In[ ]:=

$$S = \left\{\alpha_1\,(1-x) - \frac{\beta_1\,x\,(\nu\,y)^{\gamma_1}}{K_1 + (\nu\,y)^{\gamma_1}},\ \alpha_2\,(1-y) - \frac{\beta_2\,y\,x^{\gamma_2}}{K_2 + x^{\gamma_2}}\right\};$$

$$par = \{\nu \to 1,\ \alpha_1 \to 1,\ \alpha_2 \to 1,\ \beta_1 \to 200,\ \beta_2 \to 10,\ \gamma_1 \to 4,\ \gamma_2 \to 4,\ K_1 \to 30,\ K_2 \to 1\};$$

```
{Solve[S[[1]] == 0, x], Solve[S[[2]] == 0, y]}
```

Out[ ]=

$$\left\{\left\{\left\{x \to \frac{((y\,\nu)^{\gamma_1} + K_1)\,\alpha_1}{(y\,\nu)^{\gamma_1}\,\alpha_1 + K_1\,\alpha_1 + (y\,\nu)^{\gamma_1}\,\beta_1}\right\}\right\},\ \left\{\left\{y \to \frac{(x^{\gamma_2} + K_2)\,\alpha_2}{x^{\gamma_2}\,\alpha_2 + K_2\,\alpha_2 + x^{\gamma_2}\,\beta_2}\right\}\right\}\right\}$$

In[ ]:=

x_s[y_] = Solve[S[[1]] == 0, x][[1, 1, 2]] /. par

y_s[x_] = Solve[S[[2]] == 0, y][[1, 1, 2]] /. par

```
sol = NSolve[{S[[1]] == 0, S[[2]] == 0, x > 0, y > 0} /. par]; pts = {};
For[i = 1, i ≤ Length[sol], i++,
 point = {sol[[i, 1, 2]], sol[[i, 2, 2]]};
 AppendTo[pts, point]]

Print[pts]
```

Out[ ]=

$$\frac{30 + y^4}{30 + 201\,y^4}$$

Out[ ]=

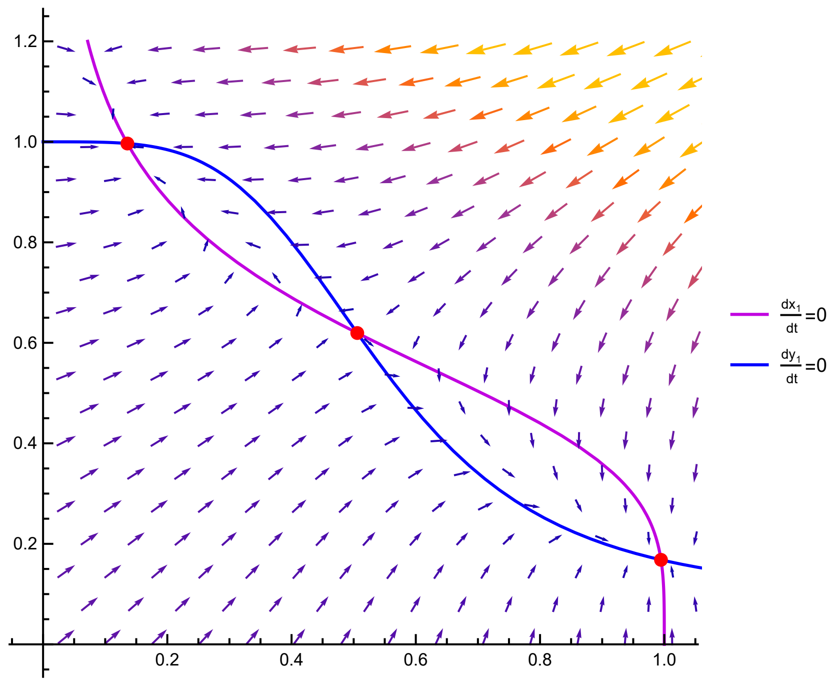
$$\frac{1 + x^4}{1 + 11\,x^4}$$

{{0.994698, 0.168157}, {0.505774, 0.619508}, {0.13573, 0.996619}}

In[]:=
```
P1 = ParametricPlot[{x_s[y], y}, {y, 0, 1.2}, PlotStyle → RGBColor[0.76, 0, 0.88]];
P2 = ParametricPlot[{x, y_s[x]}, {x, 0, 1.2}, PlotStyle → Blue];
P3 = VectorPlot[S /. par, {x, 0, 1.2}, {y, 0, 1.2}, VectorScaling → Automatic];
P4 = ListPlot[pts, PlotStyle → {Red, PointSize[0.02]}];

plot = Show[P1, P2, P3, P4, AspectRatio → 0.97];
Legended[plot, LineLegend[{RGBColor[0.76, 0, 0.88], Blue},
  {"dx_1/dt=0", "dy_1/dt=0"}, LabelStyle → {FontSize → 10}]]
```

Out[]=



In[]:=
```
n = 1; order = 1;
perturbation[ϵ_, δ_] =
  {Normal[Series[∂_x S[[1]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
   Normal[Series[∂_y S[[1]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
   Normal[Series[∂_x S[[2]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
   Normal[Series[∂_y S[[2]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]]} /.
  {x → pts[[n, 1]] + ϵ, y → pts[[n, 2]] + δ};

perturbation[0, 0]
```

Out[]=
```
{-1.00533, -0.126118, -1.69034, -5.94684}
```

In[ ]:=

```
M = {};
For[i = 1, i ≤ Length[pts], i++,
 n = i; order = 1;
 perturbation[ϵ_, δ_] =
   {Normal[Series[∂x S[[1]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
    Normal[Series[∂y S[[1]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
    Normal[Series[∂x S[[2]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]],
    Normal[Series[∂y S[[2]] /. par, {x, pts[[n, 1]], order}, {y, pts[[n, 2]], order}]]} /.
   {x → pts[[n, 1]] + ϵ, y → pts[[n, 2]] + δ};
 m = {{perturbation[0.00001, 0.00001][[1]], perturbation[0, 0][[2]]},
   {perturbation[0, 0][[3]], perturbation[0, 0][[4]]}};
 Print[m // MatrixForm];
 AppendTo[M, m]]
```

$$\begin{pmatrix} -1.00533 & -0.126118 \\ -1.69034 & -5.94684 \end{pmatrix}$$

$$\begin{pmatrix} -1.97723 & -3.1755 \\ -2.82437 & -1.61418 \end{pmatrix}$$

$$\begin{pmatrix} -7.36781 & -3.35837 \\ -0.0996147 & -1.00339 \end{pmatrix}$$

In[ ]:=

```
For[i = 1, i ≤ Length[pts], i++,
    ee = Eigenvalues[M[[i]]];
    WriteString["stdout", "Punto ", i, ": "];
    If[ee[[1]] > 0 || ee[[2]] > 0,
     WriteString["stdout", "Inestable\n"], WriteString["stdout", "Estable\n"]]]
```

```
Punto 1: Estable
Punto 2: Inestable
Punto 3: Estable
```