FNCE90056: Investment Management

Week 10 - Lecture 9: Interest Rate Risk

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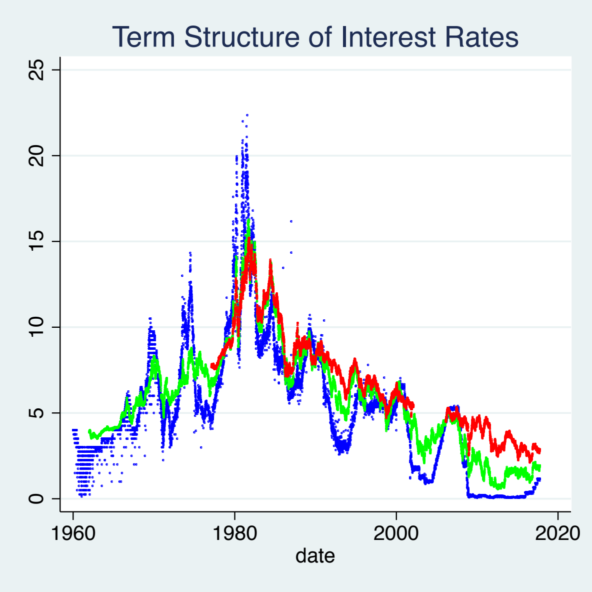
Introduction

Overview of bond portfolio risks

- **Interest rate risk**
- Credit (default) risk (lecture 6)
- Inflation risk (lecture 7)
- Exchange rate risk
- Liquidity risk



- **Interest rate risk** = uncertainty in bond prices due to interest rate change.

In the figure to the left, the blue data represent the U.S. federal funds (overnight) rate. The green data represent the 5-year risk-free rate implied by U.S. Treasuries. The red data represent the 30-year risk-free rate implied by U.S. Treasuries.

Interest rates and bond prices

Interest rates and bond prices

Today, we'll discuss **interest rate risk** in greater detail.

- If interest rates decrease, bond prices rise, but
- If interest rates increase, bond prices fall $\rightarrow$ capital loss.
- Recall: Not a concern for an investor who invests in a default risk-free, zero-coupon bond and holds the bond until maturity.

Example

Suppose that the current term structure is flat at 5%, but increases by 25 basis points to 5.25%.[1]

What happens to the prices of zero coupon bonds?

maturity	old price	new price	bond return[2]
1	95.12	94.89	-0.25%
5	77.88	76.91	-1.25%
10	60.65	59.16	-2.50%
20	36.79	34.99	-5.00%
30	22.31	20.70	-7.50%

[1] Both rates are reported assuming continuous compounding.

[2] Assuming . . . continuous compounding!

Interest rates and bond prices

Interest rate risk arises because interest rate changes affect the bond price.

We first need to understand the detail of **how interest rates and bond prices are related**.

1. The bond price is inversely related to the yield.
2. All else equal, the shorter the term to maturity, the smaller the bond price sensitivity.
3.
4.
5.

Interest rate risk

We clearly need:

- **a systematic metric to assess the riskiness of a bond portfolio to movements in interest rates (quantify the interest rate risk)**
  - (Modified) Duration
  - Macaulay duration
  - Dollar duration
  - Price value of a basis point
  - Convexity
- **a methodology to effectively manage such risk**
  - Immunisation

Duration

Definition

The **duration** of a security measures the sensitivity of its price, as a percentage of its price (dP/P), to small changes in its yield (dy).

- Recalling the definition of the yield, this is equivalent to measuring price sensitivity to small parallel shifts in the term structure.

