

# Atomic Spectroscopy and X-ray Bremsstrahlung

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This experiment aimed to determine two fundamental physical constants through different spectroscopic methods. A diffraction grating was calibrated using neon emission lines at 585.25 nm and 540.06 nm, yielding a grating spacing of  $(1.645 \pm 0.026) \times 10^{-6}$  m. Hydrogen's Balmer series wavelengths were measured as 662.3 nm (H), 496.0 nm (H), and 450.8 nm (H). Weighted linear regression of the Rydberg formula gave  $R_H = (1.0767 \pm 0.0120) \times 10^7 \text{ m}^{-1}$ , showing 1.88% agreement with the accepted value. X-ray Bremsstrahlung analysis using the Duane–Hunt law produced  $h_{\text{exp}} = (6.56 \pm 0.13) \times 10^{-34} \text{ J s}$ , within  $-0.95\%$  of the accepted value. Both results validate fundamental quantum models through independent experimental approaches.

## 1 Introduction

Determining precise values for physical constants is a key part of physics, as it allows us to model complex systems accurately. The Rydberg constant first arose in 1888 as part of an empirical formula developed by Swedish physicist Johannes Rydberg to describe the quantised electromagnetic spectra of an atom. In 1913, Niels Bohr used his model to calculate it from other, more fundamental, constants. The Rydberg constant is written as  $R_H$  for hydrogen, or  $R_\infty$  for heavy atoms. The constant represents the wavenumber of the photon required to ionise a hydrogen electron from its ground state.

One of the constants Bohr used is Planck's constant. Bremsstrahlung (“braking radiation”) is a continuous X-ray spectrum emitted when fast electrons decelerate upon collision with a metal anode. These electrons are accelerated inside an X-ray tube using a high voltage  $V$ . The Duane–Hunt law relates the cut-off wavelength of the Bremsstrahlung spectrum to  $V$ , allowing an experimental determination of  $h$ .

This report aims to independently reproduce the Rydberg constant and Planck's constant using hydrogen emission spectroscopy and X-ray Bremsstrahlung analysis.

