Lecture 9
Prediction Intervals

AR(1) one-step-ahead prediction intervals

AR(1) one-step-ahead forecasting

$$E(\, Y_{n+1} \,|\, \mathscr{Y}_n \,) = \phi_1 Y_n$$

AR(1) one-step-ahead forecasting

$$E(\, Y_{n+1} \,|\, \mathscr{Y}_n \,) = \phi_1 Y_n$$

Also assume Y_{n+1} has conditional distribution

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\, \phi_1 Y_n,\, \sigma_n^2 \,)$$

95% confidence interval for Y_{n+1} (cond. on $\mathscr{Y}_n$)

$$\phi_1 Y_n \pm 1.96 \times \sigma_n$$

AR(1) one-step-ahead forecasting

$$E(\, Y_{n+1} \,|\, \mathscr{Y}_n \,) = \phi_1 Y_n$$

Also assume Y_{n+1} has conditional distribution

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\, \phi_1 Y_n,\, \sigma_n^2 \,)$$

$\sigma_n^2 = \mathrm{var}\,(\, Y_{n+1} \,|\, \mathscr{Y}_n)$

AR(1) one-step-ahead forecasting

$$E(\,Y_{n+1} \,|\, \mathscr{Y}_n\,) = \phi_1 \boldsymbol{Y}_n$$

Also assume Y_{n+1} has conditional distribution

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\,\phi_1 \boldsymbol{Y}_n,\, \boldsymbol{\sigma}_n^2\,)$$

$$\boldsymbol{\sigma}_n^2 = \mathbf{var}\,(\,Y_{n+1} \,|\, \mathscr{Y}_n) = \mathbf{var}\,(\,\underset{\substack{\uparrow \\ \text{1-step-ahead prediction error}}}{U_{n+1}} \,|\, \mathscr{Y}_n)$$

AR(1) one-step-ahead forecasting

95% prediction interval for Y_{n+1} :

$$\widehat{\phi}_1 Y_n \pm 1.96 \times \widehat{\sigma}_n$$

$\widehat{\phi}_1$: usual parameter estimate

$$\widehat{\sigma}_n^2 = \frac{1}{n} \sum_{t=2}^{n} \widehat{U}_t^2$$

(Assumes $\mathrm{var}(\, U_t \mid \mathscr{Y}_{t-1}\,)$ is constant for all t).

AR(1) one-step-ahead prediction interval

Assuming for all t

$$Y_t \,|\, \mathscr{Y}_{t-1} \;\sim\; N(\,\phi_1 Y_{t-1}\,,\, \sigma^2\,)$$

the 95% prediction interval for Y_{n+1} is

$$\widehat{\phi}_1 Y_n \;\pm\; 1.96 \times \widehat{\sigma}.$$

- The prediction interval contains Y_{n+1} with (approx) 95% probability in repeated samples.
- Interval width indicates precision of the forecast.

General one-step-ahead prediction intervals

One-step-ahead prediction interval

Assuming for all t

$$Y_t \mid \mathscr{Y}_{t-1} \sim N(\, \mu_{t-1}(\theta) \,,\, \sigma^2 \,)$$

the 95% prediction interval for Y_{n+1} is

$$\mu_n(\hat{\theta}) \pm 1.96 \times \widehat{\sigma}.$$

- $\mu_{t-1}(\theta)$: model for the conditional mean eg. ARMA with trend, dummies etc.

One-step-ahead prediction interval

Assuming for all t

$$Y_t \,|\, \mathscr{Y}_{t-1} \;\sim\; N(\,\mu_{t-1}(\theta)\,,\,\sigma^2\,)$$

the 95% prediction interval for Y_{n+1} is

$$\mu_n(\hat{\theta}) \;\pm\; 1.96 \times \widehat{\sigma}.$$

- $\mu_{t-1}(\theta)$: model for the conditional mean eg. ARMA with trend, dummies etc.
- θ : all parameters of the model
  $\hat{\theta}$: parameter estimates

Application: 3 month BAB interest rates

AR(3) model for ΔBAB3_t, Jan-2010 to Jun-2024.

$$\Delta \text{BAB3}_t = \mu + Z_t$$
$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \phi_3 Z_{t-3} + U_t$$

$\theta = (\mu,\ \phi_1,\ \phi_2,\ \phi_3\ )'.$

$\widehat{\mu} = 0.0018$

$\widehat{\phi}_1 = 0.6387$, $\widehat{\phi}_2 = -0.1530$, $\widehat{\phi}_3 = 0.1835$

$\widehat{\sigma}^2 = 0.0120$

Application: 3 month BAB interest rates

```r
# 1-step-ahead forecast object
AR3f <- forecast(AR3, h=1)

# Point forecast
print(round(AR3f$mean,4))
```

```
          Jul
2024 0.0249
```

