Topic 7. Empirical Appraisal of the CAPM and APT

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Outline

1. Introduction

2. Testing the CAPM

   - Time series tests

   - Cross-section tests

3. Some Empirical Studies on Testing the CAPM

4. Testing the APT

Required reading: Bailey, Chapter 9

Further reading: Fama and French (2004), Fama and MacBeth (1973), Fama and French (1992,1993)

1. Introduction

- Empirical work on the CAPM and APT has two main objectives:

  (i) to test whether or not the theories should be accepted; and

  (ii) to provide information that can guide financial decisions.

- Definitive judgements are never possible in applied work. Statistical inferences need to be applied to draw sensible conclusions about how much the data support the theory.

- The art is to frame meaningful **hypotheses**, perform statistical tests using actual data, and draw justifiable conclusions.

- The CAPM and APT are often applied to aid financial decisions like portfolio selection and project evaluation.
  - Investors may be advised to buy (sell) assets that the theory predicts are ‘underpriced’ (‘overpriced’).
  - The CAPM is often used to estimate the cost of equity in capital budgeting and project evaluations (corporate finance).
- Next, we’ll first review the testing of the CAPM, describing the testing methods and discussing some empirical studies, then briefly review the testing of the APT.

2. Testing the CAPM

- Recall that the prediction of the CAPM when a risk-free asset is present (Sharpe-Lintner version of the CAPM) is:

$$\mu_j - r_0 = \beta_j(\mu_M - r_0) \quad j = 1, 2, \ldots, n \tag{1}$$

  - This prediction holds for individual assets and portfolios of assets.
  - In fact most tests of the CAPM use **portfolios** of assets in carrying out the tests.

- Tests of the CAPM investigate how well (1) fits the data. There are two main approaches to testing the CAPM.
  - Time series tests
  - Cross-section tests

2.1 Time series tests

- The time series tests interpret the CAPM prediction (1) as a linear relationship between $\mu_j - r_0$ and $\mu_M - r_0$ for a given asset j.

- In order to test this, it is necessary to find **observable counterparts** for the theoretical values of r_0, μ_j, and μ_M.
  - For r_0: r_{0t}, the observed market interest rates on a short-term government debt.
  - For μ_j: r_{jt}, the observed rates of return on asset j.
  - For μ_M: r_{Mt}, the observed rates of return on a broadly defined index of asset prices which is used to **approximate the market portfolio**.

- Substituting the terms in (1) with their observable counterparts and adding an intercept and an error term:

$$r_{jt} - r_{0t} = \alpha_j + \beta_j (r_{Mt} - r_{0t}) + \varepsilon_{jt}, \quad t = 1, 2, \ldots, T, \; j = 1, 2, \ldots, n \quad (2)$$

  – A **separate** regression equation for each asset, each can be estimated to obtain estimates of α_j and β_j.

- (2) specifies a **single-factor model** for excess returns on assets, where the single factor is excess market return.

- The CAPM predicts that the β_j's are jointly significant and the α_j's are jointly zero.

  – Further, if we include additional factors in regression (2), their regression coefficients are not significant.

- So time series tests of the CAPM focus on testing if these predictions are supported by data.

- For instance, one hypothesis that can be tested is

$$H_0: \ \alpha_1 = 0, \alpha_2 = 0, \ldots, \alpha_n = 0.$$

  (Gibbons, Ross, and Shanken (1989) provide a F test for this hypothesis)

- Comment

  – In time series tests, n is usually small relative to T.

  – Testing results are specific to the assets chosen to be put on the LHS of the regressions. The CAPM may pass the test for one set of LHS assets but fail for another.

- The Black CAPM
  - Recall the prediction of Black CAPM:

$$\mu_j = \omega + \beta_j(\mu_M - \omega) \quad j = 1, 2, \ldots, n \tag{3}$$

  - An empirical counterpart of (3):

$$r_{jt} = \alpha_j + \beta_j r_{Mt} + \varepsilon_{jt}, \;\; t = 1, 2, \ldots, T, \;\; j = 1, 2, \ldots, n \tag{4}$$

    where α_j is interpreted as $\alpha_j = \omega(1 - \beta_j)$.

