Lecture 9: Adding Fiscal Policy to the OLG model

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Adding Fiscal Policy into the Life-Cycle Model

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- Later we will relax the assumption that government spending is wasteful

Financing Government Consumption

- Suppose the govt. consumes G_t units of goods each period.

- To finance this, the govt. can raise taxes from working or retired individuals in each period, or by issuing government debt.

  - Suppose in period t, the govt. levies proportional taxes τ_t^y and τ_t^o on the consumption spending of the young and old respectively

  - The govt. can also issue **one-period** govt. bonds B_{t+1} at the end of period t, which are then repaid in period $t+1$.

  - Suppose each working individual purchases b_{t+1} govt. bonds in period t and there are N individuals in each generation

Household Budget Constraints

- Budget constraint when working:

$$(1+\tau_t^y)c_t^y + a_{t+1} + b_{t+1} = w_t + \pi_t$$

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- Budget constraint when retired:

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- Note: we assume the net rate of return on b_{t+1} and a_{t+1} are the same.

- In the absence of default, individuals view the two assets as **perfect substitutes**.

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- Substitute out A_{t+1}

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t + \pi_t - \tau_t^y c_t^y - \frac{\tau_{t+1}^o c_{t+1}^o}{1+r_{t+1}}$$

- LBC shows that PDV of lifetime consumption spending = PDV of lifetime income less PDV of tax payments

Household problem

- Note the household problem is largely still the same:
  - Households wants to make itself as happy as possible (by maximizing lifetime utility)
  - Subject to how much they can afford (subject to their lifetime budget constraint)
  - Our household still chooses how much to consume – c_t^y, c_{t+1}^o – and how much to save – a_{t+1}, b_{t+1}.
  - Our household takes prices and taxes as given when making these choices

Household utility maximization

- The household chooses c_t^y and c_{t+1}^o taking prices and taxes as given

$$\max_{\{c_t^y, c_{t+1}^o\}} U(c_t^y, c_{t+1}^o)$$

s.t.

$$(1+\tau_t^y)c_t^y + \frac{(1+\tau_{t+1}^o)c_{t+1}^o}{1+r_{t+1}} = w_t + \pi_t$$

- Can write down the Lagrangian:

$$\mathcal{L} = \max_{\{c_t^y, c_{t+1}^o\}} U(c_t^y, c_{t+1}^o) + \lambda_t \left[w_t + \pi_t - (1+\tau_t^y)\, c_t^y - \frac{\left(1+\tau_{t+1}^o\right) c_{t+1}^o}{1+r_{t+1}} \right]$$

and take FOCs wrt c_t^y, c_{t+1}^o and λ_t

Household utility maximization

- FOC wrt c_t^y:

$$\frac{\partial U(c_t^y, c_{t+1}^o)}{\partial c_t^y} = \lambda_t(1 + \tau_t^y)$$

- FOC wrt c_{t+1}^o:

$$\frac{\partial U(c_t^y, c_{t+1}^o)}{\partial c_{t+1}^o} = \lambda_t \frac{1 + \tau_{t+1}^o}{1 + r_{t+1}}$$

- FOC wrt λ_t

$$w_t + \pi_t - (1 + \tau_t^y)\, c_t^y - \frac{\left(1 + \tau_{t+1}^o\right) c_{t+1}^o}{1 + r_{t+1}} = 0$$

Household optimality conditions

- Equations characterizing household optimal choices continue to be:
  - Euler equation:

$$\frac{\partial U(c_t^y, c_{t+1}^o)}{\partial c_t^y} \frac{1}{1+\tau_t^y} = \frac{\partial U(c_t^y, c_{t+1}^o)}{\partial c_{t+1}^o} \frac{1+r_{t+1}}{1+\tau_{t+1}^o}$$

  - Lifetime budget constraint :

$$(1+\tau_t^y)\, c_t^y + \frac{\left(1+\tau_{t+1}^o\right) c_{t+1}^o}{1+r_{t+1}} = w_t + \pi_t$$