Application: 3 month BAB interest rates

```
# 1-step-ahead forecast object
```

```
        Jul
2024 0.0249
```

```
# 95% prediction interval
Lower <- round(AR3f$lower[,"95%"], 4)
Upper <- round(AR3f$upper[,"95%"], 4)
cat(paste0("(", Lower, ",", Upper, ")"))
```

```
(-0.1902,0.2401)
```

Application: 3 month BAB interest rates

```
# 95% prediction interval
Lower <- round(AR3f$lower[,"95%"], 4)
Upper <- round(AR3f$upper[,"95%"], 4)
```

(-0.1902,0.2401)

Replicating:

```
Lower <- round(AR3f$mean-1.96*sqrt(AR3$sigm
Upper <- round(AR3f$mean+1.96*sqrt(AR3$sigm
```

(-0.1902,0.2401)

AR(1) multi-step-ahead prediction intervals

AR(1) *h*-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N(\phi_1 Y_{t-1}, \sigma^2)$$

can conveniently be expressed

$$Y_t = \phi_1 Y_{t-1} + U_t, \quad U_t \mid \mathscr{Y}_{t-1} \sim N(0, \sigma^2)$$

AR(1) h-step-ahead recursive forecasts

$$Y_t \,|\, \mathscr{Y}_{t-1} \sim N(\phi_1 Y_{t-1}, \sigma^2)$$

can conveniently be expressed

$$Y_t = \phi_1 Y_{t-1} + U_t, \quad U_t \,|\, \mathscr{Y}_{t-1} \sim N(0, \sigma^2)$$

- $E(Y_t|\mathscr{Y}_{t-1}) = \phi_1 Y_{t-1}$

  $\Rightarrow$ U_t is the one-step-ahead prediction error

AR(1) h-step-ahead recursive forecasts

$$Y_t \,|\, \mathscr{Y}_{t-1} \;\sim\; N(\,\phi_1 Y_{t-1}\,,\,\sigma^2\,)$$

can conveniently be expressed

$$Y_t = \phi_1 Y_{t-1} + U_t, \quad U_t \,|\, \mathscr{Y}_{t-1} \sim N(0,\, \sigma^2)$$

- U_t is the one-step-ahead prediction error
- $E(\,U_t \,|\, \mathscr{Y}_{t-1}\,) = 0$

AR(1) h-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N(\phi_1 Y_{t-1}, \sigma^2)$$

can conveniently be expressed

$$Y_t = \phi_1 Y_{t-1} + U_t, \quad U_t \mid \mathscr{Y}_{t-1} \sim N(0, \sigma^2)$$

- U_t is the one-step-ahead prediction error
- $E(U_t \mid \mathscr{Y}_{t-1}) = 0$
- $\mathrm{var}(U_t \mid \mathscr{Y}_{t-1}) = \sigma^2$ (assumption)

AR(1) h-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N(\phi_1 Y_{t-1}, \sigma^2)$$

can conveniently be expressed

$$Y_t = \phi_1 Y_{t-1} + U_t, \quad U_t \mid \mathscr{Y}_{t-1} \sim N(0, \sigma^2)$$

- U_t is the one-step-ahead prediction error
- $E(U_t \mid \mathscr{Y}_{t-1}) = 0$
- $\mathrm{var}(U_t \mid \mathscr{Y}_{t-1}) = \sigma^2$ (assumption)
- $U_t \mid \mathscr{Y}_{t-1}$ is normally distributed. (assumption)

AR(1) *h*-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \qquad Y_{t-1} = \phi_1 Y_{t-2} + U_{t-1}$$

$$\Rightarrow \qquad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

$$\Rightarrow \quad Y_{t-2} = \phi_1 Y_{t-3} + U_{t-2}$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

$$\Rightarrow \quad Y_{t-2} = \phi_1 Y_{t-3} + U_{t-2}$$

$$\Rightarrow \quad Y_t = \phi_1^3 Y_{t-3} + \phi_1^2 U_{t-2} + \phi_1 U_{t-1} + U_t$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^3 Y_{t-3} + \phi_1^2 U_{t-2} + \phi_1 U_{t-1} + U_t$$