The (modified) duration of the asset is then defined as

$$D \equiv -\frac{\frac{dP}{P}}{dy} = -\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}y} \tag{1}$$

- The $\mathrm{d}y$ in (1) is an infinitesimal. If we want to generalise (1) for use with finite (but still very small) changes in rates, Δy, we can use the 1st-order linear approximation:

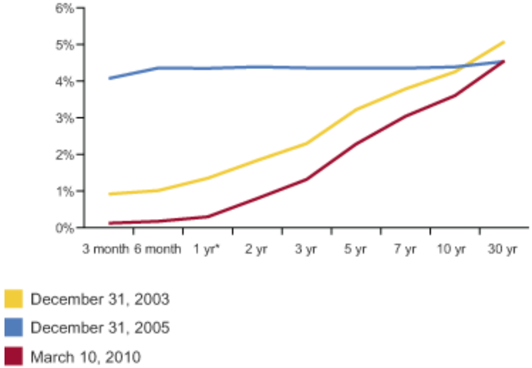
$$D \approx -\frac{1}{P}\frac{\Delta P}{\Delta y} \tag{2}$$

$$\Longrightarrow \Delta P \approx -D \cdot P \cdot \Delta y \tag{3}$$

Parallel Shifts?

How often do we see parallel shifts in the yield curve?

Rarely!!! take a look at the figure on slide 4.



Yield curve moves are rarely parallel, but duration will still be a good 1st order measure of bond risk.

Details

- Think of Δy as a small uniform (or parallel) shift in zero rates across the entire term structure.
  (in our example on slide 7, we had $\Delta y = 0.25\% = 0.0025$)

- If we know the duration of a bond, we can then immediately apply (3) to estimate the change in the bond's price.

Example

- A \$100 million bond portfolio has a duration of 10 years: $D = 10$.
- A 1 basis point (0.01%) increase in the level of interest rates therefore implies that

$$\begin{aligned}\text{change in portfolio value} = \Delta P &\approx -D \times P \times \Delta y \\ &\approx -10 \times \$100\text{m} \times 0.01\% \\ &\approx -\$100,000\end{aligned}$$

- So, for every 1 basis point increase in the term structure, the bond portfolio will lose about \$100k.

Modified Duration

The **Duration**, aka **Modified Duration** is related to the approximate percentage change in price for a given change in yield, as given by:

$$
\begin{aligned}
\text{Modified Duration} = D &= -\frac{1}{P}\frac{dP}{dy} \\
&= \frac{1}{1+y}\left[\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t}\right]\frac{1}{P}
\end{aligned} \tag{4}
$$

where dP = change in price, dy = change in yield, P = price of the bond.

What is it 'modified' from? To distinct it from Macaulay Duration, which is slightly different, but has an elegant interpretation...

Modified Duration - derivation

Recall: $$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \dots + \frac{C+Par}{(1+y)^T}$$

Therefore:

$$\frac{dP}{dy} = -\frac{C}{(1+y)^2} - \frac{2C}{(1+y)^3} - \frac{3C}{(1+y)^4} - \dots - \frac{T(C+Par)}{(1+y)^{T+1}}$$

$$= -\frac{1}{1+y}\left[\frac{C}{1+y} + \frac{2C}{(1+y)^2} + \frac{3C}{(1+y)^3} + \dots + \frac{T(C+Par)}{(1+y)^T}\right]$$

So $$\frac{dP}{dy}\frac{1}{P} = \frac{-1}{1+y}\left[\frac{C/(1+y) + 2C/(1+y)^2 + 3C/(1+y)^3 + \dots + T(C+Par)/(1+y)^T}{P}\right]$$

Macaulay Duration

The **Macaulay Duration** is defined as

$$
\begin{aligned}
\text{Macaulay Duration} &= (1+y) \times \text{Modified Duration} \\
&= (1+y) \times D \\
&= \left[\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t} \right] \frac{1}{P}
\end{aligned}
\tag{5}
$$

This is the effective time-to-maturity: a weighted average of the times until payment dates of the bond.

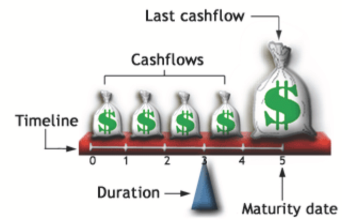
$$
\text{Macaulay Duration} = \sum_{t=1}^{T} w_t \times t \tag{6}
$$

and weights equal to the discounted payments, scaled by the price:

$$
w_t = \frac{\mathrm{CF}_t/(1+y)^t}{P} \tag{7}
$$

Macaulay Duration as a weighted average

For coupon bonds, the fulcrum shows the "average" time it takes to be repaid:



Example

Modified Duration and Macaulay Duration

Example:

Coupon Rate	Coupon Freq.	Term to Maturity	Yield	Par
8.5%	Annual	5 Years	10%	$100

- What is the Macaulay duration of this bond?
- What is the Modified duration of this bond?
- How would you interpret it?