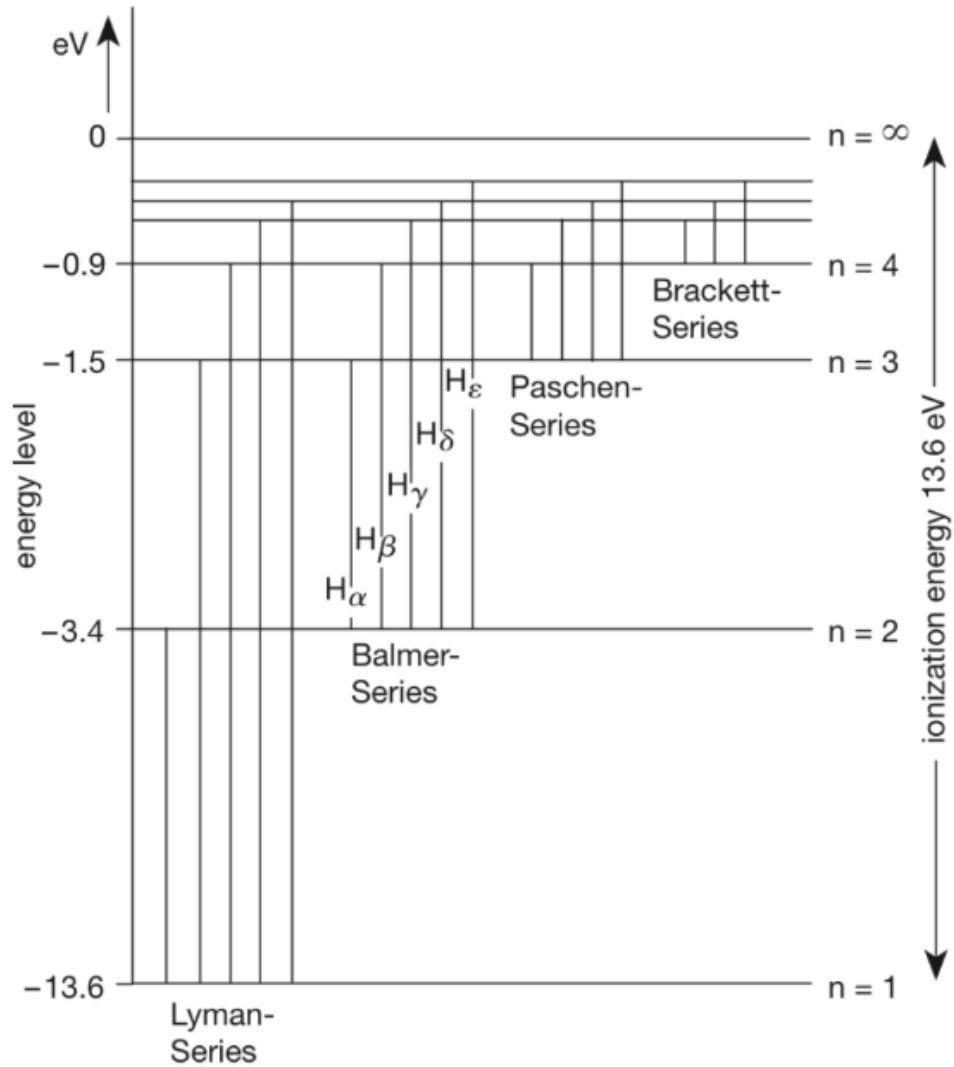
## 2 Theory

### 2.1 Atomic Spectroscopy Theory

Quantum mechanics predicts that electrons are bound to discrete energy levels, which can be determined by solving the Schrödinger equation subject to appropriate boundary conditions. In practice, the Schrödinger equation is only exactly solvable for the simplest atom: hydrogen.

When electrons are excited, they return to equilibrium by making transitions from their excited state back down to the lowest energy state available, and the energy released in doing so is released as photons. The energy of these photons is discrete and equal to the difference in energy levels of the transition, which is characteristic of the atom involved. Thus, different atoms have different atomic spectra.

A simplified diagram showing the energy levels for the electron in a hydrogen atom:



**Figure 1:** Simplified energy level diagram for the electron bound to a hydrogen nucleus.  $n$  is the principal quantum number, and  $n = \infty$  corresponds to ionisation.

Vertical lines show some of the transitions that can occur as an excited electron returns to its ground state.

The Lyman Series includes all transitions in which the final state is the ground state (principal quantum number  $n = 1$ ). In Hydrogen, the photons released as a result of these transitions are ultraviolet and thus are not observable with the naked eye.

The Balmer Series includes all transitions in which the final state has a principal quantum number  $n = 2$ . In Hydrogen, the photons released in the lowest three/four of the Balmer Series are visible (see lab book pages 28 and 29 for proof) and are the transitions I will be investigating.

In Figure 1, two further series are shown, both of which involve infrared photons.

The energy levels for a hydrogen atom are given by:

$$E_n = -\frac{R_y}{n^2} \quad (1)$$

where  $R_y$  is the Rydberg Energy Constant,  $n$  is the principal quantum number.

Niels Bohr used his model to show that  $R_y$  can be written as:

$$R_y = \frac{m_0 e^4}{2(4\pi\epsilon_0)^2 \hbar^2} = (hc)R_H \approx 13.6\text{eV} \quad (2)$$

Energy of released photons is equal to the difference in energy levels of the transition when an electron deexcites. The Balmer Series always ends on  $n = 2$ , therefore:

$$E_{\text{Balmer}} = E_n - E_2 \implies E_{\text{Balmer}} = R_y \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad (3)$$

which means that:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad (4)$$

This equation (known as the Rydberg equation) is the one I'll be using in my method to determine an experimental value for the Rydberg constant.

## 2.2 X-ray Bremsstrahlung Theory

Free electrons inside an X-ray tube are accelerated by a high voltage  $V$ , colliding with a heavy metal anode and emitting X-rays in a continuous spectrum. The maximum energy of the X-rays is equal to the energy of the incoming electrons, which relates to the voltage  $V$ . Thus:

$$eV = hv_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} \quad (5)$$

where symbols have their regular meanings.

A measurement of  $\lambda_{\text{min}}$  vs  $V$ , therefore, can be used to calculate an experimental value of Planck Constant  $h$  (provided  $e$  and  $c$  are known).

This experiment makes use of Bragg reflections from a crystal of known lattice parameter to measure  $\lambda$ . The Bragg condition for strong reflections of X-rays from a crystal is:

$$n\lambda = 2d \sin \theta \quad (6)$$

See the lab book section between pages 3 and 5 for a derivation and proof of this law.

## 3 Methods

### 3.1 Atomic Spectroscopy Method

The experimental setup consisted of the following components:

- Diffraction grating.
- Spectral Tube Box (plugged into the wall).
- Clamp.
- Clamp stand.
- Screen distance meter rule.
- Screen length meter rule.
- Spectral line position markers.
- An observing eye.

