

Astrostatistics

Saturday, 20 January 2017

Recommended Reading:

Feigelson & Babu: Chapters 1-4

Ivezic: Chapters 1-5

Schafer article

Intro to Statistics in Astronomy

Review of Probability & Statistics Foundations

Classical & Bayesian Statistical Inference

kmandel@statslab.cam.ac.uk

<https://github.com/CambridgeAstroStat/PartIII-Astrostatistics>

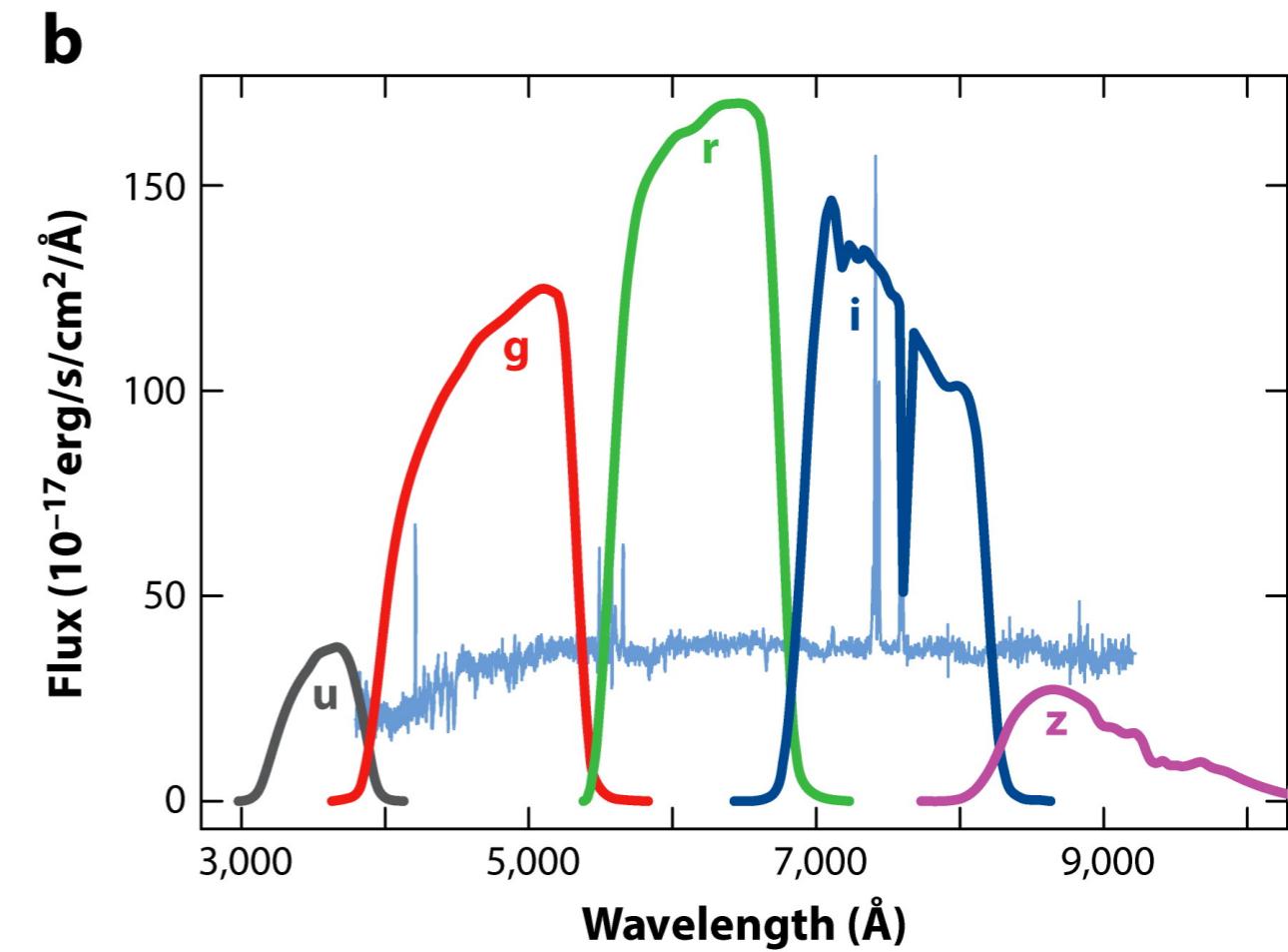
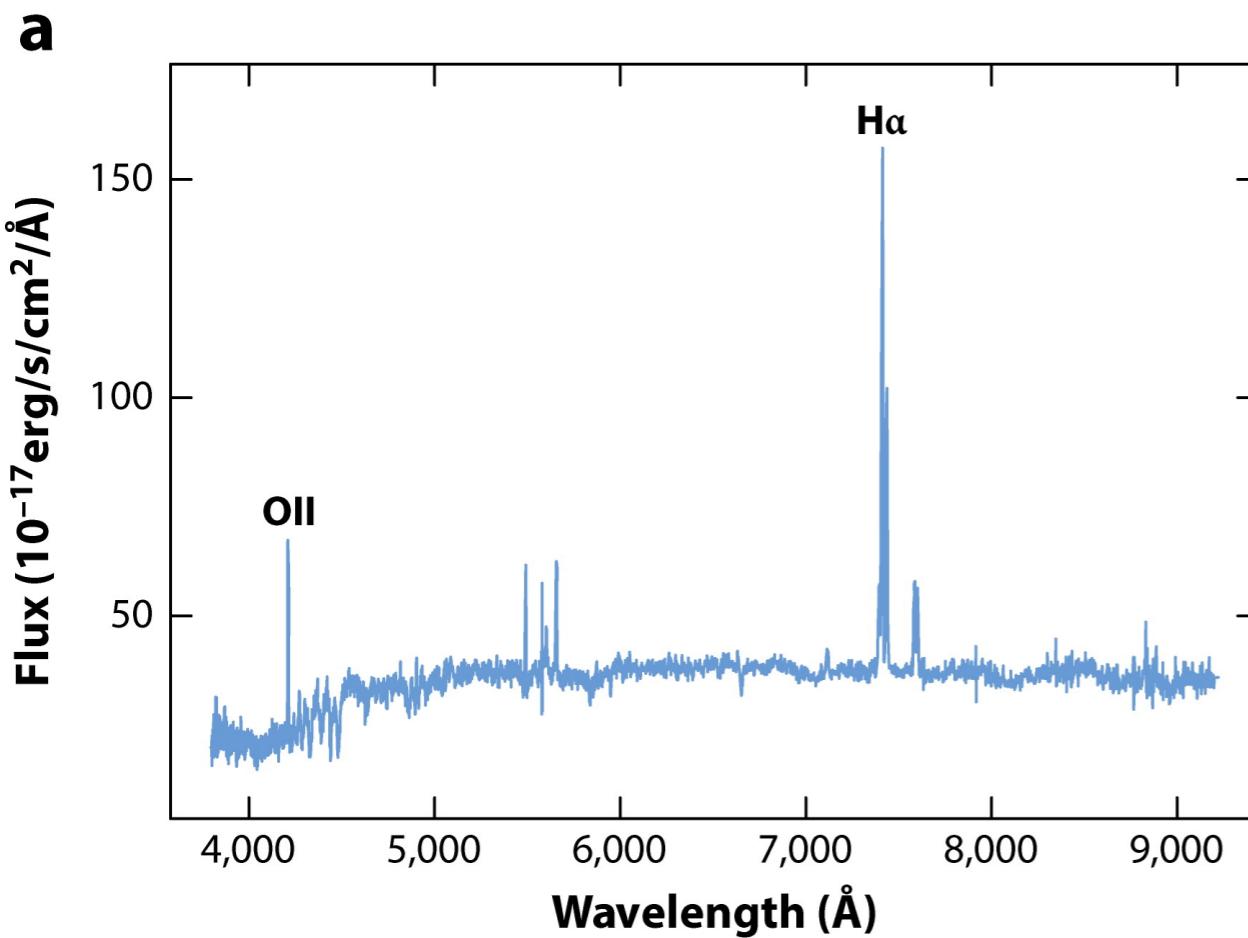
Today

- Introduction to Astronomical Data Types (for statisticians)
- Continue Motivation: Radial Velocity Case Study

What astronomers measure

- Astrometry (angular position on sky, e.g. Gaia)
- Photometry (how bright is it?)
- Spectroscopy (brightness versus wavelength)
- Time Series: Transients & Variables (e.g. stars, quasars, supernovae, exoplanets), Moving objects (e.g. asteroids)
- Spatial Variation (images, maps)
- Combinations of the above

Spectroscopy and Photometry



A Schafer CM. 2015.

