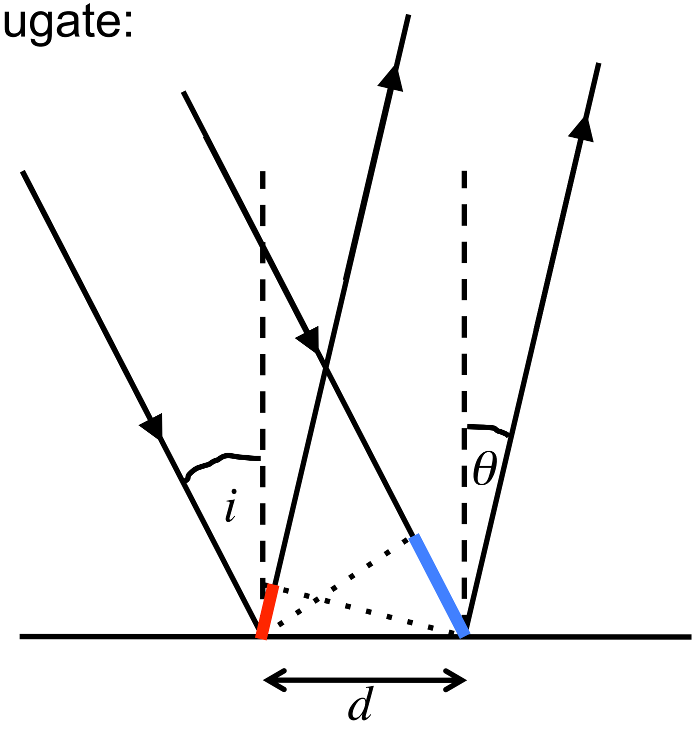
$$Ae^{i\phi} = \alpha \frac{(1-e^{iN\delta})}{(1-e^{i\delta})}$$

Multiply amplitude and phase by its complex conjugate:

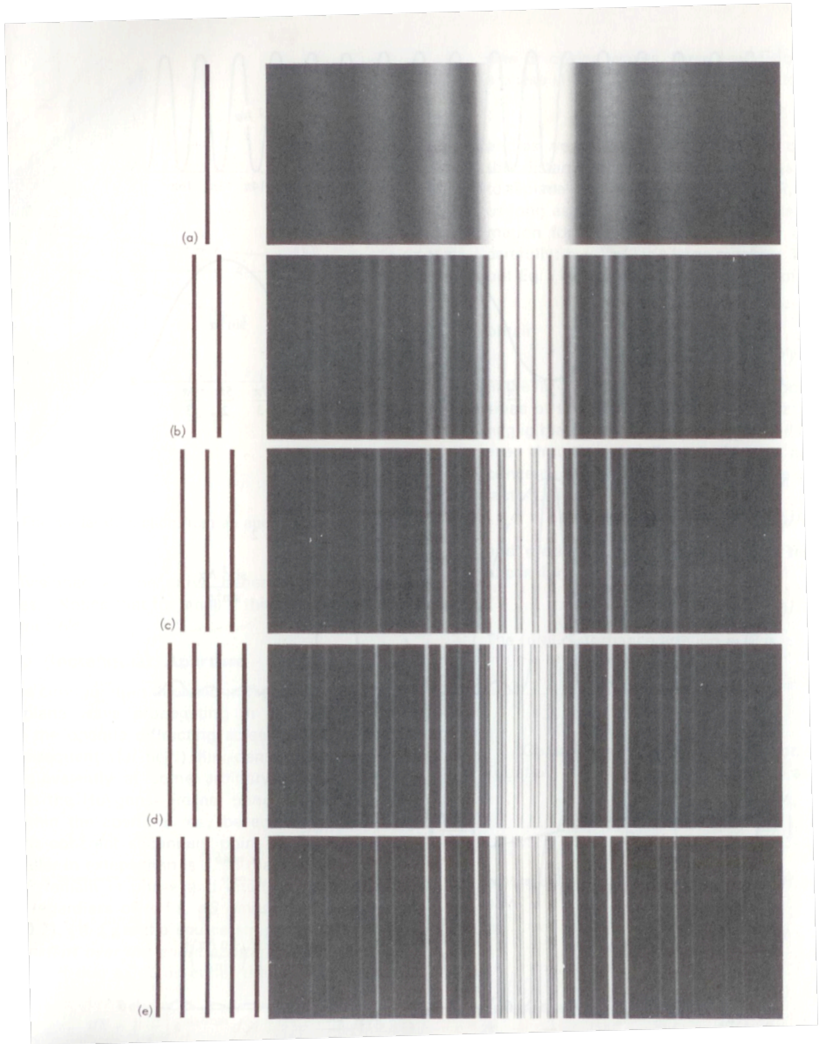
$$I = \alpha^2 \frac{(1-e^{iN\delta})(1-e^{-iN\delta})}{(1-e^{i\delta})(1-e^{-i\delta})}$$