The complete setup of these components is demonstrated in the Schematic Diagrams of Experimental Setup.

### 3.1.1 Conditions/Parameters of Experiment

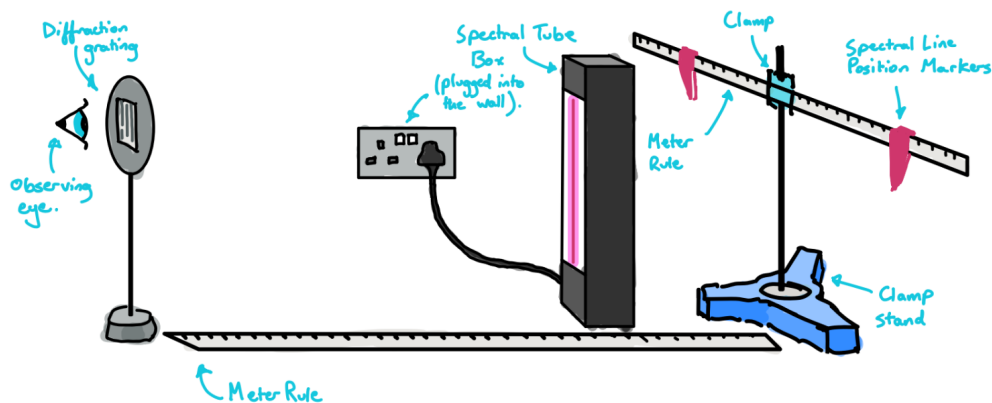
The parameters (measured) parts of this experiment are as follows:

- $d$  - screen distance.
- $l$  - screen length (labelled as  $l_L$  and  $l_R$  to indicate left and right of the spectral tube).
- $\alpha$  - angle of diffraction, which is calculated using measurements of screen distance  $d$  and screen length  $l$ :

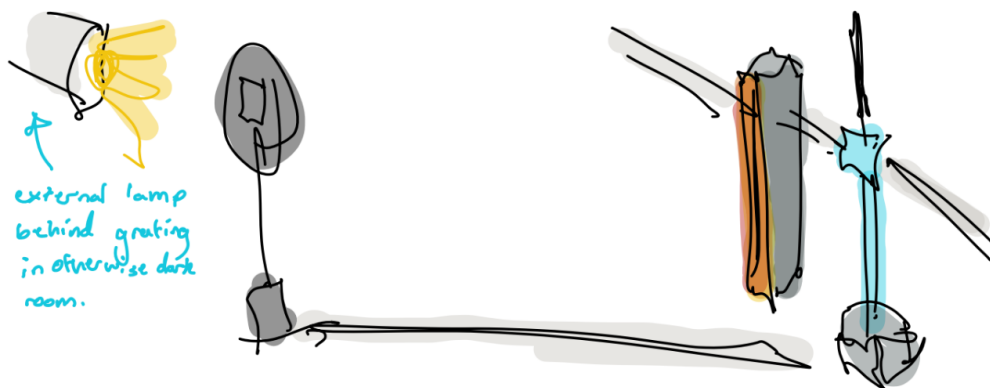
$$\sin \alpha = \frac{l}{\sqrt{d^2 + l^2}} \quad (7)$$

- $p$  - order of diffraction.
- Central position of ruler - used to calculate the screen length for each spectral line for each given line position along the screen length meter rule.

