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Extensions to GARCH models

- GARCH extensions tend to play around w/ a few key assumptions:
  - i. the impact of shocks on volatility depend on magnitudes, not their sign
    - ↳ i.e. h_t depends on ε_{t-i}^2, not ε_{t-i}.
  - ii. For positive cond. variances, all α_i and β_j coefficients are restricted to be non-negative & their sum must be less than 1. this is because stationarity requires:

$$\boxed{\sum \alpha_i + \sum \beta_j < 1}$$

Integrated GARCH (IGARCH)

- Relaxes assumption ii, insted stipulating coeffs must sum to 1.
  - ↳ Implying a unit root in the conditional variance. Therefore any shock to the conditional variance has a persistent effect, likely driven by some thick-tail distribution.
- In this model β_1 is not estimated b/c $\beta_1 = 1 - \alpha_1$
  - ↳ You should check this is the case: $H_0: \alpha_1 + \beta_1 = 1$ on GARCH model.

Threshold GARCH (TGARCH)

→ Allows for asymmetry where, e.g, bad news ($\varepsilon_t < 0$) has a larger impact than 'good news' ($\varepsilon_t > 0$)

$$y = \mu_t + \varepsilon_t, \quad h = \alpha_0 \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \eta_i d_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

$d = 1$ if $\varepsilon_{t-i} < 0$. α_i & β_i satisfy other std. reqs → η is meant to be pos.

∴ A significantly positive η implies negative shock are more impactful than positive.

Exponential GARCH (EGARCH)

→ The opposite of TGARCH

$$y = \mu_t + \varepsilon_t, \quad \ln h_t = \alpha_0 + \sum \alpha_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} + \sum \gamma \frac{|\varepsilon_{t-i}|}{\sqrt{h_{t-i}}} + \sum \beta_j \ln h_{t-j}$$

$$E_{t-1}(\ln h_t) = \alpha_0 + \alpha_1 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma_1 \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \beta_1 \ln h_{t-1}$$

↳ The effect of an $\varepsilon_{t-1} > 0$ shock on expect log volatility is $\alpha_1 + \gamma_1$.
  ↳ for $\varepsilon_{t-1} < 0$, it is $-\alpha_1 + \gamma_1$

→ The former is smaller than the latter if $\alpha_1 < 0$

→ No need to sign restrict β_j b/c logs are always positive

GARCH-in-mean (GARCH-M)

→ Allows the mean of $\{y_t\}$ to depend on conditional var.
  ↳ Useful for modelling asset mkts where risk-averse agents require risk premium. → higher avg. returns for holding a risky asset.

If y_t is return for risky asset:

$$y_t = \underbrace{\beta + \delta h_t}_{\text{expec. risk prem}} + \varepsilon_t, \quad \delta > 0, \quad \underbrace{h_t = \alpha_0 + \sum \alpha_i \varepsilon_{t-i}^2 + \sum \beta_j h_{t-j}}_{\text{std. GARCH process}}$$

expec. risk prem is constant if cond. var. is constant → otherwise $E(y_t)$ is increasing func. of h