AR(1) h-step-ahead recursive forecasts

$$Y_t = \phi_1 Y_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^2 Y_{t-2} + \phi_1 U_{t-1} + U_t$$

$$\Rightarrow \quad Y_t = \phi_1^3 Y_{t-3} + \phi_1^2 U_{t-2} + \phi_1 U_{t-1} + U_t$$

$$\vdots$$

$$Y_t = \phi_1^h Y_{t-h} + \sum_{j=0}^{h-1} \phi_1^j U_{t-j}$$

AR(1) h-step-ahead recursive forecasts

$$Y_t = \phi_1^h \mathbf{Y}_{t-h} + \sum_{j=0}^{h-1} \phi_1^j U_{t-j}$$

h-step-ahead forecast:

$$E(\, Y_t \,|\, \mathscr{Y}_{t-h} \,) = \phi_1^h \mathbf{Y}_{t-h}$$

since

$$E(\, U_{t-j} \,|\, \mathscr{Y}_{t-h} \,) = 0, \quad j = 0, 1, \ldots, h-1.$$

AR(1) h-step-ahead recursive forecasts

$$Y_t = \phi_1^h Y_{t-h} + \sum_{j=0}^{h-1} \phi_1^j U_{t-j}$$

h-step-ahead forecast:

$$E(\, Y_t \mid \mathscr{Y}_{t-h} \,) = \phi_1^h Y_{t-h}$$

h-step-ahead forecast error:

$$U_{t \mid t-h} = Y_t - E(\, Y_t \mid \mathscr{Y}_{t-h} \,)$$
$$= \sum_{j=0}^{h-1} \phi_1^j U_{t-j}$$

AR(1) h-step-ahead recursive forecasts

$$Y_t = \phi_1^h Y_{t-h} + \sum_{j=0}^{h-1} \phi_1^j U_{t-j}$$

h-step-ahead forecast:

$$E(\, Y_t \,|\, \mathscr{Y}_{t-h} \,) = \phi_1^h Y_{t-h}$$

h-step-ahead forecast error:

$$U_{t\,|\,t-h} = \textstyle\sum_{j=0}^{h-1} \phi_1^j \, U_{t-j}$$

$$\operatorname{var}\left(U_{t\,|\,t-h}\right) = \textstyle\sum_{j=0}^{h-1} \phi_1^{2j} \, \sigma^2$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_t \,|\, \mathscr{Y}_{t-1} \sim N(\,\phi_1 Y_{t-1}\,,\,\sigma^2\,)$$

$$\Rightarrow \quad Y_t \,|\, \mathscr{Y}_{t-h} \sim N\left(\,\phi_1^h Y_{t-h}\,,\,\sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j}\,\right)$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_{n+1} \,|\, \mathscr{Y}_n \;\sim\; N(\,\phi_1 Y_n\,,\,\sigma^2\,)$$

$$\Rightarrow \quad Y_{n+h} \,|\, \mathscr{Y}_n \;\sim\; N\left(\,\phi_1^h Y_n\,,\,\sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j}\,\right)$$

AR(1) h-step-ahead recursive forecasts

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\, \phi_1 Y_n \,,\, \sigma^2 \,)$$

$$\Rightarrow \quad Y_{n+h} \,|\, \mathscr{Y}_n \sim N\left(\phi_1^h \boldsymbol{Y}_n \,,\, \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

- Point forecast:

$$\widehat{E}(\, Y_{n+h} \,|\, \mathscr{Y}_n \,) = \widehat{\phi}_1^h \boldsymbol{Y}_n$$

AR(1) h-step-ahead recursive forecasts

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\phi_1 Y_n , \sigma^2)$$

$$\Rightarrow \quad Y_{n+h} \,|\, \mathscr{Y}_n \sim N\left(\phi_1^h Y_n , \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

- Point forecast:

$$\widehat{E}(Y_{n+h} \,|\, \mathscr{Y}_n) = \widehat{\phi}_1^h Y_n$$

- 95% prediction interval:

$$\widehat{\phi}_1^h Y_n \pm 1.96 \times \sqrt{\widehat{\sigma}^2 \sum_{j=0}^{h-1} \widehat{\phi}_1^{2j}}$$

AR(1) *h*-step-ahead recursive forecasts

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\, \phi_1 Y_n \,,\, \sigma^2 \,)$$

$$\Rightarrow \quad Y_{n+h} \,|\, \mathscr{Y}_n \sim N\left(\, \phi_1^h Y_n \,,\, \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

$$\Rightarrow \quad U_{n+h \,|\, n} \,|\, \mathscr{Y}_n \sim N\left(\, 0 \,,\, \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

AR(1) h-step-ahead recursive forecasts

$$Y_{n+1} \,|\, \mathscr{Y}_n \sim N(\, \phi_1 Y_n \,,\, \sigma^2 \,)$$

$$\Rightarrow \quad Y_{n+h} \,|\, \mathscr{Y}_n \sim N\left(\phi_1^h Y_n \,,\, \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

$$\Rightarrow \quad U_{n+h\,|\,n} \,|\, \mathscr{Y}_n \sim N\left(0 \,,\, \sigma^2 \sum_{j=0}^{h-1} \phi_1^{2j} \right)$$

- Variance of h-step-ahead forecast error is increasing with h.
- Prediction interval width increases with h.

