# Lecture 7: Long run growth

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# Last class

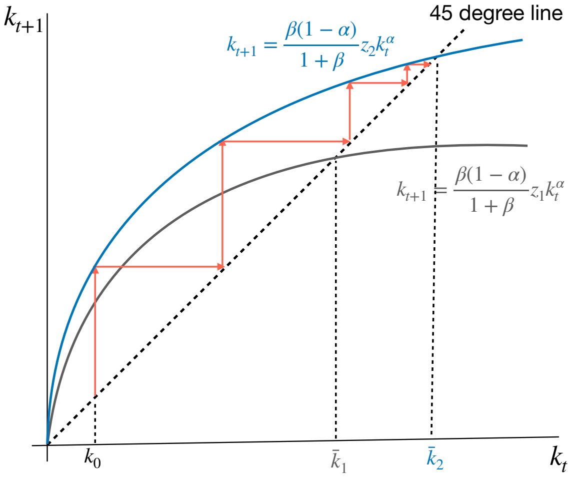
- We looked at the predictions of the OLG model regarding growth along the transition path
- As the economy moves towards its steady state, $k_t \to \bar{k}$, $y_t$,$c_t$,$w_t$ increase, while $R_t$ falls.
- Starting at a point $k_0 < \bar{k}$, our model predicted that the welfare of each generation is improving as $k \to \bar{k}$
- At steady state, $\bar{k}$, output per capita, consumption per capita, welfare etc, are constant

# Long-run growth

# Growth mechanics

- Barring no shocks, an economy starting at $k_0 < \bar{k}$ grows via capital accumulation until it reaches $\bar{k}$
- The economy can reach a higher steady state level when TFP, $z$, permanently increases
- What does the dynamics of capital accumulation and that of aggregate variables look like when TFP observes a one-time permanent increase?

# Increases in $z$ drive long-run growth

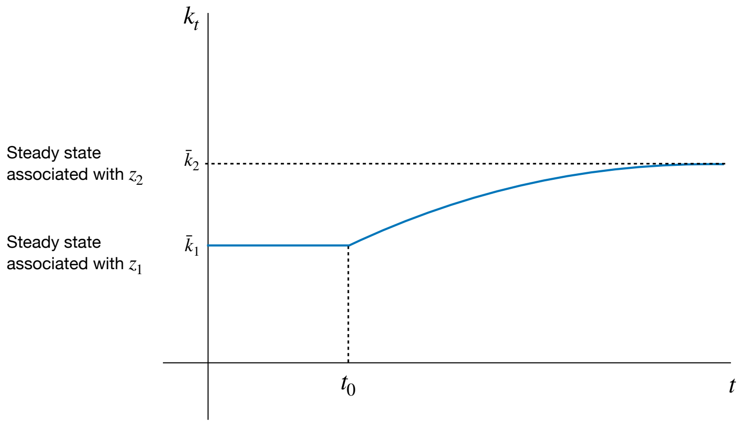



- Increases in $z$ lead to higher steady state $\bar{k}$.

Suppose the economy is initially at steady state and then observes a permanent increase in $z$ on date $t_0$. Which aggregate variables rise on impact at date $t_0$?

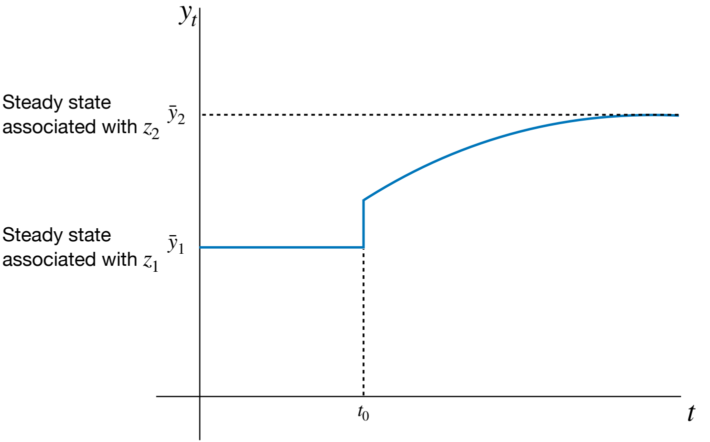
# Pre-determined variables vs. jump variables

- On impact, capital supply per person does not rise (it is pre-determined!)
- Investment, however, can rise and thus capital per person next period is higher



# Pre-determined variables vs. jump variables

- On impact, output per person does jump with rise in TFP $z$




What other variables jump on impact?

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  - With a constant population size, but continuing improvements in TFP $z_t$, the transition curve will continually **shift up**
  - An increase in $z \uparrow$ MPL and MPK, higher MPL $\uparrow$ saving and capital accumulation.
  - Thus, positive LR TFP growth can sustain positive LR growth in income per capita

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- These predictions while similar to Solow-Swan, were derived in a model where individuals were making their own consumption and savings choices

## So what can we use our OLG growth model for?

# What can we use these growth models for?

- Solow-Swan model is a descriptive model. Originally developed to explain why there are differences in growth across countries
    - The Solow-Swan model tries to ascribe these differences to factors like a country's population growth rate, savings rate, depreciation rate and technological progress

- The OLG growth model allows us to analyze how policies can affect growth, especially if these policies affect the consumption-savings behaviour of households
    - In particular, because the model we study analyzes how households make their decisions, it allows us to study how policies affect *endogenous choice* variables.

# A question we can ask using our OLG growth model

Is there a Pareto-optimal $\bar{k}$?