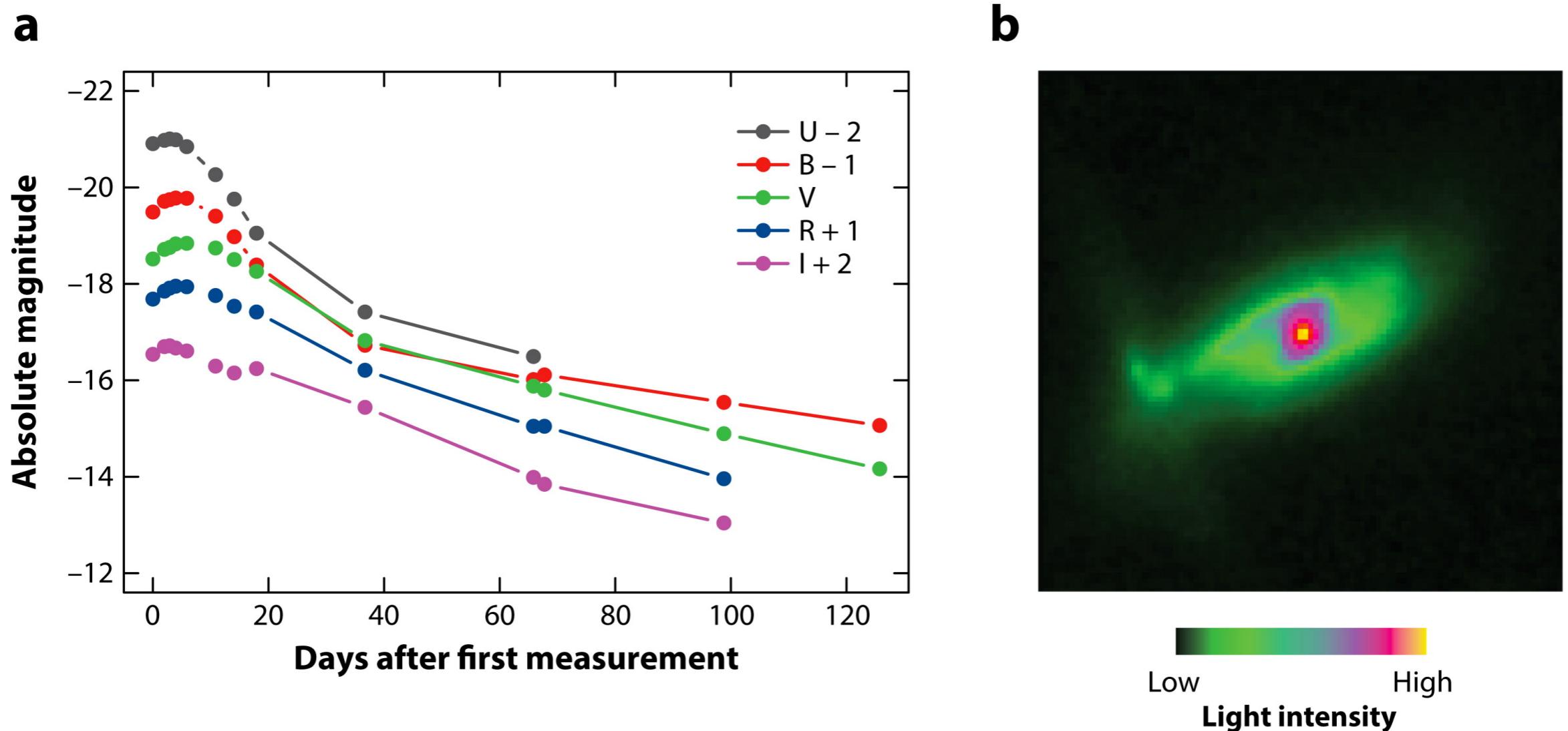
R Annu. Rev. Stat. Appl. 2:141–62

Galaxy Spectrum

Galaxy Photometry

(Flux / Magnitude)

Temporal & Spatial Variation



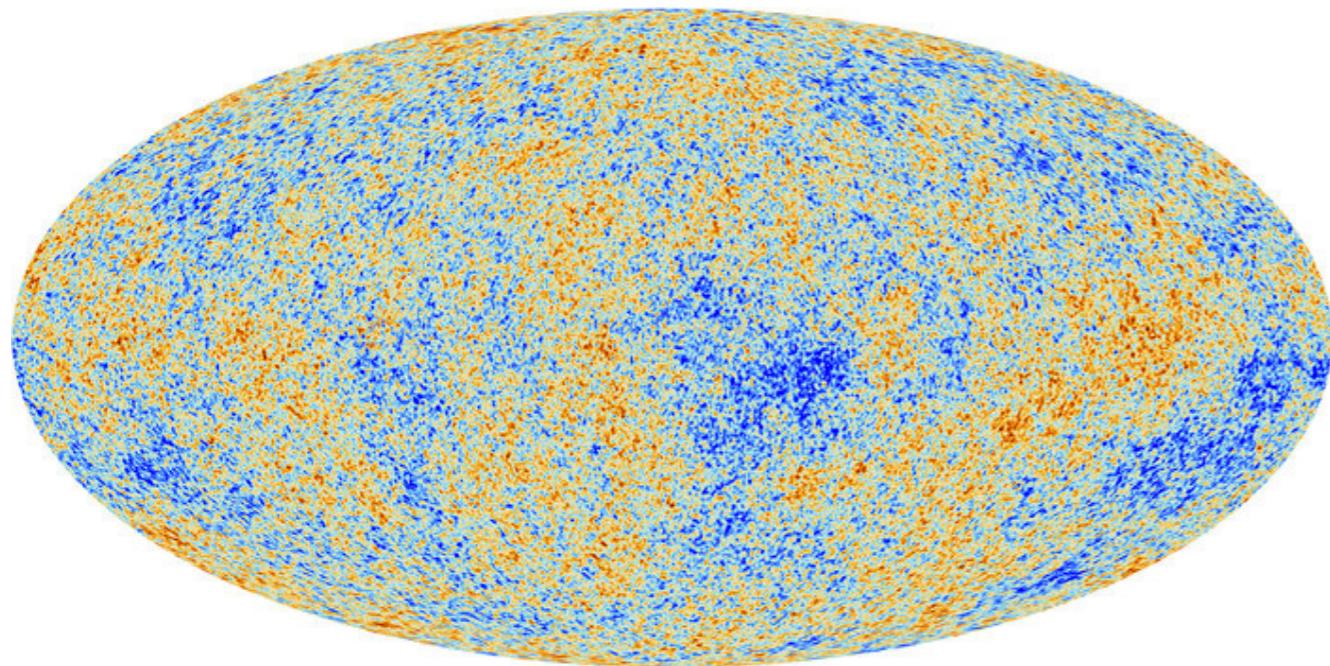
Schafer CM. 2015.

Annu. Rev. Stat. Appl. 2:141–62

Time Series (Light Curve)
Supernova

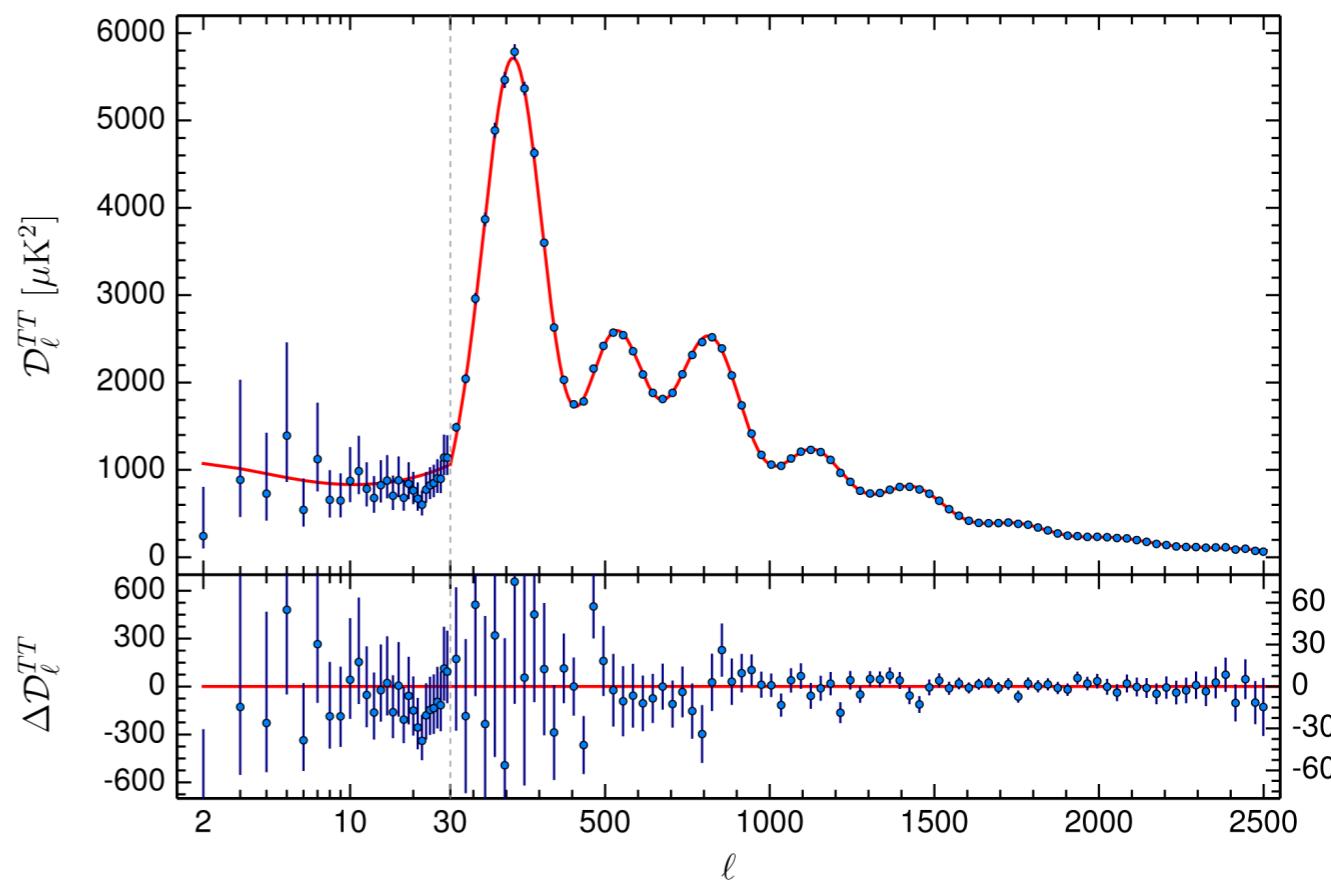
Galaxy Image
(Intensity Map)

