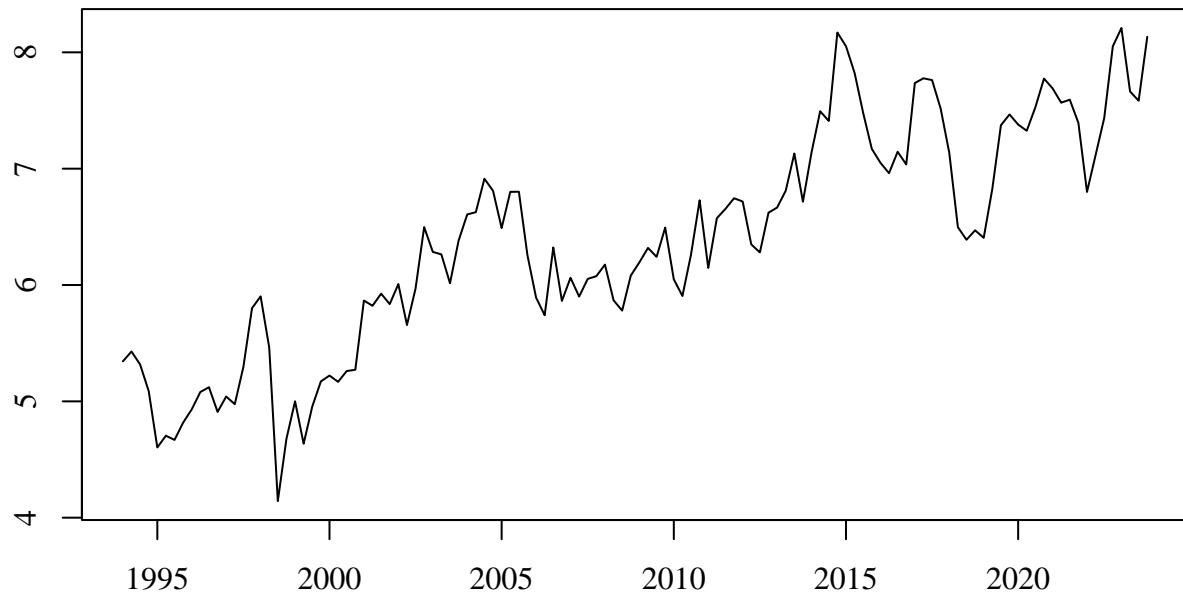
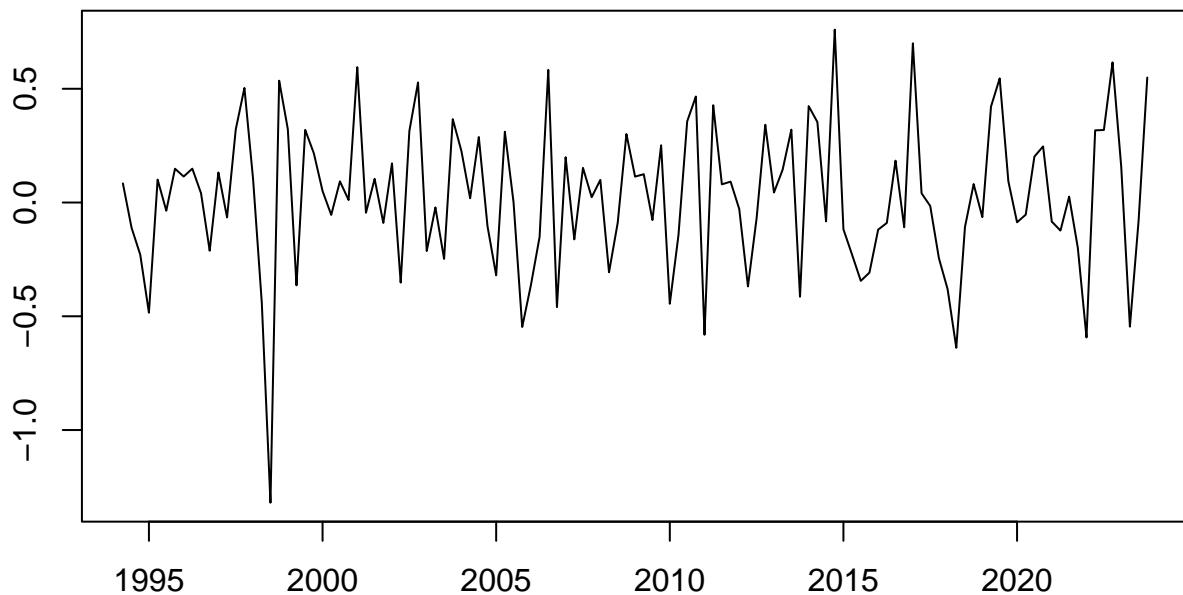


**Question 2.**

Following is a plot of a quarterly time series with 120 observations, from 1994q1 to 2023q4.