General multi-step-ahead prediction intervals

ARMA 1-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N\left(\sum_{j=1}^{t-1} \lambda_j^{(1)} Y_{t-j}\,,\, \sigma^2 \right)$$

where $\lambda_j^{(1)}$ are functions of the ARMA coefficients.

- AR(1): $\lambda_1^{(1)} = \phi_1$ and $\lambda_j^{(1)} = 0$ for $j > 1$
- AR(2): $\lambda_1^{(1)} = \phi_1$, $\lambda_2^{(1)} = \phi_2$, $\lambda_j^{(1)} = 0$ for $j > 2$
- MA(1): $\lambda_j^{(1)} = -(-\theta_1)^j$

ARMA h-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N\left(\sum_{j=1}^{t-1} \lambda_j^{(1)} Y_{t-j}\,,\ \sigma^2\right)$$

$$\Rightarrow Y_t \mid \mathscr{Y}_{t-h} \sim N\left(\sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j}\,,\ \sigma^2 \sum_{j=0}^{h-1} \eta_j^{(h)\,2}\right)$$

- $\lambda_j^{(h)}, \eta_j^{(h)}$ are functions of the ARMA coefficients.

- Eg. AR(1): $\lambda_h^{(h)} = \phi_1^h$, $\lambda_j^{(h)} = 0$ for $j > h$
  $\eta_j^{(h)} = \phi_1^j$ for $j = 0, \ldots, h-1$

ARMA h-step-ahead recursive forecasts

$$Y_t \mid \mathscr{Y}_{t-1} \sim N\left(\sum_{j=1}^{t-1} \lambda_j^{(1)} Y_{t-j}\,,\ \sigma^2\right)$$

$$\Rightarrow Y_t \mid \mathscr{Y}_{t-h} \sim N\left(\sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j}\,,\ \sigma^2 \sum_{j=0}^{h-1} {\eta_j^{(h)}}^2\right)$$

- $\lambda_j^{(h)}, \eta_j^{(h)}$ are functions of the ARMA coefficients.
- forecast handles calculating these.

Application: 3 month BAB interest rates

```r
# 12-step-ahead forecast object
AR3f <- forecast(AR3, h=12)

# Point forecast
Point <- AR3f$mean
```

```
        Jul    Aug    Sep    Oct    Nov    Dec
2024 0.0249 0.0101 0.0087 0.0092 0.0070 0.0052
        Jan    Feb    Mar    Apr    May    Jun
2025 0.0046 0.0040 0.0034 0.0030 0.0027 0.0025
```