  - The Black CAPM predicts that the β_j's are jointly significant, while $\left(\frac{\alpha_j}{1-\beta_j}\right)$'s are the same for all assets.

2.2 Cross-section tests

- The cross-section tests also begin with the Sharpe-Lintner CAPM prediction:

$$\mu_j - r_0 = (\mu_M - r_0)\beta_j, \quad j = 1, 2, \ldots, n \tag{1}$$

  However, (1) is interpreted as a linear relationship between $\mu_j - r_0$ and β_j which holds for all assets.

- So cross-section tests of the CAPM are tests of the SML – whether assets' expected returns depend **linearly and solely** on their betas (in a cross-section of assets).

- What are the observable counterparts for $\mu_j - r_0$ and β_j?
  - For $\mu_j - r_0$: $\bar{r}_j - \bar{r}_0 \equiv \frac{1}{T}\sum_{t=1}^{T}(r_{jt} - r_{0t})$, the sample average of the observed excess returns on asset j.

- For β_j: $\widehat{\beta}_j$, the estimated betas can be obtained using the definition of beta (σ_{jM}/σ_M^2) or from time series regressions as given in (2).

- The empirical counterpart of the SML is written as

$$\bar{r}_j - \bar{r}_0 = \gamma_0 + \gamma_1 \widehat{\beta}_j + \eta_j, \quad j = 1, 2, \ldots, n \tag{5}$$

Note that (5) specifies **one regression** for n assets.

- The Sharpe-Lintner CAPM predicts that $\gamma_0 = 0$ and $\gamma_1 = \mu_M - r_0$. Hence we can formulate the **hypothesis** as

$$H_0 : \gamma_0 = 0, \ \gamma_1 = \bar{r}_M - \bar{r}_0$$

Alternatively, we can test a weaker hypothesis:

whether $\gamma_0 = 0$ and $\gamma_1 > 0$ hold

- The Black CAPM
  - The empirical counterpart of the SML in the Black CAPM, $\mu_j = \omega + (\mu_M - \omega)\beta_j$, can be written as

$$\bar{r}_j = \gamma_0 + \gamma_1 \widehat{\beta}_j + \eta_j, \quad j = 1, 2, \ldots, n \tag{6}$$

  - The **hypothesis** can be formulated as

$$H_0 : \ \gamma_0 = \bar{\omega}, \ \gamma_1 = \bar{r}_M - \bar{\omega},$$

    where $\bar{\omega}$ is the sample average return on any zero-beta portfolio.

  - To construct a zero-beta portfolio, we need to choose a set of assets and construct a portfolio such that the portfolio's return has close to zero correlation with the market return.

  - Alternatively, we can test whether $\gamma_1 > 0$ holds.

- Question: If we run a regression like (5) (or (6)), what would be evidence supporting/against the Sharpe-Lintner CAPM or the Black CAPM? (Discussion)

- In short, cross-section tests of the CAPM involve two stages.
  - **First stage**: Obtain estimates of betas using the definition of beta or from time series regressions like (2) or (4).
  - **Second stage**: Regress sample average excess returns (or returns) on estimated betas, as shown in (5) or (6).

- In empirical studies, other explanatory variables are often added to (5) or (6) to examine whether the additional variables help explain expected returns on assets.
  - If the CAPM is the right model, the regression coefficients on the additional variables should not be significantly different from zero.
- Measurement errors in the estimates of betas cause an **errors-in-variables** problem, which can bias the estimation of the SML.
  - To reduce this problem, empirical studies usually use portfolios rather than individual assets, as estimates of betas for diversified portfolios are more precise (why?).

3. Some Empirical Studies on Testing the CAPM

- There have been a lot of empirical tests of the CAPM and mixed results have been found. We review several influential studies.

3.1 Black, Jensen and Scholes (1972)

- Data: monthly data on stocks traded on the New York Stock Exchange over the years 1926 to 1965.
- Choices of assets
  - Using portfolios rather than individual stocks.
  - 10 portfolios are formed by **sorting stocks on their betas** estimated using the **first 5 years of data**.
  - This sorting procedure is now standard in empirical tests.