Example

Modified Duration and Macaulay Duration

Time (t)	Cash flow ($)	PV of cash flow ($)	"Weight"	"Weight" × Time
1	$8.50	$7.727272	0.081931	0.081931
2	$8.50	$7.024793	0.074483	0.148966
3	$8.50	$6.386176	0.067712	0.203136
4	$8.50	$5.805614	0.061556	0.246224
5	$108.50	$67.369963	0.714318	3.571590
Total		$94.313818	1.000000	4.251847

Modified Duration = 3.87

Macaulay Duration

Properties of duration

Coupon c	Price P_c	Duration D	Interest Rate r_2	Price P_c	Duration D
0	61.03	10.00	1%	147.47	8.13
2%	76.62	8.95	3%	125.75	7.95
4%	92.21	8.26	5%	107.79	7.76
6%	107.79	7.76	7%	92.89	7.56
8%	123.38	7.39	9%	80.49	7.35
10%	138.97	7.11	11%	70.12	7.12
12%	154.56	6.88	13%	61.44	6.90

Suppose we change the coupon rate or the interest rate, assuming a flat yield curve. Thinking about Macaulay Duration:

- **Duration decreases as the coupon rate rises**, because there's less weight on the principal payment at maturity.
- **Duration decreases as the interest rate rises**, because the later cashflows are discounted more heavily.

Interest rates and bond prices

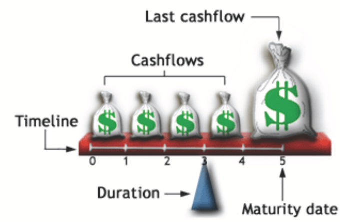
Interest rate risk arises because interest rate changes affect the bond price.

We first need to understand the detail of **how interest rates and bond prices are related**.

1. The bond price is inversely related to the yield.
2. All else equal, the shorter the term to maturity, the smaller the bond price sensitivity.
3. All else equal, the higher the coupon rate, the smaller the bond price sensitivity.
4. All else equal, the greater the yield, the smaller the bond price sensitivity.
5. The relationship between bond price and yield is not linear: it is convex.

Duration as a weighted average

For coupon bonds, the fulcrum shows the “average” time it takes to be repaid:



[On point 2:] Over time, as cash-flows occur (a coupon payment), duration changes:

Measuring Portfolio Duration

Duration of a bond portfolio (of N securities) is the weighted average of the durations of the individual bonds.

$$D_{\text{portfolio}} = \sum_{i=1}^{N} w_i \times D_i \tag{8}$$

where D_i and w_i denote the duration of, and portfolio weight of, security i; and, portfolio weight for a bond is the market value of bond divided by the market value of the portfolio.

Example: N=2

$$\begin{aligned}
P = P_1 + P_2 \quad \Rightarrow \quad D \quad &\equiv \quad -\frac{1}{P}\frac{dP}{dy} = -\frac{1}{P}\frac{d}{dy}[P_1 + P_2] \\
&= \quad -\frac{1}{P}\frac{dP_1}{dy} - \frac{1}{P}\frac{dP_2}{dy} \\
&= \quad -\frac{P_1}{P}\frac{1}{P_1}\frac{dP_1}{dy} - \frac{P_2}{P}\frac{1}{P_2}\frac{dP_2}{dy} \\
&= \quad \frac{P_1}{P}D_1 + \frac{P_2}{P}D_2
\end{aligned}$$

Complications

- The interpretation of duration as the “average time of cash flow payments” falls apart for variable coupons, conditional payments, and other contractual features for bonds.
- Important: it is the interpretation that does not apply universally. The definition of duration in (1) — sensitivity of price to changes in interest rates — is always correct.

