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Mathematics 335 - Roulette Simulation
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Casino Project Report

Introduction

Casinos are notorious for giving refuge to those who wish to take the opportunity to increase their wealth. One game in particular, Roulette, provides the player with multiple choices to make their fortunes. In Roulette, the main component is a wheel divided into equally sized slots, where each slot can be a numerical value and a color. The wheel is spun and a ball is placed in, and eventually seats into a slot. The player has multiple options to bet on, including:

1. The outcome is red/black
2. The outcome is an odd/even number
3. The outcome is within a range of values
4. The outcome is a numerical value
5. The outcome is within a column of the table (3,6,9,12,15,...36)

Here is a typical setup of the betting possibilities:

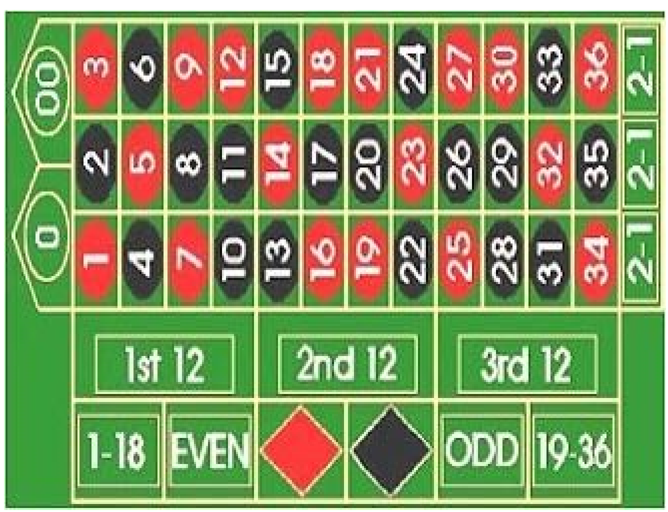

Figure 1 - Roulette Betting Table

Questions Answered

After some analysis of the project, we have refined our work to answer a few questions regarding the simplified betting schemes of roulette. Our intention is to pay our $48,450 tuition for a total of **$96,900** with the profit gained by the casino and analyzed the following with the number of gamblers equal to 2,531:

- The expected value that the casino will receive from a random bid of $1 is
  $\frac{1}{19}$
- The allowance per gambler to get the casino to expectedly payout our tuition is
  ~730
- By adding *n* more green spaces, and keeping the number of gamblers constant, the expected value that the casino will receive from a random bid of $1 is calculated as
  $\frac{2+n}{38+n}$
- By manipulating the size of the 00 slot by *n* units, and keeping the number of gamblers constant, the expected value that the casino will receive from a random bid of $1 is calculated as
  $\frac{1}{(37+n)*(50)}$ [14n + 86]
- By manipulating the payout odds for the "single" bet category to paying out *n*, and keeping the number of gamblers constant, the expected that the casino will receive from a random bid of $1 is calculated as
  $\frac{1}{50*38}$ [24 + 38(37 - n)]

After solving these experiments analytically, we found that our simulation agrees with our calculations. Our calculations for the answered listed above can be found in the Calculations section of this report.

Approach

For this project, we used **Java** for implementing the simulation. The general Class Diagram is shown as Figure 2. In the program's current state, the roulette board can be modified to add new values, to add a high probability that the ball lands in a slot, and finally to change the payouts per bet-type.

The output of the program for a single trial is the balance of the casino. A positive balance indicates that the casino had gained money, and a negative balance means that the casino lost money. The output of the simulation is the average balance over all trials.

