10 Structural Vector Autoregressive models

There are two major diffs b/w VAR & SVAR models

1. The error terms in an SVAR are uncorrelated with eachother, whereas for VAR models this might not be the case.
2. A VAR model can be estimated eqn by eqn by OLS because each eqn has the same predetermined variables on the RHS.
   ↳ Due to contemporaneous relationships among endogenous variables, SVAR models must be recovered from corresponding VAR if possible.

Under what conditions can we recover the SVAR?

VAR

$$y_t = a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + u_{1t}$$
$$z_t = a_{20} + a_{21} y_{t-1} + a_{22} z_{t-1} + u_{2t}$$

SVAR

$$y_t = b_0 - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}$$
$$z_t = b_2 - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}$$

→ On top of coeffs, VAR system estimates variances of u terms & their covariances → ∴ 9 terms

→ The SVAR system only estimates variances of ε → ∴ 10 terms

∴ SVAR is unidentifiable unless we restrict one variable

The Cholesky decomposition is the simplest way to restrict:

→ Sometimes it's reasonable to assume r/ships are asymmetric b/c y_t has a contemp. effect on z but not vice versa (i.e. $b_{12} = 0$)

→ Making this assumption reduces the number of params to 9!

→ However this changes the r/ship b/w u_t & ε_t

$$u_t = B^{-1}\varepsilon_t = \begin{bmatrix} 1 & 0 \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ -b_{21}\varepsilon_{yt} + \varepsilon_{zt} \end{bmatrix}$$

note: not b_{12}!

→ Therefore, you're able to estimate the SVAR with OLS. However, you need to be cognizant of this new ε_t & u_t r/ship.

$$\hat{\varepsilon}_{yt} = \hat{u}_{yt}, \quad \hat{\varepsilon}_{2t} = \hat{b}_{21}\hat{\varepsilon}_{yt} + \hat{u}_{2t} = \hat{b}_{21}\hat{u}_{1t} + \hat{v}_{2t}$$

Notes:

- Cholesky decomposition assumes y_t "precedes" z_t. Therefore, it's only useful when theory suggests it's appropriate to do so.
  - ↳ You must check how robust results are to alternative ordering
- The importance of ordering depends on the correlation b/w the error terms in the VAR system.
  - ↳ The weaker the correlation b/w v_{rt} terms the less important ordering is.
  - ↳ If you have several endog. variables, corrs will not be low & ∴ impractical to experiment w/ all orders.
- Recursive SVAR models can be too restrictive in terms of economic theory
  - ↳ Restrictions can be v arbitrary as you begin developing large systems.