Spatial Variation of Intensity



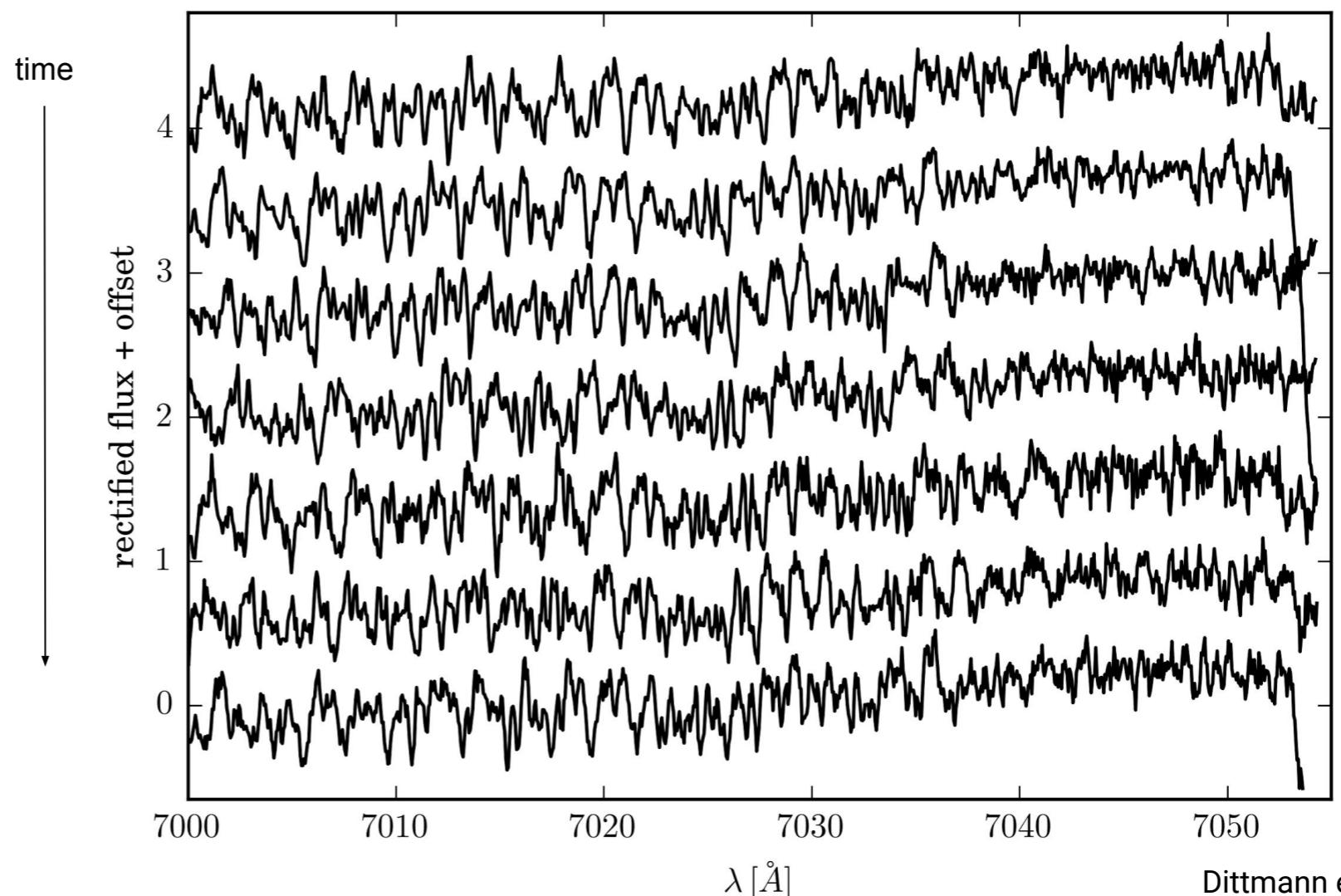
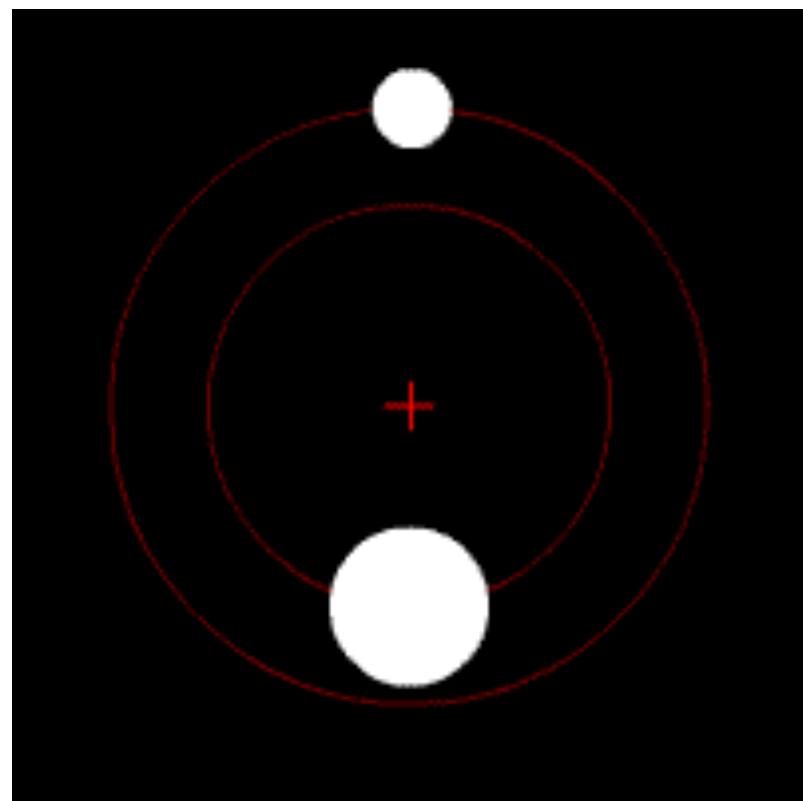
Cosmic Microwave
Background (Planck)
~ Gaussian Random Field
(mean = 2.7 K,
std dev $\sim 10^{-5}$)

Power Spectrum
(~Fourier Transform of
Correlation Function)