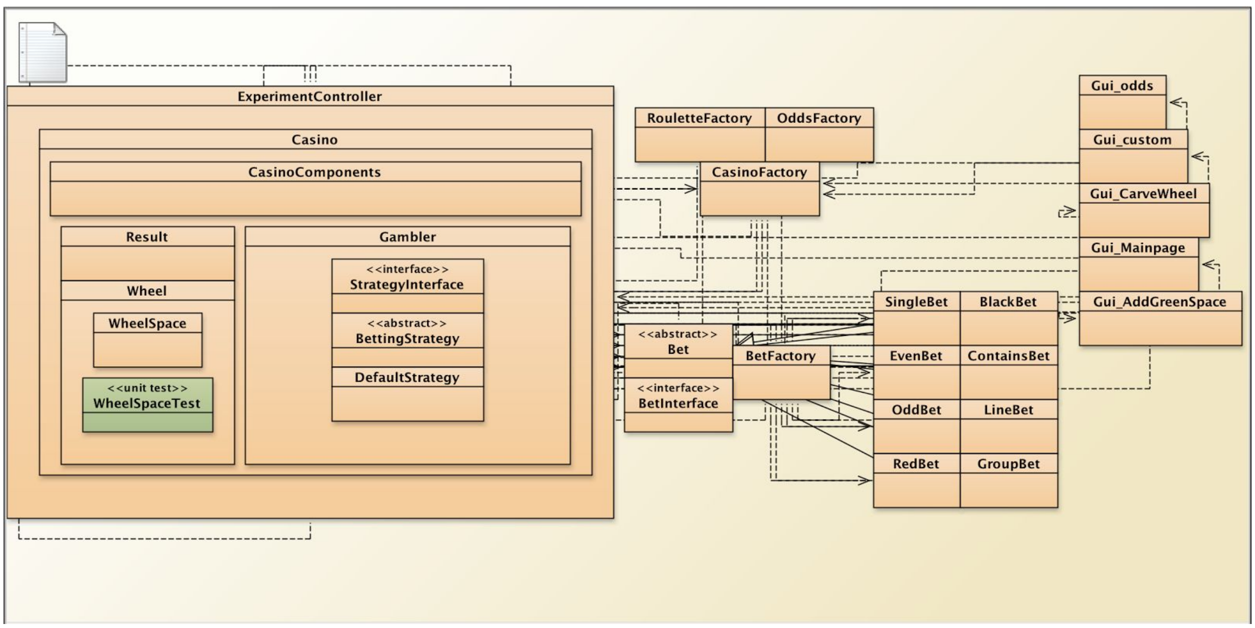

Figure 2.1 - Class Diagram

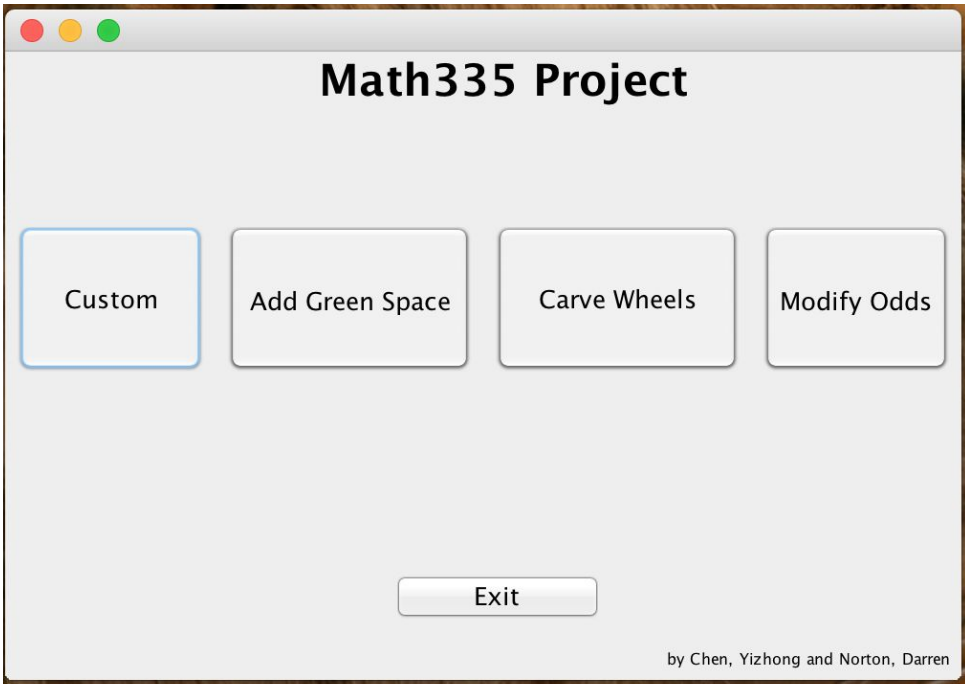

Figure 2.2 - Gui_MainPage Diagram

Experiment Setup

Constraints

We establish a few constraints about the gamblers in the casino. There are many reasons to do this, but we find that by making assumptions and constraints, our analysis and calculations become much less convoluted.

Uniformly Random Bet Selection

The first constraint we establish is that gamblers will randomly pick a spot on the roulette table and place their bet there. This translates to our experiment that there are 50 spots on the standard roulette board:

- 38 spots for “single” bets (0,00,1,2,3,4,5,...)
- 1 spot for a red bet
- 1 spot for a black bet
- 1 spot for an even bet
- 1 spot for an odd bet
- 1 spot for the first half (1-18)
- 1 spot for the second half (19-36)
- 1 spot for the first third (1-12)
- 1 spot for the second third (13-24)
- 1 spot for the third third (25-36)
- 1 spot for the first line (1,4,7,...)
- 1 spot for the second line (2,5,8,...)
- 1 spot for the third line (3,6,9,...)

This list implies that single bets are more likely, but as it turns out that doesn’t matter for most of our experiment. Given by our calculations, see the Standard Case in Appendix B, the expected value for any of these bets will be the same. The only time in which the uniformly random betting scheme would matter in Experiment 5 and is discussed there.

Uniformly Random Spending

The second constraint we make is that gamblers will select a random amount of money to spend on a bet. This constraint is meant to add some variety to the program to closer mimic reality. This also adds a more entertaining element of watching the casino go extremely negative due to a better going all in on a single bet and winning. By calculating the expected value per dollar spent, and utilizing the fact that most of the time the expected value for each betting scheme is the same, we know that this constraint is appropriate to the project.

Gamblers Spend Their Whole Allowance

To introduce a property of practicality, we constrain the gamblers to spend all of their allowance in the casino. This would allow someone using our software to further extend our analysis by making an inference on how much money their gambling population wishes to spend. To imagine this, you can think of a person going to a casino and setting a hard-limit for their spending values. In reality, this limit is often broken, but we do not wish to represent this in our code. By keeping the money spent in the casino constant per trial, we reduce some variability between trials, making them more comparable.

Gamblers Spend At Least $1 Per Bet

This constraint enforces that the gambler will spend at least $1 per bet, if they have the money. This will just prevent wasteful iterations of the program. Given that we know intuitively that spinning a roulette wheel each trial is independent, this constraint will have no effect on our data.

The Outcome of the Wheel is Selected From a Summation of Unit Wheel Space Sizes

To make adjusting the probability of the ball landing in certain slots easier, we programmed the wheel such that it will roll a die with the same number of sides as the sum of the “sizes” of each wheel slot. Therefore, a slot with size = 2 has 2 chances in the process, whereas a slot with size = 1 has only 1. When we change the size, we are not changing the probability by a unit of 1, but some fraction of the unit as determined to the relative size of the wheel.

Experiments

Experiment 1 Expected Value Measurements

Motivation

For our first experiment, we need to determine the expected earning of casino per $1 bet that gambler spent on the roulette game.

Approach

Our approach to solving this problem is to vary number of trials, number of gamblers and number of allowance for each gambler with the constant wheel and odds. The expected value E(X) = 0.05263158

Parameters

- 200000 trials for each round
- Gamblers = 1, 1, 100
- Allowance = 1, 100, 1

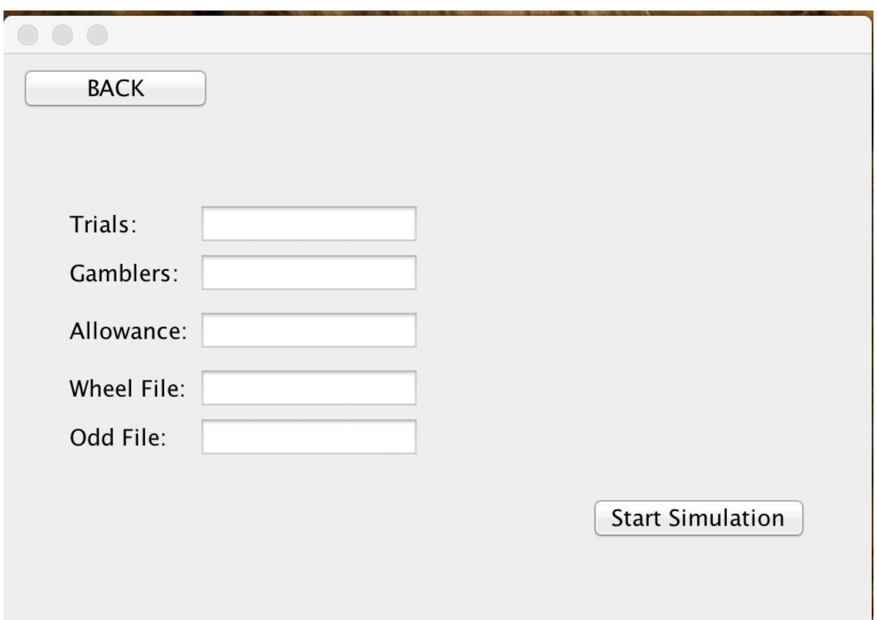

Figure 1.1 - Gui_Custom(Experiment 1)

