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(G)ARCH models

- (G)ARCH models are employed b/c sometimes having a constant variance limits interpretation for some very volatile series. They drop this assumption by modelling ε_t as an ARMA process.
  ↳ If it's reasonable to adopt a GARCH model, the squared correlograms will look very AR, indicating the series has a substantial dependence in the volatility of the returns.

The process for estimating a model w/ GARCH errors:

i. Estimate the mean eqn (mult. regress. or ARIMA) for y_t

ii. Take the squared residuals, e_t^2, and estimate variance eqn

Once you specify the mean eqn you can formally test for conditional heteroskedasticity using the Lagrange Multiplier (LM) test.

H_0: no ARCH effects of order $1, \dots, q$. H_a: some ARCH effects of same order.

Finalising this hypothesis test is a two-step process:

i. Estimate e_t^2 w/ OLS: $e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-q}^2 + \xi_t$
   ↳ No ARCH effects of order 1 to q if insig.

ii. Take R^2 from regression & compute LM statistic
   ↳ $LM = TR^2$ → T = usable sample size
   ↳ Under H_0, LM converges to a chi-square dist. w/ df = q

→ Reject H_0 if LM is sufficiently large (use test to check if GARCH specif. is sufficient)