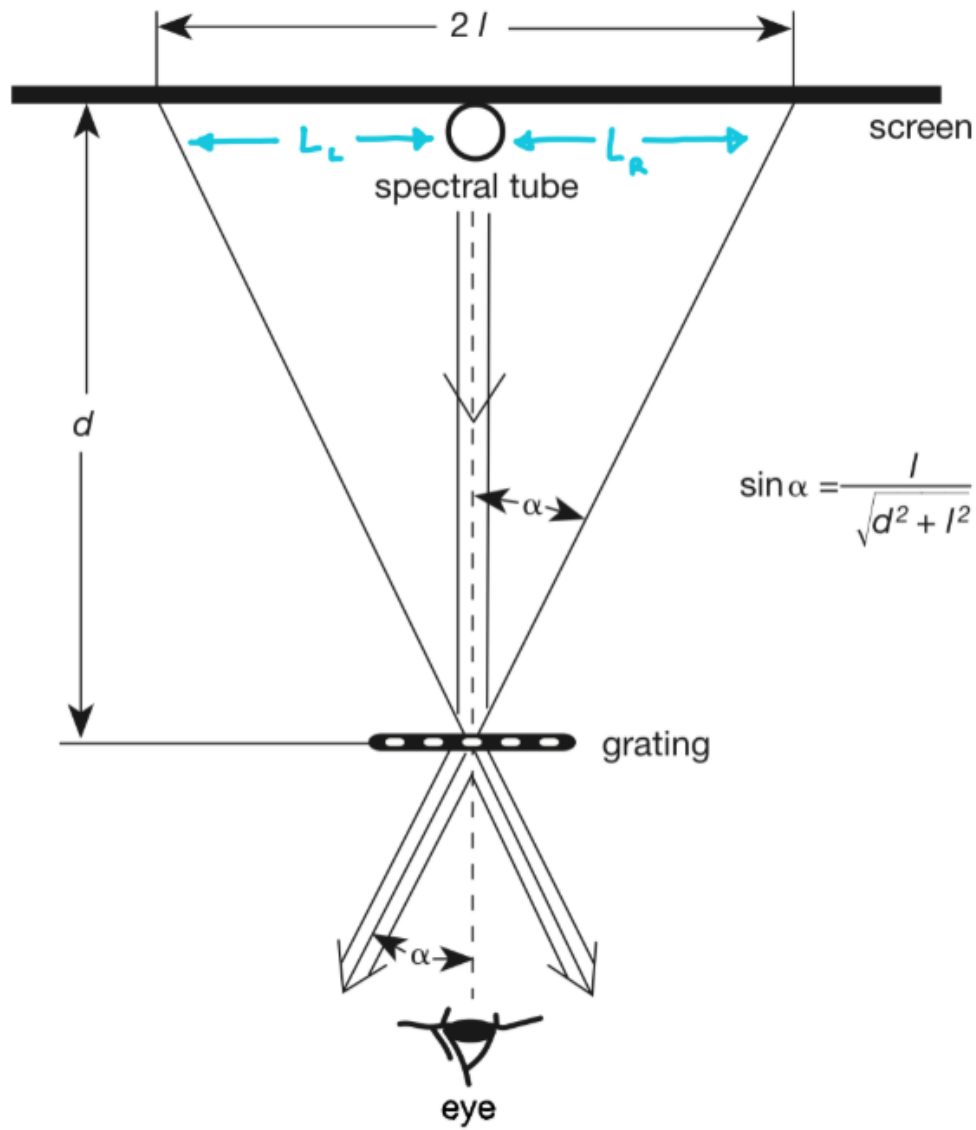
### 3.1.2 Schematic Diagrams



**Figure 2:** Three-quarter view of Experimental setup for Atomic Spectroscopy Experiment, showing position of listed equipment.



**Figure 3:** Position of external lamp behind the grating so the Spectral Line Position Markers are visible.



**Figure 4:** Top-down view of Experimental setup for Atomic Spectroscopy Experiment, showing the position of listed equipment and measured parameters of the experiment.

### 3.1.3 Steps

1. First, using a neon lamp (whose spectral wavelengths are known) to calibrate an experimental value for the diffraction grating constant.
2. To do this, we set up the equipment as shown in the diagrams, with a neon gas tube in the Spectral Tube Box.
3. Then put eye at angled positions looking at the spectra from gas tube through diffraction grating, then slide spectral line position markers to position of yellow and green emission lines (there are two green emission lines, I positioned the marker to the line closest to the red end of the spectrum).
4. Measure and record screen lengths for observed emission lines on both the left and right.
5. Using known conditions/parameters of the experiment and the diffraction grating equation, calculate a value for the diffraction grating constant.

6. Repeat steps 2, 3, 4 with hydrogen gas in Spectral Tube Box, measuring screen lengths for red, green and blue emission lines.
7. Using calculated values for screen length, diffraction grating constant and rearranged diffraction grating equation, calculated experimental values for wavelengths of spectral lines.
8. Plot a graph of wavelengths against principal quantum numbers  $n$ , and use that graph to obtain an experimental value for the Rydberg Constant.
9. Compare experimental and accepted values of  $R_H$ , validating experimental methods and physical principles.