$$= \alpha^2 \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

$$\propto \left(\frac{\sin^2\beta}{\beta}\right)\left(\frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}\right)$$

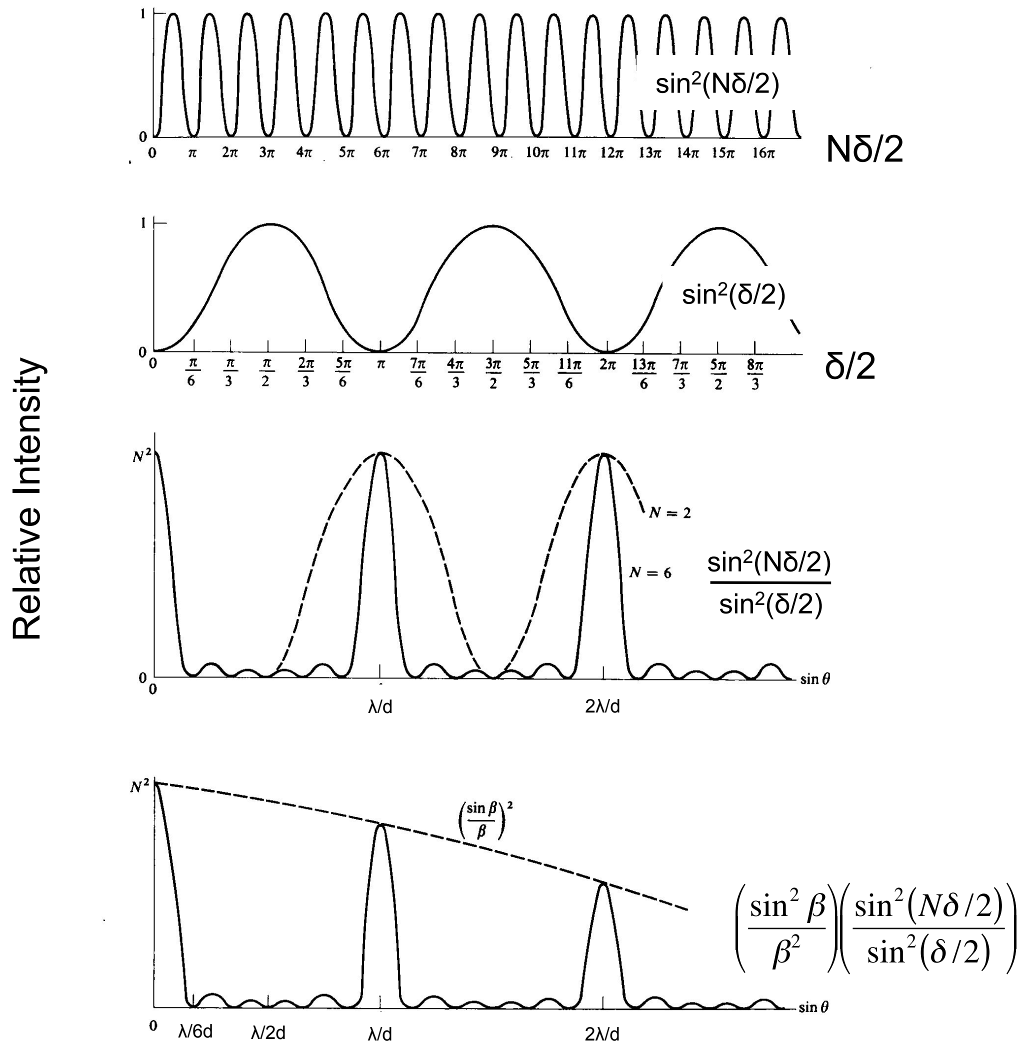


Increasing number of slits, N



- Primary peaks get narrower and brighter
- Secondary peaks fade

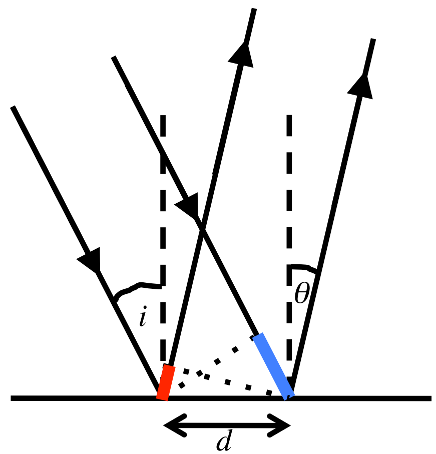
Diffraction Pattern from N Slits/grooves



$$I \propto \left(\frac{\sin^2 \beta}{\beta^2}\right)\left(\frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}\right)$$