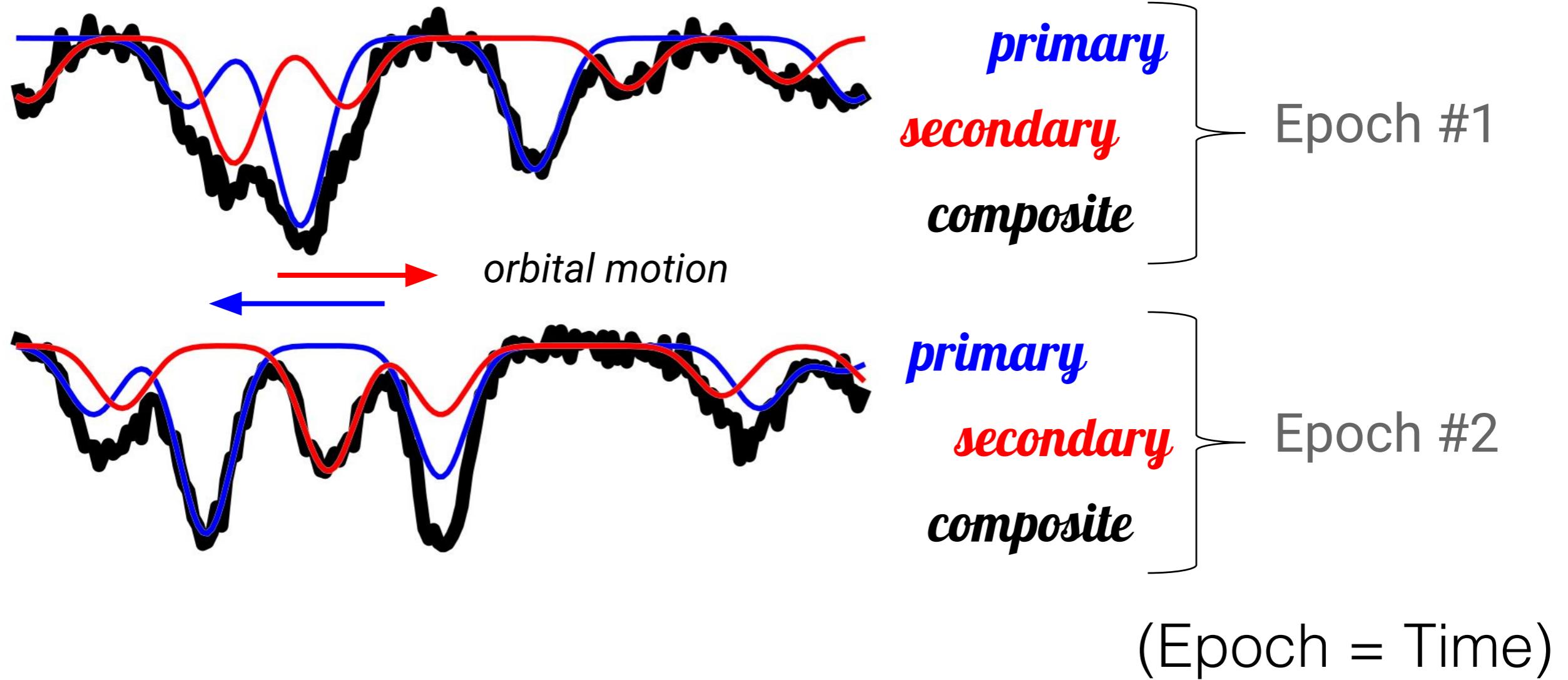


Astrostatistics Case Studies:
Disentangling Time Series Spectra with Gaussian
Processes: Applications to Radial Velocity Analysis
(Czekala et al. 2017, ApJ, 840, 49. arXiv:1702.05652)

Raw Observations of the LP661-13 M4 Binary



Spectroscopic Binary Stars



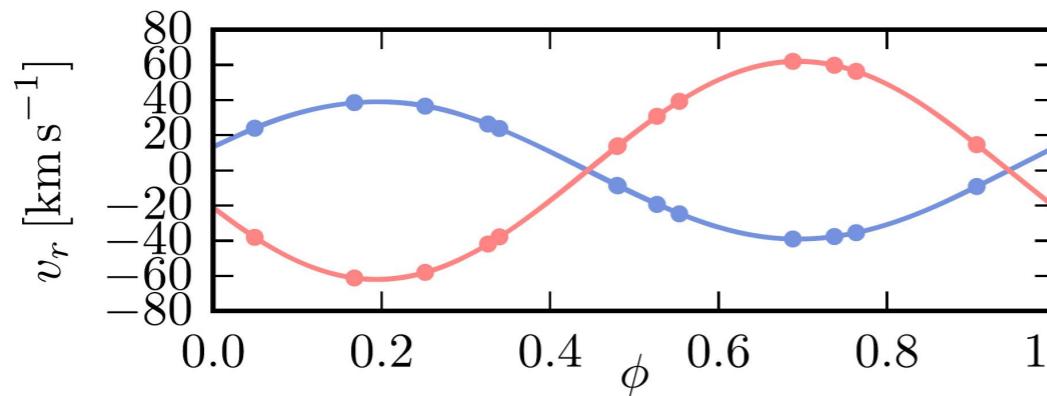
We only observe the “noisy” sum of two (latent) spectra.
Latent (underlying) spectra are unknown functions
Observed spectrum = Measured Data

Forward Model = Generates Data

Problem setup

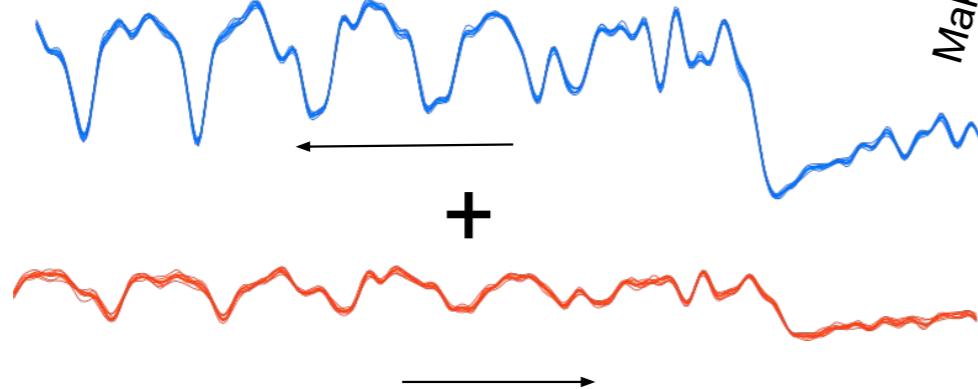
Orbit: period,
eccentricity,
phase, etc.

?



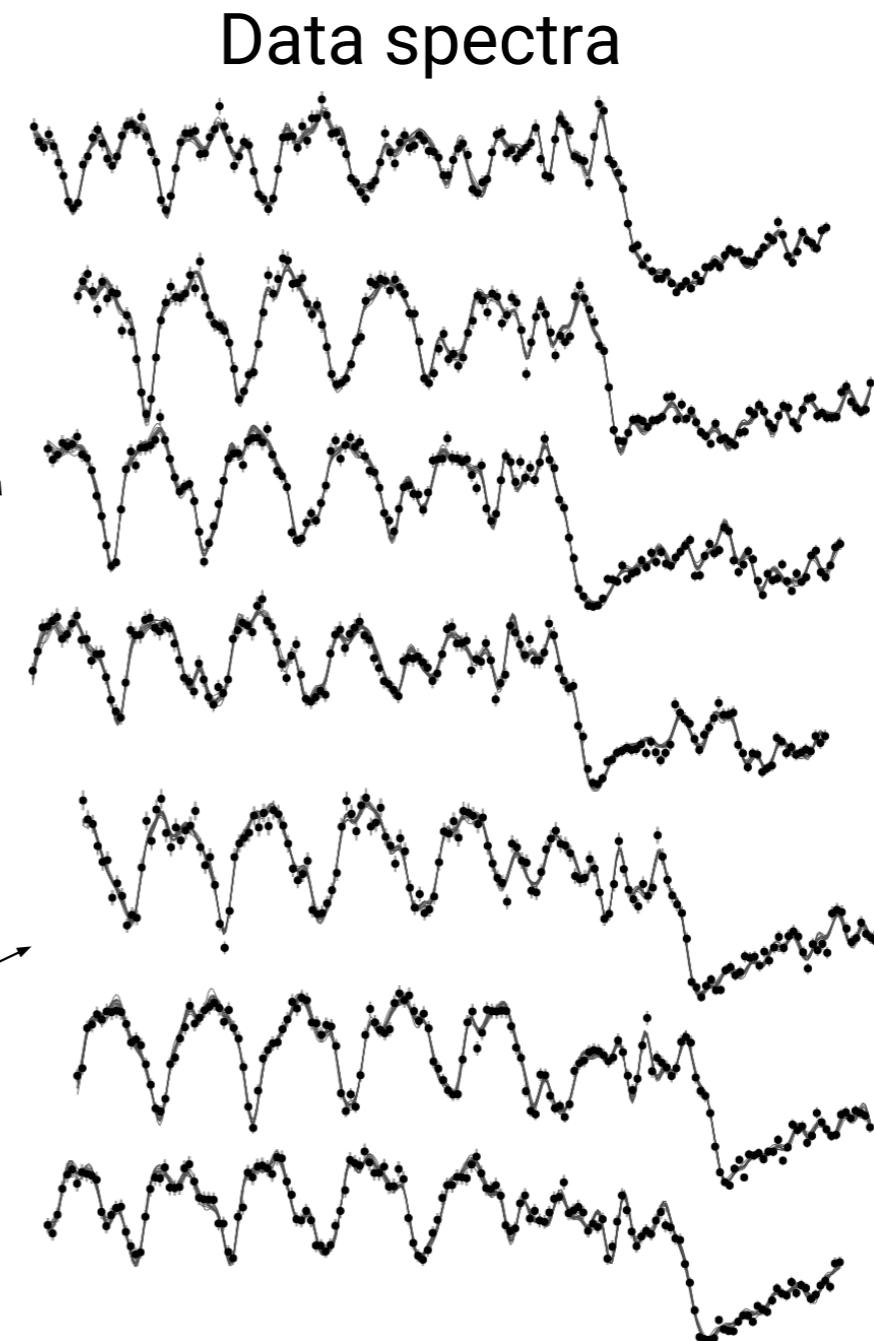
Model
spectra

?



