ECON30024
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Solution to Tutorial 3

1. Quiz 3 questions

   (1) Drew is assumed to choose a portfolio according to the Expected Utility Hypothesis (EUH). Which of the following statements is TRUE?

   (a) Drew's preference does not take into account the uncertainty associated with the payoffs of assets.

   (b) Drew's decisions are incompatible with maximization of a mean-variance objective function.

   (c) Drew acts as if a probability is assigned to each state and orders these actions according to the expected value of the utility function.

   (d) Drew can choose an optimum portfolio that maximizes his expected utility only if he is told the true probability for each possible state of the world.

   (e) None of the rest is true.

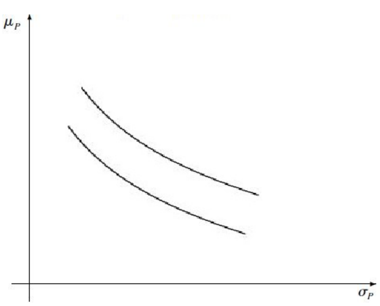
   Answer: (c)

   (a) is false because the EUH is a special case of the state-preference approach

   (b) is false because the mean-variance objective function is a special case of the expected utility, so decisions under the EUH can be compatible with decisions with a mean-variance objective.

   (d) is false because an investor makes portfolio decisions based on her own belief about the states of the world, which is not required to be the true probabilities.

   (2) Consider the following indifference curves for an investor:



   Which of the following statements is TRUE?

   (a) This investor is risk averse and the higher line corresponds to the higher level of utility.

   (b) This investor is risk loving and the higher line corresponds to the higher level of utility.

(c) This investor is risk averse and the lower line corresponds to the higher level of utility.

(d) This investor is risk loving and the lower line corresponds to the higher level of utility.

(e) None of the rest is true.

Answer: (b)

First, the investor is risk loving, as the indifference curve is downward sloping; Second, because the investor is risk loving, given μ_P, higher σ_P leads to higher utility, so the higher line corresponds to higher level of utility.

(3) Allison has the following utility function $u(W) = W - W^2$ for $0 \leq W < \frac{1}{2}$[1]. What is her attitude to risk?

(a) Risk neutral for low levels of wealth and risk averse at higher levels of wealth.

(b) Risk averse with increasing absolute risk aversion.

(c) Risk averse with decreasing absolute risk aversion.

(d) Risk averse with constant relative risk aversion.

(e) None of the above is true.

Answer: (b)

First, Allison is risk averse because

$$u'(W) = 1 - 2W \quad \Rightarrow u''(W) = -2 < 0.$$

Second,

$$R_A(W) = -\frac{u''(W)}{u'(W)} = \frac{2}{1-2W} \quad \Rightarrow R'_A(W) = -\frac{2}{(1-2W)^2}(-2) = \frac{4}{(1-2W)^2} > 0$$

so Allison has increasing ARA.

(4) If an individual has increasing ARA, then they have ___ RRA?

(a) increasing

(b) decreasing

Answer: (a)

If an individual has increasing ARA, then as their wealth increases they will invest less of their wealth in the risky asset. Therefore on a percentage basis, they must also invest less in the risky asset. Therefore, the individual also has increasing RRA.

(5) Consider the mean-variance utility:

$$G(\mu_P, \sigma_P^2) = \mu_P - \alpha\sigma_P^2.$$

[1] This restriction on possible values of W is to ensure $u(W)$ is strictly increasing and $R_A(W)$ is positive, as can be seen in the answer to this question. I didn't put this restriction before and a student pointed out the potential issue without this restriction. Much appreciated.

In Excel, draw the indifference curves for $G = 10$ with $\alpha = 1$ (call it Curve 1) and $\alpha = 2$ (call it Curve 2). Which of the following statements is FALSE?

(a) Curve 1 and Curve 2 are both upward sloping.

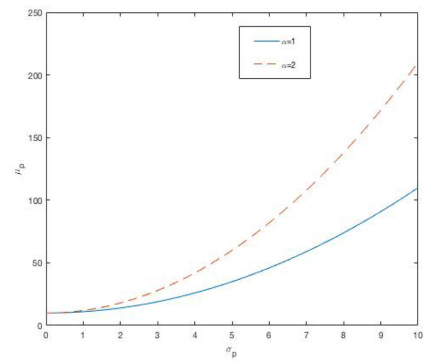
(b) Curve 2 is steeper than Curve 1.

(c) Curve 1 is steeper than Curve 2.

(d) Curve 1 and Curve 2 are both convex-shaped.