$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi d(\sin i + \sin\theta)}{\lambda}$$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta$$



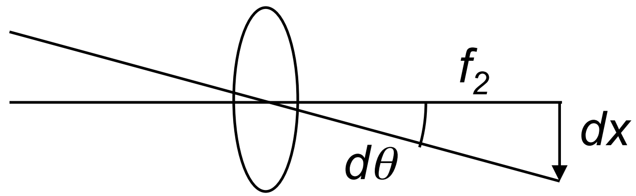
Dispersion

The Grating Equation (i.e. the condition for constructive interference in the diffracted beam) tells us that the angular location of interference fringes is wavelength dependent:

$$m\lambda = d(\sin i + \sin\theta)$$

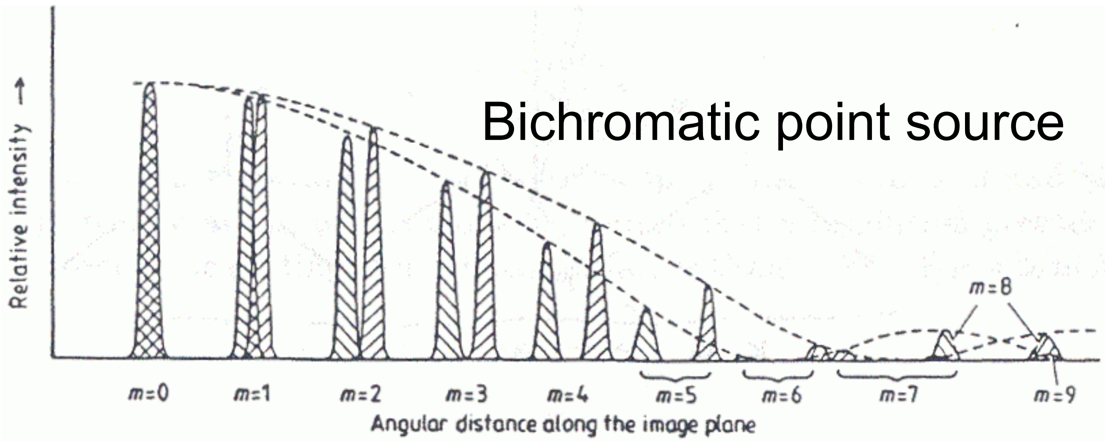
Differentiating the Grating Equation gives us the Dispersion Equation:

$$\frac{d\lambda}{d\theta} = \frac{\cos\theta}{m} d \qquad \frac{d\lambda}{dx} = \frac{d\lambda}{d\theta}\frac{d\theta}{dx} = \frac{\cos\theta}{mf_2} d$$

