ECOM90024
Forecasting in Economics and Business
Tutorial 6

1.) Consider the variables Y_t and X_t such that Y_t is described by an AR(1) model,

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

while X_t is described by the following restricted ARMA(4,1) model,

$$X_t = \beta X_{t-4} + u_t + \theta u_{t-1}$$

where both ε_t and u_t are white noise series and $|\phi| < 1$, $|\beta| < 1$ and $|\theta| < 1$ so that the stationarity and invertibility of Y_t and X_t are guaranteed.

Show that the variable $Z_t = Y_t + X_t$ can be described by an ARMA(5,4) model. (*Hint: The lag operator will be useful here*!)

2.) Consider the general MA(∞) representation for a stationary time series Y_t, that is,

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots +$$

and suppose that the parameters $\theta_1, \theta_2, \ldots$ are known.

a.) The 1 step ahead forecast error is defined as:

$$\varepsilon_{t+1|t} = Y_{t+1} - E[Y_{t+1}|\Omega_t]$$

$$\Omega_t = \{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$$

What are the forecast errors for 3 and 4 steps ahead?

b.) What is the covariance between the 3 and 4 step ahead errors?

3.) Suppose that the time series Y_t is governed by the following process,

$$Y_t = Y_{t-1} + \varepsilon_t$$

Where ε_t is a white noise series with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t^2] = \sigma^2$ for all t. Also suppose that that Y_t is observed every six months, but that it is aggregated to annually observed time series X_T by taking the sum of the two observations of Y in year T. Show that X_T can be described by

$$X_T = X_{T-1} + u_T$$

Where u_T is an MA(1) process with first order autocorrelation equal to $^1/_6$. (*Hint: Let periods* t *and* $t-1$ *be in year* T)

4.) For each of the following stationary time series processes

a.) $Y_t = \mu + \beta Y_{t-1} + u_t$

b.) $Y_t = \mu + u_t + 0.6u_{t-1} + 0.2u_{t-2}$

Where u_t is a white noise process with $E[u_t] = 0$ and $E[u_t^2] = \sigma^2$

i.) Derive the unconditional mean $E[Y_t]$
ii.) Derive the unconditional variance $Var(Y_t)$
iii.) Derive the first-order autocovariance $Cov(Y_t, Y_{t-1})$