Velocity shifts

Make composite spectra

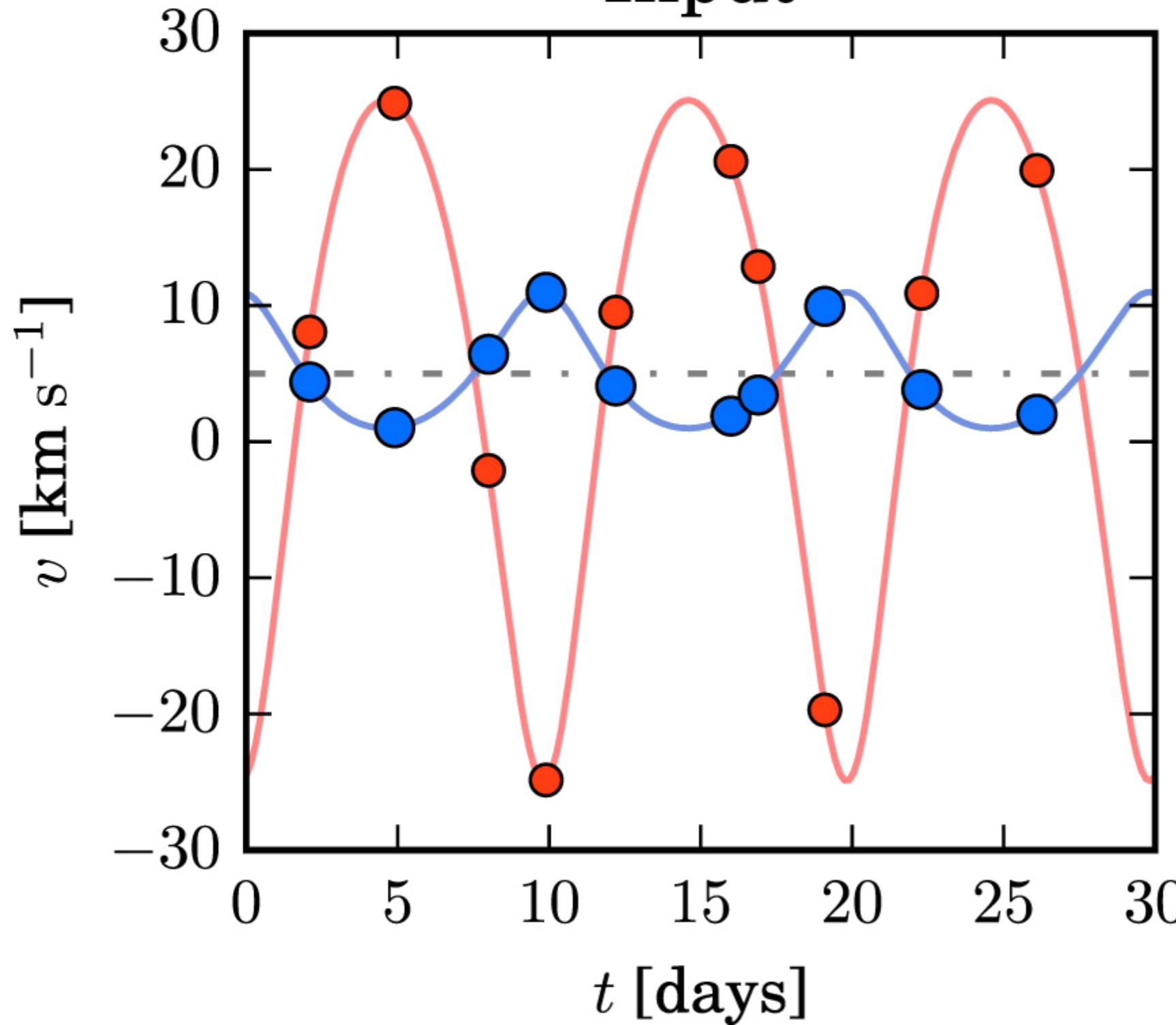


<https://www.youtube.com/watch?v=kHjN42ft6aU>

Goal: Go Backwards and Infer the Component Spectra & Orbital Parameters from noisy, observed (composite) spectra time series

Orbital Parametric Model

Input



- Seven Parameters:
- Mass Ratio
 - Velocity Amplitude
 - eccentricity
 - Arg of Periastron
 - Epoch of Periastron
 - Orbital Period
 - Systemic Velocity

Nonparametric Bayes

Gaussian processes

We will model the latent stellar spectrum f_λ as a Gaussian process

$$f_\lambda \sim \text{GP}(\mu(\lambda), k(\lambda, \lambda'))$$

A function is said to have a Gaussian process if for any collection of inputs the random vector \mathbf{f} has a multivariate Gaussian distribution with mean $\mathbf{\mu}$ and covariance matrix given by k evaluated over ***lambda***

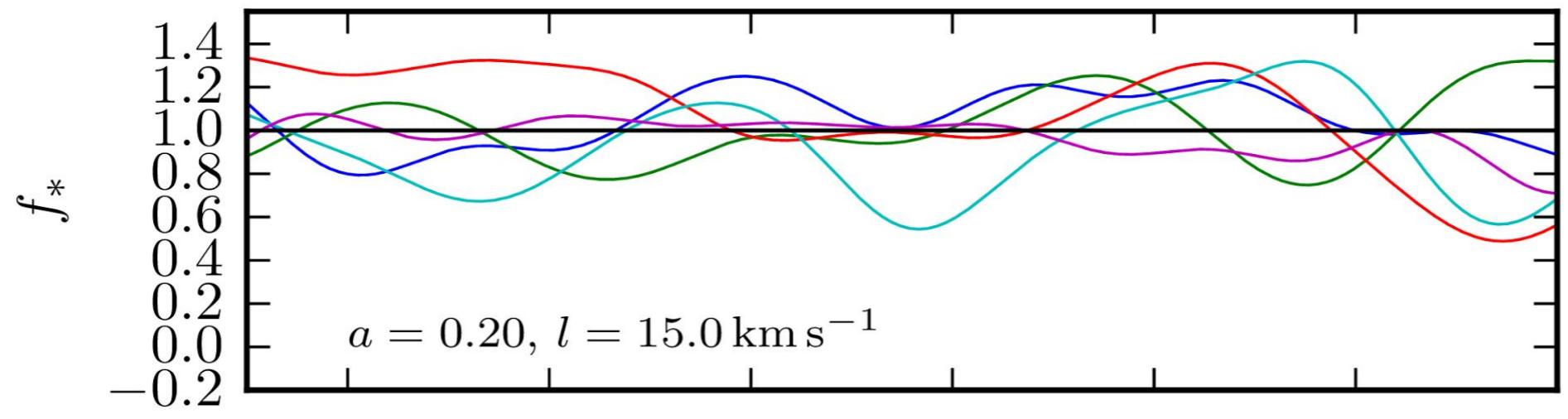
For a covariance kernel, we will use the commonly used squared exponential kernel, which relates pixels in the spectrum based upon their distance in log-wavelength (\propto velocity)

$$k_{ij}(r_{ij} | a, l) = a^2 \exp\left(-\frac{r_{ij}^2}{2l^2}\right)$$

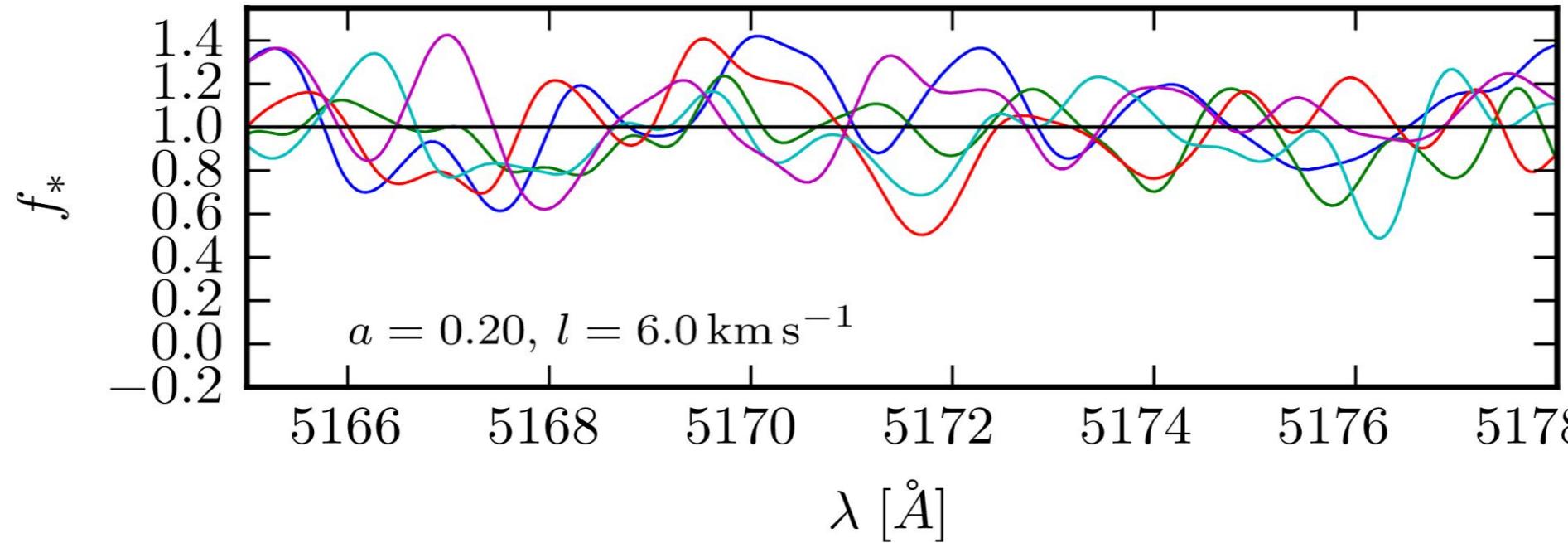
Gaussian Process = a prior on functions (latent spectra)

