

Solution to Tutorial 2

1. (1) The main difference between the three forms of market efficiency is that
- (a) the definition of efficiency differs.
 - (b) the definition of excess return differs.
 - (c) the definition of prices differs.
 - (d) the definition of information differs.
 - (e) they were discovered by different people.

Answer: (d)

- (2) If you believe in the _____ form of the EMH, you believe that stock prices reflect all relevant information including historical stock prices and current public information about the firm, but not information that is available only to insiders.
- (a) semi-strong
 - (b) strong
 - (c) weak
 - (d) none of the above

Answer: (a)

- (3) If securities market is efficient, which of the following statements is FALSE?
- (a) Security prices react quickly to new information
 - (b) Security prices are seldom far above or below their justified levels
 - (c) Security analysts will not enable investors to realize superior returns consistently
 - (d) One cannot make money

Answer: (d)

- (4) Proponents of the EMH typically advocate
- (a) an active trading strategy.
 - (b) investing in an index fund.
 - (c) fundamental analysis
 - (d) technical analysis

Answer: (b)

- (5) Assume that there is evidence that the prices of shares of MayLi Inc. follow a random walk. Which of the following statements is FALSE?
- (a) The evidence implies that knowledge of the current and past rates of return on MayLi Inc's shares are of no use in predicting future rates of return on the shares.

- (b) The evidence shows that the market is weak form efficient.
- (c) The evidence demonstrates that the market is semi-strong form efficient.
- (d) The evidence implies that technical analysis on MayLi Inc's shares is of no use in predicting future prices of its shares.

Answer: (c)

2. The purpose of this question is to deepen your understanding of a random walk process so that you understand what it means when we say asset prices follow a random walk. It also practices some formulas we have reviewed in math review 1.

- (a) To simulate the $\{p_t\}$ sequence based on equation (1), we need to draw realisations of the $\{\varepsilon_t\}$ sequence from a Normal distribution with mean zero and variance 0.01. This can be done using the random number generator in Excel or any other software. Once we have $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{500}$, we can calculate p_1, p_2, \dots, p_{500} as follows:

$$\begin{aligned}
 p_1 &= p_0 + \varepsilon_1 \\
 p_2 &= p_1 + \varepsilon_2 \\
 p_3 &= p_2 + \varepsilon_3 \\
 &\dots \\
 p_{500} &= p_{499} + \varepsilon_{500}
 \end{aligned}$$

Let $P_t = e^{p_t}$, for all $t = 0, 1, \dots, 500$, and plot the $\{P_t\}$ sequence. Note that your figure might look very different from my figure, as the simulated path of asset prices differs in each simulation.

- (b) Using (1) and successive substitution, we have

$$\begin{aligned}
 p_1 &= p_0 + \varepsilon_1 \\
 p_2 &= p_1 + \varepsilon_2 = p_0 + \varepsilon_1 + \varepsilon_2 \\
 p_3 &= p_2 + \varepsilon_3 = p_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \\
 &\dots \\
 p_t &= p_0 + \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t
 \end{aligned}$$

The expression for p_t implies that the asset price in period t carries all past shocks to asset prices up to period t .

- (c) Using (1),

$$E(p_{t+1}|p_t) = E(p_t + \varepsilon_{t+1}|p_t) = E(p_t|p_t) + E(\varepsilon_{t+1}|p_t) = p_t + 0 = p_t,$$

where $E(\varepsilon_{t+1}|p_t) = 0$ follows from the assumption that $\{\varepsilon_t\}$ is a sequence of i.i.d. shocks and the expression of p_t in (a):

$$E(\varepsilon_{t+1}|p_t) = E(\varepsilon_{t+1}|p_0 + \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t) = 0.$$

In fact, we can show that

$$E(p_{t+1}|\Omega_t) = E(p_t + \varepsilon_{t+1}|\Omega_t) = p_t + E(\varepsilon_{t+1}|\Omega_t) = p_t + 0 = p_t.$$

That is, prices follow a Martingale.

(d) Using (1),

$$\text{var}(p_{t+1}|p_t) = \text{var}((p_t + \varepsilon_{t+1})|p_t) = \text{var}(\varepsilon_{t+1}|p_t) = \text{var}(\varepsilon_{t+1}) = \sigma^2.$$

Using the result in (a),

$$\text{var}(p_{t+1}) = \text{var}(p_0 + \varepsilon_1 + \dots + \varepsilon_{t+1}) = \text{var}(\varepsilon_1) + \dots + \text{var}(\varepsilon_{t+1}) = (t+1)\sigma^2.$$

Note the difference between the conditional variance and unconditional variance. The unconditional variance can be interpreted as the variance of p_{t+1} at date 0, which increases towards infinity as $t \rightarrow \infty$, suggesting that when you try to predict asset price at date 0, it would become increasingly difficult to predict asset prices over a longer horizon.

(e) Using (1),

$$\begin{aligned} \text{cov}(p_{t+1}, p_t|P_t) &= \text{cov}(p_t + \varepsilon_{t+1}, P_t|P_t) \\ &= \text{cov}(P_t, P_t|P_t) + \text{cov}(\varepsilon_{t+1}, p_t|p_t) \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

This results states that conditional on p_t , p_{t+1} is uncorrelated with p_t as their covariance is zero. That is, the variations or changes in future prices are uncorrelated with today's price.

3. Key points to answer this question:

- Give the definition: A financial market is said to be efficient if it fully and correctly reflects all relevant information in determining asset prices.
- Testing the EMH requires specifying the set of information that determines asset prices (Ω_t) and a model of asset prices. The model and Ω_t yield predictions on asset prices or returns, which serve as the criteria for market efficiency. Based on the criteria, testable hypothesis is formulated and tested using data collected from the information set and a statistical procedure.
- Testing EMH is a joint hypothesis test: testing efficiency conditional on the model and testing validity of the model used. It is impossible to separate appraisals of market efficiency from the assumed model of asset prices.
- Conclusions about efficiency must be conditional on the model that provides the criteria for efficiency - the conclusions can never be incontrovertible.

4. Discussion question:

- (a) If asset markets are efficient (even in the strong form), arbitrage opportunities can still exist, but cannot persist. As new information arrives in the market (public or private information), arbitrage opportunities may arise. However, if asset markets are efficient, the arbitrage opportunities would disappear immediately as asset prices would adjust instantly to reflect the new information.¹ In other words, instant arbitrage opportunity does not contradict market efficiency.

In reality, asset markets are hardly fully efficient. Arbitrage opportunities may persist for a while, depending on the degree of market efficiency, presence of financial frictions, and other factors.

- (b) Presence of asset market anomalies do not necessarily mean asset markets are informationally inefficient.

Anomalies occur when asset prices or returns deviate from what would be expected based on conventional models of asset pricing, such as the CAPM.

If an asset market anomaly is not consistent with the pattern predicted by a conventional model of asset pricing, while you believe this conventional model represents market efficiency, then this market anomaly provides evidence against market efficiency. However, another possibility is that the conventional model is not adequate in explaining asset prices; the anomaly can be explained by other mechanism that's not captured by the conventional model (for example, think about the January effect, the weather effect).

¹Even if the information is privately known only by some investors, as soon as they start trading based on their private information, their trading activities would become public information and other investors would react accordingly. As a result, the arbitrage opportunities would quickly disappear.