The Duane-Hunt Law: Determination of the Planck constant

Before starting this experiment please read the important safety information associated with it provided on ELE.

Introduction

The x-ray apparatus provided is used to study crystal diffraction, the spectrum of x-rays from an x-ray tube, and the energy levels of the atomic shell.

Free electrons inside an x-ray tube are accelerated by a high voltage V to the heavy-metal anode. As they hit the anode their rapid deceleration causes then to emit x-rays in a continuous spectrum (known as bremsstrahlung, or 'braking radiation'). The maximum energy of x-rays in this spectrum is equal to the energy of the incoming electrons. Hence

$$eV = h\nu_{max}$$
$$= \frac{hc}{\lambda_{min}} \tag{1}$$

where the symbols have their usual meanings. This is known as the Duane-Hunt law. A measurement of λ_{min} *vs* V therefore provides a measurement of the Planck constant h (provides that e and c are known).

There are other features in the x-ray spectrum which relate to atomic transitions in the material of the anode. However these are not relevant to the experiment you will be carrying out.

The experiment makes use of Bragg reflections from a crystal of known lattice parameter in order to measure λ. You will recall that the Bragg condition for strong reflections of x-rays from a crystal is that

$$n\lambda = 2d\sin\theta . \tag{2}$$

Since this equation is central to the operation of the experiment, you should provide a proof of it either in you Laboratory notebook or in the experiment report.
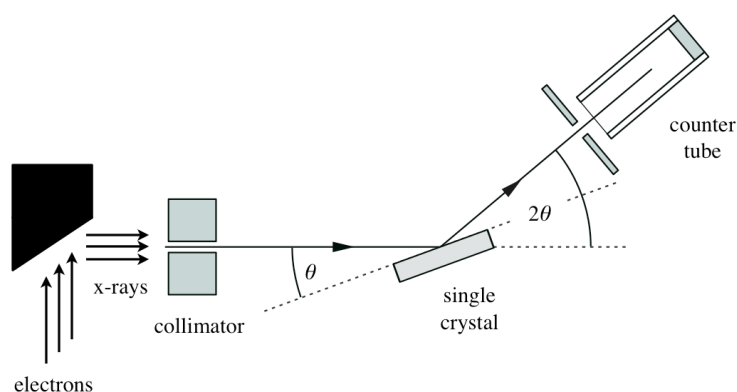

Figure 1: schematic diagram of the diffraction experiment. Monochromatic x-rays are produced by Bragg-scattering from a known crystal, and their intensity is measured using a counter tube.

Figure 1 shows a schematic diagram of the experiment. X-rays produced by the x-ray tube are collimated and impinge on the (100) face of a single crystal of sodium chloride which is tilted at an angle θ with respect to the x-ray beam. The x-rays scatter from the crystal in all directions, and those that are scattered through 2θ are detected by the counter tube (Geiger-

Müller tube). Intense reflections only occur for x-ray wavelengths satisfying the Bragg condition. Therefore measurements of intensity *vs* the angle θ can be converted to a spectrum of intensity *vs* wavelength, using equation (2) and assuming $n = 1$ and $d = 0.283$ nm (half the lattice parameter of sodium chloride – see diagram opposite).

Notes on the apparatus

Figure 2: crystal structure of sodium chloride.

The right-hand chamber of the x-ray apparatus contains the rotation gear (goniometer) for the experiment. The goniometer is driven by stepper motors and you should not attempt to rotate it by hand, nor should the mechanism be blocked in any way. The x-ray apparatus can measure angle-dependent x-ray scattering in one of three modes:
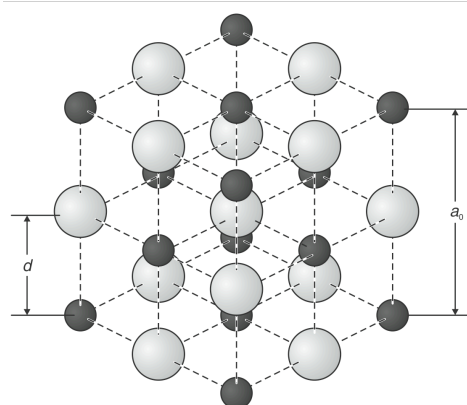

Key:
a: collimator mount
b: sensor holder
c: goniometer ribbon cable
d: goniometer guide rod
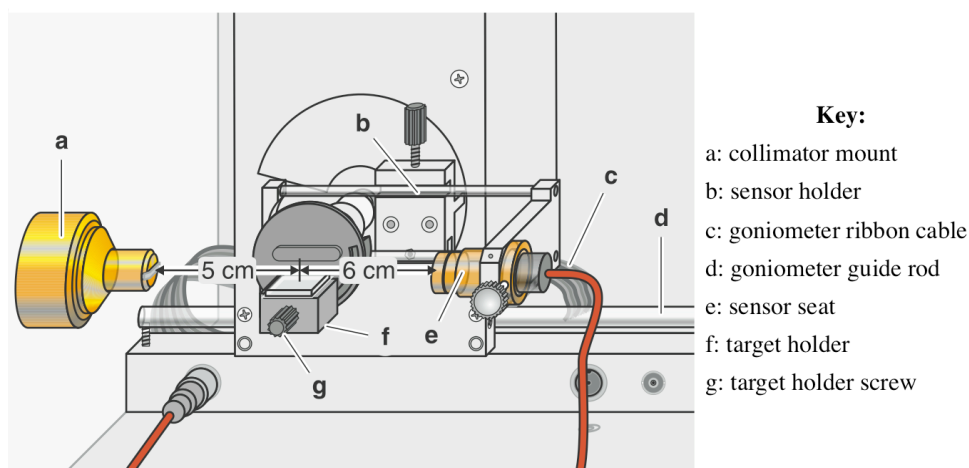e: sensor seat
f: target holder
g: target holder screw

Figure 3: Experiment set-up for the Bragg-scattering x-ray monochromator.