Dollar Duration

Dollar Duration

- Our definition of duration assumed that the asset or portfolio has a non-zero value. However, several fixed income securities (swaps, forwards, etc.) as well as long-short strategies have a value of zero (at initiation).
- To compute duration on such claims, we instead use **Dollar Duration**:

$$\text{Dollar Duration} = D^{\$} = -\frac{\mathrm{d}P}{\mathrm{d}y} \tag{9}$$

- That is, the dollar duration corresponds to the sensitivity of the price (level), P, to changes in the yield, y.
- For a non-zero-valued security or portfolio with price P, the relation between (modified) duration and dollar duration is:

$$D^{\$} = P \times D \tag{10}$$

Price value of a basis point

Our previous example allows us to compute a common measure of interest rate risk, the **price value of a basis point**, also known as **PVBP** or **PV01**.

The price value of a basis point of a security is defined as

$$\text{PVBP} = -D^{\$} \times dy = -D^{\$} \times 0.01\% \tag{11}$$

This represents the decrease in the security's value from an increase in yield of 0.01%.

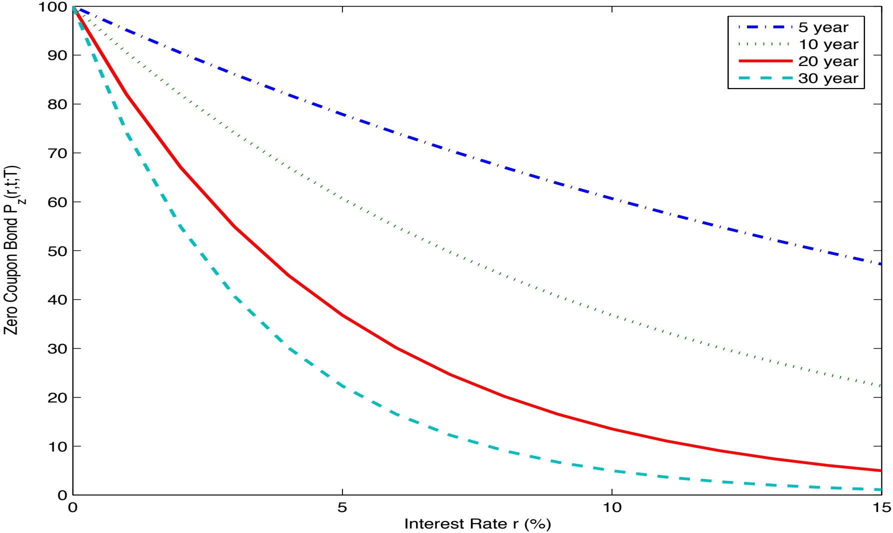
Unlike duration itself, dollar duration and PVBP are not independent of the size of the bond — a characteristic that is otherwise irrelevant in our economic analyses.

Convexity

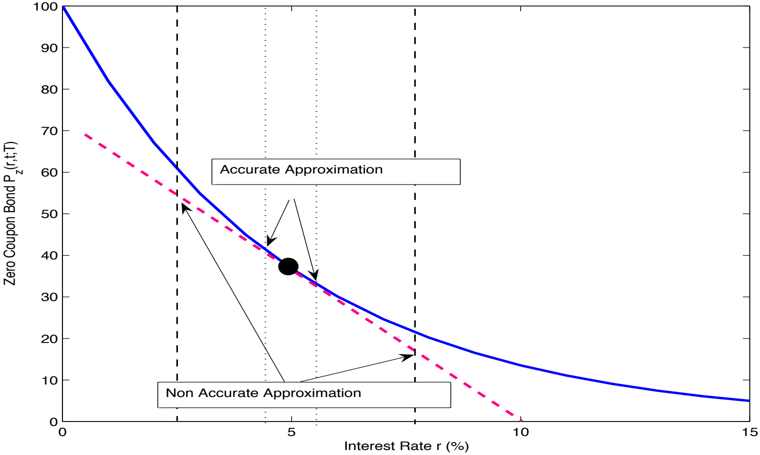
Comments on Duration

- Duration recognises that term to maturity is an inadequate measure of the time period of an investment.
- We also need to take into account the pattern of cash flows within the term to maturity.
- Extreme example: A 50-year bond which pays \$100 after 1 year and \$1 after 50 years.
  - The term to maturity of this bond is 50 years, but would you really think of it as a 50-year investment?
  - The duration of this bond would be slightly more than 1 year.
- For small changes in the yield, duration does a good job in estimating the change in price.
- **When the changes in the yield is large, what happens?**