Answer: (c)

The two indifference curves for $\alpha = 1$ and $\alpha = 2$ are shown in Figure 1 below:



2. The indifference curves for $\alpha = 1$ and $\alpha = 2$ are shown in Figure 1 above.

   Indifference curves of a mean-variance utility have the following properties. Risk aversion is the reason underlying all these properties.

   - Given expected return (μ_P), higher risk (higher σ_P) leads to lower utility.
   - Given risk (σ_P), higher expected return (higher μ_P) leads to higher utility.
   - The indifference curves are upward slopping, i.e., μ_P increases with σ_P along an indifference curve or mathematically, $\mu'_P(\sigma_P) > 0$. This implies that higher expected return is required to compensate for higher risk, to yield the same level of utility.
   - The indifference curves are convex, i.e., μ_P increases with σ_P at an increasing speed or mathematically, $\mu''_P(\sigma_P) > 0$. This implies that as risk increases the compensation by higher expected return becomes increasingly difficult–a higher increase in expected return is required to compensate for the same increase in risk.
   - The more risk averse (higher α), the steeper the indifference curves are, implying that if at a given risk level, a higher expected return is required to yield the same level of utility if investors are more risk averse.

3. (a) By definition, the two assets' rates of return are given by

$$r_1 = \frac{v_1 - p_1}{p_1} = \frac{v_1}{p_1} - 1, \; r_2 = \frac{v_2 - p_2}{p_2} = \frac{v_2}{p_2} - 1.$$

The proportions of initial wealth invested in each of the two assets are given by

$$a_1 = \frac{p_1 x_1}{A},\ a_2 = \frac{p_2 x_2}{A}.$$

Note that $a_1 + a_2 = \frac{p_1 x_1 + p_2 x_2}{A} = 1$.

(b) Terminal wealth is given by

$$\begin{aligned} W &= v_1 x_1 + v_2 x_2 = (1+r_1)p_1 x_1 + (1+r_2)p_2 x_2 \\ &= (1+r_1)a_1 A + (1+r_2)a_2 A = (a_1+a_2)A + (a_1 r_1 + a_2 r_2)A \\ &= A + (a_1 r_1 + a_2 r_2)A. \end{aligned} \quad (1)$$

By definition, the rate of return on the portfolio is defined as

$$r_P \equiv \frac{W - A}{A}.$$

From (1),

$$\frac{W-A}{A} = a_1 r_1 + a_2 r_2,$$

and hence

$$r_P = a_1 r_1 + a_2 r_2.$$

(c) The expected rate of return on the portfolio:

$$\mu_P = E(r_P) = E(a_1 r_1 + a_2 r_2) = a_1 E(r_1) + a_2 E(r_2) = a_1 \mu_1 + a_2 \mu_2.$$

The variance of the rate of return on the portfolio:

$$\begin{aligned} \sigma_P^2 &= var(r_P) = var(a_1 r_1 + a_2 r_2) \\ &= a_1^2 var(r_1) + a_2^2 var(r_2) + 2a_1 a_2 cov(r_1, r_2) \\ &= a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + 2a_1 a_2 \sigma_1 \sigma_2 \rho_{12}, \end{aligned}$$

where ρ_{12} denotes the correlation coefficient between r_1 and r_2.

(d) Recall that $W = (1 + r_P)A$, so

$$E(W) = (1 + E(r_P))A = (1 + \mu_P)A,$$

$$var(W) = var((1 + r_P)A) = A^2 var(r_P) = A^2 \sigma_P^2.$$

4. (a) The expected utility form implies quite a few underlying assumptions
   - Investors know the states of the world.
   - They are able to assign a probability to each state of the world.
   - They know the payoff from the portfolio chosen (consequence of their action) in each state of the world.

(The assumptions above do not require the states, probabilities and payoffs assigned by an investor are objective or true, so these assumptions are reasonable for a rational investor.)

– The probabilities they assign to each state and their payoffs in each state are independent (This assumption is controversial as more optimistic (pessimistic) people may assign a higher probability to a good (bad) state that gives higher (lower) payoff).

– They evaluates the payoffs in each state in the same way. (This is also controversial as \$100 dollars may value more to an investor in a bad state than in a good state.)

(b) There are a lot of other aspects of the portfolio which investors may view as important. For example, a pessimistic investor might care about the worst rate of return (i.e., the minimum value of the rate of return) on the portfolio and how likely this would happen. An optimistic investor might care about the maximum value of the rate of return. Investors may also care about the shape of the distribution of return. A negatively skewed distribution or a distribution that exhibits fat left tail indicates high likelihood of realising below-average returns and hence high risk. These considerations are not captured by the mean-variance objective.