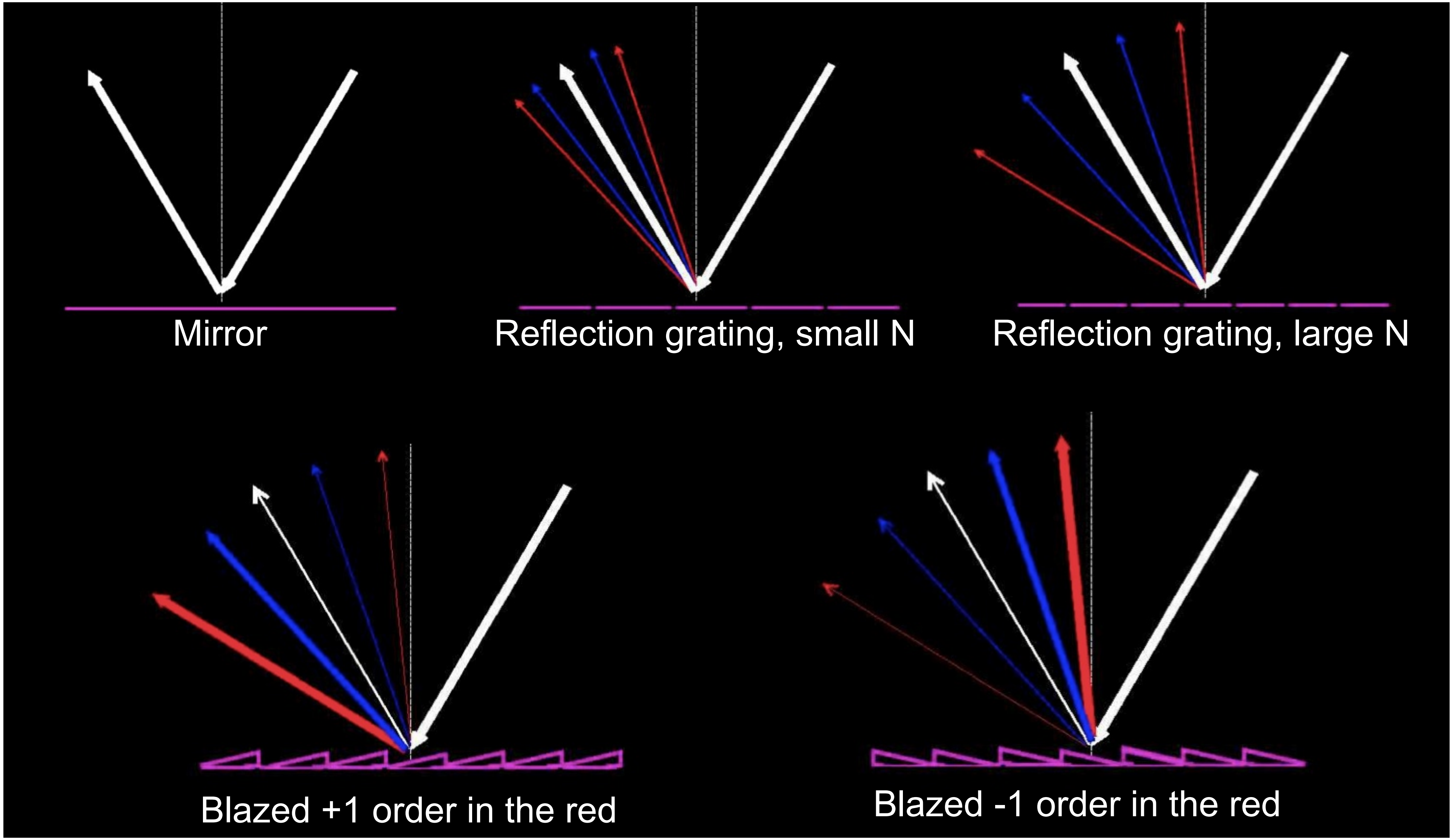


Re-arranging the Dispersion Equation shows that there is no dispersion in the zeroth order (m=0):

$$\Delta\theta = \frac{m(\Delta\lambda)}{d\cos\theta}$$



Blazed Gratings



Spectral Resolving Power

Resolving power is the smallest possible wavelength difference that can be resolved, and is defined as:

$$R = \frac{\lambda}{\Delta\lambda}$$

Differentiate the expression for phase difference to show that we can write R in terms of phase difference:

$$R = \frac{\delta}{\Delta\delta}$$

Homework: what about the minus sign?

$$\delta = \frac{2\pi p}{\lambda} = \frac{2\pi d(\sin i + \sin\theta)}{\lambda} \qquad (\Delta\delta)\lambda + \delta(\Delta\lambda) = 0$$