Table 1. BJS (1972) Portfolios (1931-1965)

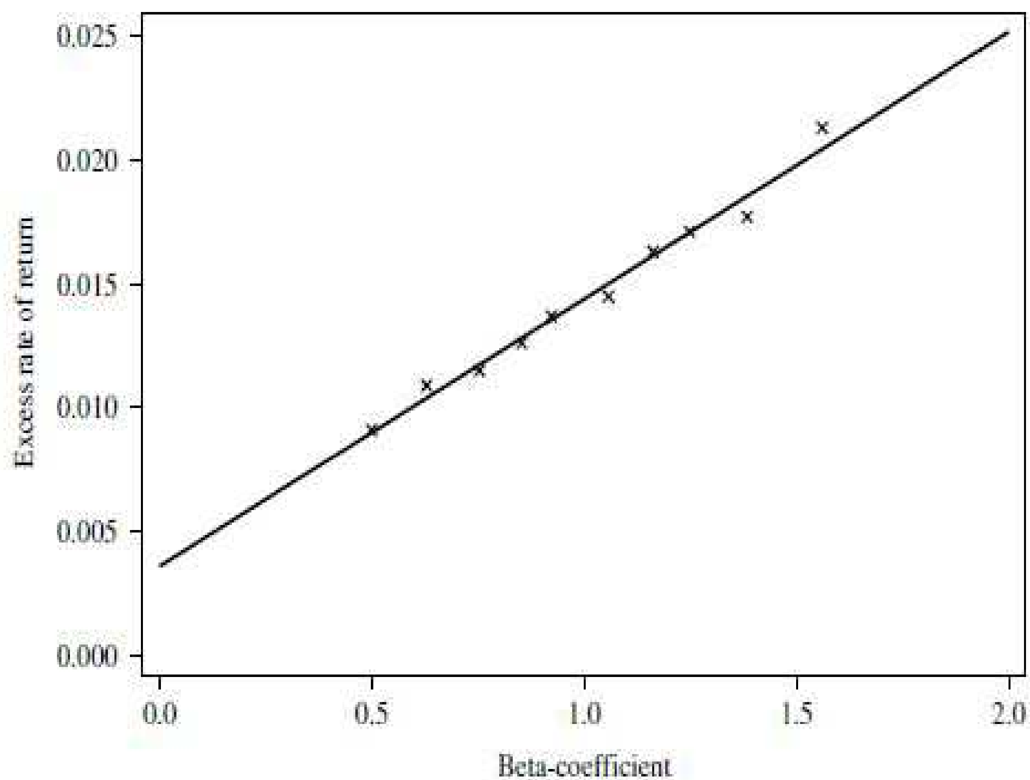
	Excess return (%)	Beta-coefficient
Portfolio 1	0.021	1.561
Portfolio 2	0.018	1.384
Portfolio 3	0.017	1.248
Portfolio 4	0.016	1.163
Portfolio 5	0.015	1.057
Portfolio 6	0.014	0.923
Portfolio 7	0.013	0.853
Portfolio 8	0.012	0.753
Portfolio 9	0.011	0.629
Portfolio 10	0.009	0.499

- Testing method
  - Both time series tests and a cross-section test, using data in the subsequent periods (1931-65).
- Results
  - Figure 1 plots the 10 portfolios' average excess returns against their estimated betas, with the OLS fitted line:

$$\text{average excess return } = 0.0036 + 0.0108 \times \text{estimated betas}$$

  - The intercept (γ_0) is significantly different from zero.
  - The slope (γ_1) is significant, but smaller than $\bar{r}_M - \bar{r}_0$.
  - What do you conclude?

Figure 1. BJS (1972) cross-section test of the CAPM



3.2 Fama and MacBeth (1973)

- Data: monthly returns for all common stocks traded on NYSE during Jan. 1926 to June 1968.
- Instead of estimating a single cross-section regression of average monthly returns on betas, they estimate **month-by-month cross-section regressions** of monthly returns on betas.
  - The times-series means of the monthly regression coefficients, together with their standard errors and autocorrelations, are used to test the CAPM predictions.
  - The regressions also include additional explanatory variables.
- Formally, a cross-section regression as follows is estimated for each month t:

$$r_{jt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_{jt-1} + \gamma_{2t}\widehat{\beta}^2_{jt-1} + \gamma_{3t}\bar{s}_{jt-1} + \eta_{jt},\ j = 1, \ldots, 20$$

– The study considers 9 subperiods, each consisting of a portfolio selection period, an initial estimation period, and a testing period.