The first difference of the time series shown here:



- (a) Purely from visual inspection, do you think the original time series is stationary?  
What about the first difference time series? Explain.

A Dickey-Fuller test was computed for the original time series, including a trend and two lagged differenced in the test equation. For reference, the command was:

```
adf <- ur.df(Y, type=c("trend"), lags=2, selectlags="Fixed")
```

The following summary results were obtained:

```
#####
# Augmented Dickey-Fuller Test Unit Root Test #
#####

Estimate Std. Error t value Pr(>|t|)

(Intercept) 1.453860 0.360766 4.030 0.000102 ***
z.lag.1     -0.296664 0.073029 -4.062 9.05e-05 ***
tt          0.007527 0.001964 3.833 0.000210 ***
z.diff.lag1 0.141602 0.095048 1.490 0.139089
z.diff.lag2 -0.014406 0.095089 -0.152 0.879851

Critical values for test statistics:
    1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
```

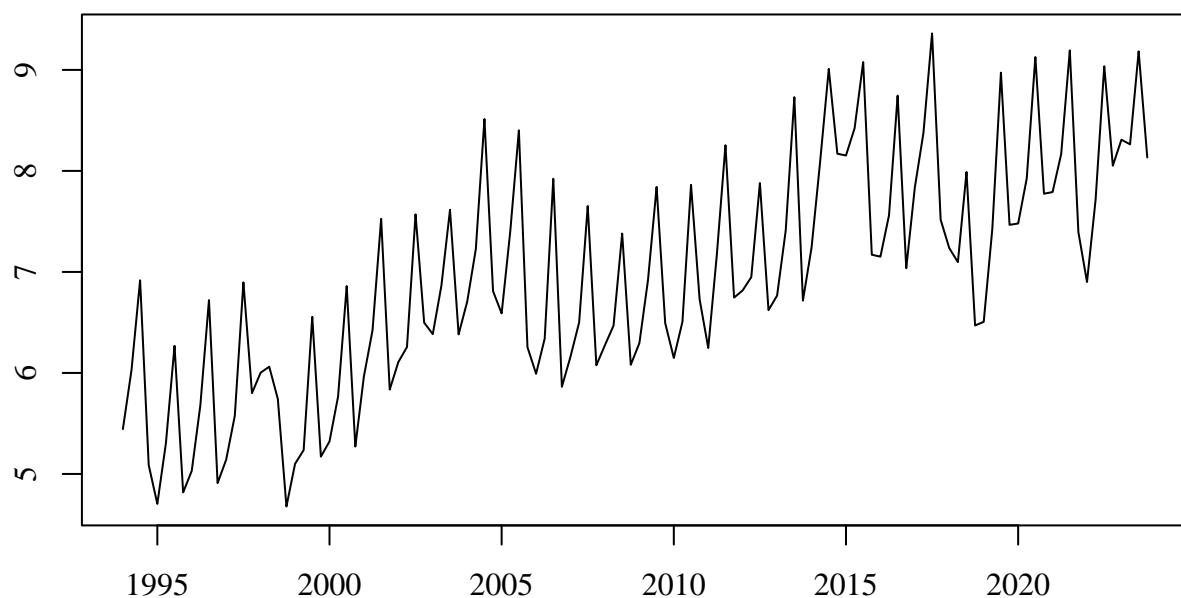
**(b)** Write out the estimated ADF test regression in equation form.

**(c)** Carry out the unit root test at the 5% level of significance, including specifying the hypotheses, test statistic, decision rule and conclusion.

**(d)** Suppose we carried out a unit root test for the differenced time series. What do you think the conclusion would be, and why? What other feature we have discussed in the subject would be relevant for the differenced time series?

- (e) Suppose the unit root regression above has produced an ADF  $t$  statistic of  $-3.13$ . What would the  $p$ -value be for this statistic? Explain.

Here is another closely quarterly time series over the same time period:



- (f) What additional feature is evident in this time series that is not in the original time series?

- (g) Write out the form of the ADF test equation you would want to estimate to test for a unit root in this time series.

- (h) Give an outline of the steps you would implement to obtain the appropriate critical value for the unit root in the previous part. (Do not give R code, just simple descriptions of the computational steps required.)