ECOM40006/ECOM90013 Econometrics 3
Department of Economics
University of Melbourne

Week 11 Tutorial Exercise

Semester 1, 2025

1. Ask any questions that you may have about the lectures, etc. If there is still time then please attempt the following questions.

2. Find Method of Moments estimators for the parameter θ, based on a simple random sample $X_1, X_2, \ldots, X_n$, in the following models:

   (a) The Bernoulli Distribution.

   $$f(x) = \theta^x (1-\theta)^{1-x}, \quad 0 \le \theta \le 1; x \in \{0, 1\}.$$

   Hint: $\mathrm{E}[X] = \theta$. In an ideal world you would prove this for yourself.

   (b) The Geometric Distribution.

   $$f(x) = \theta(1-\theta)^x, \quad 0 < \theta \le 1; x \in \{0, 1, 2, \ldots\}$$

   Hint: $\mathrm{E}[X] = (1-\theta)/\theta$. In an ideal world you would prove this for yourself for which an additional hint is to differentiate both sides of the identity $\sum_{x=0}^{\infty} f(x) = 1$ with respect to θ.

   (c) The Beta Distribution. $\theta = (\alpha, \beta)'$.

   $$f(x) = \frac{1}{\mathrm{B}(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad \alpha > 0, \beta > 0; \mathrm{B}(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}; 0 < x < 1.$$

   Hint: Here $\mathrm{E}[X] = \alpha/(\alpha+\beta)$ and $\mathrm{E}[X^2] = \alpha(\alpha+1)/[(\alpha+\beta)(\alpha+\beta+1)]$. Ideally you should derive these values for yourself.

   (d) The Pareto Distribution. $\theta = (\theta_1, \theta_2)'$

   $$f(x) = \frac{\theta_1 \theta_2^{\theta_1}}{x^{\theta_1+1}}, \quad \theta_1 > 0, \theta_2 > 0; \theta_2 < x < \infty.$$

   Observation: This all gets a bit messy but does simplify towards the end. As such, this question is really only for the super keen.

As a general hint, remember that probability mass/density functions, $f(x)$ say, are typically of the form:

$$f(x) = \text{normalizing constant} \times \text{kernel} = c \times k(x),$$

in an obvious notation, where the kernel of the density depends on the random variable and the normalizing constant does not. Consequently, because probability mass/density functions must sum/integrate to unity we know that

$$\int_X k(x)\,\mathrm{d}x = \frac{1}{c},$$

where $k(x)$ denotes the kernel of the probability mass/density function of a random variable X and $\int_X k(x)\,\mathrm{d}x$ should be read as the sum of the probabilities for all values x in the support of $k(x)$ if X is a discrete random variable and as the integral over the support of $k(x)$ if X is a continuous random variable.

3. Please attempt at least Question 2 from the Week 10 Tutorial Exercise.