– 20 portfolios are formed by sorting individuals stocks by their betas using the data in portfolio selection period.

– Data in initial estimation period are used to estimate the initial betas for each portfolio j. For each month t in the testing period, $\widehat{\beta}_{jt-1}$'s are re-estimated using data in initial estimation and testing periods up to $t-1$.

– Two additional variables are added, where $\bar{s}_{jt-1}$ is a measure of the idiosyncratic risk of portfolio j.

- The cross-section regressions yield estimates $(\widehat{\gamma}_{0t}, \widehat{\gamma}_{1t}, \widehat{\gamma}_{2t}, \widehat{\gamma}_{3t})$ for each t. These estimates are averaged to obtain the FM estimates for the subperiod:

$$\bar{\widehat{\gamma}}_0 = \frac{1}{T}\sum_{t=1}^{T} \widehat{\gamma}_{0t}, \quad \bar{\widehat{\gamma}}_1 = \frac{1}{T}\sum_{t=1}^{T} \widehat{\gamma}_{1t}, \quad \text{and so on}$$

- Their findings support the CAPM: the following hypotheses are all accepted.
  - Positive return-risk trade off: $\bar{\widehat{\gamma}}_1 > 0$
  - Linearity: $\bar{\widehat{\gamma}}_2 = 0$
  - No systematic effect of non-beta risk: $\bar{\widehat{\gamma}}_3 = 0$
- The approach of FM (repeated sampling of the regression coefficients through month-by-month cross-section regressions) also becomes standard in the literature.

3.3 Other studies

- There have been a large empirical literature that have identified determinants of asset returns other than beta, questioning the validity of the CAPM.
- Variables that have been found to be statistically significant include:
  - the earnings to price ratio, E/P, or profits to price ratio;
  - the debt to equity ratio, an index of the company's leverage;
  - the size of the company, as measured by the market value of its equity, ME;
  - the ratio of the book value of the firm's equity to its market value, i.e. BE/ME.
- One such influential study is the **Fama-French three-factor model** (to be reviewed in Section 4).

3.4 Roll's critique

- Roll's critique (1977) is a famous critique on the validity of empirical tests of the CAPM.
- Roll argues that the CAPM is untestable.
- The problem is that it is impossible to create or observe a truly diversified market portfolio.
  - A true 'market portfolio' would include every investment in every market, including commodities, collectibles, and virtually anything with marketable value.
  - It's impossible to observe the returns on all possible investment opportunities.
  - A market index, such as the S&P 500, is only a proxy for a fully-diversified portfolio.

- Roll also pointed out that any mean-variance **efficient portfolio** satisfies the CAPM equation exactly.
  - The CAPM prediction is obtained by applying the necessary condition for efficient portfolios to the market portfolio (see Topic 5).
  - So every test of the CAPM is just a test of whether the portfolio that represents the market portfolio is efficient.
- In view of Roll's critique, we should
  - acknowledge that every test of the CAPM is conditional upon the portfolio chosen to represent the market;
  - always try to construct the empirical market portfolio to correspond as closely as possible with the theoretical market portfolio.

3.5 Summary

- In testing the CAPM, time series regressions are employed to obtain estimates of betas for assets or portfolios. Cross-section regressions then investigate how well the expected returns on different assets are correlated with their estimated betas.
- Some studies find a positively sloped SML, and others find a flat SML.
- Some other variables are found to be important in determining expected returns on assets, such as firm size, earnings to price ratio, book to market value, etc.
- This has led to tests of multifactor models and the APT.

4. Testing the APT

4.1 Basic idea

- As discussed earlier, many tests of the CAPM find that variables other than beta are correlated with average returns on assets.