Bond prices are nonlinear functions of interest rates



Assessing the duration approximation



For a more accurate measure of the impact a change in the term structure has on bond prices, we must account for the convexity of the price, which can be measured by taking the 2nd derivative.

Convexity

The **convexity** of an asset with price P measures the curvature of its price with respect to its yield, y, as a percentage of its price.

$$\text{Convexity} = \frac{1}{P}\frac{\mathrm{d}^2 P}{\mathrm{d}y^2} \tag{12}$$

We can also define **dollar convexity** as

$$\text{Dollar convexity} = \frac{\mathrm{d}^2 P}{\mathrm{d}y^2} \tag{13}$$

for use when the asset's price might be zero, similar to the dollar duration discussed on slide 27.

Bond price approximation

Convexity, in conjunction with duration, gives us a better approximation of the impact of changes of interest rates on bond prices.

An approximation of the percentage impact of interest rates on the price of a security is given by

$$
\begin{aligned}
\frac{\Delta P}{P} &\approx \left[\frac{dP}{dy}\frac{1}{P}\Delta y\right] + \left[\frac{1}{2}\frac{d^2 P}{dy^2}\frac{1}{P}(\Delta y)^2\right] && (14) \\
&= -\text{Duration} \times (\Delta y) \; + \; \frac{1}{2} \times \text{Convexity} \times (\Delta y)^2 && (15)
\end{aligned}
$$

If we ignore the second term (or, equivalently, assume zero convexity), then we have the simple duration approximation.

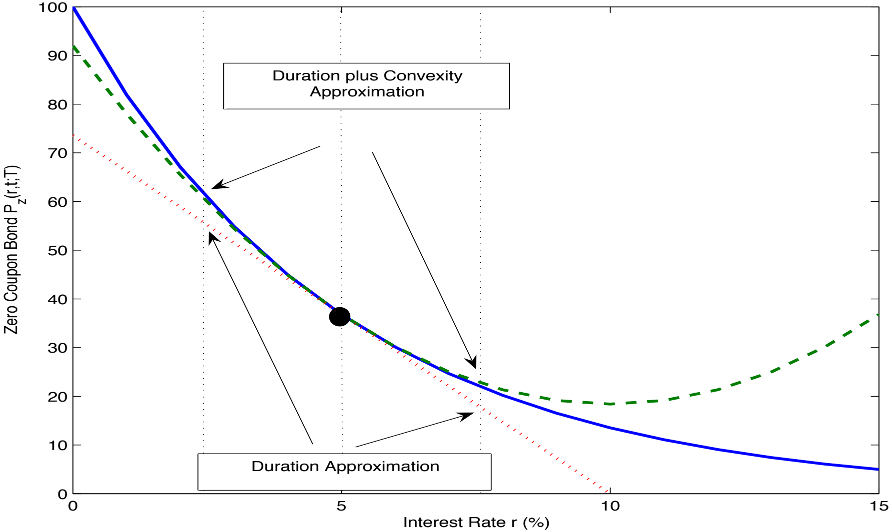
Adding the second, convexity adjustment, term gives us a better approximation. You may recognize (14) as a 2nd-order Taylor approximation.

Implications

- The practical importance of convexity grows as larger interest rate moves are considered.
- Since $(\mathrm{d}y)^2$ is always positive, large rate changes can only increase the return if the fixed income security has positive convexity.
- Why can't we profit, if the price estimate using duration is too low?
  - *The market knows this!* Thus, prices are higher, and yields lower, for bonds having more convexity.
- Positions with high convexity are effectively bets on higher-than-expected yield volatility, not the direction of interest rates.[3]

[3] The effect of duration, however, is clearly a directional exposure to interest rate changes.