At a primary peak in the interference pattern:

$$\delta = 2m\pi$$

The adjacent minimum is a separated by a small phase difference of:

$$\Delta\delta = \frac{2\pi}{N}$$

Resolving power of a grating is therefore:

$$R = mN$$

Maximum Resolving Power

Resolving power of a grating is therefore:

$$R = mN$$

Use the Grating Equation to substitute for m:

$$m\lambda = d(\sin i + \sin\theta)$$

$$R = \frac{Nd}{\lambda}(\sin i + \sin\theta)$$

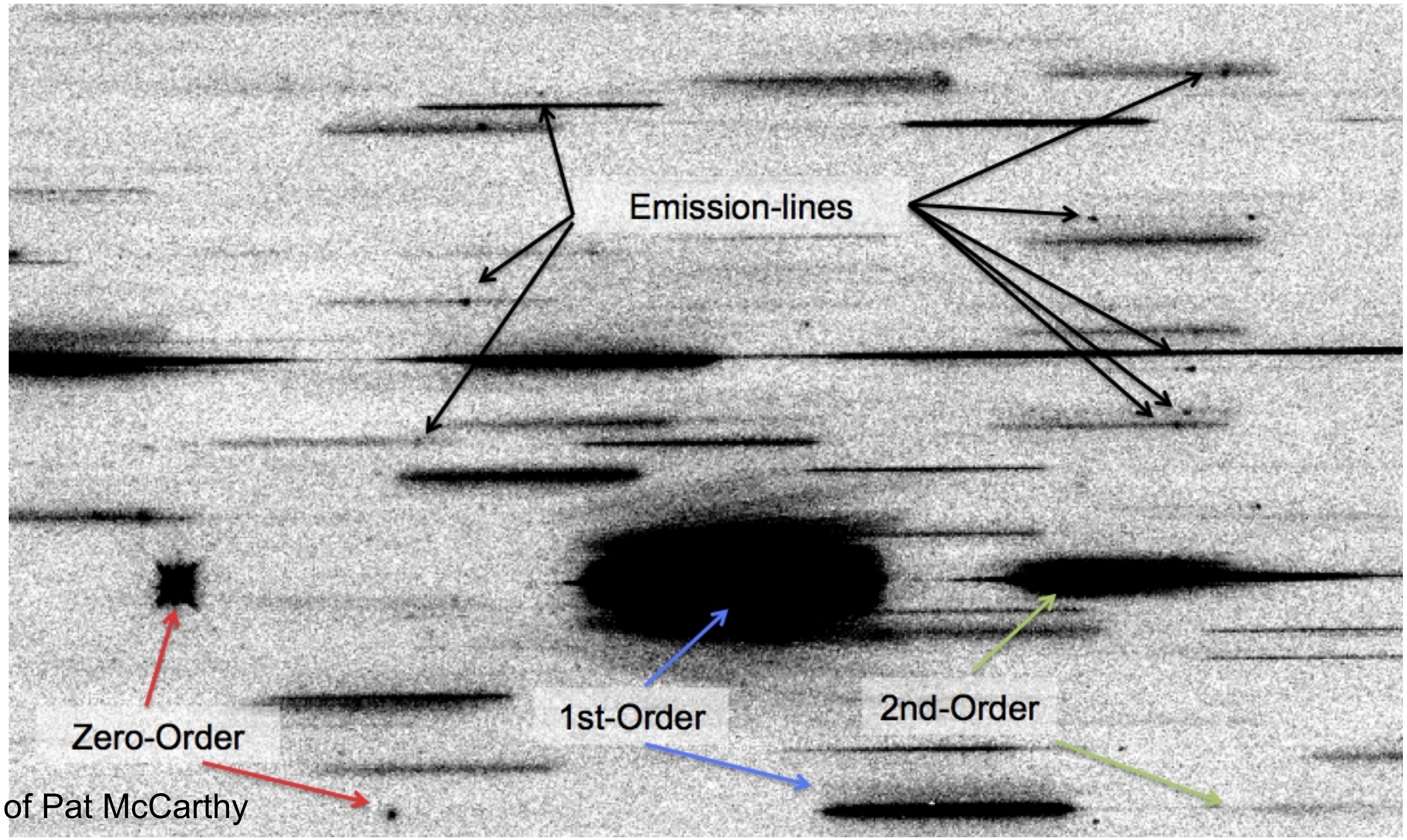
Maximum resolving power is therefore:

$$R_{\max} = \frac{2Nd}{\lambda} = \frac{2w}{\lambda}$$

Maximum resolving power therefore depends only on the illuminated width of the grating, w=Nd, and the wavelength of light.
[Remember: N=number of slits/grooves; d=separation of slits/grooves]