- This finding has motivated researchers to propose and test multifactor models and APT for expected returns.

- The empirical work bears a close resemblance to that on the CAPM. Both time series and cross-section tests are employed.
  - The main difference is that other explanatory variables are introduced in addition to, or instead of, the market rate of return.

- Time series studies are often made of the multifactor models described in Topic 6.
  - Consider, for example, a two-factor model:

$$r_{jt} = b_{j0} + b_{j1}F_{1t} + b_{j2}F_{2t} + \varepsilon_{jt}, \;\; j = 1, \ldots, n \;\; t = 1, \ldots, T. \quad (7)$$

    (can put $r_{jt} - r_{0t}$ on the LHS)
  - (7) specifies a separate time series regression for each asset, yielding a set of estimates $(\widehat{b}_{j0}, \widehat{b}_{j1}, \widehat{b}_{j2})$, $j = 1, \ldots, n$.
- By applying the arbitrage principle to the multifactor model (7), the APT predicts a cross-section relationship:

$$\mu_j = \lambda_0 + \lambda_1 b_{j1} + \lambda_2 b_{j2}, \quad j = 1, 2, \ldots, n, \quad (8)$$

  where λ_0, λ_1, λ_2 are parameters to be estimated.

- Similar as testing the CAPM, the cross-section regression for testing (8) could be expressed as

$$\bar{r}_j = \lambda_0 + \lambda_1 \widehat{b}_{j1} + \lambda_2 \widehat{b}_{j2} + \nu_j, \quad j = 1, 2, \ldots, n, \tag{9}$$

  where $\widehat{b}_{j1}$ and $\widehat{b}_{j2}$ are estimates from the time series regression for asset j, $\bar{r}_j$ is the sample average return on asset j and ν_j is an unobserved random error.

- Then values of λ_0, λ_1, and λ_2 can be estimated and tested.

- Factor selection
  - guidance from economic theory
  - out-of-sample forecasting

4.2 Fama and French (1992)

- The idea of Fama-French three-factor model originates in Fama and French (1992), which is in fact a test of the CAPM.
- They started with the observation that two classes of stocks have tended to do better than the market as a whole:
  - **small caps**, i.e., stocks of companies with low capitalisation (ME) or smaller size.
  - stocks with a high book-to-market ratio (BE/ME), often called **value stocks**
- So they added variables such as ME and BE/ME, in addition to beta, to explain the cross-sectional variations in average stock returns.

- They use the cross-sectional regression approach of Fama and MacBeth (1973).
  - Each month the cross-section of returns on **individual stocks** are regressed on their betas, MEs, BE/MEs, leverage, E/Ps, etc.
  - Individual stocks' MEs, BE/MEs, and other variables are precisely measured using publicly traded non-financial firms' data.
  - Betas are estimated for portfolios (from time series regressions) and then assign a portfolio's beta to each stock in the portfolio.

- Table VI in Fama and French (1992)

Variable	7/63–12/90 (330 Mos.) Mean	Std	t(Mn)	7/63–12/76 (162 Mos.) Mean	Std	t(Mn)	1/77–12/90 (168 Mos.) Mean	Std	t(Mn)
				NYSE Value-Weighted (VW) and Equal-Weighted (EW) Portfolio Returns					
VW	0.81	4.47	3.27	0.56	4.26	1.67	1.04	4.66	2.89
EW	0.97	5.49	3.19	0.77	5.70	1.72	1.15	5.28	2.82
				$R_{it} = a + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$					
a	1.77	8.51	3.77	1.86	10.10	2.33	1.69	6.67	3.27
b_2	−0.11	1.02	−1.99	−0.16	1.25	−1.62	−0.07	0.73	−1.16
b_3	0.35	1.45	4.43	0.36	1.53	2.96	0.35	1.37	3.30
				$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$					
a	2.07	5.75	6.55	1.73	6.22	3.54	2.40	5.25	5.92
b_1	−0.17	5.12	−0.62	0.10	5.33	0.25	−0.44	4.91	−1.17
b_2	−0.12	0.89	−2.52	−0.15	1.03	−1.91	−0.09	0.74	−1.64
b_3	0.33	1.24	4.80	0.34	1.36	3.17	0.31	1.10	3.67

- They conclude that “market β seems to have no role in explaining the average returns on United States stocks”.

