## ECON30009/90080 - TUTORIAL 7

This Version: Semester 2, 2025

This tutorial is designed to get you used to solving search models of unemployment. To aquaint ourselve with the model, we will consider static (one-period) versions of the search model in this tutorial, although you should also familiarize yourself with solving the two-period model.

## Question 1

Consider the search model of unemployment. The economy only lasts for one period. Households get utility from consuming their income at the end of the period. We will assume log utility from consumption. Conditional on being employed, the household inelastically supplies one unit of labour to the firm and receives an exogenous wage equal to  $\bar{w}$ . If non-employed, the household produces home goods worth h. There is no disutility to working. There is a measure 1 of households in the population.

A job is a single firm-worker pair. A firm needs a worker to produce. Specifically, a matched firm-worker pair produces output  $y > \bar{w} > h$ . An unmatched firm must pay a vacancy posting cost of  $\kappa$  to post a vacancy. The unmatched firm receives a lump-sum subsidy S, for each vacancy it creates. Search is random and new firms fill their vacancies with probability  $q(\theta)$  and households find jobs with probability  $p(\theta)$  where  $\theta = v/u$ . Let the matching function in this economy be given by

$$M = \xi \frac{uv}{(u^{\alpha} + v^{\alpha})^{1/\alpha}}$$

where  $0 < \alpha < 1$ . All individuals are initially non-employed in the economy at the start of the period.

Suppose there exists a government that wants to incentivize job creation by providing a subsidy S to firms for each vacancy created. This implies the total amount of subsidies are equal to Sv. The government runs a balanced budget and finances the subsidy by levying a lump-sum tax,  $\tau$ , on all households.

- a Write down the government budget constraint
- b Write down the value of the non-employed, the value of the employed and value of the matched firm at the end of the period.
- c Write down the value of a vacancy at the start of the period.
- d Assume free entry of firms. Show that  $\theta$  and thus vacancies v are positively related to the subsidy S
- e Show that the unemployment rate is declining with the subsidy S.

## Question 2

Consider the search model of unemployment and again assume that the economy only lasts one period. Households get utility from consuming their income at the end of the period. We will assume log utility from consumption. There is a measure 1 of households in the population, but now there are two types of households, A and B. Type A households, if employed, produce  $y_A > y_B$ , where  $y_B$  refers to the output produced by type B if employed. Both types of households observe the same level of home-production h if non-employed. The proportion of type A households in the economy is given by  $\eta_A$  while the proportion of type B households in the economy is given by  $\eta_B = 1 - \eta_A$ . Employed households earn a wage of  $\overline{w}y_j^{\gamma}$  where  $0 < \gamma < 1$ ,  $0 < \overline{w} < 1$  and  $j \in \{A, B\}$ . Further,  $\overline{w}y_j^{\gamma} > h$  so that trivially all households want to participate in the labour market. The rest of the set-up of the model is the same as in Question 1 of this tutorial, except there is no government, and therefore no tax or subsidy.

- a Write down the end-of-period value of the non-employed. Write down the end-of-period value of the employed for each type  $j \in \{A, B\}$ . Write down the value of the firm matched to a type j worker at the end of the period.
- b Write down the value of a vacancy. Assume free entry and solve for  $\theta$  and thus v.
- c How does the composition of job-seekers affect the number of vacancies in the economy? Do firms create more vacancies when  $\eta_A$  is higher? Provide some brief intuition with your answer.