Slit-less Spectroscopy

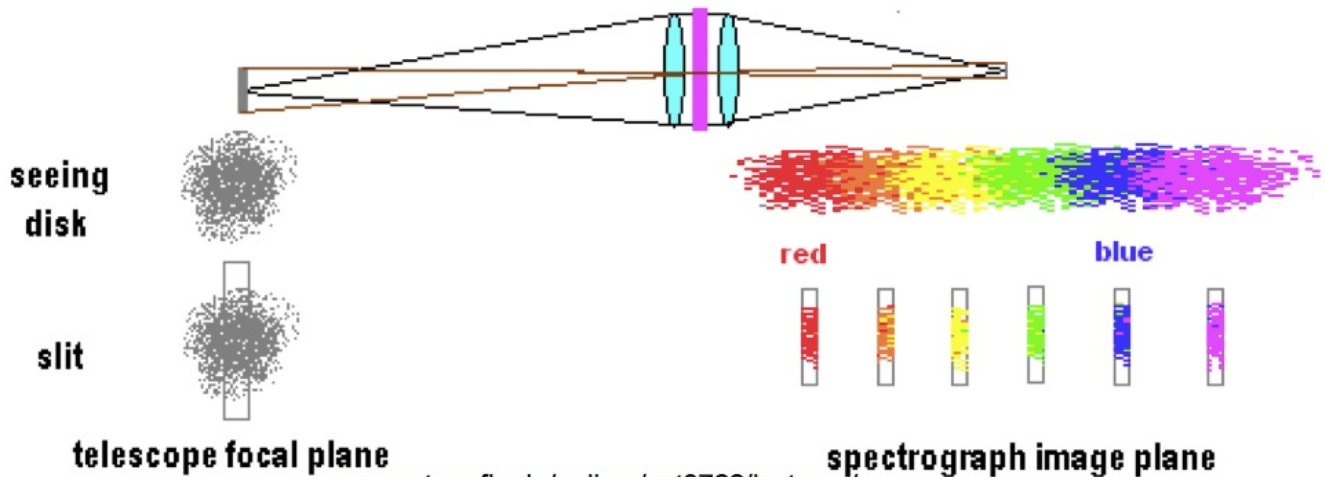
- Slit-less spectroscopy is possible, but the spectra overlap and can be hard to analyse/interpret