Assessing the convexity approximation



Convexity of a portfolio

- The **convexity of a portfolio of securities** is given by the weighted average of convexity of individual securities, with the weight being the fraction of the portfolio's wealth invested in each security.
- Convexity of a portfolio works exactly the same as duration of a portfolio.

Immunisation

Duration application

Consider a defined-benefit pension fund that faces a known liability in the future and would like to hedge against interest rate risk.

- One method of hedging this liability risk is to buy bonds that match the liability's cash flows. This is simply known as **cash-flow matching**.
- Another method of hedging this liability risk is to buy bonds that match the liability's duration. This is known as **immunisation**.
  - For changes in rates, the assets and liabilities change by the same amount so that the net position is hedged.
  - How well does this idea work?
- Note that bond immunisation is commonly employed in **liability-driven investing**.

Example: set-up

- An insurance company must make a payment of \$19,487 in 7 years. The interest rate is 10%, so the present value of the obligation is \$10,000. The company's portfolio manager wishes to fund the obligation using 3-year zero-coupon bonds and perpetuities paying annual coupons.
- We focus on zeros and perpetuities to keep the algebra simple.
- How can the manager immunise the obligation?

Duration of a zero and perpetuity (coupon C, yield y)

	T-year Zero	Perpetuity
Price, P	$\frac{C}{(1+y)^T}$	$\frac{C}{y}$
Dollar duration $= -\frac{dP}{dy}$	$\frac{T \cdot C}{(1+y)^{T+1}}$	$\frac{C}{y^2}$
Modified duration $= -\frac{1}{P}\frac{dP}{dy}$	$\frac{T}{1+y}$	$\frac{1}{y}$
Macaulay duration $= (1+y) \times$ Modified duration	T	$\frac{1+y}{y}$

Example: set-up

- Immunisation requires that the duration of the portfolio of assets to equal the duration of the liability.
- Calculate the Macaulay duration of the liability. In this case, it is 7 years.
- Calculate the Macaulay duration of the asset portfolio:
  - 3-year zero-coupon bond: 3 years
  - Perpetuity: $(1+\text{yield})/\text{yield} = 1.1/0.1 = 11$ Years
  - Portfolio duration: $w*3 + (1\text{-}w)*11$, where w is the fraction of the portfolio invested in the 3-year zero-coupon bond.

Example: set-up

- Find the asset mix that sets the duration of assets equal to the 7-year duration of liability: w*3 + (1-w)*11 = 7 years.
- That implies that w = 0.5.
- The obligation has a present value of $10,000, the manager will purchase $5,000 of the zero-coupon bond and $5,000 of the perpetuities.
- The duration of the portfolio is equal to the duration of the liability.
- The present value of the cash flow from the portfolio equals to the present value of the future liability.

Example: commentary

- For small yield changes across all maturities, we are hedged.
- For large yield changes, the position is not perfectly hedged since bond values are non-linearly related to zero rates.
- Also, duration-hedging performs poorly for non-parallel shifts in the term structure. Of course, most of the time the term structure shifts in a non-parallel way.
- How to fix this? **Key rate** duration-hedging is commonly used, which accommodates term structure moves other than parallel shifts in the curve.
- Duration-matching is not a "set and forget" strategy.
  - When term-to-maturity changes, duration changes, so the portfolio has to be rebalanced.
  - Rebalancing tradeoffs: More frequent rebalancing increases transaction costs, thereby reducing the likelihood of achieving the target yield. Less frequent rebalancing will result in the duration wandering from the target duration, which will also reduce the likelihood of achieving the target yield.

Summary

1. All bonds are subject to interest rate risk, with longer-term bonds, low-coupon rate bonds, and low-yield bonds typically more sensitive.
2. PVBP is the change in the price of the bond if the required yield changes by 1 basis point.
3. The average 'life' of a bond is measured with Macaulay duration.
4. Both Macaulay and modified duration measures are indicators of the sensitivity of a bond's price to a change in its yield (interest rates).
5. Immunisation requires matching both duration and present value.
6. Immunisation also requires rebalancing adjustments.