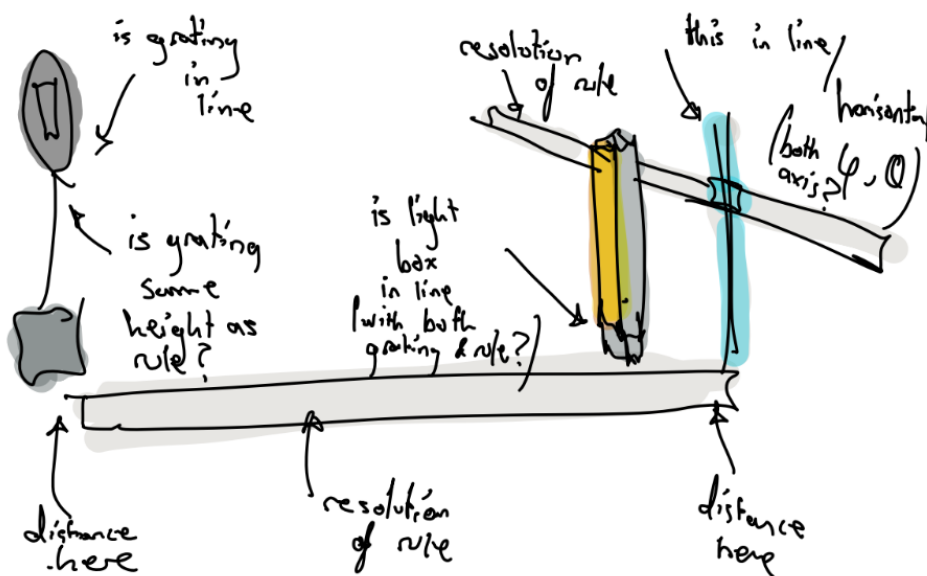
### 3.1.4 Safety

Gas discharge tubes require between  $1kV$  and  $5kV$  to operate. The power supply delivers this voltage, but it is current-limited to  $2.5mA$ , which is well below the safe limit.

The discharge tubes are fragile and made of glass. Therefore, I made sure to get the demonstrator to handle them when needed, held them on the wider sections as more robust, less likely to snap.

### 3.1.5 Sources of Error

There are several sources of error that I had to consider in this experimental setup, the significance of which will be addressed in the discussion section.



**Figure 5:** Sources of uncertainty in the setup.

In addition to the sources listed in 5, I also considered parallax in readings and when reading screen lengths off the screen length meter rule. These errors were then propagated through the use of the partial derivative error propagation equation. For the equation of the form:

$$f = f(x, y, z, \dots) \quad (8)$$

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} \sigma_z\right)^2 + \dots} \quad (9)$$

This general equation was applied to the reading of screen length  $l$ , calculations of  $\sin \alpha$  (7) using the diffraction grating equation (6), and the Rydberg Equation (3) to find the final uncertainty in the final experimental value for Rydberg Constant. For full details of this error analysis, please refer to the sections between pages 35 – 46 in the attached lab book.

## 3.2 X-ray Bremsstrahlung Method

The X-ray spectra were acquired using a computer-controlled X-ray spectrometer equipped with a molybdenum target tube and a NaCl crystal serving as a monochromator. The tube high voltage  $U$  and the emission current  $I$  were controlled from the front panel. In this configuration, the displayed angle corresponds directly to the Bragg angle  $\theta$  required for analysis.

Before data collection, the tube voltage was set to  $U = 35$  kV and the emission current set to  $I = 1.00$  mA. The measurement parameters were configured as:

$$\Delta t = 10 \text{ s} \quad (\text{integration time per step}), \quad \Delta \beta = 0.1^\circ \quad (\text{angular step width}).$$

The scan limits were chosen to isolate the relevant part of the Bremsstrahlung spectrum:

$$\beta_{\min} = 2.5^\circ, \quad \beta_{\max} = 10.0^\circ.$$

Two preliminary scans were performed: one using the above parameters, and a faster scan with  $\Delta t = 1$  s to reduce acquisition time. Both scans showed clear peaks near  $\beta \approx 6.4^\circ$  and  $\beta \approx 7.2^\circ$  the expected characteristic  $K_\alpha$  and  $K_\beta$  lines of molybdenum. This confirmed that the goniometer zero was correctly aligned and that the spectrometer was operating normally.

Full data collection was then carried out for tube voltages in the range

$$U = 15\text{--}35 \text{ kV} \quad \text{in increments of } 2.5 \text{ kV},$$

giving nine datasets in total. Set the goniometer to the COUPLED key, activating  $2\theta$  coupling of the target and sensor. Then set the lower limit of the target angle to 2.5 and the upper limit to 12.5.

