Lecture 20: A simple static NK model

ECON30009/90080 Macroeconomics

Semester 2, 2025

Last Class

- Found that RBC model with money exhibits classical dichotomy. Monetary policy only affects nominal variables. Real variables can be analyzed separately
- Empirically, monetary policy observed to have an impact on real variables in data
- Key feature why monetary policy has no impact in RBC with money is because of flexible prices.

$$1 + i_t = \Pi_t R_t$$

  Any rise in nominal interest rate had no impact on real interest rate because prices moved to absorb all changes.

Back to Keynes

- Simple fix? Put in nominal rigidities
- In particular, we will be considering a model where there is nominal wage rigidities: fixed nominal wage (all other prices flexible)

$$w_t = \frac{\overline{W}}{P_t}$$

- Nominal wage is fixed, but real wage can change with the price level.
- What matters for firms' bottom line: real wages

Static NK model of nominal wage rigidity

Set-up

- Static (one-period) economy
- Household gets utility from consumption and real money balances.
- Household has no disutility from labour, ready to supply any amounts of labour that firm requires
- Household born with physical asset $a = 1$.
- Household receives wage income, rental income, dividends and transfer from government
- Measure 1 of households in population

Set-up

- Firms produce output according to $Y = zK^{\alpha}L^{1-\alpha}$
- Firms rent capital at rental rate R_t and hire labour to produce. There is perfect competition among firms.
- Nominal wage norm $W = \overline{W}$ implies that real wage is given by $w = \overline{W}/P$.
- Nominal wage is perfectly rigid (constant at $\overline{W}$). Does not respond to aggregate conditions

Set-up

- Monetary authority/Govt sets exogenous money supply rule:

$$M^s = \overline{M}$$

Households

Household utility

- Household has following utility:

$$U(c, m) = \epsilon \ln c + \gamma \ln m$$

  where $m = M/P$ and $0 < \gamma < 1$ and $\epsilon > 0$

- Household gets utility from consumption and real money balances

- ϵ and γ are parameters that affect how much the household prefers consumption over having real money balances

- Variations in ϵ proxy for demand shocks in our simple 1 period model

Household budget constraint

- Household has the following budget constraint in nominal terms:

$$Pc + M = PRa + \overline{W}\ell + P\pi + P\tau$$

- Note that ℓ is not necessarily equal to 1. Household supplies as much labour as firm requires. ℓ is endogenous.

- Although ℓ is endogenous, it is not chosen by the household (household has no disutility from labour, willing to supply any amounts of it)

- ℓ is determined in equilibrium (by the firm)

Household budget constraint

- Household has the following budget constraint in nominal terms:

$$Pc + M = PRa + \overline{W}\ell + P\pi + P\tau$$

- Divide by P to make the budget constraint real:

$$c + m = Ra + w\ell + \pi + \tau$$

Household utility maximization problem

- Household's problem is given by:

$$\max_{c,m} \epsilon \ln c + \gamma \ln m$$

s.t.

$$c + m = Ra + w\ell + \pi + \tau$$

Household utility maximization problem

- Household's problem is given by:

$$\max_{c,m} \epsilon \ln c + \gamma \ln m$$

s.t.

$$c + m = Ra + w\ell + \pi + \tau$$

- Can write this as a Lagrangian:

$$\mathcal{L} = \epsilon \ln c + \gamma \ln m + \lambda \left[Ra + w\ell + \pi - c + \tau - m \right]$$

Household optimality conditions

- Optimal trade-off between real money balances and consumption:

$$\frac{\gamma}{m} = \frac{\epsilon}{c}$$

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- Optimal trade-off between real money balances and consumption:

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- and budget constraint

$$c + m = Ra + w\ell + \pi + \tau$$

Firms

Firm's problem and optimality conditions

- Firms take prices as given. Nominal wage norm $\overline{W}$ is exogenous.
- Firm's problem is:

$$\max_{K,L} zK^{\alpha}L^{1-\alpha} - RK - \frac{\overline{W}}{P}L$$

- Optimality condition wrt K:

$$R = \alpha z K^{\alpha-1} L^{1-\alpha}$$

Firm's optimality conditions

- Firm's problem is:

$$\max_{K,L} zK^{\alpha}L^{1-\alpha} - RK - \frac{\overline{W}}{P}L$$

- Optimality condition wrt L:

$$\frac{\overline{W}}{P} = (1-\alpha)\, zK^{\alpha}L^{-\alpha}$$

- Re-arrange to make L the subject:

$$L^{d} = \left[\frac{P\,(1-\alpha)\, z_1 K_1^{\alpha}}{\overline{W}}\right]^{1/\alpha}$$

Price P affects firm's labour demand through its impact on real wages.