Data

```
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
YOU ARE RUNNING A CUSTOMIZED SIM
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
INPUT PARAMETERS
TRIALS: 200000
GAMBLERS: 1
ALLOWANCE: 1
WHEEL FILE: config_wheel_1.txt
ODDS FILE: config_odds_1.txt

Simulation over after 200000 trials.
Average balance for casino: 0.052605
Variance: 26.556418906094674
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&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
YOU ARE RUNNING A CUSTOMIZED SIM
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
INPUT PARAMETERS
TRIALS: 200000
GAMBLERS: 1
ALLOWANCE: 100
WHEEL FILE: config_wheel_1.txt
ODDS FILE: config_odds_1.txt
```

Analysis

For the first sub-experiment, there is 1 gambler with $1 allowance. The expected average balance for casino should be: 0.05263158. The simulation shows the value to be 0.052605. For the second sub-experiment, there is 1 gambler with $100 allowance. The expected average balance for casino should be: 5.263158. The simulation shows 5.34612. This confirms our calculation that the expected value grows by a linear factor of allowance. For the third sub-experiment, there are 100 gamblers with $1 allowance. The expected average balance for the casino should be: 5.263158. The simulation gets 5.13878, once again showing that our calculations that the expected value should grow with a linear factor of the number of gamblers.

Experiment 2 Standard Case Allowance Test

Motivation

For our second experiment, we need to know the minimum average allowance to earn the tuition for two students if all students in Lafayette College play our roulette gambling.

Approach

Our approach to solving this problem is to umber of allowance for 2531 gamblers with the constant wheel and odds. The expected value E(X) = 730

Parameters

- 200000 trials
- Gamblers = 2531
- Allowance = 730

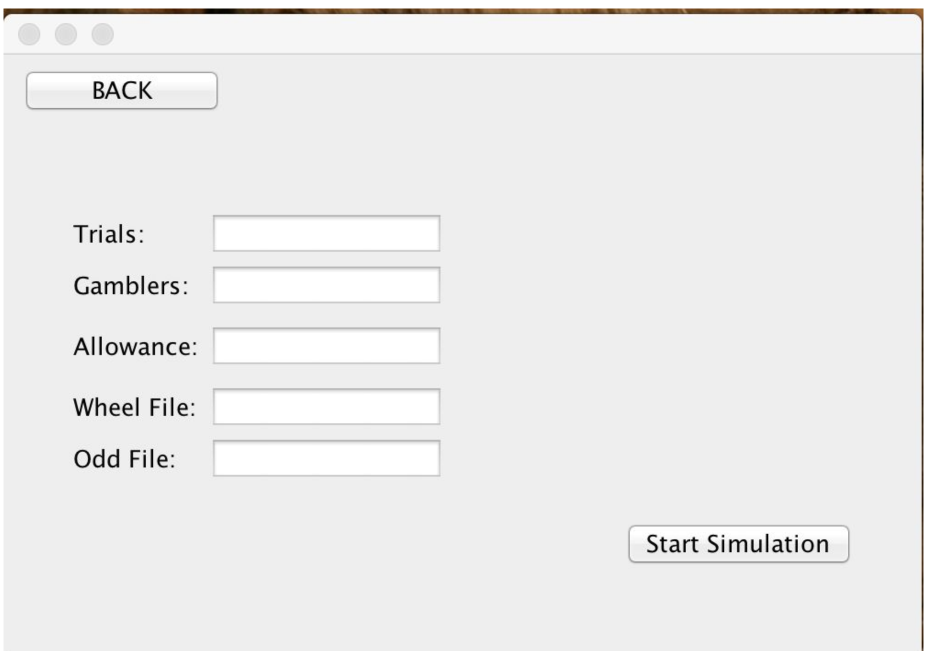

Figure 2.1 - Gui_Custom(Experiment 2)

Data

Simulation over after 200000 trials.

Average balance for casino: 5.34612
Variance: 130177.42658406105
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&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
YOU ARE RUNNING A CUSTOMIZED SIM
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
INPUT PARAMETERS
TRIALS: 200000
GAMBLERS: 100
ALLOWANCE: 1
WHEEL FILE: config_wheel_1.txt
ODDS FILE: config_odds_1.txt

Simulation over after 200000 trials.
Average balance for casino: 5.13878
Variance: 2666.5266307442494
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YOU ARE RUNNING A CUSTOMIZED SIM
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INPUT PARAMETERS
TRIALS: 200000
GAMBLERS: 2531
ALLOWANCE: 730
WHEEL FILE: config_wheel_1.txt
ODDS FILE: config_odds_1.txt

Simulation over after 200000 trials.
Average balance for casino: 97175.3373
Variance: 2.9643916694105907E10
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

Analysis

There are 2531 gamblers with $730 allowance. Expected average balance for the casino should be: 97234.684. The simulation yields 97175.337. We feel that the results we get from simulation are close enough to expectation.. This shows the correctness of our simulation. We can conclude that in order to earn tuition for us (Darren and Yizhong), every student in Lafayette needs to spend $730 in our business.

Experiment 3 Adding Green Spaces

Motivation

For our third experiment, we decided to understand how the expected value per $1 bet changes in respect to the number of green spaces on the roulette wheel. The green spaces on the board are quite important as they are the elements that provide the house edge making Roulette a profitable game for casinos.

Approach

Our approach to solving this problem was to vary the number of green spaces available on the wheel and observe how the expected value changes. From our Calculations in Appendix B, we expect E(X) = $\frac{2+n}{38+n}$.

Parameters

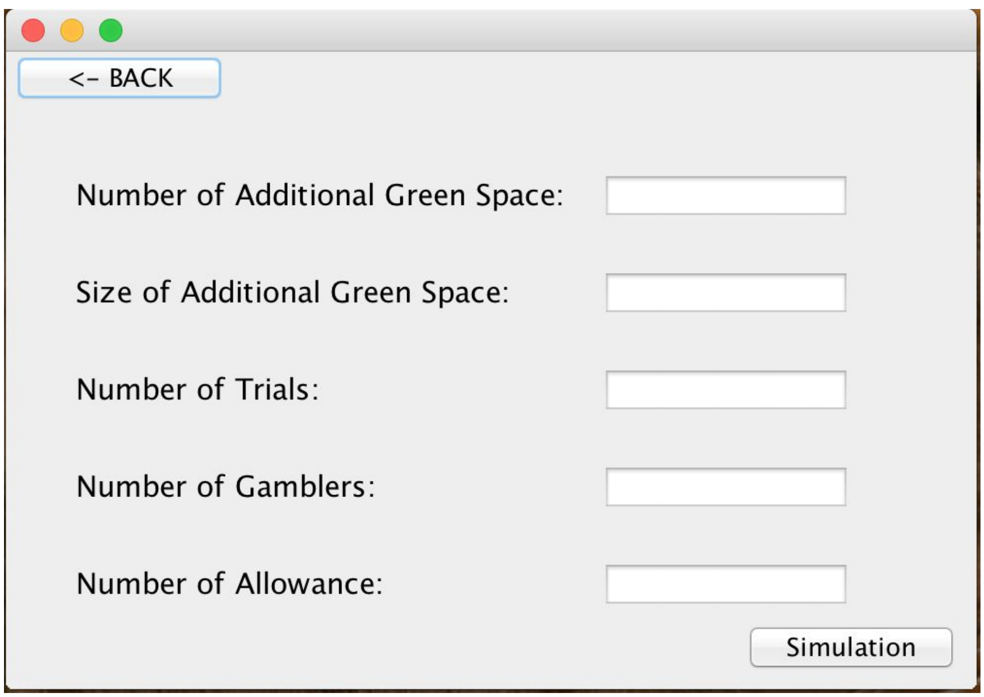

Figure 3.1 - Gui_AddGreenSpace(Experiment 3)

