

## Solution to Tutorial 1

### 1. Quiz 1 questions

- (1) What are the functions of financial markets?
- (a) Facilitating flow of funds from suppliers to demanders
  - (b) Intermediating credit between lenders and borrowers
  - (d) Discovering fair prices of assets
  - (c) Diversifying risks for investors
  - (e) All of the rest

Answer: (e)

- (2) Suppose that an asset has three possible payoffs with probabilities:

Payoff:	\$1	\$4	\$5
Probability:	0.3	0.5	0.2

What is the expected payoff on a unit of the asset and what is the standard deviation of the payoff on a unit of the asset?

Answer: 3.3 and 1.552.

Refer to the math review note 1 posted under Week 1 Module for the formulae for calculating the expectation and variance of a random variable.

Denote the payoff of the asset by  $V$ . Then, the expected payoff is:

$$E(V) = 1 \times 0.3 + 4 \times 0.5 + 5 \times 0.2 = 3.3.$$

The variance of  $V$  is defined by:

$$\text{var}(V) = E[(V - E(V))^2] = E(V^2) - (E[V])^2.$$

Hence

$$\text{var}(V) = (1^2 \times 0.3 + 4^2 \times 0.5 + 5^2 \times 0.2) - (3.3)^2 = 13.3 - 10.89 = 2.41$$

Standard deviation:  $\text{std}(V) = \sqrt{\text{var}(V)} = \sqrt{2.41} = 1.552$ .

- (3) An asset with market value at date  $t$  equal to \$360 will pay a dividend of \$34 at date  $t + 1$ . Its market value at  $t + 1$  will be \$380 (with certainty). What is the rate of return on the asset (without compounding) from  $t$  to  $t + 1$ ?

Answer: By definition, the rate of return on the asset is given by

$$r_{t+1} = \frac{p_{t+1} + d_{t+1} - p_t}{p_t} = \frac{380 + 34 - 360}{360} = 15\%.$$

(4) Which of the following statements is FALSE?

- (a) The price of an asset is ultimately determined by the supply and demand for this asset.
- (b) An investor's demand for assets is determined by her optimal portfolio selection decision.
- (c) Investors need to form expectations about the future payoffs of assets when making their portfolio selection decisions today.
- (d) Investors make their portfolio selection decisions only based on the existing prices of assets and their preferences and budget constraints.

Answer: (d), investors also need to form expectations about the future payoffs of assets so as to predict the returns of the assets. This is a difference from standard utility maximisation problem in microeconomics. The reason is that portfolio selection is an investment decision, and the demand for assets depend on the returns on assets.

(5) Is this statement true or false? When market frictions are pervasive, we are not able to draw clear implications on asset prices from the arbitrage principle alone.

Answer: True, refer to Q3 below.

2. The rate of return on the asset is defined by  $\frac{V-P}{P} = \frac{V}{P} - 1$ . Hence, the expected rate of return on the asset is:

$$E \left[ \frac{V}{P} - 1 \right] = \frac{E(V)}{P} - 1 = \frac{3.3}{2} - 1 = 0.65 = 65\%.$$

In the above equation, the first equality follows from the fact that current price  $P$  is observable and hence is not a random variable while the future payoff  $V$  is a random variable to be realised in the future. The variance of the rate of return on the asset is:

$$\text{var} \left[ \frac{V}{P} - 1 \right] = \text{var} \left[ \frac{V}{P} \right] = \frac{\text{var}(V)}{P^2} = \frac{2.41}{4} = 0.6025.$$

(The standard deviation of the rate of return is:  $\sqrt{0.6025} \approx 0.776$ .)

3. (a) Yes there exists an arbitrage strategy: borrow  $x$  pounds, exchange for  $1.5x$  dollars, then exchange for  $120 \times 1.5x = 180x$  Japanese yen, then use  $150x$  Japanese yen to exchange for  $x$  pounds and pay back the loan. The profit is  $30x$  Japanese yen. As  $x$  can be infinitely large (borrowing is not constrained as the financial markets are frictionless), this strategy can achieve unlimited profit.