For each voltage, the following procedure was used:

1. Set the tube high voltage  $U$  on the control panel.
2. Keep the emission current fixed at  $I = 1.00$  mA.
3. Press the SCAN key to run an automated angular scan from  $\beta_{\min}$  to  $\beta_{\max}$ .
4. Allow the data to transfer to the attached PC and save the scan as a data file.

After all spectra were recorded, both the tube voltage and emission current were reduced to zero for safety, and the X-ray software was closed.

This procedure produced nine Bremsstrahlung spectra  $I(\theta)$  with consistent experimental conditions, enabling the extraction of the cut-off wavelength  $\lambda_{\min}$  and determination of Planck's constant via the Duane-Hunt law.

For full, detailed methods of data and error analysis, please visit the sections between pages 11 – 26 in the attached lab book.

## 4 Results

### 4.1 Atomic Spectroscopy

#### 4.1.1 Neon Diffraction Grating Calibration

diffraction grating spacing. Screen length measurements were taken on the left and right of the central maximum, converted into diffraction angles, and inserted into the grating equation. A weighted mean was applied to account for differing uncertainties.

The final calibrated grating spacing is:

$$d = (1.645 \pm 0.026) \times 10^{-6} \text{ m.}$$

The left and right measurements agreed within their uncertainties, indicating that the grating was aligned and the geometric assumptions of the setup were valid.

#### 4.1.2 Hydrogen Spectral Lines

Hydrogen emission lines were measured for the red ( $n = 3$ ), green ( $n = 4$ ), and blue ( $n = 5$ ) Balmer transitions. Multiple left/right measurements were combined using weighted means. Table 1 shows the resulting experimental wavelengths.

Transition ( $n \rightarrow 2$ )	$\lambda_{\text{exp}}$ (nm)	Uncertainty (nm)
$3 \rightarrow 2$ (H $\alpha$ )	662.3	1.1
$4 \rightarrow 2$ (H $\beta$ )	496.0	0.9
$5 \rightarrow 2$ (H $\gamma$ )	450.8	1.2

**Table 1:** Weighted mean hydrogen wavelengths obtained from the diffraction grating.

A clear ordering  $\lambda_{3 \rightarrow 2} > \lambda_{4 \rightarrow 2} > \lambda_{5 \rightarrow 2}$  was observed, consistent with the Balmer series predictions.

#### 4.1.3 Determination of the Rydberg Constant

The Balmer formula was linearised by plotting the inverse wavelength against the term  $(\frac{1}{4} - \frac{1}{n^2})$ , producing a straight line whose gradient corresponds to the Rydberg constant  $R_H$ . A weighted linear regression yielded:

$$R_H = (1.0767 \pm 0.0120) \times 10^7 \text{ m}^{-1}.$$

This value differs from the accepted value ( $1.09737 \times 10^7 \text{ m}^{-1}$ ) by only 1.88%, demonstrating good consistency between measurement and theory.

### 4.2 X-ray Bremsstrahlung and Duane–Hunt Law

#### 4.2.1 Bremsstrahlung Spectra

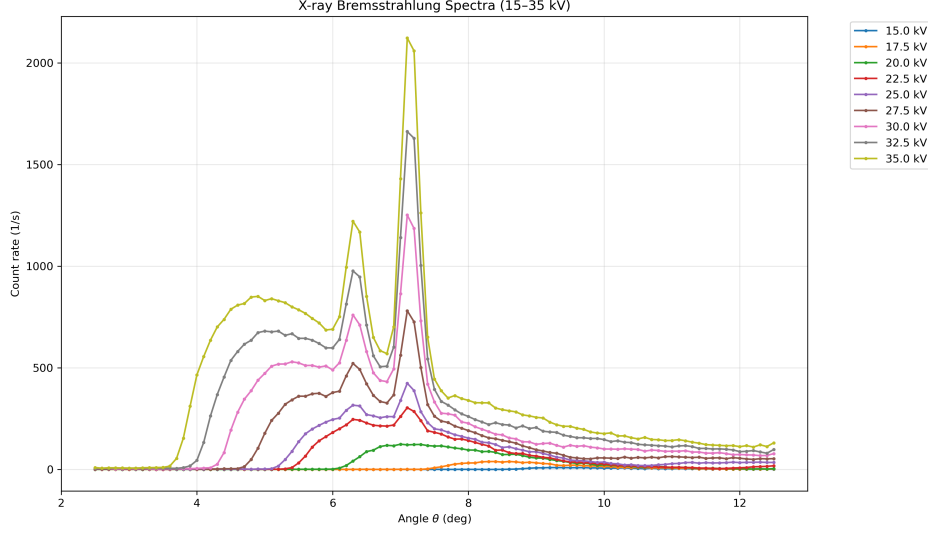
Intensity  $I(\theta)$  was measured as a function of the Bragg angle  $\theta$  for nine accelerating voltages between 15 kV and 35 kV. A representative set of spectra is shown in Fig. 6. Each spectrum exhibits a sharp rise at low angles followed by a broad continuum and two characteristic molybdenum peaks at approximately  $6.4^\circ$  and  $7.2^\circ$ .