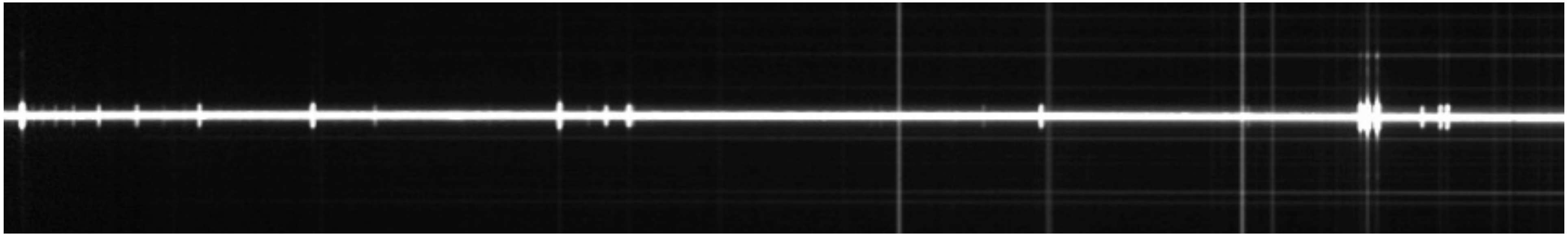
Courtesy of Pat McCarthy

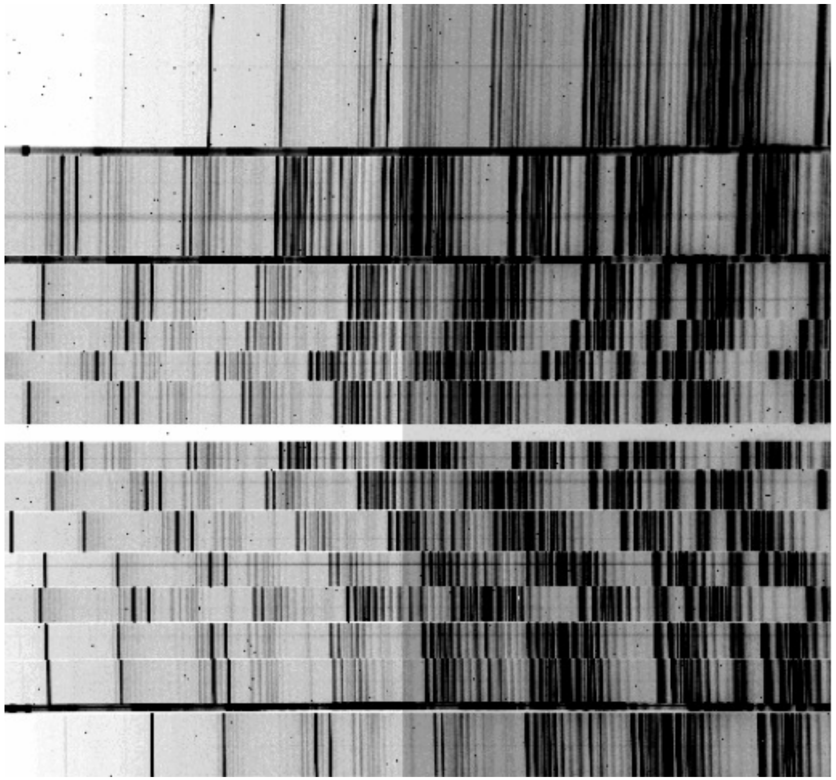
Slits, spectral resolution, losses

- Using a slit in the focal plane therefore only allows the light from the object of interest to be dispersed and recorded
- To first order the possible spectral resolution set by the slit width, ω, (in the dispersion direction), as imaged in the spectrograph
- The wavelength resolution is thus given by $\delta\lambda = \omega \, d\lambda/dx$
- Trade-off: narrower slit gives higher resolution but greater slit-losses