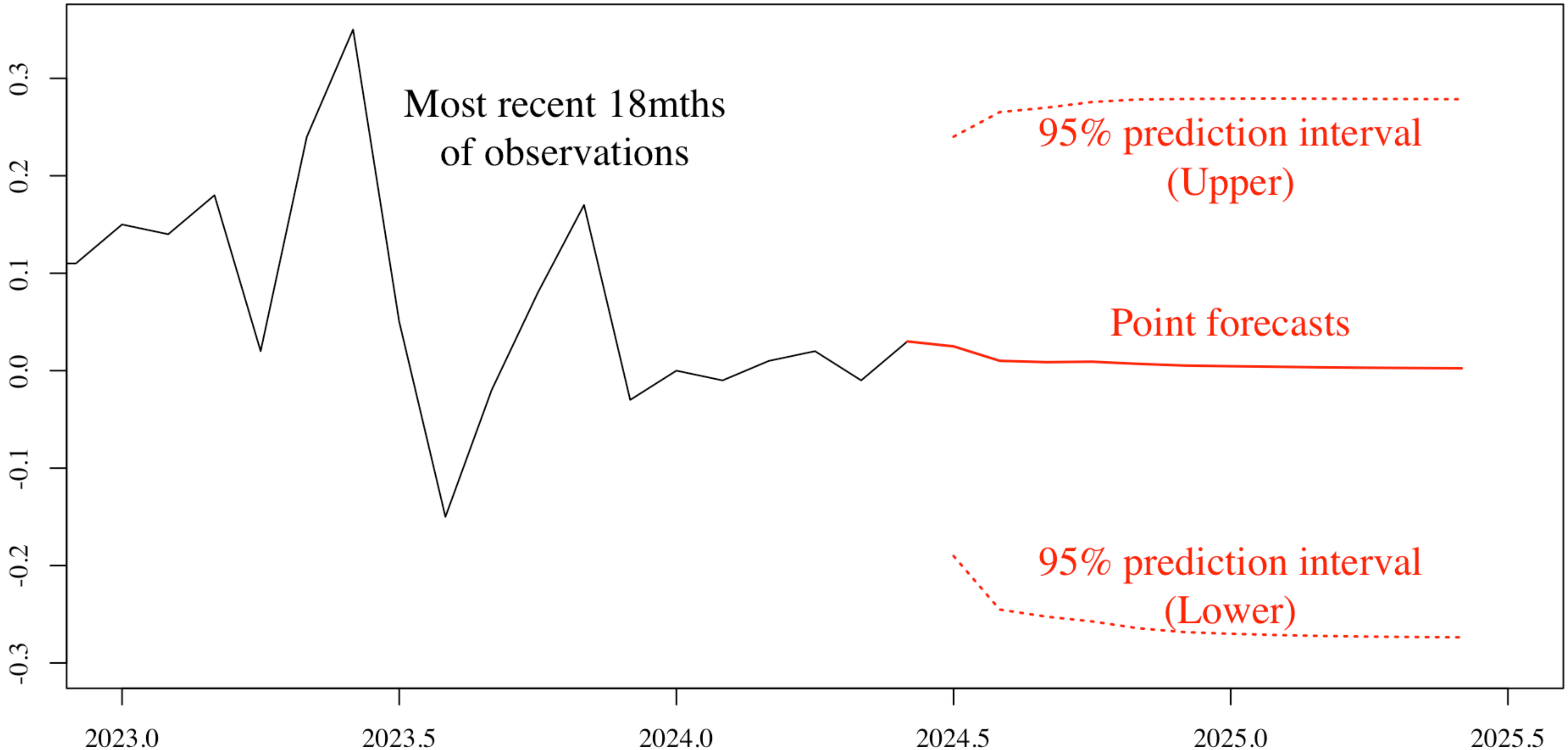
Application: 3 month BAB interest rates

```
# 12-step-ahead forecast object
AR3f <- forecast(AR3, h=12)

# Point forecast
Point <- AR3f$mean

# 95% prediction intervals
Lower <- AR3f$lower[,"95%"]
Upper <- AR3f$upper[,"95%"]
```

