ECOM90024
Forecasting in Economics and Business
Tutorial 5 SOLUTIONS

1.) Is the following MA(2) process covariance-stationary?

$$Y_t = (1 + 2.4L + 0.8L^2)\varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

If so, calculate its autocovariances and autocorrelations.

Yes, all MA(2) processes are covariance stationary since the impact of the innovations do not persist past the second lag.

To calculate the autocovariances and autocorrelations, we can use the equations from the lecture notes. That is, for an MA(2) with

$$Y_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$$

We have that the autocovariances are given by

$$\gamma(0) = (1 + \theta_1^2 + \theta_2^2)\sigma^2 = (1 + 2.4^2 + 0.8^2) = 7.4$$

$$\gamma(1) = (\theta_1 + \theta_2\theta_1)\sigma^2 = (2.4 + (2.4)(0.8)) = 4.32$$

$$\gamma(2) = \theta_2\sigma^2 = 0.8$$

Therefore the autocorrelations are given by

$$\rho(1) = \frac{4.32}{7.4} = 0.584$$

$$\rho(2) = \frac{0.8}{7.4} = 0.108$$

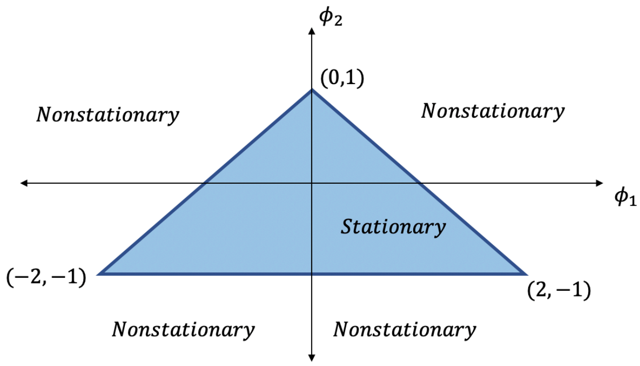
2.) Is the following AR(2) process covariance-stationary?

$$(1 - 1.1L + 0.18L^2)Y_t = \varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

If so, calculate its autocovariances and autocorrelations.

The above process has the AR coefficients $\phi_1 = 1.1$ and $\phi_2 = -0.18$. This pair of coefficients clearly lie within the triangle outlined in the lecture



Hence the roots are stable and we can conclude that the process is covariance stationary.

The first two autocovariances and the first autocorrelation are computed as:

$$\gamma_0 = \frac{(1-\phi_2)\sigma^2}{(1+\phi_2)[(1-\phi_2)^2 - \phi_1^2]} = \frac{1.18}{(0.82)(1.18^2 - 1.1^2)} = 7.89$$

$$\gamma_1 = \phi_1\gamma_0 + \phi_2\gamma_1$$

$$\gamma_1 = \frac{\phi_1}{(1-\phi_2)}\gamma_0 = \frac{1.1}{1.18}(7.89) = 7.355$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{7.355}{7.89} = 0.932$$

With these two starting values, we can then calculate the entire sequence of autocovariances and autocorrelations using their recursive form:

$$\gamma_j = \phi_1 \gamma_{j-1} + \phi_2 \gamma_{j-2}$$

$$\rho_j = \phi_1 \rho_{j-1} + \phi_2 \rho_{j-2}$$

Tau	Gamma	Rho
0	7.89	1
1	7.355	0.93219265
2	6.6703	0.84541191
3	6.01343	0.76215843
4	5.414119	0.68620013
5	4.8731135	0.61763162
6	4.38588343	0.55587876
7	3.94731134	0.50029295
8	3.55258346	0.45026406
9	3.19732576	0.40523774
10	2.87759332	0.36471398
11	2.58983401	0.32824259
12	2.33085062	0.29541833

3.) Given the following AR(2) process,

$$Y_t = 0.6Y_{t-1} - 0.08Y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

Compute the roots of the corresponding lag polynomial and verify that it is indeed a covariance stationary process.

We can write the above AR(2) process in it's lag polynomial form

$$(1 - 0.6L + 0.08L^2)Y_t = \varepsilon_t$$

We know that the AR(2) process is covariance stationary when the roots of the polynomial

$$1 - \phi_1 z - \phi_2 z^2 = 0$$

Lies outside the unit circle or equivalently, when the roots of the polynomial

$$\lambda^2 - \phi_1\lambda - \phi_2 = 0$$

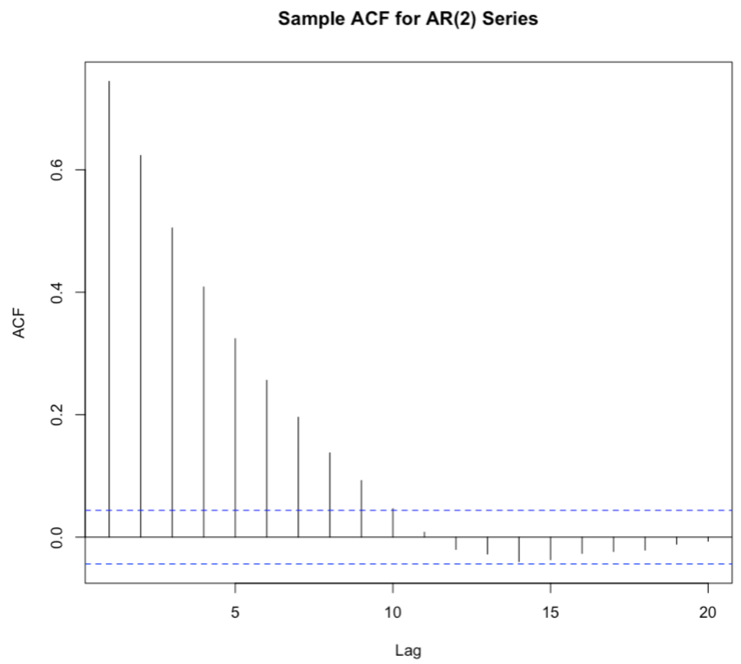
Lies inside the unit circle. Using the quadratic formula, we can see that the two roots are