Firm profit maximization

- Firm's problem is unchanged:

$$\max_{K_t, L_t} \pi_t = F(z_t, K_t, L_t) - w_t L_t - R_t K_t$$

- Optimal L and K demand implicitly given by marginal product $=$ marginal cost:

$$F_L(z_t, K_t, L_t) = w_t$$

and

$$F_K(z_t, K_t, L_t) = R_t$$

where $R_t = 1 + r_t$, gross rate of return

Government budget constraint

- Now in addition to households and firms, we have a 3rd agent: the government

- **Govt. budget constraint** is now another equilibrium condition. GBC in period t:

$$G_t + (1+r_t)B_t = \underbrace{N\tau_t^y c_t^y + N\tau_t^o c_t^o}_{T_t} + B_{t+1}$$

- And a transversality condition if the govt issues debt (govt must repay its debt):

$$\lim_{s\to\infty} \frac{B_{t+s}}{R_t R_{t+1} \dots R_{t+s}} = 0$$

Market clearing

- Households, firms and the government interact in markets
- In addition to the following markets:
  - a labour market
  - an asset (physical capital) market
  - a goods market
- Households now also trade in a **Government bonds market**
- Specifically, government supplies bonds and young individuals purchase them:

$$B_{t+1} = Nb_{t+1}$$

Equilibrium

- Equilibrium requires:
  - **Households** choose consumption and savings optimally
    - Euler Equation holds (MB of consuming today = MC of consuming today)
    - Lifetime Budget Constraint holds (must be afforable)
  - **Firms** choose capital and labour optimally (maximize profits)
  - The **government**'s budget constraint and transversality condition holds
  - All (labour, asset, goods and government bonds) **markets clear**

An example to explore different fiscal policies

Financing government spending

- Governments have a few way to finance their spending in each period. We will consider the implications if G_t in each period is fully financed by:
  - A tax only on the young $\tau_t^y > 0, \tau_t^o = 0, B_{t+1} = 0$ for all t
  - A tax only on the old $\tau_t^y = 0, \tau_t^o > 0, B_{t+1} = 0$ for all t
  - A mix of tax instruments and debt
- Moreover, the type of tax (proportional vs lump-sum) also has different implications for the economy

Some assumptions

- As before, we will assume log utility:

$$U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

- Assume output is produced using a Cobb-Douglas production function:

$$F(z_t, K_t, L_t) = z_t K_t^\alpha L_t^{1-\alpha}$$

- And capital depreciates completely after use in production, $\delta = 1$

$$K_{t+1} = (1-\delta)K_t + I_t \implies K_{t+1} = I_t \quad \text{when } \delta = 1$$

- For simplicity, we will assume that $G_t = G \implies g_t = G_t/N = g$ and $z_t = z$

Case 1: Proportional tax on old consumption only

Government Budget Constraint:

$$g_t + (1 + r_t)b_t = \tau_t^y c_t^y + \tau_t^o c_t^o + b_{t+1}$$

- The government runs a balanced budget: govt spending **completely** paid for by the proportional tax on old consumption

- This implies

$$\tau_t^y c_t^y = 0, \quad \text{and} \quad \tau_t^o c_t^o = g \quad \text{and} \quad b_{t+1}, b_t = 0$$

- So in per-capita terms, government budget constraint becomes:

$$g = \tau_t^o c_t^o$$

Case 1: Proportional tax on old consumption only

Firm's problem and optimality conditions are exactly the same as before

$$\max_{K_t, L_t} zK_t^{\alpha} L_t^{1-\alpha} - w_t L_t - R_t K_t$$

- Optimal labour demand satisfies:

$$(1-\alpha)z\left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha)zk_t^{\alpha} w_t$$

- Optimal capital demand satisfies:

$$\alpha z\left(\frac{K_t}{L_t}\right)^{\alpha-1} = \alpha z k_t^{\alpha-1} = R_t$$

- And firms earn zero profit, $\pi_t = 0$

Case 1: Proportional tax on old consumption only

Household budget constraints when $\tau_t^y c_t^y = 0, b_{t+1} = 0$:

- Budget constraint when young :

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$$(1 + \tau_{t+1}^o) c_{t+1}^o = (1 + r_{t+1}) a_{t+1}$$

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$$(1 + \tau_{t+1}^o) c_{t+1}^o = (1 + r_{t+1}) a_{t+1}$$

- So LBC is:

$$\boxed{c_t^y + \frac{(1 + \tau_{t+1}^o) c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t}$$

Case 1: Proportional tax on old consumption only

Household problem when $\tau_t^y c_t^y = 0, b_{t+1} = 0$:

$$\mathcal{L} = \max \ln c_t^y + \beta \ln c_{t+1}^o + \lambda_t \left[w_t + \pi_t - c_t^y - \frac{\left(1 + \tau_{t+1}^o\right) c_{t+1}^o}{1 + r_{t+1}} \right]$$

- ☐ Euler equation

$$\frac{1}{c_t^y} = \frac{\beta(1 + r_{t+1})}{c_{t+1}^o \left(1 + \tau_{t+1}^o\right)} \implies (1 + \tau_{t+1}^o) c_{t+1}^o = \beta(1 + r_{t+1}) c_t^y$$

- ☐ LBC

$$c_t^y + \frac{\left(1 + \tau_{t+1}^o\right) c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t$$

Case 1: Proportional tax on old consumption only

In equilibrium:

- Substitute Euler equation into LBC

$$c_t^y = \frac{1}{1+\beta}\left(w_t + \pi_t\right)$$

- From budget constraint of young and capital market clearing, we also know:

$$k_{t+1} = a_{t+1} = w_t + \pi_t - c_t^y = \frac{\beta}{1+\beta}\left(w_t + \pi_t\right)$$

- We know w_t and π_t from firm's optimality conditions:

$$k_{t+1} = \frac{\beta}{1+\beta}\left(1-\alpha\right) z k_t^{\alpha}$$

Case 1: Proportional tax on old consumption only

Transition equation

$$\boxed{k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)\, z k_t^{\alpha}}$$

- This is the same transition equation we saw in class when there was no government !
- So in this case, the proportional tax on the consumption of the old does not affect the level of investment and thus the growth path of capital per person.
- $\Longrightarrow$ the growth path of y_t is also unaffected by the introduction of government spending financed by a proportional tax on the consumption of the old

Case 1: Proportional tax on old consumption only

Welfare

- But consumption of the old is certainly affected by the proportion tax, τ^o_{t+1}
- From budget constraint of the old and using capital market clearing

$$(1+\tau^o_t)c^o_t = R_t k_t$$

- And using firm's optimality condition to sub for R_t:

$$c^o_t + \tau^o_t c^o_t = \alpha z k^\alpha_t$$

- And using GBC:

$$c^o_t = \alpha z k^\alpha_t - g$$

Case 1: Proportional tax on old consumption only

Welfare

- The decline in consumption by the old exactly offsets the increase in government consumption.
- Each generation observes lower consumption when old than they would without the introduction of g.
- So welfare is lower since utility from consumption when old is smaller.

Case 2: Proportional tax on young consumption only

Government Budget Constraint:

$$g_t + (1 + r_t)b_t = \tau_t^y c_t^y + \tau_t^o c_t^o + b_{t+1}$$

- The govt runs a balanced budget: govt spending each period is **completely** paid for by a proportional tax on young consumption

- This implies

$$\tau_t^y c_t^y = g, \quad \text{and} \quad \tau_t^o c_t^o = 0 \quad \text{and} \quad b_{t+1}, b_t = 0$$

- So in per-capita terms, government budget constraint becomes:

$$g = \tau_t^y c_t^y$$

Case 2: Proportional tax on young consumption only

- Firm's problem and optimality conditions are exactly the same as before

Case 2: Proportional tax on young consumption only

Household budget constraints when $\tau^o_{t+1} c^o_{t+1} = 0, b_{t+1} = 0$:

- Budget constraint when young :

$$(1 + \tau^y_t) c^y_t + a_{t+1} = w_t + \pi_t$$

Case 2: Proportional tax on young consumption only

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- So LBC is:

$$\boxed{(1 + \tau^y_t) c^y_t + \frac{c^o_{t+1}}{1 + r_{t+1}} = w_t + \pi_t}$$

Case 2: Proportional tax on young consumption only

Household problem when $\tau^o_{t+1} c^o_{t+1} = 0, b_{t+1} = 0$:

$$\mathcal{L} = \max \ln c^y_t + \beta \ln c^o_{t+1} + \lambda_t \left[w_t + \pi_t - (1 + \tau^y_t) c^y_t - \frac{c^o_{t+1}}{1 + r_{t+1}} \right]$$

- ☐ Euler equation

$$\frac{1}{(1 + \tau^y_t) c^y_t} = \frac{\beta(1 + r_{t+1})}{c^o_{t+1}}$$

- ☐ LBC

$$(1 + \tau^y_t) c^y_t + \frac{c^o_{t+1}}{1 + r_{t+1}} = w_t + \pi_t$$

Case 2: Proportional tax on young consumption only

In equilibrium

- Make $\frac{c^o_{t+1}}{1+r_{t+1}}$ subject of Euler equation and plug into LBC:

$$(1+\tau^y_t)c^y_t = \frac{1}{1+\beta}(w_t + \pi_t)$$

- Use GBC and the wage from firm's optimality condition

$$c^y_t = \frac{1}{1+\beta}(1-\alpha)zk^\alpha_t - g$$

- Govt spending g completely off-set by decline in consumption of young.

Case 2: Proportional tax on young consumption only

In equilibrium

- From budget constraint of young and capital market clearing:

$$k_{t+1} = w_t + \pi_t - (1 + \tau_t^y) c_t^y$$

- which plugging in for w and GBC and form of c_t^y,

$$\begin{aligned} k_{t+1} &= (1-\alpha) z k_t^\alpha - \frac{1}{1+\beta}(1-\alpha) z k_t^\alpha \\ &= \frac{\beta}{\beta+1}(1-\alpha) z k_t^\alpha \end{aligned}$$

- Same transition equation as before. Because decline in c_t^y exactly offsets g, gross investment still the same, and thus growth path of k_t,y_t unchanged.

Case 2: Proportional tax on young consumption only

Welfare

- Consumption of young in this case is lower with $g > 0$
- Each generation observes lower consumption when young than they would without the introduction of g
- Welfare is lower since utility from consumption when young is smaller

Proportional taxes on consumption spending

- We examined two different cases where the government ran a balanced budget and financed its spending . . .
  - either with a proportional tax on consumption spending of the old
  - or with a proportional tax on consumption spending of the young
  - Both cases affected not just the household's LBC but also his/her optimal intertemporal trade-off in consumption spending, i.e., the household Euler equation
  - Notably, when it became more costly to consume in a particular period, the household optimally lowered that period's consumption

Proportional taxes on consumption spending

- Thus far, government spending looks like it overall reduces welfare.
- But we made an important assumption that government spending in the two examples that we did:
  - In particular, we assumed government spending is *wasteful* and takes away resources which could have been allocated to households in the economy
- Welfare might be very different if instead assumed the spending on g went towards increasing households' utility from consuming a *public good*
- In your tutorial 5, you will consider a case where government spending goes towards the provision of a public good and analyze welfare under that case

Tax policies and implications

Tax on young vs. old

- So far, we've seen that a proportional tax completely on c_t^y or a proportional tax completely on c_{t+1}^o lowers the corresponding type of consumption
- Growth paths of k_t and y_t unaffected because decline in corresponding consumption offset increase in government spending
- However, this result is dependent on the type of tax instrument used
- In your tutorial 5, you will prove that government spending financed completely by a lump-sum tax on the young can actually affect growth paths!

Impact of fiscal policy

- Assumptions about household preferences also affects implications of fiscal policies
- Key takeaway: Different tax policies and different types of government spending will have different outcomes on the economy!

Wrapping up

- This class: a look at tax policies in the OLG model
- Next class: public capital formation and intro to social security