Data

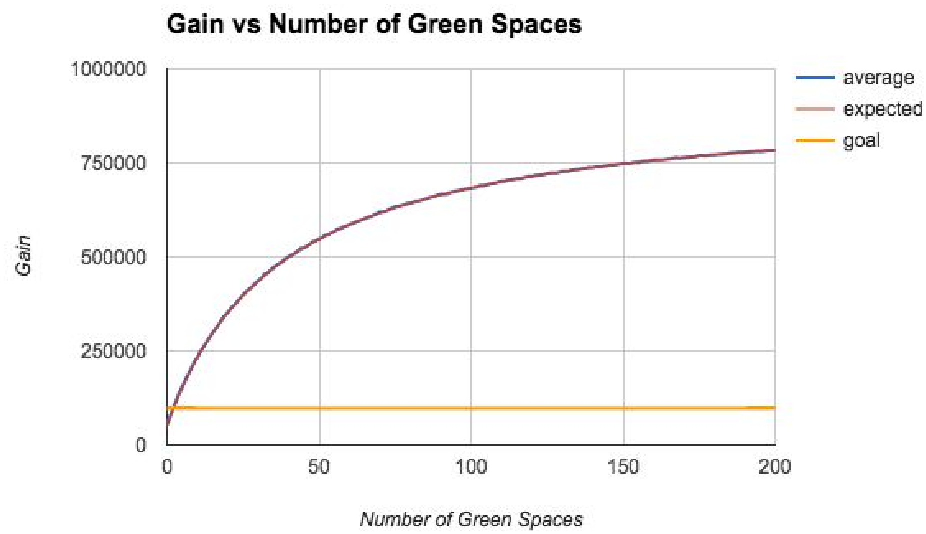

Figure 3.2 - Gain vs Number of Green Spaces

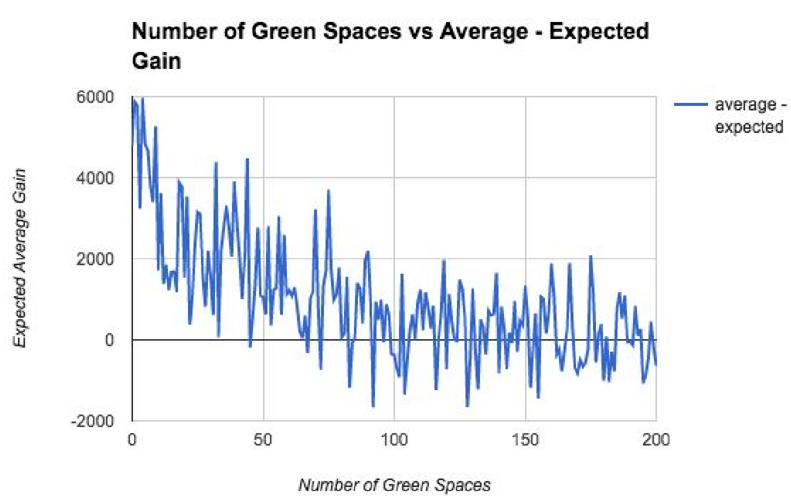

Figure 3.3 - Number of Green Spaces vs Average-Expected Gain

Analysis

From Figure 3.2 we see that adding green spaces can be quite effective in increasing the average gain. We reach our goal tuition using half of the number of gamblers only after adding 3 green spaces. However, adding green spaces is quite obvious to gamblers that they are being taken for a ride, so we would not advise doing this in our casino.

In a quick comparison to our calculations, labeled as “expected” we see that the growth rate of the lines are quite similar. Closer inspection via Figure 3.3 shows that the difference we observe between the measured and calculated results is fair. We believe that the oscillations can be explained by the naturally high variance in the simulation.

Experiment 4 Carving the Wheel

Motivation

For our fourth experiment, we decided to understand how the expected value per $1 bet changes in respect to carve the wheel differently (some spaces may have a different probability to hit).

Approach

Our approach to solving this problem was to carve one space of wheel to different sizes and observe how the expected value changes. From our Calculations in Appendix B, we expect E(X) = $\frac{1}{(37+n)*(50)}$ [14n + 86].

Parameters

- Average over 5000 trials for each n (n = number of additional green spaces)
- Gamblers = 2531
- Allowance = 365 (Half of the value from experiment 2, allows us to show growth)
- Max n = 200
- Increment = 1

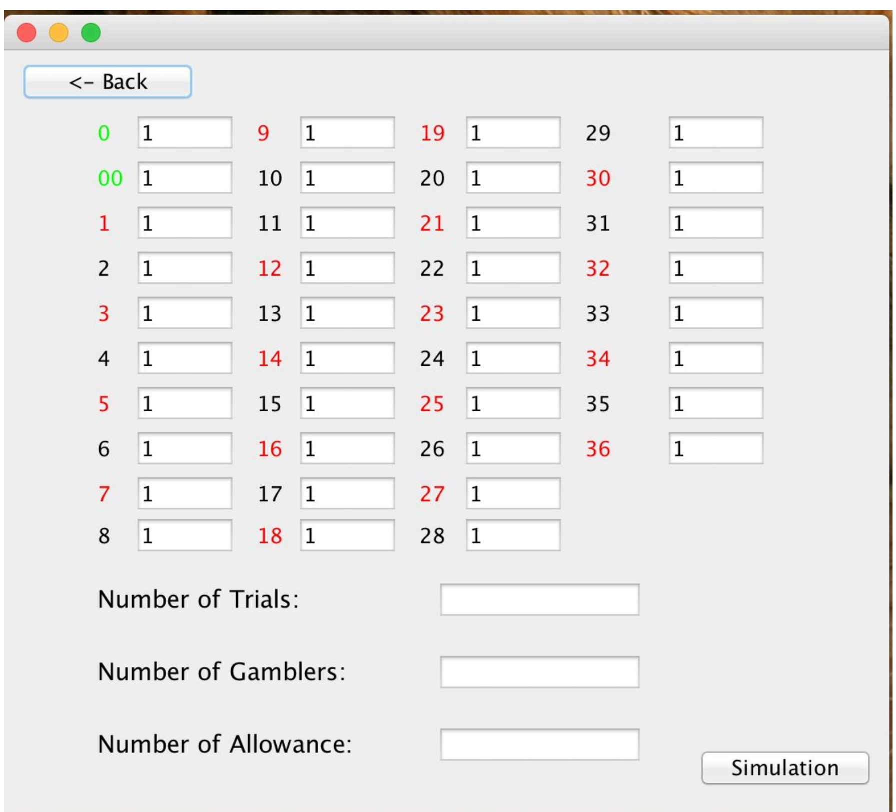

Figure 4.1 - Gui_CarveWheel (Experiment 4)

