ECOM40006/ECOM90013 Econometrics 3
Department of Economics
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Week 5 Tutorial Exercise

Semester 1, 2025

1. Ask any questions that you have about the lectures, etc.

2. Suppose that for some estimator $\hat{\theta}$ of the scalar parameter θ, the limiting distribution is known to be
$$n^{1/2}(\hat{\theta} - \theta) \xrightarrow{d} \mathrm{N}(0, 1).$$
Construct a 95% confidence interval for e^{θ}.

3. It is common for wage equations to be estimated with some measure of the level of education as one of the expanatory variables. To allow for a non-linear response, this variable often enters the equation in both level and squared forms. So, for the i-th individual the equation may look something like
$$wages_i = \beta educ_i + \delta educ_i^2 + x_i'\theta + u_i, i = 1, \ldots, n,$$
where w_i denotes the wage of the i-th individual, $educ_i$ their level of educational attainment, and x_i is a vector of observations on all the other expanators in the equation (including the intercept). Note that we have said nothing about how any of these variables are measured. In any event, the postulated model allows for a quadratic relationshp between wages and education and an obvious question to ask is where is the turning point in the relationship between wages and education given the x's. Elementary mathematics tells us that this occurs where $educ_i = -\beta/(2\delta)$. Suppose that the joint asymptotic distribution of $[\hat{\beta}, \hat{\delta}, \hat{\theta}']'$ is of the form
$$\begin{bmatrix} \hat{\beta} \\ \hat{\delta} \\ \hat{\theta} \end{bmatrix} \underset{a}{\sim} \mathrm{N}\left(\begin{bmatrix} \beta \\ \delta \\ \theta \end{bmatrix}, n^{-1} \begin{bmatrix} \sigma_\beta^2 & \sigma_{\beta\delta}^2 & \Sigma_{\beta\theta}' \\ \sigma_{\beta\delta}^2 & \sigma_\delta^2 & \Sigma_{\delta\theta}' \\ \Sigma_{\beta\theta} & \Sigma_{\delta\theta} & \Sigma_\theta \end{bmatrix} \right).$$

   (a) What is the marginal asymptotic distribution of $[\hat{\beta}, \hat{\delta}]$?

   (b) Using your previous answer, find the asymptotic distribution for the turning point of the wage equation as a function of education.