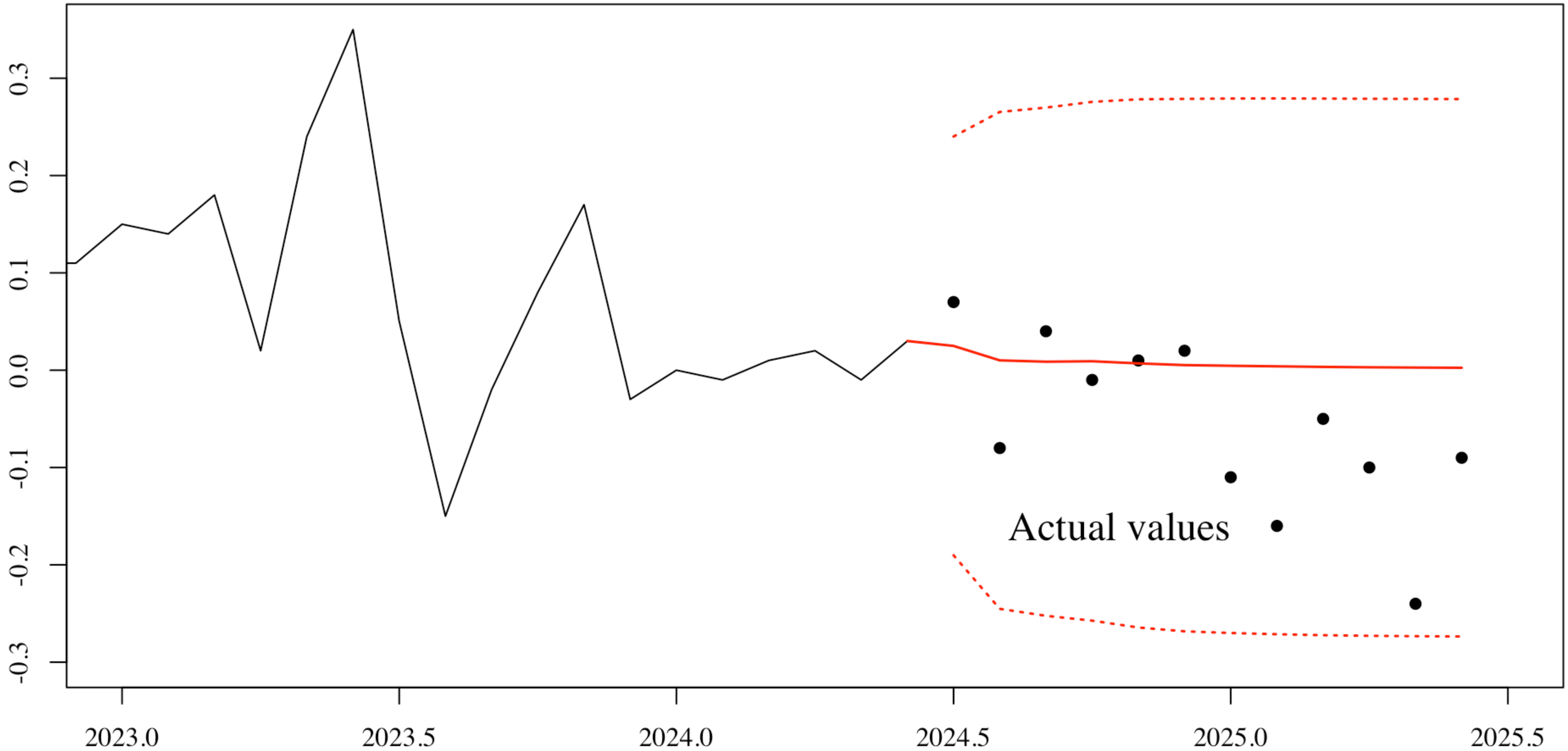
Application: 3 month BAB interest rates

	Lower	Point	Upper
Jul 2024	−0.190	0.025	0.240
Aug 2024	−0.245	0.010	0.265
Sep 2024	−0.252	0.009	0.270
Oct 2024	−0.257	0.009	0.276
Nov 2024	−0.264	0.007	0.278
Dec 2024	−0.268	0.005	0.279
Jan 2025	−0.270	0.005	0.279

Application: 3 month BAB interest rates



Application: 3 month BAB interest rates



Relaxing normality:

bootstrap prediction intervals

The normality assumption

$$Y_t \mid \mathscr{Y}_{t-1} \sim N\left(\sum_{j=1}^{t-1} \lambda_j^{(1)} Y_{t-j} \,,\; \sigma^2 \right)$$

$$\Rightarrow Y_t \mid \mathscr{Y}_{t-h} \sim N\left(\sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} \,,\; \sigma^2 \sum_{j=0}^{h-1} \eta_j^{(h)\,2} \right)$$

Justifies the usual 95% confidence interval:

$$\text{Point} \pm 1.96 \times \text{s.d.}$$

The normality assumption

$$Y_t \mid \mathscr{Y}_{t-h} \sim N\left(\sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j}\,,\ \sigma^2 \sum_{j=0}^{h-1} \eta_j^{(h)\,2}\right)$$

is based from on a recursive representation

$$Y_t = \sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} + \sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$$

derived from the ARMA model.

The normality assumption

$$Y_t = \sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} + \sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$$

The conditional normality assumption for Y_t is equivalent to a normality assumption for U_t.

- Can we tell if U_t is importantly non-normal?

- And what to do about it?

The normality assumption

$$Y_t = \underbrace{\sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j}}_{\substack{\text{Mean of } Y_t \\ \text{conditional} \\ \text{on } \mathscr{Y}_{t-j}}} + \underbrace{\sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}}_{\substack{\text{Unknown/} \\ \text{stochastic} \\ \text{given } \mathscr{Y}_{t-j}}}$$

The normality assumption

$$Y_t = \sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} + \sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$$

$$U_t \sim N(0, \sigma^2) \quad \Rightarrow \quad \sim N\left(0\,,\, \sigma^2 \sum_{j=0}^{h-1} \eta_j^{(h)\,2}\right)$$

- Can we tell if U_t is importantly non-normal?

  Look at a histogram of residuals $\widehat{U}_t$.

The normality assumption

$$Y_t = \sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} + \sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$$

$$U_t \sim N(0, \sigma^2) \quad \Rightarrow \quad \sim N\left(0\,,\, \sigma^2 \sum_{j=0}^{h-1} \eta_j^{(h)\,2}\right)$$

- And what to do about it?
  Use the residuals to simulate/"bootstrap" the *h*-step ahead prediction error distribution.

Bootstrap for prediction errors

To approximate the distribution of $\sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$.