Data

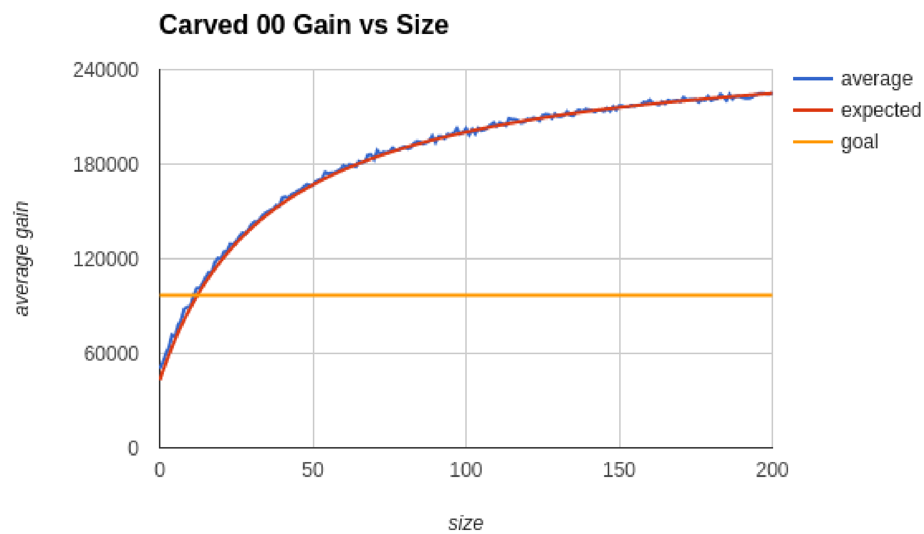

Figure 4.2 - Carved Size vs Gain

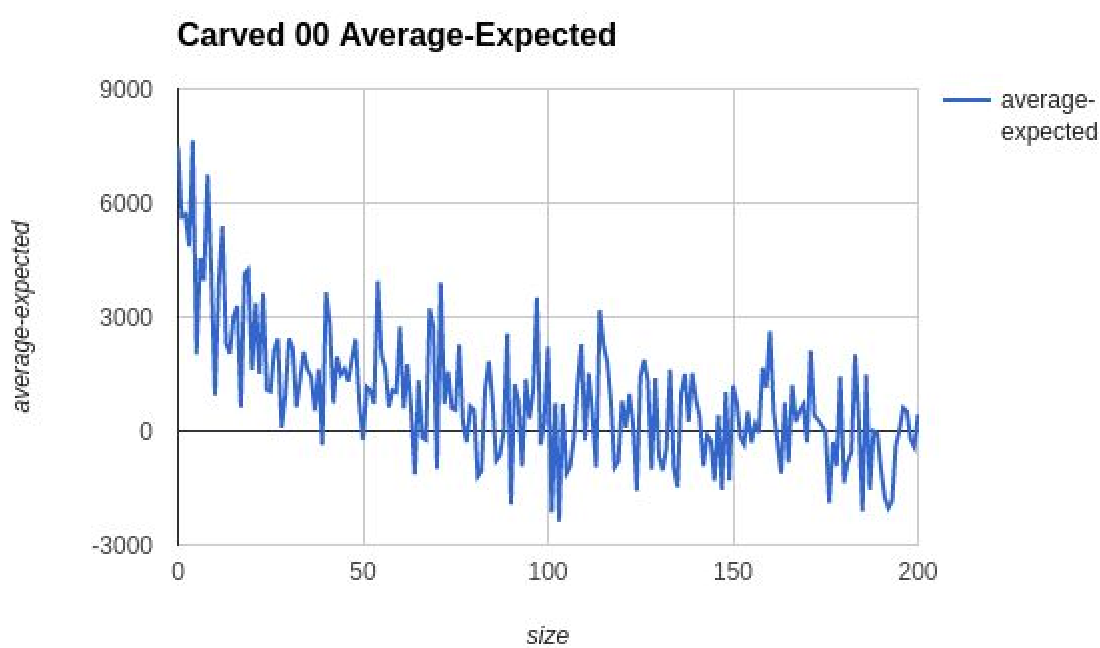

Figure 4.3 - Carved size vs Average-Expected Gain

Analysis

With a similar growth rate to adding green spaces, carving the wheel proves to be effective at increasing the average gain. At initial conception, this might seem unintuitive, being that every gambler has a chance to bet to win at a high percentage. While this is true, there is still the issue that every bet scheme's expected value is adjusted due to the carving. For example, we carved the 00 deeper to increase the probability that the ball lands in it. This inherently affected all of the other betting schemes by lowering their probabilities. Check out our Calculations section in Appendix B for exactly how the carving process influences that.

Looking at Figure 4.2, it appears that again the average and calculation are close. We achieve our goal tuition after changing the 00 slot to a slot of size 11. We would remark about this in a visual aspect, but we aren't exactly sure what this would look like. We try to imagine that this would be something fairly obvious.

Figure 4.3 gets a close up on how much we were off by in our experiment, but we feel again that the high variance of the simulation would cause these differences.

Experiment 5 Changing the Odds

Motivation

For our fifth experiment, we decided to understand how the expected value per $1 bet changes in respect to change the odds differently (casino pays gamblers differently when they win).

Approach

Our approach to solving this problem was to change one odd to different amounts and observe how the expected value changes. From our Calculations in Appendix B, we expect E(X) = $\frac{1}{50 * 38}$ [24 + 38(37 - n)].