Or put differently, how much should we as a society save?

# Pareto optimality

- A **pareto optimal** outcome is one such that there is no way to reallocate resources to make an individual better off without making someone else worse off.

- How do we find the pareto optimum? Consider the social planner's problem

# A social planner

- What's a social planner?
  - A hypothetical construct: a social planner is a benevolent dictator who decides how much people consume, work and save
- What is the social planner's goal?
  - Make people as happy as possible (maximize the utility of **all** generations) given the technological constraints of the economy.

# A social planner

- In choosing the pareto optimum, the social planner **does not use markets**
  - This means that prices do not show up in the social planner's problem
- Rather, the planner chooses how to allocate (assign) factors to production, consumption and investment so as to maximize the utility of **all** households
- Planner is only subject to feasibility (technological constraints).
  - Feasibility example: the planner cannot assign more goods to consumption than what is currently being produced.

# A social planner's problem

- Suppose as before, $\delta = 1$ (full depreciation) and no population growth
- Suppose further that $z_t = z$
- And that the social planner puts equal weight on all generations
- We want to find the pareto optimal $\bar{k}$ that maximizes the steady state utility of all generations.

# A social planner's problem

Goal: find pareto optimal $\bar{k}$ that maximizes household's lifetime utility in the long-run

- The social planner seeks to maximize households' lifetime utility in the LR

$$W = \max U(\bar{c}^y, \bar{c}^o)$$

subject to technological constraints :

$$\bar{c}^y + \bar{c}^o + \bar{k} = \bar{y}$$

- Note 1: at steady state, consumption of young and old is not changing across time
- Note 2: we have written the technological or resource constraint in per-capita terms

# A social planner's problem: Example

- Suppose $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ and Cobb-Douglas production function.
- We can re-write the social planner's problem with a Lagrangian:

$$\mathcal{L} = \max \ln \bar{c}^y + \beta \ln \bar{c}^o + \lambda \left[ z\bar{k}^\alpha - \bar{c}^y - \bar{c}^o - \bar{k} \right]$$

- Note: $\lambda$ in the planner's problem now represents the shadow value of relaxing the resource constraint
  - In other words, $\lambda$ represents how much happier households would be if the planner had more resources to allocate towards consumption
- The planner can choose $\bar{c}^y, \bar{c}^o$ and $\bar{k}$

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  - FOC wrt $\lambda$

$$z\bar{k}^\alpha - \bar{c}^y - \bar{c}^o - \bar{k} = 0$$

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- Optimal LR choice of gross investment, $\bar{k}$, satisfies:

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- Optimal choices are feasible (resource constraint)

$$\boxed{z\bar{k}^{\alpha} = \bar{c}^y + \bar{c}^o + \bar{k}}$$

# Question

Is the steady-state in our market economy in our OLG model pareto-efficient?

# Social planner

- Denote the social planner's choice of $\bar{k}$ as $\bar{k}^{SP}$
- Pareto-optimal steady state $k$ given by $\bar{k}^{SP} = [\alpha z]^{1/(1-\alpha)}$
- Denote the market economy steady state $k$ as $\bar{k}^M$
- So long as $\bar{k}^M \neq \bar{k}^{SP}$, market economy LR equilibrium is inefficient!
- Same example (utility function, production function) in market economy yielded:

$$\bar{k}^M = \left[\frac{\beta(1-\alpha)}{(1+\beta)} z\right]^{1/(1-\alpha)}$$

# Dynamic inefficiency

- In general, can get $\bar{k}^M \neq \bar{k}^{SP}$
- Each household in the market economy does not consider other generations' utility/welfare when making their own choices
- This is unlike the social planner who cares about the utility of **all** generations
- In particular, capital stock per person at the start of the period is determined by the previous generation's investment. $k_t$ is predetermined and $= a_t$
- The amount of $k_t$ at the start of the period available for production affects prices $w_t, R_t$ which in turn affects how much young today consumes and saves

# Dynamic inefficiency

- Put differently, the market economy does not achieve pareto-optimality (despite perfect competition among firms and no credit frictions, etc)

- Because there is a missing market: the young of generation $t$ cannot trade with the young in generation $t+1$ in period $t$ because the latter are not born yet!

# Over-accumulation of capital per person

- Over-accumulation of $k$ can occur: case where $\bar{k}^M > \bar{k}^{SP}$
- In this case, the inefficiency is not at the cost of lower $y$
- But the economy is inefficient because everyone can be made better off if they consumed more and saved less
- If the young invested less in $k$, they will have higher $c^y$
- Lower $k \implies$ higher MPK. $\uparrow$ in return to savings is large enough such that $c^o$ also higher
- Then everyone is better off if did not over-accumulate $k$, implying we can have a pareto-improvement.

# Over-accumulation of capital per person

- Intuitively, over-accumulation of $k$ can occur when the young save too much.
- This over-saving can happen when you expect your income to decline over your lifetime.
- But the more one saves, the lower the return to savings (because MPK is lower with higher $k$)
- A lower return to savings yields less income to spend on consumption when old.

# Over-accumulation of capital per person

- In general, the outcome in the market economy is **not** pareto-efficient
- Role for government to step in to provide incentives to change consumption and savings behavior
  - In particular, the OLG model suggests there is a role for social security to deal with over-accumulation of $k$
- But to talk about this, we need to introduce a new agent into the model: the government

# Wrapping up

- This class: pareto optimum $\bar{k}$
- Introducing government and fiscal policy