Gaussian Process model for a single, stationary star

(Zoomed) draws from the prior



l



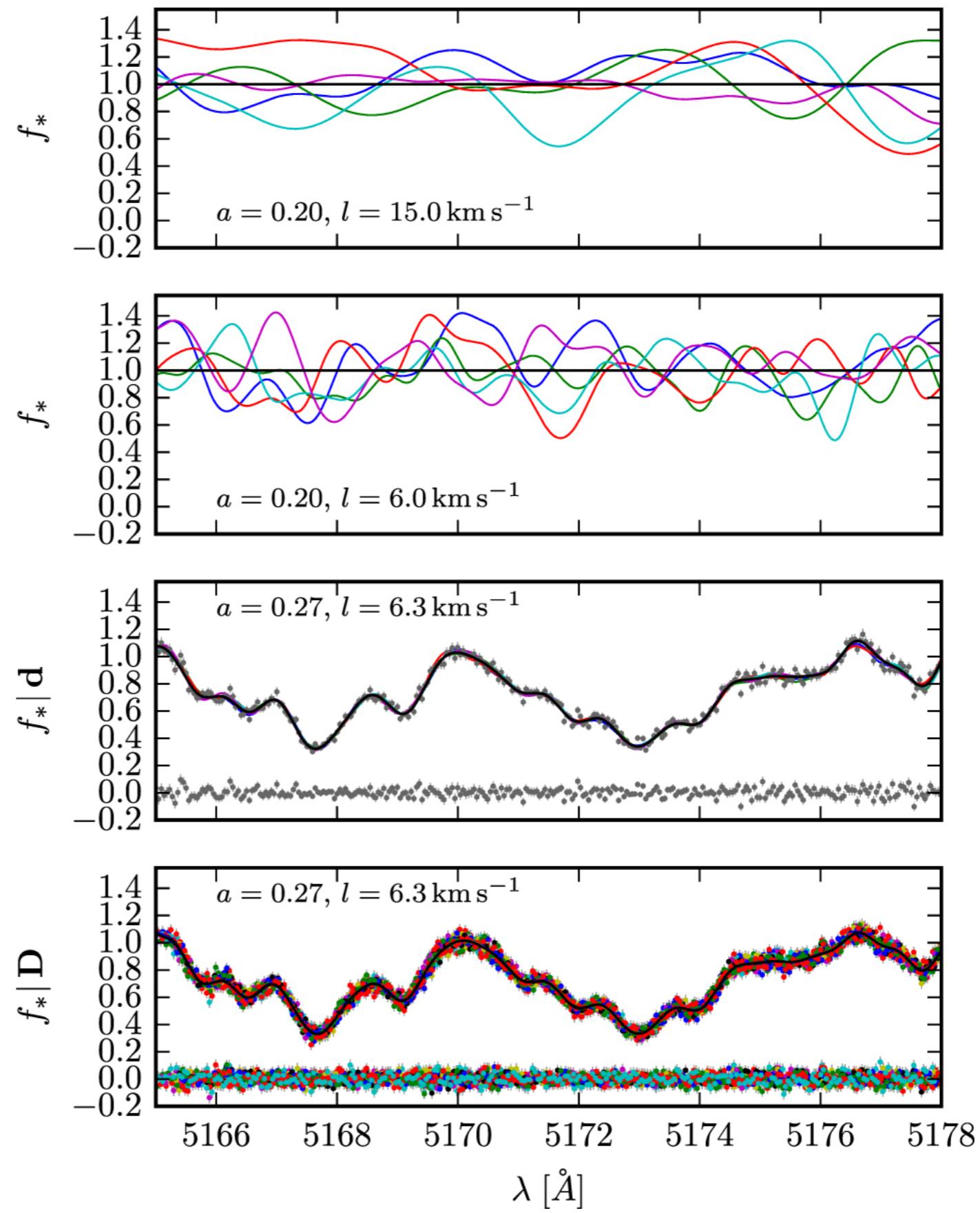
l

Inference = Which function is most consistent with the data?

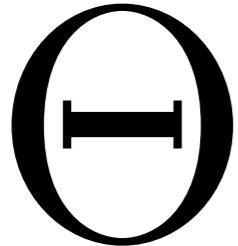
Gaussian Process: Priors & Posteriors

GP prior
(long/short length scales)

GP Posterior
(conditioned on
data spectrum \mathbf{d})
Inference of latent
spectrum



Known Unknowns



7-dim Orbital Parameters = Period, Phase, eccentricity, Velocity Amplitude

$f(\lambda), g(\lambda)$

(∞ -dim) Latent Functions = the unobserved component spectra of the primary (f) and secondary (g) stars

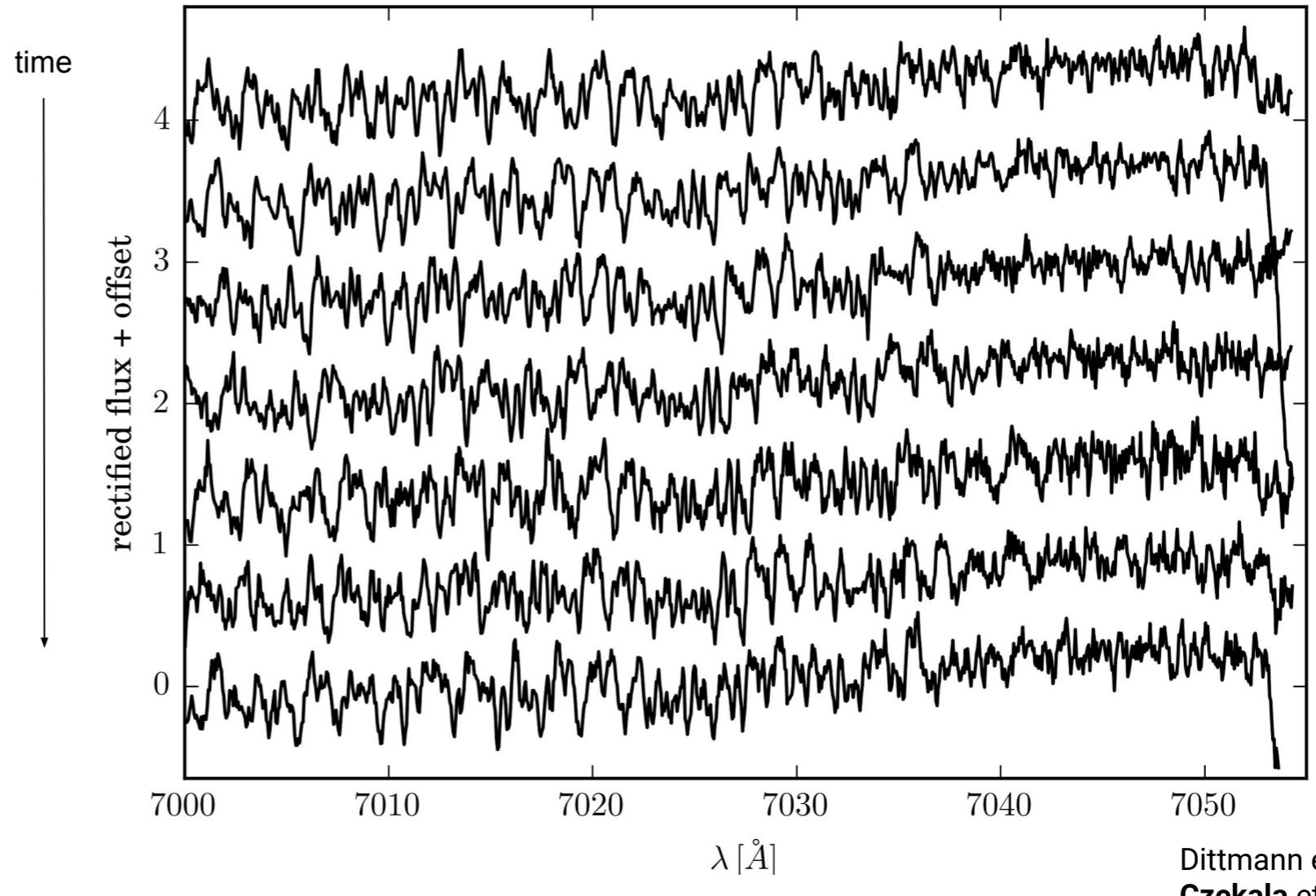
$\alpha =$
 (a_f, l_f, a_g, l_g)