1) In sensor mode, the mount holding the target crystal (f in Figure 3) is held fixed while the sensor (the counter tube, e) is rotated around it.

2) In target mode, the target crystal is rotated while the sensor remains fixed.

3) In coupled mode the sensor and target both rotate, the sensor rotating twice the angle of the target to maintain the geometry of Figure 1.

For these experiments, the coupled mode is used. However this only works if the zero angle condition has been set accurately: when the goniometer is driven to zero, the collimator slit, the target crystal and the detector should all line up as in Figure 3. If this is not the case you should consult the technician.

The x-ray apparatus provides the counter tube (Geiger-Müller tube) with its supply voltage of about 400 V and displays its count rate. The count rate and goniometer position are also reported to the PC *via* a USB connection and the x-ray software loaded on the computer. When an x-ray is detected, there is a momentary dip in this voltage. The power supply unit converts each dip into a pulse which can then be counted.

The experiment

Confirm that the sodium chloride crystal is mounted in the target holder (f) and that the counter tube (e) is installed. Start the software package XRay.exe, a shortcut to which is on

the computer's desktop. This package has extensive context-sensitive help which you should read in order to familiarize yourself with its features.

On the control panel of the x-ray apparatus (Figure 5) set the tube high voltage (U) to 35 kV, and the emission current to 1 mA. Settings are made by first pressing the button for the parameter required and then rotating the dial until the correct value is shown in the display. Set the measuring time per angular step Δt to 10 s, and the angular step $\Delta\beta$ to 0.1°.

Key:

b1: display

b2: adjustment dial

b3: parameter selector keys

b4: scan mode keys

b5: operation keys

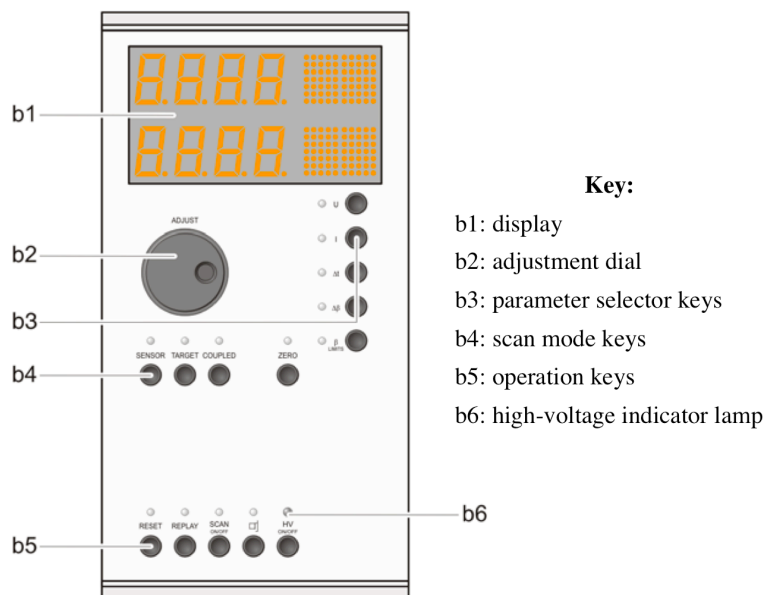b6: high-voltage indicator lamp

Figure 4: x-ray apparatus control panel

Press the COUPLED key to activate 2θ coupling of target and sensor and set the lower limit of the target angle to 2.5° and the upper limit to 10°(the lower limit is set by pressing the β limit once; the upper limit by pressing the same key a second time).

Start measurement and data transfer to the PC by pressing the SCAN key. When you have finished measuring, save the measurement series under an appropriate name.

Note: at the beginning of each session using the x-ray machine it is a useful precaution to check the accurate zeroing of the goniometer by doing a quick scan as above, but with a reduced exposure time and reduced angle range. Using an x-ray machine with a molybedenum tube, peaks should occur in the spectrum at angles of 6.4° and 7.2°. If they do not ask your demonstrator, or the laboratory technician, to reset the zero of the goniometer.

Measure spectra for a range of tube high voltages from 15 kV to 35 kV, setting the emission current I = 1.00 mA, the measuring time per angular step Δt = 10 s and the angular step width $\Delta\beta$ = 0.1°. Press the COUPLED key to activate 2θ coupling of target and sensor and set the lower limit of the target angle to 2.5° and the upper limit to 12.5°. Adjust these parameters if necessary to observe the dependence of the cut-off wavelength λ_{min} on tube high voltage. Plot an appropriate graph to obtain the Planck constant.

Bibliography

1.   Hook JR and Hall HE: *Solid State Physics*. John Wiley & Sons (Second Edition, 1991)

2.   Kaye GWC and Laby TH: *Tables of Physical and Chemical Constants*. Longman (Fifteenth Edition, 1986), or online: http://www.kayelaby.npl.co.uk (accessed 07/02/2013)