ECOM40006/ECOM90013 Econometrics 3
Department of Economics
University of Melbourne

Assignment 3

Semester 1, 2025

Let $Y_1, Y_2, \ldots, Y_n$ denote a simple random sample from a population with probability density function

$$f(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \theta > 0, \\ 0, & \text{otherwise.} \end{cases}$$

1. Show that the sample mean $\overline{Y}$ is a consistent estimator of $\theta/(\theta+1)$. [7 marks]

   Hint: First derive the mean of the population and then remember that laws of large numbers are your friends.

2. Derive a consistent method of moments estimator, $\tilde{\theta}$ say, for θ. [1 mark]

3. Specify the log-likelihood function for this sample. [1 mark]

4. Derive the maximum likelihood estimator, $\hat{\theta}$ say, for θ and prove that it is, indeed a *maximum* likelihood estimator. [3 marks]

5. Derive the Fisher information for the sample. [2 marks]

6. Suppose that someone wishes to test the null hypothesis null hypothesis $H_0 : \theta = 1$ against the alternative that $H_1 : \theta \neq 1$. State the true population density function and describe in words the implication for the population when this null hypothesis is true. [2 marks]

7. Derive likelihood ratio, Lagrange multiplier and Wald tests for the hypotheses of Question 6. In each case provide the decision rule that you would use in practice to apply the test, including any critical value(s) you may need. [12 marks]

8. Without appeal to the generic properties of maximum likelihood estimators, prove that $\hat{\theta}$ is consistent for θ. [6 marks]

Your answers to the Assignment should be submitted via the LMS no later than 4:30pm, Thursday 22 May. Your mark for this assignment may contribute up to 10% towards your final mark in the subject.

No late assignments will be accepted and an incomplete exercise is better than nothing.