4-dim GP hyperparameters = controlling the amplitude and smoothness of Gaussian Process prior on latent spectra

Knowns (Data)

Raw Observations of the LP661-13 M4 Binary

D =



Dittmann et al. 17
Czekala et al. 17a

Bayesian Inference

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

In this case:

$$\begin{aligned} P(\Theta, f, g, \alpha | D) &\propto \\ P(D | \Theta, f, g, \alpha) \times P(\Theta, f, g, \alpha) \end{aligned}$$

a probability density on (4+7+ ∞)-dim parameter space

Bayesian Computation

1. Run an MCMC (e.g. affine-invariant ensemble sampler) on the 4+7 small dimensional marginal posterior

$$P(\Theta, \alpha | D) = \int df dg P(\Theta, f, g, \alpha | D)$$

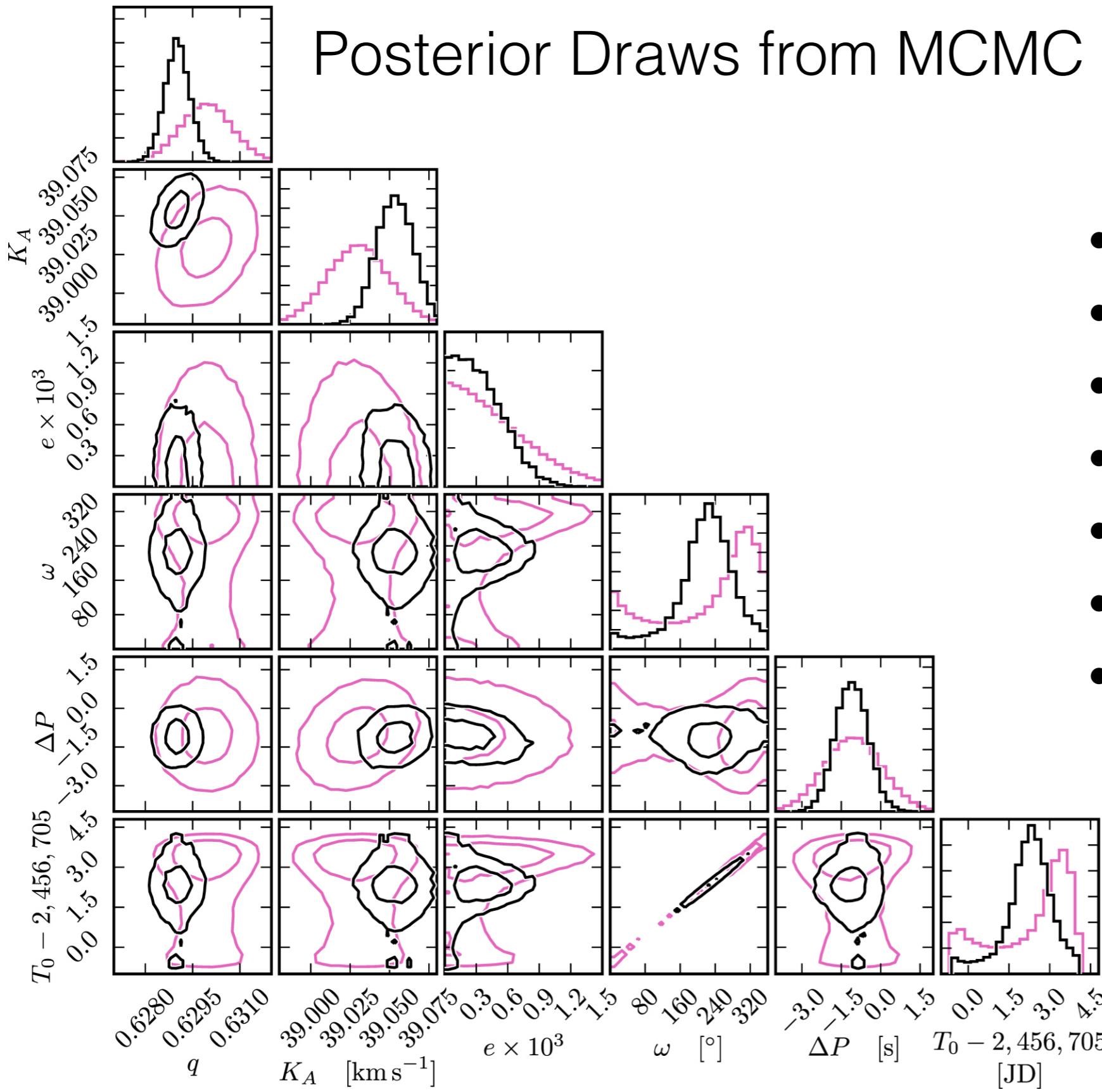
Generates samples: $\Theta_i, \alpha_i \sim P(\Theta_i, \alpha_i | D)$

2. Draw high-dim (**f**, **g**) spectra from the posterior predictive distribution

$$f_i, g_i \sim P(f, g | \Theta_i, \alpha_i, D)$$

Application to the Mid-M-Dwarf Binary LP661-13

Posterior Draws from MCMC



Seven Orbital Parameters:

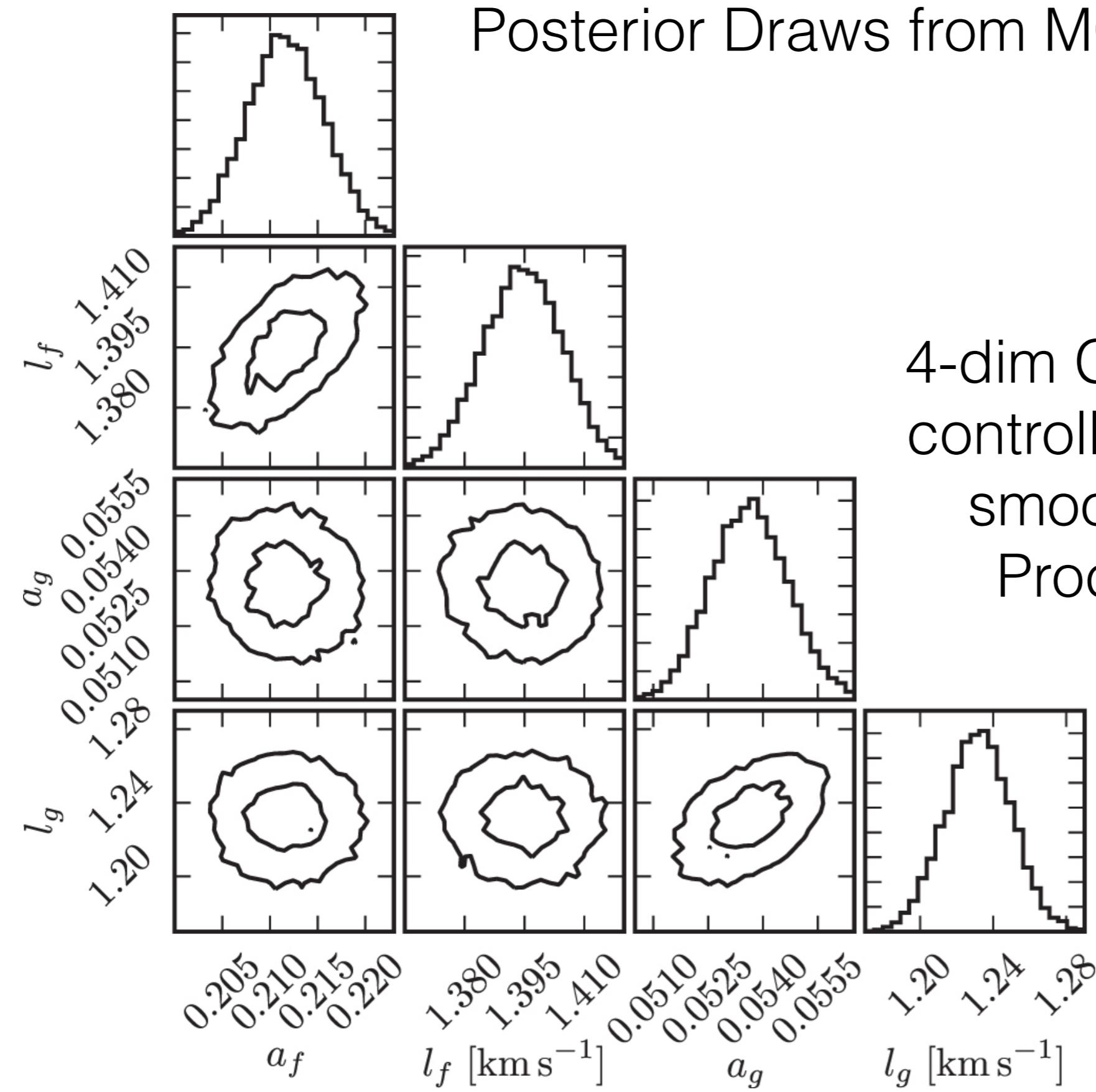
- Mass Ratio
- Velocity Amplitude
- eccentricity
- Arg of Periastron
- Epoch of Periastron
- Orbital Period
- Systemic Velocity