1. Obtain residuals $\widehat{U}_t,\ t = 1, \ldots, n$.

2. From these residuals, draw a random sample (with replacement) of size h denoted

$$U_t^*,\ U_{t-1}^*,\ \ldots,\ U_{t-h+1}^*.$$

3. Compute $\sum_{j=0}^{h-1} \hat{\eta}_j^{(h)} U_{t-j}^*$

4. Repeat 2,3 “many times”.

5. Compute quantiles $q_{0.025}^*,\ q_{0.975}^*$.

Bootstrap prediction interval

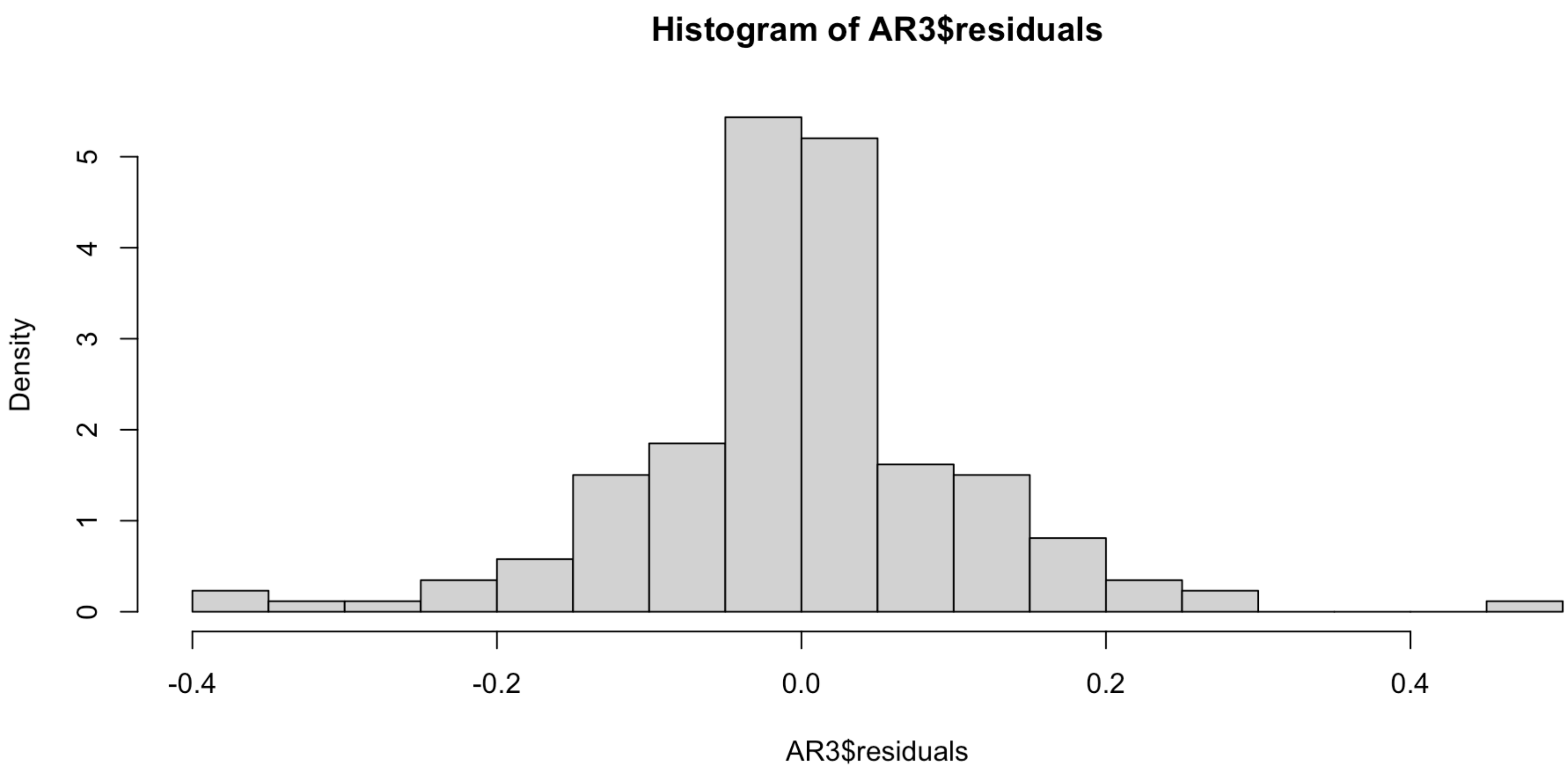
$$Y_t = \sum_{j=h}^{t-1} \lambda_j^{(h)} Y_{t-j} + \sum_{j=0}^{h-1} \eta_j^{(h)} U_{t-j}$$

$$\left(\sum_{j=h}^{t-1} \widehat{\lambda}_j^{(h)} Y_{t-j} + q^*_{0.025} \,, \right.$$

$$\left. \sum_{j=h}^{t-1} \widehat{\lambda}_j^{(h)} Y_{t-j} + q^*_{0.975} \right)$$