#### 4.2.2 Cut-off Angle Extraction

The cut-off angle  $\theta_{\text{min}}$  was determined by smoothing each spectrum, locating the steepest rising edge in the first 40% of the angle range, and fitting a straight line to this region. Low-voltage spectra (15 kV and 17.5 kV) lacked a sufficiently strong rising edge and were excluded. Table 2 summarises the extracted  $\theta_{\text{min}}$  and corresponding  $\lambda_{\text{min}}$  values from Bragg’s law.

Values for 15 kV and 17.5 kV were recorded but excluded from the analysis. At these low voltages, the Bremsstrahlung intensity was too weak to produce a discernible rising edge, giving unphysically large uncertainties in  $\theta_{\text{min}}$  and  $\lambda_{\text{min}}$ . Such points carry negligible statistical weight and are not reliable measurements of the cut-off wavelength. For full details, refer to the section of the lab book between pages 39 – 46.





**Figure 6:** Measured X-ray spectra  $I(\theta)$  for tube voltages of 15–35 kV.

Voltage (kV)	$\theta_{\min}$ (deg)	$\lambda_{\min}$ (m)	$\Delta\lambda_{\min}$ (m)
20.0	5.963	$5.88 \times 10^{-11}$	$1.01 \times 10^{-11}$
22.5	5.278	$5.21 \times 10^{-11}$	$3.32 \times 10^{-12}$
25.0	5.001	$4.93 \times 10^{-11}$	$2.96 \times 10^{-12}$
27.5	4.583	$4.52 \times 10^{-11}$	$2.33 \times 10^{-12}$
30.0	4.167	$4.11 \times 10^{-11}$	$1.94 \times 10^{-12}$
32.5	3.855	$3.81 \times 10^{-11}$	$1.72 \times 10^{-12}$
35.0	3.558	$3.51 \times 10^{-11}$	$1.53 \times 10^{-12}$

**Table 2:** Extracted cut-off angles and wavelengths from Bragg reflection.

#### 4.2.3 Weighted Linear Regression and Planck Constant

Plotting  $1/\lambda_{\min}$  against tube voltage  $V$  yielded a linear relationship consistent with the Duane–Hunt law.

With residuals:

A weighted regression gave a gradient of

$$m = (8.14 \pm 0.17) \times 10^5 \text{ m}^{-1}\text{V}^{-1}.$$

Using

$$h = \frac{e}{mc},$$

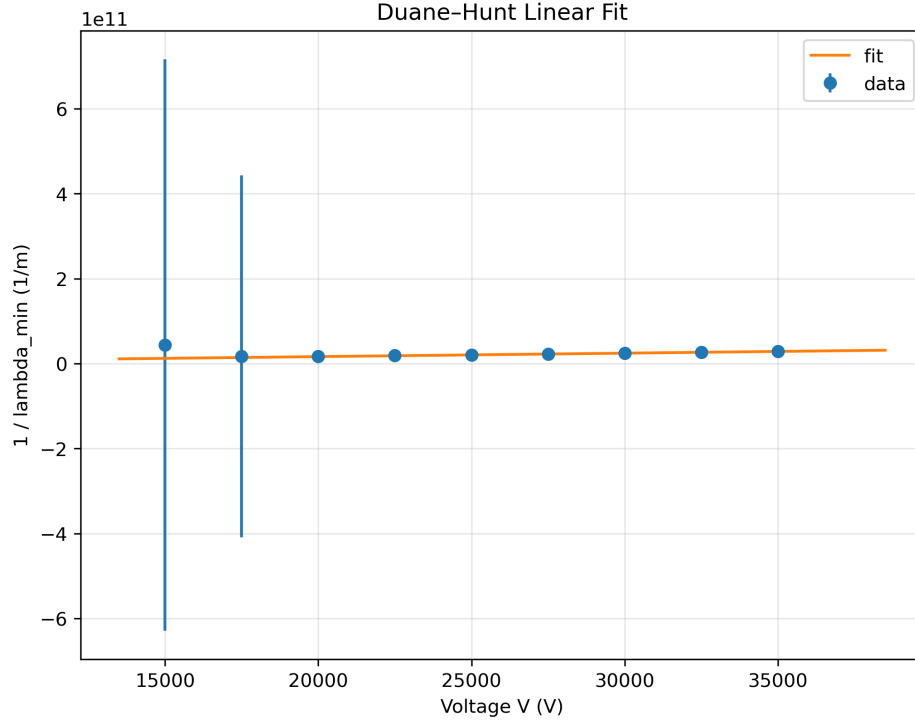
With known values of  $e$  and  $c$ , the experimental value of Planck’s constant was found to be

