The Cosmic Time in Terms of the Redshift

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In cosmology one labels the time t since the Big Bang in terms of the redshift of light emitted at t, as we see it now. In this Note we derive a formula that relates t to z which is valid for all redshifts. One can go back in time as far as one wishes, but not to the Big Bang at which the redshift tends to infinity.

Keywords: cosmic time, redshift

I. INTRODUCTION

In a recent paper by Renyue Cen and Jeremiah P. Ostriker [4], and in the review by Bertram Schwarzschild [6], the important problem of the missing baryons was discussed. That is, the fraction of baryons observed at high redshift is about twice that observed at low redshift. Cen and Ostriker performed a hydrodynamic computer simulation of cosmic evolution from about 2 billion years after the Big Bang to the present. The precise time scale depends on the details of the cosmological model from which it is derived. According to Schwarzschild [6] cosmologists relate the cosmic time t to cosmological redshift z by roughly $t \approx 14 \text{Gyr}/(1+z)^{3/2}$. Here t denotes the time since the Big Bang, that depends on the redshift z of light emitted at time t, as we see it now.

The Cen-Ostriker simulation covered the interval from z=0 to 3. In the simulation, the intergalactic gas gets steadily hotter after z=3. More and more of it is shock heated as it repeatedly falls into gravitational potential wells of accumulated nonbaryonic "dark" matter. As the gas gets hotter, less and less of it remains un-ionized, so that Lyman α absorption lines become increasingly harder to see. They suggest that as a result we cannot see about 50% of the baryonic matter. Cen and Ostriker conclude that the missing baryons reside in a 'warm-hot intergalactic medium'. It's clear, however, that for further investigation of the problem one needs a more accurate formula for the relationship between t and z that is valid for all redshift values. This may allow simulations of this kind closer to the time of the Big Bang.

*Electronic address: carmelim@bgu.ac.il †Electronic address: john@physics.uwa.edu.au ‡Electronic address: f.oliveira@jach.hawaii.edu In this Note we derive such a formula:

$$t = \frac{2H_0^{-1}}{1 + (1+z)^2} \tag{1}$$

where H_0 is the Hubble parameter. For an appropriate choice of H_0 (70 km/s-Mpc), we obtain

$$t \approx \frac{28}{1 + (1+z)^2} \text{Gyr.}$$
 (2)

The formula is valid for all z. The dependence on z is essentially different from that in the simulated formula. However, for z from 0 to 3 (see Fig. 1) the values of t in both cases are very close and their difference essentially negligible.

II. COSMOLOGICAL LINE ELEMENT

The Universe expands, of course, by the Hubble law $x = H_0^{-1}v$. But one cannot use this law directly to obtain a relation between z and t. So we start by assuming that the Universe is empty of gravitation. One can then describe the property of expansion as a null-vector in the flat four dimensions of space and expanding velocity v [1, 2, 3].

The cosmological line element [1, 2] is given by $ds^2 = \tau^2 dv^2 - (dx^2 + dy^2 + dz^2)$, where τ is the Big Bang time, the reciprocal of the Hubble parameter H_0 in the limit of zero distances, and it is a constant in this epoch of time. Its value is given by $\tau = 12.486$ Gyr (Carmeli [3], p.138, Eq. (A.66)). When ds = 0 one gets the Hubble expansion with no gravity. This line element should be compared with the Minkowskian line element in standard relativity, $ds^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$. When ds = 0 in the latter case, one gets the equation of motion for the propagation of light.

Space and time coordinates transform according to the

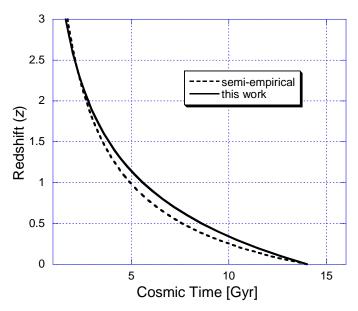


FIG. 1: Cosmic time as a function of redshift (z) where the solid line represents Eq. (2) (which is identical with Eq. (11)) derived in this paper and the broken line represents $t \approx 14 \text{Gyr}/(1+z)^{3/2}$ for the semi-empirical model. The equation (2) is valid over all z and is further shown in Figs. 2 and 3.

Lorentz transformation,

$$x' = (x - vt) / (1 - v^2/c^2)^{1/2},$$
 (3a)

$$t' = (t - vx/c^2) / (1 - v^2/c^2)^{1/2},$$
 (3b)

in ordinary physics. In cosmology the coordinates transform by the cosmological transformation (See [1, 2] and Sect. 2.11, p. 15 of [3]),

$$x' = (x - tv) / (1 - t^2/\tau^2)^{1/2},$$
 (4a)

$$v' = (v - xt/\tau^2) / (1 - t^2/\tau^2)^{1/2}$$
. (4b)

where t is the cosmic time with respect to us now.

Comparing the above transformations shows that the cosmological one can formally be obtained from the Lorentz transformation by changing t to v and c to τ ($v/c \to t/\tau$). Thus the transfer from ordinary physics to the expanding Universe, under the above assumption of empty space, for null four-vectors is simply achieved by replacing v/c by t/τ , where t is the cosmic time measured with respect to us now.