This is not the unique arbitrage strategy, you may try to find other arbitrage strategies that can also achieve unlimited profits.

- (b) These exchange rates cannot persist. The presence of arbitrage opportunities (as we have shown above) imply that some currency must be overvalued or undervalued against another currency. One possibility is that dollar is over-valued against Japanese yen. Investors would exchange dollars for Japanese yen to take

the advantage of this arbitrage opportunity, which would lead to depreciation of dollar against Japanese Yen until the arbitrage opportunity disappears. When the dollar/yen exchange rate becomes 1 dollar = 100 yen, the arbitrage opportunity would disappear.

Again, this is not the only case that's possible. It's also possible that under the current exchange rates, pound is undervalued against yen, then pound would appreciate against yen until  $\text{£}1 = \text{¥}180$ . The bottom line is that the exchange rates must adjust until there is absence of arbitrage opportunities.

Note that this question provides another example of using the arbitrage principle to draw implications for asset prices (exchange rates in this question) when asset market is frictionless.

- (c) In that case, we cannot conclude whether there exists an arbitrage opportunity from the observed exchange rates, because different transaction costs can affect the effective exchange rates among these currencies. For instance, the seemingly overvaluation of dollar against yen might be there to justify a higher (possibly hidden) transaction cost when exchanging dollar for yen. Other forms of financial frictions, such as constraints or regulations on the exchange of any of these currencies, can also influence the observed exchange rates. Consequently, we cannot infer how exchange rates would change based on the arbitrage principle alone.
- (d) The statement in Quiz question (5) is correct: When market frictions are pervasive, we are not able to draw clear implications on asset prices from the arbitrage principle alone. Financial frictions such as transaction costs, borrowing constraints, and institutional restrictions on certain types of trading can influence the prices and returns of assets in complicated ways. So they need to be taken into account when we try to draw implications on asset prices using the arbitrage principle. Refer back to part (c) for an example.
- (e) This statement is false. When frictions are pervasive in an asset market, the arbitrage principle can still hold, i.e., it's possible that there are no arbitrage opportunities in the market. It's just that we cannot draw clear implications of the arbitrage principle on asset prices, as asset prices are also influenced by financial frictions.

No arbitrage is a minimal desired attribute of a well functioning financial market. When an arbitrage opportunity is present, investors can make unlimited profit out of it. So arbitrage opportunities may be present temporarily, but cannot persist.

#### 4. Brief answers

- (a) Properly determined asset prices can enable efficient allocation of financial resources, which then contribute to efficient allocation of real resources.
- (b) Refer to Greenwood and Scharfstein (2013), and the following reading:  
<https://www.ecb.europa.eu/press/key/date/2014/html/sp140902.en.html>  
Some key points:

- The financial sector has gradually extended its scope towards “non-intermediation” financial activities, such as trading. Although active trading contributes to price discovery which contributes to efficient allocation of funds and then real resources, but how big the contribution is relative to the resources consumed in active trading is questionable.
- Much of the spike in financial intermediation before the GFC was associated with household credit. Household credit might contribute less to long term growth compared with corporate credit.
- Too much young talent has been attracted to the financial industry. Though earning higher wage, talented young people might contribute more to the society if working in other sectors.
- The growth in finance can reflect increased risk-taking and leverage which increases the probability of a systemic meltdown. Financial institutions were indeed highly leveraged before the GFC.
- The complexity and interconnectedness of financial institutions tend to increase with the size of the overall financial system. This complexity makes it hard for individual investors to participate, and for regulators to impose proper regulations.

To summarise, the effect of finance on growth can be nonlinear; beyond a threshold size, the effect of finance on long-term economic growth can weaken and even become negative. However, what is the optimal size of a financial sector remains a complex question yet to be answered.