Parameters

- Average over 5000 trials for each n (n = number of additional green spaces)
- Gamblers = 2531
- Allowance = 365 (Half of the value from experiment 2, allows us to show growth)
- Max n = 200
- Increment = 1

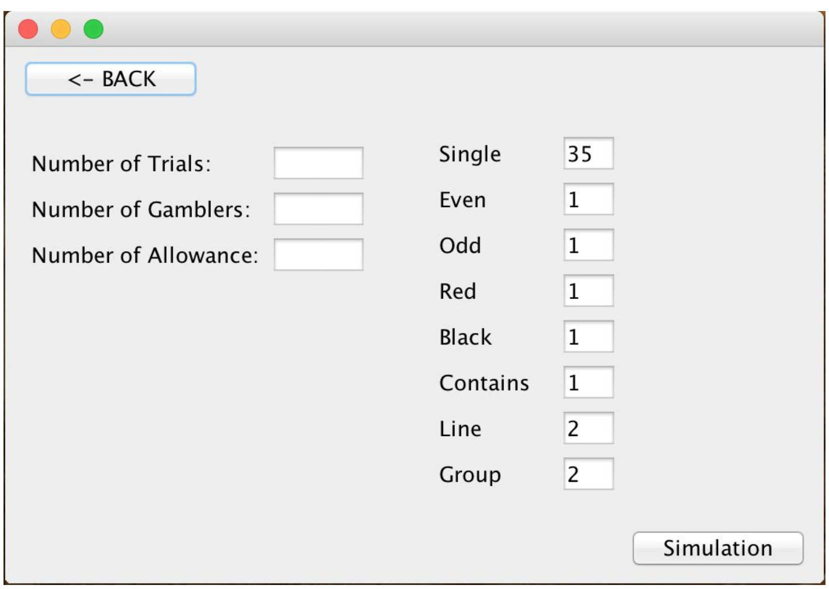

Figure 5.1 - Gui_Odds (Experiment 5)

Data

Figure 5.2 Average gain vs payout odd with lines representing the measured values and the calculated values, and our goal tuition

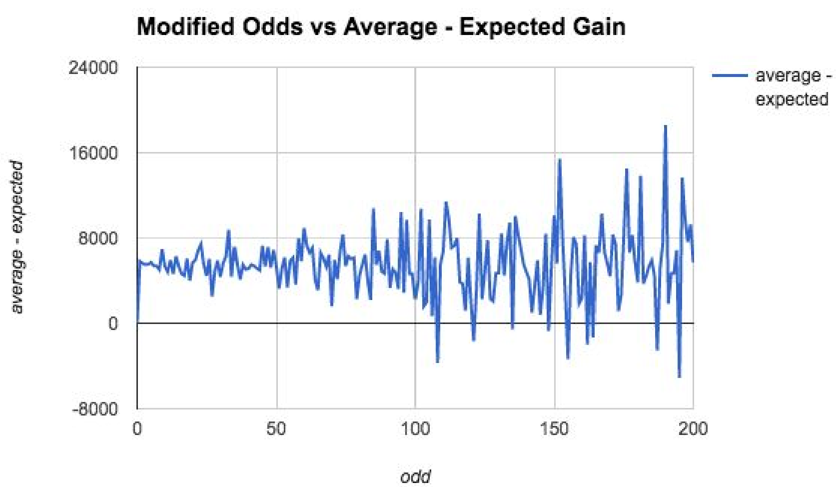

Figure 5.3 The difference between the average and expected

Analysis

In Figure 5.1 We see that the measured values appear to be close to the calculated values. Alongside this, we see that we reach our goal between the payout odds being 32 - 33. What this means is that with half of the allotted allowance, we need to kick the payout rate for the 00 value from 35 to 32 to achieve our goal. Without taking a closer look at the probabilities, this would look the least suspicious out of the other alternatives to the naked eye.

We take a closer inspection on the difference between the average and calculated by examining the arithmetic difference in Figure 5.2. It appears that the slope of our calculated equation and the measure results is close, only shifted by a vertical bias.

Closing Remarks

As we close out this report, we would like to discuss our results in the real world context. Casinos financially support themselves by having house edge. They could possibly acquire their house edge through promiscuous methods like carving a roulette wheel or providing unfair odds. Being educated in probability could allow one to spot these exploits, or even benefit from them in the future. We believe that the most important part of this project is trying to not only conjure up, but analyze a myriad of ways to achieve more favorable outcomes. We were pleasantly surprised with how accurate our predictions were, showing the real power of probability knowledge.

Appendix A Useful Information and Formulae

What's the Deal with G(b,w)?

Let G(b, w) be the function mapping bets to payouts, where w = true denotes that the casino won that bet, and b = the payout type (single, red, even, …)

Here is an example of G for the standard roulette wheel:

Bet Type (*b*)	Satisfying values	W = true	W = false
Single	Whatever value was selected 0, 00, 1, 2, ...	1	35
even	All even numbers not including 0, 00	1	1
odd	All odd numbers not include 0, 00	1	1
red	All red numbers	1	1
black	All black numbers	1	1
contains	The first half (1 - 18) Or the second half (19 - 36)	1	1
line	The first line (1,4,7,...) Or the second line (2,5,8,...) Or the third line (3,7,11,...)	1	2
group	The first third (1 - 12) Or the second third (13 - 24) Or the third third of (25 - 36)	1	2

Useful Formulae

$$E(X) = \sum_{S} x_i \, p_i$$

$$p_{casinoWins} = \frac{\textit{number of slots in wheel where casino wins}}{\textit{number of slots in wheel}}$$

$$p_{casinoLoses} = 1 - p_{casinoWins}$$

$$E(X) = E[E(X|B)] = \sum_{bets} [E(X|B = b) * p(B = b)]$$

Expectation of gain, conditional to betting scheme

$$E(X|B = bet) = (p_{casinoWins} * G(bet, true)) - (p_{casinoLoses} * G(bet, false))$$

Appendix B Calculations

Standard Case

Calculating Expected winnings from each betting scheme

E(X|B = single) = ($\frac{37}{38}$ * G(single, true)) - ($\frac{1}{38}$ * G(single, false))
E(X|B = single) = ($\frac{37}{38}$ * 1) - ($\frac{1}{38}$ * 35)
E(X|B = single) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = red) = ($\frac{20}{38}$ * G(red, true)) - ($\frac{18}{38}$ * G(red, false))
E(X|B = red) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = red) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = black) = ($\frac{20}{38}$ * G(black, true)) - ($\frac{18}{38}$ * G(black, false))
E(X|B = black) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = black) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = odd) = ($\frac{20}{38}$ * G(odd, true)) - ($\frac{18}{38}$ * G(odd, false))
E(X|B = odd) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = odd) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = even) = ($\frac{20}{38}$ * G(even, true)) - ($\frac{18}{38}$ * G(even, false))
E(X|B = even) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = even) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = firstHalf) = ($\frac{20}{38}$ * G(firstHalf, true)) - ($\frac{18}{38}$ * G(firstHalf, false))
E(X|B = firstHalf) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = firstHalf) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = secondHalf) = ($\frac{20}{38}$ * G(secondHalf, true)) - ($\frac{18}{38}$ * G(secondHalf, false))
E(X|B = secondHalf) = ($\frac{20}{38}$ * 1) - ($\frac{18}{38}$ * 1)
E(X|B = secondHalf) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = firstThird) = ($\frac{26}{38}$ * G(firstThird, true)) - ($\frac{12}{38}$ * G(firstThird, false))