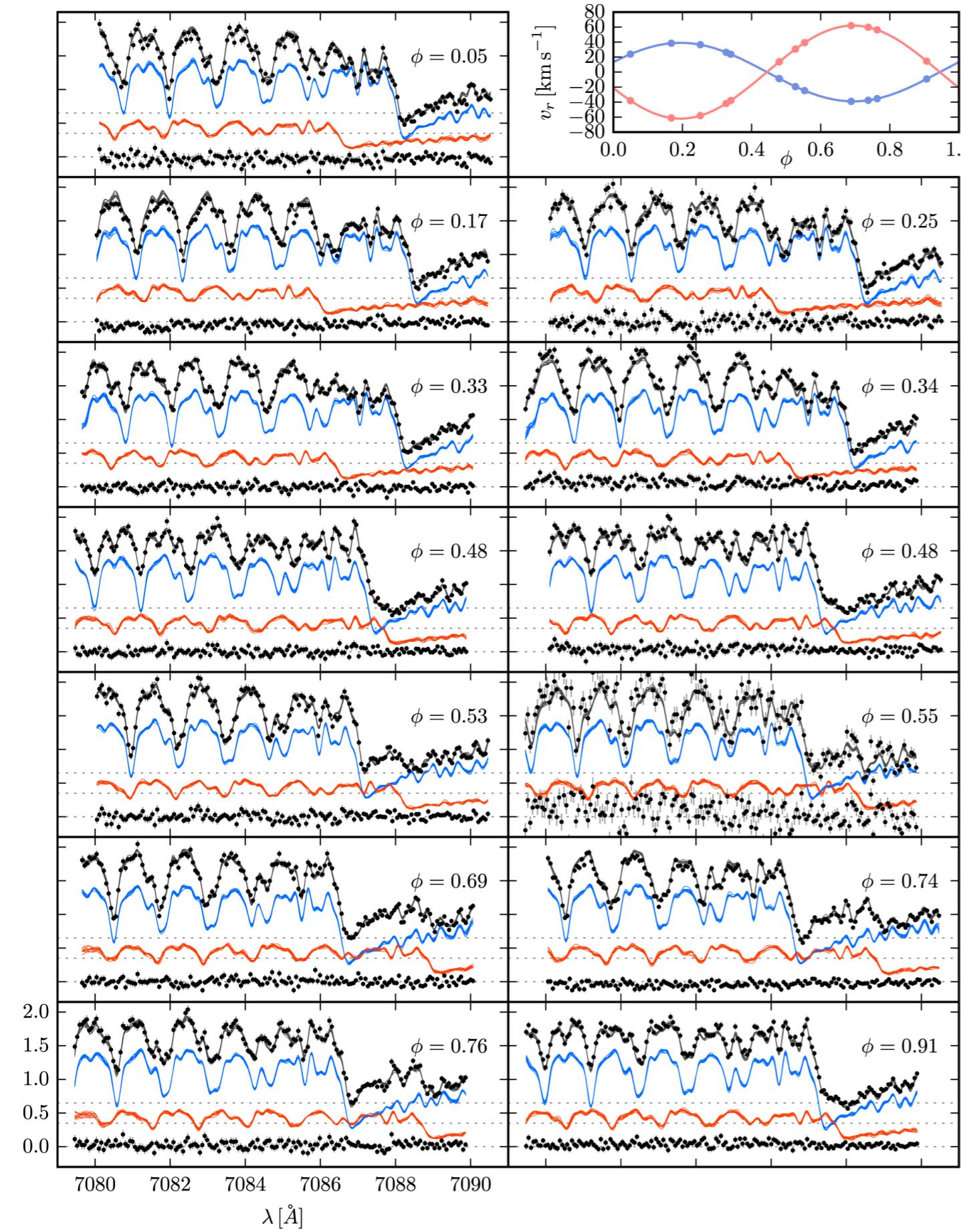
Application to the Mid-M-Dwarf Binary LP661-13

Posterior Draws from MCMC $\alpha =$

$$(a_f, l_f, a_g, l_g)$$

4-dim GP hyperparameters =
controlling the amplitude and
smoothness of Gaussian
Process prior on latent
spectra





Posterior Inference of
Component Spectra
(f, g)

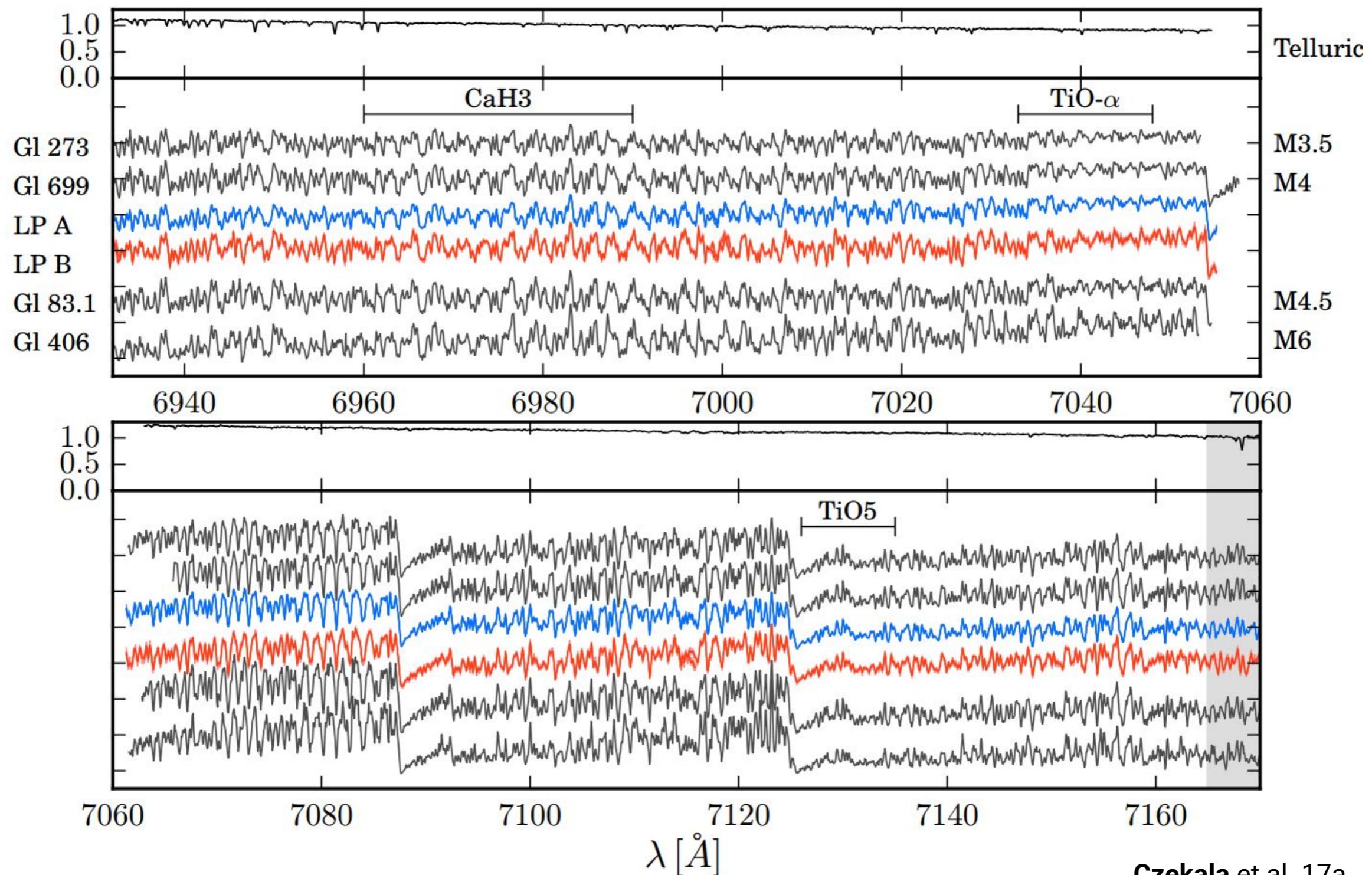
Compared to 10 epochs of
observed spectra **(data)**

Model Checking!
Checking Fit against Data

Model Checking!

Checking Fit against Domain Knowledge (astrophysics)!

Disentangled spectra match other single standard stars



Astrostatistics Case Study 1: Disentangling Time Series Spectra with Gaussian Processes: Applications to Radial Velocity Analysis (Czekala et al. 2017, arXiv:1702.05652)

<http://psoap.readthedocs.io/en/latest/>

- Statistics:
 - Parametric Modelling (Orbit)
 - Nonparametric Modelling (Gaussian Process Spectrum)
 - Bayesian Inference
 - Markov Chain Monte Carlo
- Astronomy:
 - Applications to Radial Velocity Analysis of Stars/Exoplanets