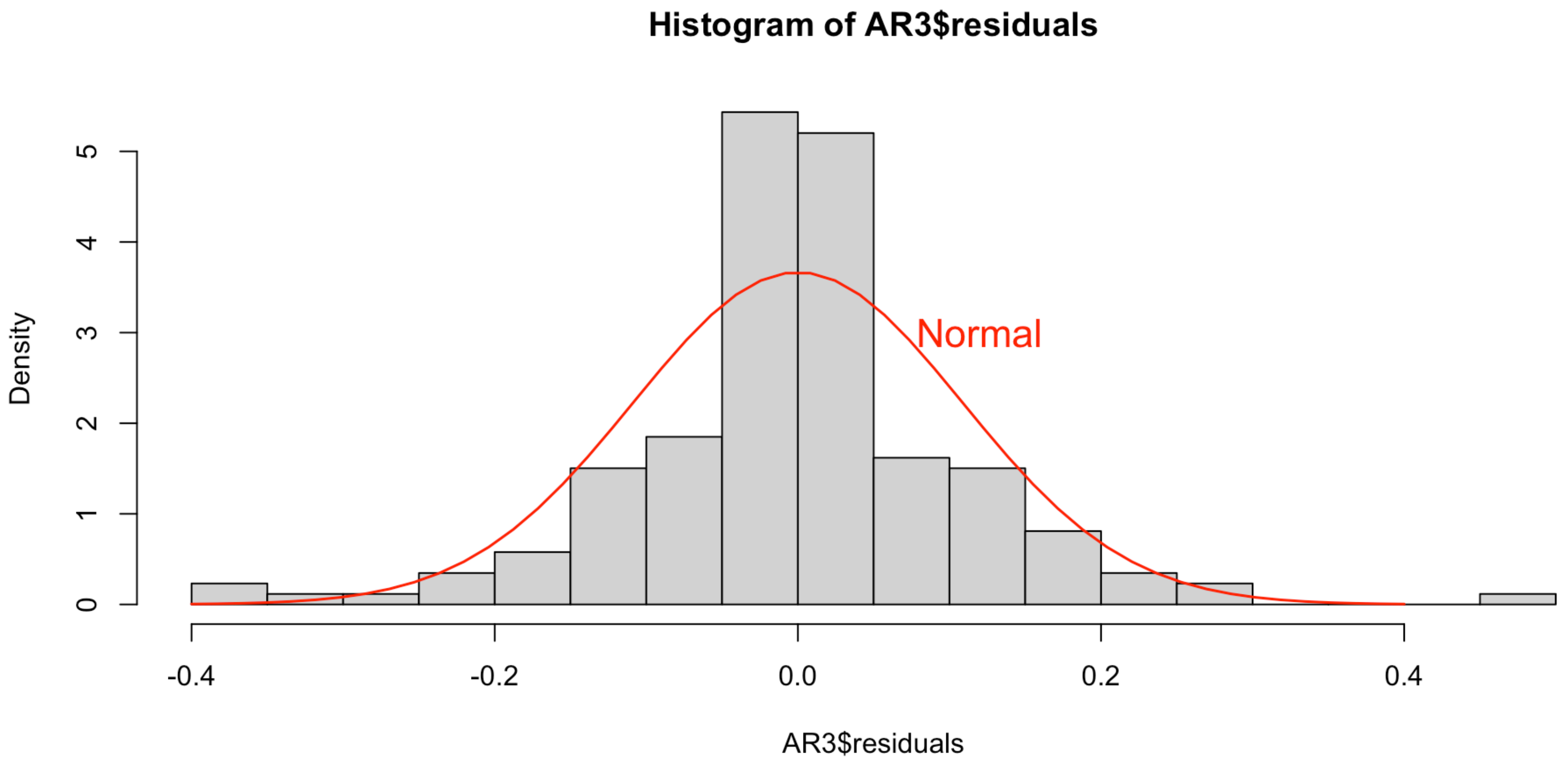
Application: 3 month BAB interest rates

```
AR3 <- Arima(DY, order=c(3,0,0))
hist(AR3$residuals, breaks=25, freq=FALSE)
```



Application: 3 month BAB interest rates

```r
AR3 <- Arima(DY, order=c(3,0,0))
hist(AR3$residuals, breaks=25, freq=FALSE)
```



Application: 3 month BAB interest rates

```r
# h-step-ahead forecasts, BS prediction int
AR3fBS <- forecast(AR3, h=12, bootstrap=TRU
LowerBS <- AR3fBS$lower[,"95%"]
UpperBS <- AR3fBS$upper[,"95%"]
```

	Time	LowerBS	Point	UpperBS
1	2024.500	-0.223	0.025	0.266
2	2024.583	-0.260	0.010	0.264
3	2024.667	-0.279	0.009	0.270
4	2024.750	-0.266	0.009	0.289
5	2024.833	-0.281	0.007	0.288
6	2024.917	-0.283	0.005	0.291
7	2025.000	-0.297	0.005	0.282

Application: 3 month BAB interest rates