Govt/Monetary Authority

Government/Monetary authority

- Exogenous money supply rule:

$$M^s = \overline{M}$$

- Govt runs a balanced budget, revenue generated from creating money transferred to households:

$$P\tau = M^s = \overline{M}$$

Equilibrium

- Capital, K, is pre-determined in the model (affected by physical asset a that household is born with). R_t adjusts to clear capital market
- Nominal wage is sticky: wage price does not adjust to make labour market clear.
- How much labour is used in production is **determined by the firm's labour demand**

$$L^d = \left[\frac{P\left(1-\alpha\right)zK^{\alpha}}{\overline{W}}\right]^{1/\alpha}$$

- Money supply rule affects P and thus, real wages, and thus in turn L^d.

In equilibrium

- Money supply equal to money demand in equilibium

$$Pm = M^s = \overline{M}$$

In equilibrium

- Money supply equal to money demand in equilibium

$$Pm = M^s = \overline{M}$$

- We know optimal money demand is given by:

$$m = \frac{\gamma}{\epsilon} c$$

so

$$P = \frac{\overline{M}}{m} = \frac{\epsilon \overline{M}}{\gamma c}$$

In equilibrium

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- And from goods market clearing:

$$c = zK^{\alpha} L^{1-\alpha}$$

In equilibrium

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$$P = \frac{\overline{M}}{m} = \frac{\epsilon \overline{M}}{\gamma c}$$

- And from goods market clearing:

$$c = zK^{\alpha} L^{1-\alpha}$$

- further we know $L = L^d$

Price P in equilibrium

- Sub for c and for $L = L^d = \left[\frac{P(1-\alpha)zK^\alpha}{\overline{W}}\right]^{1/\alpha}$.
  And since $N = 1$,$a = 1 \implies K = 1$, we have:

$$P = \frac{1}{z}\left[\frac{\epsilon\overline{M}}{\gamma}\right]^{\alpha}\left[\frac{\overline{W}}{1-\alpha}\right]^{1-\alpha}$$

- Monetary policy (changes in money supply rule) can impact real variables in this model through the price level

AS curve

- Labour demand is increasing in P (lower real wage)

$$L^d = \left[\frac{P\,(1-\alpha)\,zK^{\alpha}}{\overline{W}}\right]^{1/\alpha}$$

- Output supplied:

$$Y = zK^{\alpha}L^{1-\alpha}$$

  where $L = L^d$ in equilibrium

- AS (aggregate supply) curve is upward sloping in (P, Y) graph, as output supplied is affected by labour used in production

AD curve

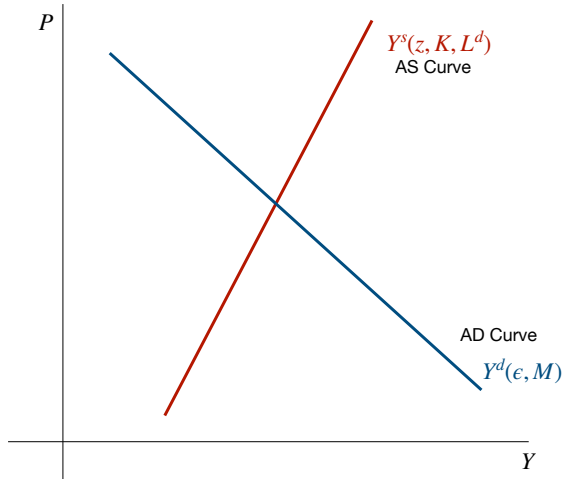
- Goods market clearing: output = consumption (no investment since 1 period model)
- From optimal trade-off between consumption and real money balances, we have:

$$c = \frac{\epsilon}{\gamma} m = \frac{\epsilon}{\gamma} \frac{\overline{M}}{P}$$

- Since consumption= output, AD (aggregate demand) curve is downward sloping in P

AS-AD curves

Putting things together:



Question: what happens in a recession?