$$h_{\text{exp}} = (6.56 \pm 0.13) \times 10^{-34} \text{ J s},$$

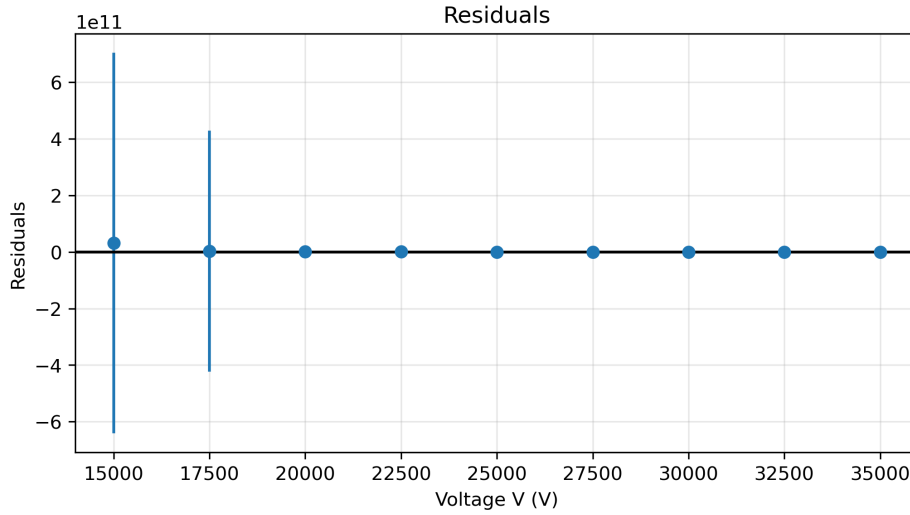
which is within  $-0.95\%$  of the accepted value.

## 5 Discussion

The neon calibration produced a grating spacing consistent with expected commercial values, and the close agreement between the left/right measurements validated geometric alignment. Hydrogen wavelengths followed the expected Balmer ordering. The linearity of the Rydberg plot and the small regression uncertainty indicate that systematic effects—parallax, alignment, ruler resolution—were well controlled. The final 1.88% deviation from the accepted  $R_H$  is small and within typical undergraduate experiment performance.



**Figure 7:** Weighted linear regression for Voltage  $V$  against  $\frac{1}{\lambda}$



**Figure 8:** Residuals for Voltage  $V$  against  $\frac{1}{\lambda}$

The X-ray component showed stronger systematic limitations. At low voltages, the rising edge of the Bremsstrahlung spectrum was too weak to extract a reliable  $\theta_{\min}$ . Removing the 15 kV and 17.5 kV datasets was therefore justified. At higher voltages, the cut-off region was well resolved and uncertainties were dominated by angular resolution, the choice of fitting region, and crystal imperfections (mosaic spread). Despite these, the extracted value of Planck's constant agreed within 0.95% of the accepted value—excellent given the sensitivity of the method to small angular errors.

Overall, both techniques confirmed their respective quantum models: quantised bound-state energy levels for hydrogen, and photon–electron energy equivalence for X-ray emission.

## 6 Conclusion

The two spectroscopic techniques successfully yielded accurate experimental values for fundamental constants. Hydrogen spectroscopy gave:

$$R_H = (1.0767 \pm 0.0120) \times 10^7 \text{ m}^{-1} \quad (1.88\% \text{ deviation})$$

X-ray Bremsstrahlung gave:

$$h_{\text{exp}} = (6.56 \pm 0.13) \times 10^{-34} \text{ J s} \quad (0.95\% \text{ deviation})$$

Both results lie close to accepted values, demonstrating the validity of the models and the effectiveness of the experimental procedures.

## References

- [1] W. Duane and F. L. Hunt, “On X-ray Wavelengths,” *Physical Review*, vol. 6, pp. 166–172, 1915.