E(X|B = firstThird) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = firstThird) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = secondThird) = ($\frac{26}{38}$ * G(secondThird, true)) - ($\frac{12}{38}$ * G(secondThird, false))
E(X|B = secondThird) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = secondThird) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = thirdThird) = ($\frac{26}{38}$ * G(thirdThird, true)) - ($\frac{12}{38}$ * G(thirdThird, false))
E(X|B = thirdThird) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = thirdThird) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = firstLine) = ($\frac{26}{38}$ * G(firstLine, true)) - ($\frac{12}{38}$ * G(firstLIne, false))
E(X|B = firstLine) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = firstLine) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = secondLine) = ($\frac{26}{38}$ * G(secondLine, true)) - ($\frac{12}{38}$ * G(secondLine, false))
E(X|B = secondLine) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = secondLine) = $\frac{2}{38}$ = $\frac{1}{19}$

E(X|B = thirdLine) = ($\frac{26}{38}$ * G(thirdLine, true)) - ($\frac{12}{38}$ * G(thirdLine, false))
E(X|B = thirdLine) = ($\frac{26}{38}$ * 1) - ($\frac{12}{38}$ * 2)
E(X|B = thirdLine) = $\frac{2}{38}$ = $\frac{1}{19}$

Overall expected value given uniformly distributed random bets

E(X) = E[E(X|B)]

E(X) = $\sum_{bets}$ [E(X|B = b) * p(B = b)]

E(X) = $\frac{1}{38+6+6} \sum_{bets}$ [E(X|B = b)]

E(X) = $\frac{1}{38+6+6} * \frac{1}{19} * (38 + 6 + 6)$

E(X) = $\frac{1}{19}$

For the standard case, we assume that we gain 5.26 cents per $1 bid.

Expected value for bigger bids?

We expect that the expected value for bigger bids simply be a multiple of the expected value of bids of $1.

E(a * X) = a * E(X)

Expected value for a number of gamblers?

Similarly, we believe that the expected value for the casino over a number of gamblers will be a multiple of the expected value of 1 gambler.

E(b * X) = b * E(X)

Combine the two?

What if we provide the circumstance that there are *a* guests, bring *b* dollars? Again linearity ensues.

E(a * b * X) = a * b * E(X)

How much allowance do we need to pay tuition in the standard case?

Our goal is to reach $96,900 using 2,531 gamblers, so our problem of finding the allowance b to reach our goal is of the form:

E(2,531 * b * X) = 96,900

2,531 * b * E(X) = 96,900
2,531 * b * $\frac{1}{19}$ = 96,900
b = $\frac{19 * 96{,}900}{2531}$
b = 727.42
Let's round that up to 730

Adding Green Spaces

Expected value for n additional green spaces

E(X|B = single) = ($\frac{37+n}{38+n}$ * G(single, true)) - ($\frac{1}{38+n}$ * G(single, false))

E(X|B = single) = ($\frac{37+n}{38+n}$ * 1) - ($\frac{1}{38+n}$ * 35)

E(X|B = single) = $\frac{2+n}{38+n}$

E(X|B = red) = ($\frac{20+n}{38+n}$ * G(red, true)) - ($\frac{18}{38+n}$ * G(red, false))

E(X|B = red) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = red) = $\frac{2+n}{38+n}$

E(X|B = black) = ($\frac{20+n}{38+n}$ * G(black, true)) - ($\frac{18}{38+n}$ * G(black, false))

E(X|B = black) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = black) = $\frac{2+n}{38+n}$

E(X|B = odd) = ($\frac{20+n}{38+n}$ * G(odd, true)) - ($\frac{18}{38+n}$ * G(odd, false))

E(X|B = odd) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = odd) = $\frac{2+n}{38+n}$

E(X|B = even) = ($\frac{20+n}{38+n}$ * G(even, true)) - ($\frac{18}{38+n}$ * G(even, false))

E(X|B = even) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = even) = $\frac{2+n}{38+n}$

E(X|B = firstHalf) = ($\frac{20+n}{38+n}$ * G(firstHalf, true)) - ($\frac{18}{38+n}$ * G(firstHalf, false))

E(X|B = firstHalf) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = firstHalf) = $\frac{2+n}{38+n}$

E(X|B = secondHalf) = ($\frac{20+n}{38+n}$ * G(secondHalf, true)) - ($\frac{18}{38+n}$ * G(secondHalf, false))

E(X|B = secondHalf) = ($\frac{20+n}{38+n}$ * 1) - ($\frac{18}{38+n}$ * 1)

E(X|B = secondHalf) = $\frac{2+n}{38+n}$

E(X|B = firstThird) = ($\frac{26+n}{38+n}$ * G(firstThird, true)) - ($\frac{12}{38+n}$ * G(firstThird, false))

E(X|B = firstThird) = ($\frac{26+n}{38+n}$ * 1) - ($\frac{12}{38+n}$ * 2)

E(X|B = firstThird) = $\frac{2+n}{38+n}$

E(X|B = secondThird) = ($\frac{26+n}{38+n}$ * G(secondThird, true)) - ($\frac{12}{38+n}$ * G(secondThird, false))

E(X|B = secondThird) = ($\frac{26+n}{38+n}$ * 1) - ($\frac{12}{38+n}$ * 2)

E(X|B = secondThird) = $\frac{2+n}{38+n}$

E(X|B = thirdThird) = ($\frac{26+n}{38+n}$ * G(thirdThird, true)) - ($\frac{12}{38+n}$ * G(thirdThird, false))

$E(X|B = thirdThird) = (\frac{26+n}{38+n} * 1) - (\frac{12}{38+n} * 2)$

$E(X|B = thirdThird) = \frac{2+n}{38+n}$

$E(X|B = firstLine) = (\frac{26+n}{38+n} * G(firstLine, true)) - (\frac{12}{38+n} * G(firstLine, false))$

$E(X|B = firstLine) = (\frac{26+n}{38+n} * 1) - (\frac{12}{38+n} * 2)$

$E(X|B = firstLine) = \frac{2+n}{38+n}$

$E(X|B = secondLine) = (\frac{26+n}{38+n} * G(secondLine, true)) - (\frac{12}{38+n} * G(secondLine, false))$

$E(X|B = secondLine) = (\frac{26+n}{38+n} * 1) - (\frac{12}{38+n} * 2)$

$E(X|B = secondLine) = \frac{2+n}{38+n}$

$E(X|B = thirdLine) = (\frac{26+n}{38+n} * G(thirdLine, true)) - (\frac{12}{38+n} * G(thirdLine, false))$