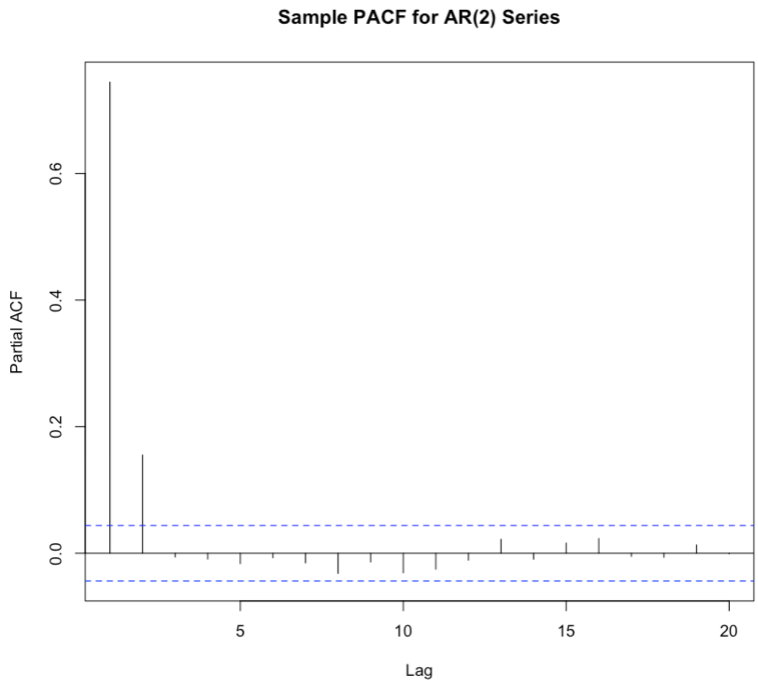
$$\lambda_1 = \frac{\phi_1 + \sqrt{\phi_1^2 + 4\phi_2}}{2} = 0.4$$
$$\lambda_2 = \frac{\phi_1 - \sqrt{\phi_1^2 + 4\phi_2}}{2} = 0.2$$

Therefore both of the roots are less than unity and so we can conclude that the AR(2) process is covariance stationary.

4.) The data file ar2.csv contains 2000 observations that have been simulated from an AR(2) process whose coefficients you do not know.

a.) Using ***R***, compute and plot the sample autocorrelations and partial autocorrelations. Do they accord with the dependence structure of an AR(2) process?





We can see clearly from the sample PACF that there is a cut-off from the second lag. This is consistent with the dependence structure of an AR(2) mode.

b.) Using your knowledge of the Yule-Walker equations, compute estimates of the AR(2) coefficients.

From the derivation of the autocorrelation function of the AR(2) model, we have that the Yule-Walker equations are given by

$$\rho_1 = \frac{\phi_1}{1-\phi_2}$$

$$\rho_2 = \phi_1\rho_1 + \phi_2$$

Therefore, we can rewrite these two equations in terms of the autoregressive coefficients. From the first equation we have that

$$\phi_1 = \rho_1(1-\phi_2)$$

Substituting this into the second equation we have that

$$\rho_2 = \rho_1^2(1-\phi_2) + \phi_2$$

Solving for ϕ_2, we have

$$\phi_2 = \frac{\rho_1^2 - \rho_2}{\rho_1^2 - 1}$$

Since $\hat{\rho}_1 = 0.745$ and $\hat{\rho}_2 = 0.624$, we obtain

$$\hat{\phi}_2 = \frac{(0.745)^2 - 0.624}{(0.745)^2 + 1} = \frac{-0.068975}{-.444975} = 0.1553$$

$$\hat{\phi}_1 = 0.745(1 - 0.1553) = 0.6289$$

c.) Using ***R***, verify your computation in part b.) by computing OLS estimates of the AR(2) coefficients.

Computing the OLS estimates should generate very similar values to the calculation performed in the previous part. These numbers will not be exactly the same since the OLS estimation is conducted with a slightly smaller set of observations (i.e. T-3).

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Call:
lm(formula = ar2l0 ~ 0 + ar2l1 + ar2l2)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6081 -0.6242  0.0658  0.6907  3.1554 

Coefficients:
      Estimate Std. Error t value Pr(>|t|)    
ar2l1  0.63079    0.02211  28.532  < 2e-16 ***
ar2l2  0.15701    0.02211   7.102  1.7e-12 ***
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.9898 on 1996 degrees of freedom
Multiple R-squared:  0.5707,	Adjusted R-squared:  0.5703 
F-statistic:  1327 on 2 and 1996 DF,  p-value: < 2.2e-16
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