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Asset price bubbles & tests

$$P_t(1+R) = E[P_{t+1} + D_{t+1}] = \underbrace{\sum_{i=1}^h \beta^i E_t(D_{t+i})}_{\text{PV of asset}} + \underbrace{\beta^h E_t(P_{t+h})}_{\text{Bubble ("B_t")}}$$

asset price constant rf. IR cond. exp. value using available info Dividend payment

If no bubble, $B_t = 0$, otherwise:

$$B_t = \beta E_t(P_{t+1}) \therefore E_t(B_{t+h}) = (1+R)^h B_t = \text{exponential for } h > 1$$

tests

→ Right-tail ADF = $H_0: \phi = 1$, $H_a: \phi_1 > 1$
 ↳ issue: when bubble collapses during sample, low prob. of rejecting H_0

→ Supremum ADF (SADF) = same H_0 & H_a as above
 ↳ Based on recursive calcs of ADF test stats with an expanding sample window. Test stats are calculated from progressive sub-samples, SADF = the largest (supremum) of them.

→ Generalised SADF (GSADF) = H_0 : same, H_a : mult. period. collapsing bubbles
 ↳ The sample window is gradually "increased" from $t=0$ one at a time & SADF is performed at each starting point. → GSADF is their supremum.

↳ If any test rejects H_0 , you can use date stamping to start/end point of bubbles.

↳ start point is the first time where test stat > crit value → ends at first failure.