Try this first on your own

- Suppose TFP falls in a recession. Which curve would move?
- What would happen to the price level in the economy? (Show using both the graph and equation we've derived)
- What would happen to the level of consumption in the economy?
- What can monetary policy do to counteract a fall in TFP?

A fall in TFP

- Suppose $K = 1$. Output $Y = zL^{1-\alpha}$. Fall in TFP makes all inputs less productive and output falls: **AS curve shifts inward**

- In response to decline in TFP z, P rises:

$$P \uparrow = \frac{1}{z\downarrow}\left[\frac{\epsilon\overline{M}}{\gamma}\right]^{\alpha}\left[\frac{\overline{W}}{1-\alpha}\right]^{1-\alpha}$$

- Because P rises, real wage is lower. Lower w makes L^d increase, but not enough to mitigate fall in output due to fall in z

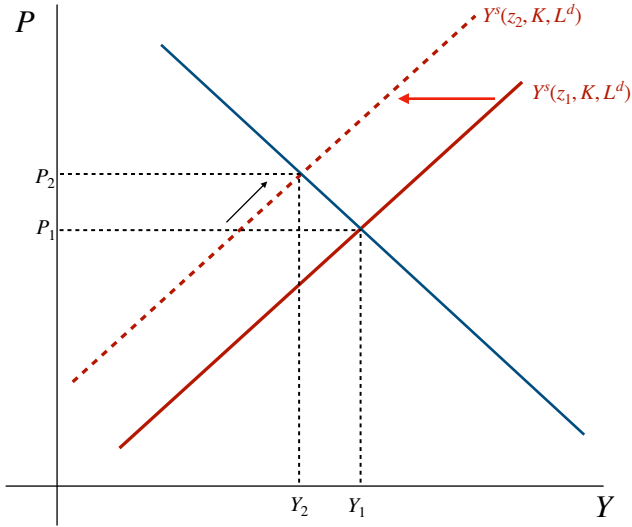
A fall in TFP

- Overall output is lower. Households have less income to spend $\implies$ less consumption

- Note that the fall in consumption demand shows up as a *movement* along the AD curve.

$$c \downarrow = \frac{\epsilon}{\gamma} m = \frac{\epsilon}{\gamma} \frac{\overline{M}}{P \uparrow}$$

A fall in TFP



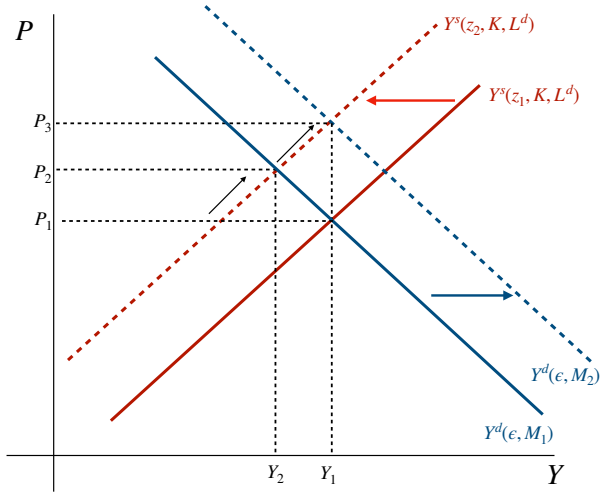
Expansionary monetary policy

- In our simple **static** NK model, expansionary monetary policy is equivalent to raising money supply $\bar{M}$
- This has the effect of raising the price level **and** consumption demand

$$c = \frac{\epsilon}{\gamma} m = \frac{\epsilon}{\gamma} \frac{\overline{M}}{P} = z \left(\frac{\epsilon \overline{M}}{\gamma} \right)^{1-\alpha} \left[\frac{\overline{W}}{1-\alpha} \right]^{\alpha-1}$$

- Consumption demand increases since increased money supply results in higher transfer to household and thus additional income
- Higher price leads to lower real wage, movement up along AS curve.

Expansionary monetary policy, graphically



Wrapping up

- Monetary policy has impact on both GDP and prices in NK model
- Simple addition was 1 sticky price: fixed nominal wage
- Next class: what does government spending shock (fiscal) do in simple NK model? what does a demand shock do?