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Granger Causality

Granger causality means precedence: one TS variable consistently & predictably changes before another does.

Formally: ↳ Two-way/feedback Granger causality requires two corresponding one-way causal rships ($Z \leftrightarrow Y$)

↳ Z is Granger causal to Y ($Z \to Y$) only if y_{t+1} can be predicted better when the info. set contains current/lagged Z.

cond. f/cast error variance → $\{\sigma^2(e_{t+1}|\Omega_t)\} < \sigma^2(e_{t+1}|\Omega_t - \{Z_{t-i}\})$; e = f/cast error

↳ denoted as $Z - Y$

↳ A variable is instantaneously Granger causal if y_{t+1} can be better predicted when the info set contains current/lagged/future Z.

$$\sigma^2(e_{t+1}|\Omega_t + Z_{t+1}) < \sigma^2(e_{t+1}|\Omega_t)$$

this is a mutual rship, i.e. $Z - Y$ implies $Y - Z$.

Notes:

1. Granger causality definitions refer to one period ahead. For bivariate r/ships this is sufficient, but for multivariate cases it might be insufficient. → X might not cause Y_{t+1}, but it causes Z which in turn cause Y_{t+2} → Implying a two-period ahead causal r/ship
2. Granger causality b/w two stationary variables can be tested for in individual eqns. Though VAR models are typically used to test in all directions.
3. Granger causality can depend on data frequency. Daily data probably highly AR unlike annual data.
4. In a multivariate system, a variable is endogenous if the other variables jointly cause it - otherwise it's exogenous

Testing for Granger causality

Two approaches: F-test or Wald chi-square test on all lags of a variable(s) jointly.

↳ H_0: all lags have zero coefficients, H_a: some lag(s) has (have) non zero coeffs

↳ The F-test requires normally distributed error terms, but in large samples is equivalent to the Wald test because: $\lim_{m_2 \to \infty} m_1 F \sim \chi^2_{m_1}$

Tips for interpreting the printout:

→ The final test for each variable ([var.]<=ALL) indicates the endogeneity of the variable

↳ Either test details causality amongst specific variables

→ Sometimes the p-values of the F-test and Wald chi-square test will disagree. In this case you need to be cognizant of the F-test requiring normally distributed error terms.

↳ You need to therefore conduct a normality.test() on the VAR system.

↳ If the JB test has a sufficiently small p-value, reject H_0 that errors are normally distributed and rely on the Wald chi-square test.