**ECOM90024**
**Forecasting in Economics and Business**
**Tutorial 8**

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1.) Let $\varepsilon_t$ be a sequence of innovations that behaves according to an ARCH(1) process,

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

$$\upsilon_t \sim_{i.i.d.} (0,1)$$

$$\alpha_0 > 0, \alpha_1 \geq 0$$

a.) Show that $\varepsilon_t$ is serially uncorrelated. That is, verify that all of its autocorrelations (apart from its zero-th autocorrelation) are zero.

Suppose now that $\varepsilon_t$ behaves according to an ARCH($m$) process. Will $\varepsilon_t$ be still serially uncorrelated?

b.) Suppose that $\varepsilon_t$, in addition to behaving according to an ARCH($m$) process, also represents the innovations to an ARMA($p, q$) process,

$$Y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \alpha + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

What are the forecasting implications of incorporating an ARCH($m$) structure into the time series behaviour of $Y_t$?

With a plain ARMA process, the forecast intervals depends only on the forecast horizon.

When we incorporate an ARCH or GARCH structure to our innovations, then our forecast horizons will also depend on the information.

2.) The file "btc.csv" contains observations of the daily closing price of Bitcoin from 08/05/2016 to 08/05/2019. Using ***R*** you are required to do the following:

a.) Generate the daily returns on Bitcoin as the log difference of the daily price.

b.) Verify using the sample ACF and PACF, as well as appropriately specified Box tests that the daily returns are serially uncorrelated.

c.) Using the methods described in the lecture, test for the presence of ARCH effects in the daily Bitcoin returns.

d.) Estimate an appropriate ARCH model for the returns. Verify that your ARCH specification is adequate by plotting the standardized residuals from your ARCH estimation.