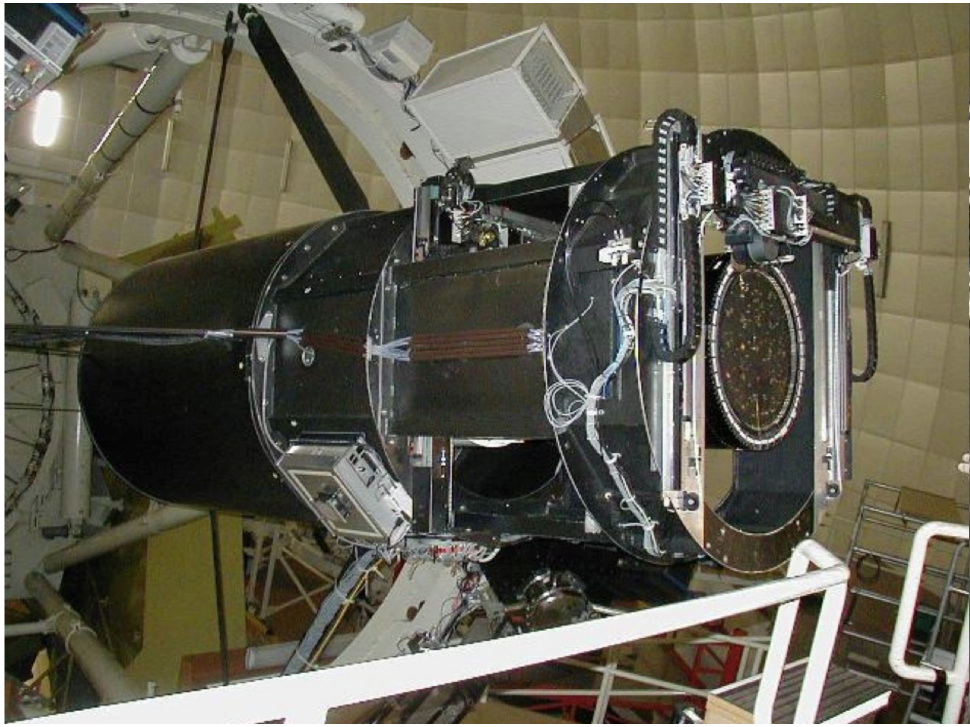
Long-slit and multi-slit observations

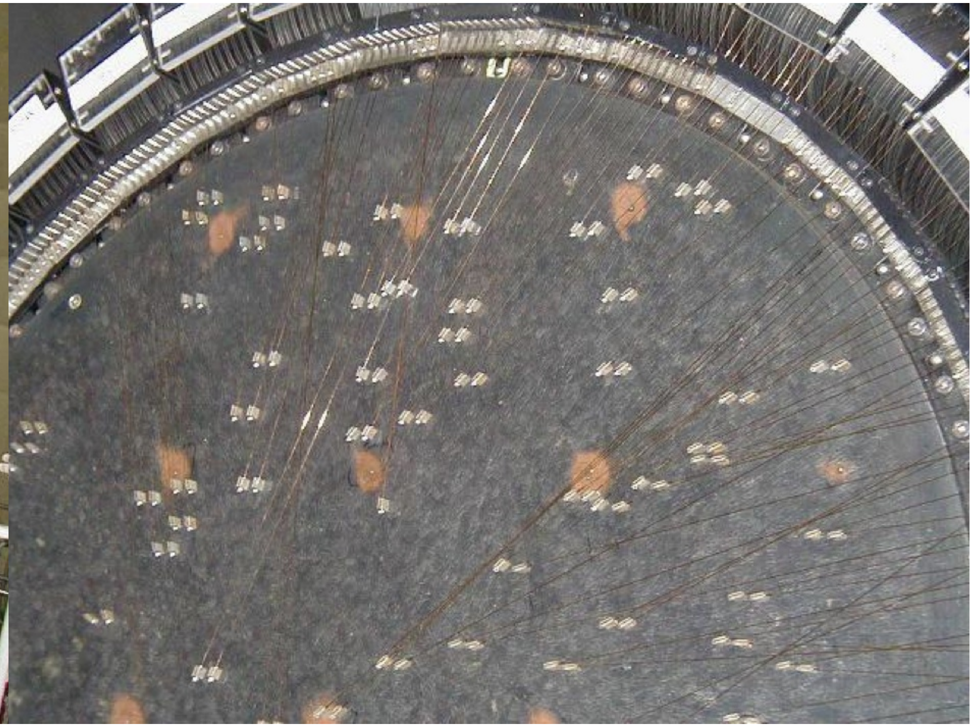


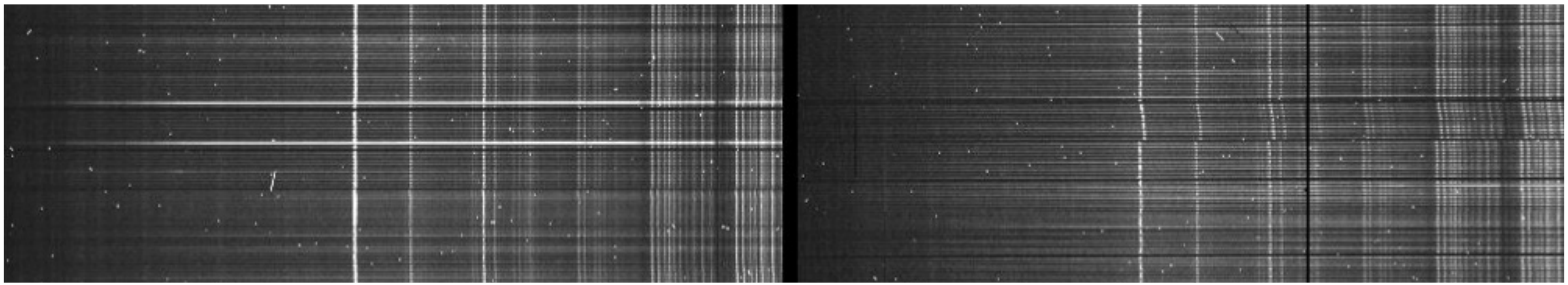


Multi-fibre Spectroscopy

Example: 2dF Spectrograph







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