$E(X|B = thirdLine) = (\frac{26+n}{38+n} * 1) - (\frac{12}{38+n} * 2)$

$E(X|B = thirdLine) = \frac{2+n}{38+n}$

$E(X) = E[E(X|B)]$

$E(X) = \sum_{bets} [E(X|B = b) * p(B = b)]$

$E(X) = \frac{1}{38+6+6+n} \sum_{bets} [E(X|B = b)]$

$E(X) = \frac{1}{38+6+6+n} * \frac{2+n}{38+n} * (38 + 6 + 6 + n)$

$E(X) = \frac{2+n}{38+n}$

Changing the size of a green space

Expected value for changing the size of a green space (00) to n

$E(X|B = \text{single}_{not00}) = (\frac{36+n}{37+n} * G(\text{single, true})) - (\frac{1}{37+n} * G(\text{single, false}))$

$E(X|B = \text{single}_{not00}) = (\frac{36+n}{37+n} * 1) - (\frac{1}{37+n} * 35)$

$E(X|B = \text{single}_{not00}) = \frac{1+n}{37+n}$

$E(X|B = \text{single}_{00}) = (\frac{37}{37+n} * G(\text{single, true})) - (\frac{n}{37+n} * G(\text{single, false}))$

$E(X|B = \text{single}_{00}) = (\frac{37}{37+n} * 1) - (\frac{n}{37+n} * 35)$

$E(X|B = \text{single}_{00}) = \frac{37-35n}{37+n}$

$E(X|B = \text{red}) = (\frac{19+n}{37+n} * G(\text{red, true})) - (\frac{18}{37+n} * G(\text{red, false}))$

$E(X|B = \text{red}) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = \text{red}) = \frac{1+n}{37+n}$

$E(X|B = \text{black}) = (\frac{19+n}{37+n} * G(\text{black, true})) - (\frac{18}{37+n} * G(\text{black, false}))$

$E(X|B = \text{black}) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = \text{black}) = \frac{1+n}{37+n}$

$E(X|B = \text{odd}) = (\frac{19+n}{37+n} * G(\text{odd, true})) - (\frac{18}{37+n} * G(\text{odd, false}))$

$E(X|B = odd) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = odd) = \frac{1+n}{37+n}$

$E(X|B = even) = (\frac{19+n}{37+n} * G(even, true)) - (\frac{18}{37+n} * G(even, false))$

$E(X|B = even) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = even) = \frac{1+n}{37+n}$

$E(X|B = firstHalf) = (\frac{19+n}{37+n} * G(firstHalf, true)) - (\frac{18}{37+n} * G(firstHalf, false))$

$E(X|B = firstHalf) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = firstHalf) = \frac{1+n}{37+n}$

$E(X|B = secondHalf) = (\frac{19+n}{37+n} * G(secondHalf, true)) - (\frac{18}{37+n} * G(secondHalf, false))$

$E(X|B = secondHalf) = (\frac{19+n}{37+n} * 1) - (\frac{18}{37+n} * 1)$

$E(X|B = secondHalf) = \frac{1+n}{37+n}$

$E(X|B = firstThird) = (\frac{25+n}{37+n} * G(firstThird, true)) - (\frac{12}{37+n} * G(firstThird, false))$

$E(X|B = firstThird) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = firstThird) = \frac{1+n}{37+n}$

$E(X|B = secondThird) = (\frac{25+n}{37+n} * G(secondThird, true)) - (\frac{12}{37+n} * G(secondThird, false))$

$E(X|B = secondThird) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = secondThird) = \frac{1+n}{37+n}$

$E(X|B = thirdThird) = (\frac{25+n}{37+n} * G(thirdThird, true)) - (\frac{12}{37+n} * G(thirdThird, false))$

$E(X|B = thirdThird) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = thirdThird) = \frac{1+n}{37+n}$

$E(X|B = firstLine) = (\frac{25+n}{37+n} * G(firstLine, true)) - (\frac{12}{37+n} * G(firstLine, false))$

$E(X|B = firstLine) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = firstLine) = \frac{1+n}{37+n}$

$E(X|B = secondLine) = (\frac{25+n}{37+n} * G(secondLine, true)) - (\frac{12}{37+n} * G(secondLine, false))$

$E(X|B = secondLine) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = secondLine) = \frac{1+n}{37+n}$

$E(X|B = thirdLine) = (\frac{25+n}{37+n} * G(thirdLine, true)) - (\frac{12}{37+n} * G(thirdLine, false))$

$E(X|B = thirdLine) = (\frac{25+n}{37+n} * 1) - (\frac{12}{37+n} * 2)$

$E(X|B = thirdLine) = \frac{1+n}{37+n}$

$E(X) = E[E(X|B)]$

$E(X) = \sum_{bets} [E(X|B = b) * p(B = b)]$

$$E(X) = \sum_{non\ single\ bets} [E(X|B = \text{non-single})] + \sum_{single\ bets\ not\ 00} [E(X|B = \text{single}_{not00})]$$

$$+ \sum_{00\ bet} [E(X|B = \text{single}_{00})]$$

$$E(X) = [\ \frac{1+n}{37+n} * \frac{12}{50}\ ] + [\ \frac{1+n}{37+n} * \frac{37}{50}\ ] + [\ \frac{37-35n}{37+n} * \frac{1}{50}\ ]$$

$$E(X) = \frac{1}{(37+n)*(50)}\ [12 + 12n + 37 + 37n + 37 - 35n]$$

$$E(X) = \frac{1}{(37+n)*(50)}\ [14n + 86]$$

As a quick sanity check, let's plug in n = 1 for the standard wheel.

$$E(X) = \frac{1}{(37+1)*50} [14 + 86]$$

$$E(X) = \frac{1}{38*50} [100]$$

$$E(X) = \frac{100}{38*50} = \frac{2}{38} = \frac{1}{19}$$

Changing the odds for the “single” bet

Expected value for changing G(single, false) to n

$E(X|B = \text{single}) = (\frac{37}{38} * G(\text{single, true})) - (\frac{1}{38} * G(\text{single, false}))$

$E(X|B = \text{single}) = (\frac{37}{38} * 1) - (\frac{1}{38} * (n))$

$E(X|B = \text{single}) = \frac{37 - n}{38}$

$E(X) = E[E(X|B)]$

$E(X) = \sum_{bets} [E(X|B = b) * p(B = b)]$

$E(X) = \sum_{non\ single\ bets} [E(X|B = \text{non-single})] + \sum_{single\ bets} [E(X|B = \text{single})]$

$E(X) = [\frac{1}{19} * \frac{12}{50}] + [(\frac{37 - n}{38}) * \frac{38}{50}]$

$E(X) = \frac{1}{50} (\frac{12}{19} + \frac{38 * (37 - n)}{38})$

$E(X) = \frac{1}{50 * 38} [24 + 38(37 - n)]$

As a quick sanity check, let’s plug in n = 35 for the standard wheel.

$E(X) = \frac{1}{50 * 38} [24 + 38(37 - 35)]$

$E(X) = \frac{1}{50 * 38} [24 + 38(2)]$

$E(X) = \frac{1}{50 * 38} [100]$

$E(X) = \frac{2}{38} = \frac{1}{19}$