Thus classical physical laws in which velocities appear can be transferred to cosmology by replacing the velocity by the cosmic time measured with respect to us now $(v/c \to t/\tau)$. For example, one can use the apparatus of four dimensions (ct, x, y, z) well known in electrodynamics. Using the wave four-vector (ω, \mathbf{k}) one can easily derive the transformation of ω and \mathbf{k} from one coordinate system to another. This then gives the Doppler effect.

A charged particle receding from the observer with a velocity v and emitting electromagnetic waves will experience a frequency shift given by (see, for example, L.

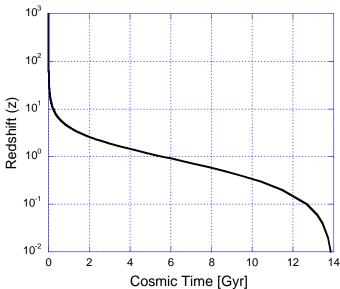


FIG. 2: Cosmic time as a function of redshift z from Eq. (11) with time measured since the Big Bang.

Landau and E. Lifshitz, p.121 [5])

$$\omega = \omega' \sqrt{\frac{1 + v/c}{1 - v/c}},\tag{5}$$

where ω' and ω are the frequencies of the emitted radiation received from the particle at velocity v and at rest, respectively. And thus a redshift is obtained from

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}}. (6)$$

In our case τ replaces c and t replaces v (v/c goes over to t/τ), thus getting

$$1 + z = \sqrt{\frac{1 + t/\tau}{1 - t/\tau}}. (7)$$

Rearranging we get

$$\frac{t}{\tau} = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}. (8)$$

Now because the time t is measured backwards, we need to make the substitutions such that t is measured forward from the Big Bang, i.e., $t \to \tau - t$. Hence we obtain

$$t = \frac{2\tau}{1 + (1+z)^2}. (9)$$

where t now denotes the time since the Big Bang.

The above formula was derived for the flat space. In order to extend the result to the present state where Hubble's law determines the expansion, we adopt the method

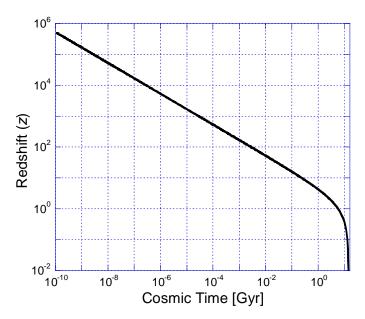


FIG. 3: Cosmic time as a function of redshift z from Eq. (11) in a logarithmic scale.

used in classical general relativity theory. In that case when one goes from flat space to curved space, one simply replaces the Minkowskian metric by the Riemannian metric (see, for example, Sect. 6.1, p. 122 of Trautman et al [7]). In our case this is manifested by the Hubble law which replaces the flat space expansion, thus replacing τ by H_0^{-1} seems to be the proper thing to do. Accordingly, we get

$$t = \frac{2H_0^{-1}}{1 + (1+z)^2}. (10)$$

If we assume $H_0 = 70 \text{ km/s-Mpc}$ we finally obtain

$$t \approx \frac{28}{1 + (1+z)^2} \text{Gyr.} \tag{11}$$

In Fig. 2 we give the dependence of the cosmic time on the redshift for the entire range of z. In order to visualize the dependence of cosmic time on redshift (z)

in the early Universe, equation (11) is shown in Fig. 3 on a logarithmic time axis. Here t is the time measured from the Big Bang.

III. CONCLUDING REMARKS

It is worth mentioning the physical assumptions behind the above mathematical formalism. First we have the principle of cosmological relativity according to which the laws of physics are the same at all cosmic times. This is an extension of Einstein's principle of relativity according to which the laws of physics are the same in all coordinate systems moving with constant velocities.

In cosmology the concept of time (t=x/v) replaces that of velocity (v=x/t) in ordinary special relativity. Second, we have the principle that the Big Bang time τ is always constant with the same numerical value, no matter at what cosmic time it is measured. This is obviously comparable to the assumption of the constancy of the speed of light c in special relativity.

Velocity in expanding Universe is not absolute just as time is not absolute in special relativity. Velocity now depends on at what cosmic time an object (or a person) is located; the more backward in time, the slower velocity progresses, the more distances contract, and the heavier the object becomes. In the limit that the cosmic time of a massive object approaches zero, velocities and distances contract to nothing, and the object's energy becomes infinite.

In Einstein's special relativity, as is well known, things depend on the velocity: The faster the object moves, the slower time progresses, the more distances contract, and the heavier the object becomes.

In this Note we have derived a simple formula, valid for all redshift values in the Universe. The formula relates the cosmic time t since the Big Bang, for an earth observer at the present epoch, to the measured redshift z of light emitted at time t. It is hoped that the formula will be useful for identifying objects at the early Universe since we can go back in time as far as we desire but not to the Big Bang event at which the redshift becomes infinity.

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