Lecture 3: A Two Period Consumption-Savings Problem and Permanent Income Hypothesis

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Last class

- We had our first look at simple two-period consumption-savings problem.
- We said that the household had an objective: to maximize his/her utility from consumption (most basic decision of every household, how and what to spend on)
- but that the household was subject to constraints (finite resources to spend)
- We showed that the optimal consumption bundle satisfied two conditions:
  - Affordable (on the budget constraint!)
  - Balanced MRS (rate at which you are willing to substitute c^y for c^o) against interest rate (opportunity cost of c^y)

Today

- We want to explore a bit more about our household's optimal consumption bundle tells us.

An example

- Suppose preferences are given by:

$$U(c^y, c^o) = \ln c^y + \beta \ln c^o$$

where β is a parameter representing the discount factor, and $0 < \beta < 1$

- Then problem becomes:

$$\max_{c^y, c^o} \ln c^y + \beta \ln c^o$$

$$\text{s.t.} \qquad c^y + \frac{c^o}{1+r} = y^y + \frac{y^o}{1+r}$$

Solve for c^y given r, y^y, y^o and β

An example

- Lagrangian:

$$\max_{c^y, c^o, \lambda} \mathcal{L}(c^y, c^o, \lambda) = \ln c^y + \beta \ln c^o + \lambda \left[y^y + \frac{y^o}{1+r} - c^y - \frac{c^o}{1+r} \right]$$

- FOCs:

$$\begin{aligned}
(c^y): \quad & \tfrac{1}{c^y} - \lambda && = 0 \\
(c^o): \quad & \tfrac{\beta}{c^o} - \tfrac{\lambda}{1+r} && = 0 \\
(\lambda): \quad & y^y + \tfrac{y^o}{1+r} - c^y - \tfrac{c^o}{1+r} && = 0
\end{aligned}$$

An example

- 2 key optimality conditions:
  - Euler equation:
$$\frac{1}{c^y} = \frac{\beta(1+r)}{c^o}$$
  - Lifetime budget constraint:
$$y^y(1+r) + y^o - (1+r)c^y = c^o$$

An example

- Plug budget constraint into Euler equation to solve for c^y

$$c^y = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- Now that we have c^y, use Euler to get c^o:

$$c^o = \frac{\beta(1+r)}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- c^y and c^o both depend on lifetime income and not just on current income

An example

- Optimal consumption when young:

$$c^y = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- Solution to household problem gives us her individual demand schedule: i.e., how much she will consume given any interest rate r and her income.

- Consumption today when young c^y is declining in r.

- Intuitively, higher r, means consuming one unit today $=$ larger foregone savings and thus consumption tomorrow.

An example

- Optimal consumption when old:

$$c^o = \frac{\beta\,(1+r)}{1+\beta}y^y + \frac{\beta}{1+\beta}y^o$$

- Consumption today when old, c^o, is increasing in r.

- Holding all else constant, a higher r means a higher return on savings and thus more resources available to consume from when old

An example

- Optimal consumption when young and old:

$$c^y = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right) \quad \text{and} \quad c^o = \frac{\beta\,(1+r)}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- c^y, c^o are functions of **lifetime** income (PDV of income is on RHS of equation)

- This feature, that consumption responds to **lifetime** income contrasts with Keynesian consumption function
$\implies C_t = \bar{C} + bY_t$

Permanent Income Hypothesis

- **Consumption spending decisions are based on permanent income**

- It doesn't matter if you receive all the income today or tomorrow **if** its equal to the same lifetime income

  - Case 1: Only receive income when young, $y^y = y, y^o = 0$.

  - Case 2: Only receive income when old, and $y^o = y(1+r)$, $y^y = 0$.

  - Notice that both cases give us the same lifetime income:

$$\text{Lifetime income} = y^y + \frac{y^o}{1+r} = y$$

Permanent Income Hypothesis

- From our example earlier, we had:

$$c^y = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right) \quad \text{and} \quad c^o = \frac{\beta(1+r)}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- Consumption patterns are completely unchanged so long as lifetime income is unchanged.

- To attain their desired consumption levels in period despite current income being zero in that particular period, individuals either save or dis-save

Saving and dis-saving

- What would you do if you received \$1000 today?
- Spend all \$1000 on ...

Model predictions about savings behavior

- To better understand the model's predictions regarding savings behavior, let's consider what happens if:
    - Case 1: only income when young increases by Δ
    - Case 2: income in each period increases by Δ

Back to our example: a special case $\beta = \frac{1}{1+r}$

- To simplify the analysis we will consider a special case where $\beta = \frac{1}{1+r}$

- In this special case, $c^y = c^o$:

$$c^y = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

and

$$c^o = \frac{\beta(1+r)}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right) = \frac{1}{1+\beta}\left(y^y + \frac{y^o}{1+r}\right)$$

- This is the case of perfect consumption smoothing. That is, the households wants the same amounts of consumption in every period

Back to our example: a special case $\beta = \frac{1}{1+r}$

- Only income when young increases by Δ: $y^y + \Delta$.

$$c^y = \frac{1}{1+\beta}\left(y^y + \Delta + \frac{y^o}{1+r}\right) \implies \frac{\partial c^y}{\partial \Delta} = \frac{1}{1+\beta} = \frac{1+r}{2+r}$$

- Budget constraint when young:

$$c^y + a = y^y + \Delta$$

and

$$a = \frac{\beta}{1+\beta}(y^y + \Delta) - \frac{1}{1+\beta}\left(\frac{y^o}{1+r}\right) \implies \frac{\partial a}{\partial \Delta} = \frac{\beta}{1+\beta}$$

- Part of increase in current income Δ gets saved! Household wants to smooth consumption over his/her lifetime.

