ECON30009/90080 – Tutorial 2

This Version: Semester 2, 2025

Question 1: A 2 period consumption savings model

Suppose the household has utility given by $U(c^y, c^o) = (c^y)^\alpha (c^o)^{1-\alpha}$ where $0 < \alpha < 1$. The household also receives exogenous income y^y and y^o when young and old, respectively. There exists an asset that the household can choose to save in which has a gross rate of return $R = (1 + r)$.

a) Write down the household's problem

b) Derive the household's optimality conditions

c) Solve for optimal c^y and c^o in terms of variables the household takes as given. Compute also the ratio of consumption when old to consumption when young.

d) One of the reasons why we said we would take a micro-founded approach is that our model allows us to examine how the household would change his/her consumption choices in response to policy changes. Suppose the policy is to introduce a tax on interest income. That is, τ proportion of your returns to savings Ra is now taxed. How does c^o/c^y vary in response to the introduction of a tax on interest income? Provide an explanation for why the ratio changes in that direction

Question 2: Habit Persistence in the Household Problem

Consider the two period household consumption-savings problem. The household receives exogenous income of y^y and y^o when young and old, respectively. There exists an asset a that if you save in, gives a gross return of $R = 1+r$. Suppose the household's preferences feature **habit persistence**. Habit persistence (sometimes also known as habit formation) is the feature where household's utility from consumption not just on today's consumption but also his/her history of past consumption. That is, the household is only happier if he/she is able to consume more today than she did yesterday. Specifically, we will assume that the household's preferences are given by:

$$U(c^y, c^o) = \ln c^y + \beta \ln (c^o - \eta c^y)$$

where $0 \leq \eta \leq 1$.

a State what are the endogenous choice variables of the household.

b Set up the household's problem

c Derive the household's optimality conditions

d Solve for c^y in terms of y^y, y^o, R, β and η. Explain how c^y varies with η, which is the weight households put on past consumption when old (the habit). Give a brief intuition as to why c^y varies with η in that direction.

Question 3: A CES production function

Assume $Y = (K^\gamma + L^\gamma)^{1/\gamma}$ where Y is output, K is capital and L is labor. γ is a parameter that determines the elasticity of substitution. Does this production function satisfy all the properties we would like in a production function?

a) Show that output is increasing in its inputs

b) Show that the production function features diminishing marginal returns

c) Show that the function satisfies constant returns to scale

d) Show that K and L are complements for $0 < \gamma < 1$

e) Write down the firm's profit maximization problem taking the wage rate w and rental rate of capital R as given.

f) Suppose $0 < \gamma < 1$. Derive the firm's optimality conditions

g) Suppose $R < w$ and $\gamma = 1$. Would the firm like to use labour in production? Provide some brief intuition to your answer.