4 Deterministic & stochastic trends

φ_1 = characteristic root

$$y_t = \varphi_1 y_{t-1} + \varepsilon_t \quad \text{, if:}$$

	$\lvert\varphi_1\rvert < 1$	$\lvert\varphi_1\rvert = 1$	$\lvert\varphi_1\rvert > 1$
$E(y_t)$	0	$\pm y_0$	$\pm\infty$
$Var(y_t)$	$\frac{\sigma^2}{1-\varphi_1^2}$	$\sigma^2 t$	∞
$Cov(y_t, y_{t-h})$	$\varphi_1^h \frac{\sigma^2}{1-\varphi_1^2}$	$\sigma^2(t-h)$	$\pm\infty$

$\therefore$ Mean, variance & autocovariance time independent only if $\lvert\varphi_1\rvert < 1$

↳ All approach infinity if > 1. Although mean is stationary for $\lvert\varphi_1\rvert = 1$, other parts approach infinity

Deterministic trends:

$y_t = y_{t-1} + \varepsilon_t$ → Pure random walk

$y_t = a_0 + y_{t-1} + \varepsilon_t$ → Random walk w/ drift

$y_t = a_0 + y_{t-1} + a_2 t + \varepsilon_t$ → Random walk w/ drift & linear trend

note: random walks are difference stationary processes

↳ If stationarity achieved after n differences, series is integrated of order n (i.e. $I(n)$)