Back to our example: a special case $\beta = \frac{1}{1+r}$

- Income when young and old increases by Δ: $y^y + \Delta$ and $y^o + \Delta$:

$$c^y = \frac{1}{1+\beta}\left(y^y + \Delta + \frac{y^o + \Delta}{1+r}\right) \implies \frac{\partial c^y}{\partial \Delta} = \frac{1}{1+\beta}\frac{2+r}{1+r} = 1$$

and

$$\begin{aligned} a &= \frac{\beta}{1+\beta}(y^y + \Delta) - \frac{1}{1+\beta}\left(\frac{y^o + \Delta}{1+r}\right) \\ \implies \frac{\partial a}{\partial \Delta} &= \frac{1}{1+\beta}\frac{\beta(1+r) - 1}{1+r} = 0 \end{aligned}$$

- No change in a. Household need not save to consume more of the same amount in each period!

Saving and dis-saving

Back to our question

- What would you do if you received \$1000 today?
- Spend all \$1000 today?
- Is your answer different if you received \$1000 everyday into perpetuity?

PIH and Keynesian consumption function

- Marginal propensity to consume (MPC): the increase in consumer spending in response to an increase in disposable income

- Recall Keynesian consumption function has the form:

$$C_t = \overline{C} + \overbrace{b}^{MPC} Y_t$$

PIH and Keynesian consumption function

- Friedman (1957) argues:
  "... regressions between consumption and income are simply a reflection of the inadequacy of measured income as an indicator of long-run income status"

- In other words, the regressions have an *omitted variable*: lifetime income

- Which means estimated MPC from a regression of current C against current Y can be biased *if* current income is correlated with lifetime income

  - Problematic since estimated MPC from these regressions used to guide policy decisions and to think about multiplier effects from stimulus

Takeaways?

- How to recover MPC from data is actually kind of tricky!
- At any point in time, we don't really see lifetime income, or have good data on expectations of lifetime income
- Solution?: estimate consumption response to an *unanticipated* change in income
- This is as if lifetime income changed by the unanticipated amount

Some thoughts/questions

- What type of income profile do you think you will face?
- What does the model we covered suggest you should be doing with regards to savings a
- Does your consumption/savings pattern fit what our model would suggest?

Access to credit markets

- If $y^y << y^o$, $a < 0$, individuals optimally want to borrow against future income so as to smooth consumption over their lifetime.

- In reality, not everyone can borrow against future income

  - Uncertainty about individual's ability to repay (asymmetric information)

  - Uncertainty about individual's willingness to repay (moral hazard)

Hand-to-mouth individuals

- For some individuals, problem of a household may look like:

$$\max_{c^y, c^o} U(c^y, c^o)$$

s.t. $\quad c^y + a = y^y$

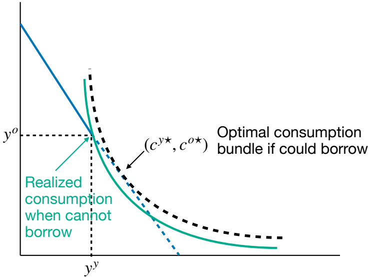
and $\quad c^o = y^o + (1+r)a$

and $\quad a \geq 0 \quad$ no borrowing constraint

- If $y^o >> y^y$, individual wants to borrow, $a < 0 \implies c^{y*} = y^y - a > y^y$

- But individual cannot borrow (no access to credit markets), so best that individual can do is consume $c^y = y^y$

Hand-to-mouth individuals



- Individual who cannot borrow is on a lower indifference curve (can't obtain her optimal consumption bundle)

- Suppose individual was given one more unit of current income y^y, how much would c^y rise by?

Back to example (again!)

Suppose $\beta = \frac{1}{1+r}$ for simplicity.

- Perfect consumption smoothing

$$\frac{1}{c^y} = \frac{\beta(1+r)}{c^o} \quad \implies c^y = c^o$$

- Further assume $y^o >> y^y$ and $a > 0$. We already observed that individual will choose $c^y = y^y$ (best she/he can do)

- Now suppose given extra income Δ when young such that $y^o > y^y + \Delta > y^y$, what is c^y?

- In this case, individual spends all of extra income Δ on c^y, i.e., $c^y = y^y + \Delta$

Continuing the example

Suppose $\beta = \frac{1}{1+r}$ for simplicity.

- Individual desire perfect consumption smoothing

$$c^y = c^o$$

- Given extra income Δ such that $y^o > y^y + \Delta > y^y$, individual spends all of extra income Δ on c^y, i.e., $c^y = y^y + \Delta$

- Observe that for this individual, $\frac{\partial c^y}{\partial \Delta} = 1$

- Different from our earlier result when individuals who were allowed to borrow and received only Δ increase in y^y today, observed $\frac{\partial c^y}{\partial \Delta} = \frac{1}{1+\beta}$

Marginal propensities to consume and wealth

- In general, we observe individuals with little-to-no liquid wealth – *hand-to-mouth* individuals – observing very high MPCs

- These are individuals who typically cannot borrow and thus who cannot obtain their optimal consumption bundle

- An extra dollar today gets them closer to their optimal consumption bundle

- Individuals who have greater amounts of wealth, who can borrow and are not credit-constrained, have lower MPCs

- These individuals tend to save part of the increase in income

The role of policy?

- 1950s-60s: Used statistical relationships between c and y to govern policy recommendations
- PIH: a critique of the purely statistical model. Simple consumption-savings model suggests that consumption dependent on life-time income
- Reality: PIH doesn't negate the role for policy. But suggests we have to be more careful about how we estimate MPCs from data
- Reality: MPCs can differ, with the hand-to-mouth observing high MPCs and consumption moving closely with current income

Roadmap

- Next class: Introduction to firm's problem
- Next week: An OLG model
- After that: General equilibrium in an OLG model