4.3 Fama and French (1993)

- Fama and French (1993) identify five common risk factors in the returns on stocks and bonds.
- The three factors chosen to represent the stock market:
  - (i) an overall **market factor**: the excess market return (the return on the value-weighted portfolio of all stocks minus the one-month treasury bill rate), denoted as $r_M - r_0$
  - (ii) a factor related to firm size (ME) (**size factor**): the difference between returns on small-stock portfolio and big-stock portfolio, denoted as SMB
  - (iii) a factor related to book-to-market ratio (**value factor**): the difference between returns on high BE/ME portfolio and low BE/ME portfolio, denoted as HML

- The two factors chosen to represent the bond market:

(iv) a factor related to **maturity**: the difference between long-term and short-term interest rates; and

(v) a factor related to **default risk**: the difference between the yield on corporate bonds (high-risk) and government bonds (low-risk).

- This study is a **time series** analysis: to what extent the five factors can account for US equity and bond returns over the period 1963 to 1991.
  - Monthly excess returns on 25 portfolios of stocks (formed on ME and BE/ME) are regressed on the three stock market factors.

- Monthly excess returns on two government bonds and five corporate bonds are regressed on the two bond market factors.
- They find that the five factors are able to explain asset returns satisfactorily, the three stock market factors being particularly relevant for equity returns and the two interest rate factors for bonds.

- In particular, their findings from the time-series regressions for each portfolio based on the three-factor model (see their Table 6, and page 21):

$$r_{jt} - r_{0t} = \alpha_j + \beta_{j1}(r_{Mt} - r_{0t}) + \beta_{j2}SMB_t + \beta_{j3}HML_t + \varepsilon_{jt},$$

  - β_{j1}'s are close to 1, β_{j2}'s and β_{j3}'s are significant for most portfolios.

- β_{j2}'s decrease monotonically for smaller to bigger ME portfolios, becoming negative for the largest ME portfolios (small caps tend to outperform).
- β_{j3}'s increase monotonically from strong negative values for the lowest BE/ME portfolios to strong positive values for the highest BE/ME portfolios (value stocks tend to outperform).
- α_j's are close to zero.
- R^2's are close to 95%.

- There is a lot of debate about whether the outperformance tendency indicates market inefficiency (recall Topic 2).

Review questions

1. What are the two main approaches to testing the CAPM?
2. In a time series test of the CAPM, what are the observable counterparts for the theoretical values of r_0, μ_j and μ_M.
3. Understand why the market portfolio can only be approximated in applied work.
4. Understand why a time series test involves a separate regression for each chosen portfolio.
5. Understand the null hypothesis in a time series test of the Sharpe-Linter CAPM and the Black CAPM?
6. Understand why a cross-section test is a test of the SML, and why it involves just one regression for all chosen portfolios.
7. What are the observable counterparts for $\mu_j - r_0$ and β_j? What are the two stages of a cross-section test?
8. What is the null hypothesis in a cross-section test of the CAPM?
9. What is the errors-in-variables problem in a cross-section test?

10. Understand why empirical tests of the CAPM and APT usually work with portfolios instead of invididual assets.
11. Have some idea of how empirical studies sort individual stocks to form portfolios, in particular, different data are used for constructing portfolios and for testing the theory.
12. Have some idea of the month-by-month cross-section regressions in Fama and MacBeth (1973), and how the the estimates of γ's are constructed.
13. What is Roll's critique for the tests of CAPM? In particular, why he argues every test of the CAPM is just a test of whether the portfolio that represents the market portfolio is mean-variance efficient?
14. Understand the similarities and differences in the testing of APT and CAPM, in particular, a cross-section test of the APT also involves a two-stage procedure.
15. What are the major determinants of average stock returns considered in Fama and French (1992)?

16. In Fama and French (1993), what are the three factors chosen to represent the stock market, and the two factors chosen to represent the bond market?

17. Have some idea of the cross-sectional regression approach in Fama and French (1992) and the time series approach in Fama and French (1993).