ECOM40006/ECOM90013 Econometrics 3
Department of Economics
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Week 10 Tutorial Exercise Solutions

Semester 1, 2025

Question 2

(a) For each of the tests LM_1–LM_4 test $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$.

Hint: When you run the auxiliary regression (3), you will find that R bases its estimate of σ^2 on the residuals from that equation rather than on e_0. Consequently, its reported t statistics are not the statistics that you need to calculate either LM_3 or LM_4. You need to construct the statistics by hand, using the residuals from the regression of $LOGSAL$ on an intercept, $GENDER$, $MINORITY$, and $JOBCAT$.

Solution:
See code in file `Ex10Q2_Rcode.R`.

To see why LM_3 and LM_4 are equal, consider again the regressions that they come from. Generically the regressions are of the form $y = x\beta + \text{error}$ and so the OLS estimator for β is $\hat{\beta} = x'y/x'x$ and $\text{Var}\left[\hat{\beta}\right] = \sigma^2/x'x$, so that the (asymptotic) t-ratio on $\hat{\beta}$ is

$$t = \frac{\hat{\beta}}{\sqrt{\widehat{\text{Var}}\left[\hat{\beta}\right]}} = \frac{\hat{\beta}}{\sqrt{\hat{\sigma}^2/x'x}} = \frac{n^{1/2}x'y}{\sqrt{(y'M_x y)(x'x)}} = \frac{n^{1/2}x'y}{\sqrt{(y'y)(x'x) - (x'y)^2}},$$

where $\hat{\sigma}^2 = n^{-1}y'M_x y$. (Remember that all these inner products are scalars so we get to cancel things out that we wouldn't otherwise be able to.) In the case of LM_3 we have, from equation (3) and the fact that $e_0 = M_{X_2}y$,

$$\hat{\phi} = \frac{(M_{X_2}x_1)'e_0}{(M_{X_2}x_1)'(M_{X_2}x_1)} = \frac{x_1'M_{X_2}M_{X_2}y}{x_1'M_{X_2}x_1} = \frac{x_1'M_{X_2}y}{x_1'M_{X_2}x_1}.$$

Moreover, your variance estimate from this equation is

$$\begin{aligned}\hat{\sigma}^2_\phi &= n^{-1}e_0'M_{M_{X_2}x_1}e_0 = n^{-1}y'M_{X_2}M_{M_{X_2}x_1}M_{X_2}y \\ &= n^{-1}y'(M_{X_2} - M_{X_2}x_1(x_1'M_{X_2}x_1)^{-1}x_1'M_{X_2})y, = n^{-1}\left[y'M_{X_2}y - \frac{(x_1'M_{X_2}y)^2}{x_1'M_{X_2}x_1}\right].\end{aligned}$$

The resulting t-ratio is

$$t_\phi = \frac{\hat{\phi}}{\sqrt{\widehat{\text{Var}}\left[\hat{\phi}\right]}} = \frac{x_1' M_{X_2} y / x_1' M_{X_2} x_1}{\sqrt{\hat{\sigma}_\phi^2 / x_1' M_{X_2} x_1}} = \frac{x_1' M_{X_2} y}{\sqrt{\hat{\sigma}_\phi^2 x_1' M_{X_2} x_1}}$$
$$= \frac{n^{1/2} x_1' M_{X_2} y}{\sqrt{(y' M_{X_2} y)(x_1' M_{X_2} x_1) - (x_1' M_{X_2} y)^2}}.$$

LM_4 is based on equation (4) and so

$$\hat{\varphi} = \frac{e_0' M_{X_2} x_1}{e_0' e_0} = \frac{y' M_{X_2} M_{X_2} x_1}{y' M_{X_2} M_{X_2} y} = \frac{y' M_{X_2} x_1}{y' M_{X_2} y} = \frac{x_1' M_{X_2} y}{y' M_{X_2} y}.$$

As for the variance estimator, it takes the form

$$\hat{\sigma}_\varphi^2 = n^{-1} x_1' M_{X_2} M_{e_0} M_{X_2} x_1 = n^{-1} x_1' (M_{X_2} - M_{X_2} y (y' M_{X_2} y)^{-1} y' M_{X_2}) x_1$$
$$= n^{-1} \left[x_1' M_{X_2} x_1 - \frac{(x_1' M_{X_2} y)^2}{y' M_{X_2} y} \right].$$

Combining these results we can construct the relevant t-ratio as

$$t_\varphi = \frac{\hat{\varphi}}{\sqrt{\widehat{\text{Var}}\left[\hat{\varphi}\right]}} = \frac{x_1' M_{X_2} y / y' M_{X_2} y}{\sqrt{\hat{\sigma}_\varphi^2 / y' M_{X_2} y}} = \frac{x_1' M_{X_2} y}{\sqrt{\hat{\sigma}_\varphi^2 y' M_{X_2} y}}$$
$$= \frac{n^{1/2} x_1' M_{X_2} y}{\sqrt{(x_1' M_{X_2} x_1)(y' M_{X_2} y) - (x_1' M_{X_2} y)^2}}.$$

We see that the formulae for t_ϕ and t_φ are identical, which is why the t-ratios are numerically the same.

(b) Use $LM1$ to test $H_0 : \beta_3 = \beta_4 = 0$.

Solution:
See code in file `Ex10Q2_Rcode.R`.

We haven't spoken of it but I hope that over the journey you have learned good habits when answering hypothesis testing questions. Good habits include:

(i) A clear statement of both null and alternative hypotheses: $H_0 : \beta_3 = \beta_4 = 0$ versus H_1 : Either one or both of β_3 and β_4 does not equal zero.

(ii) A clear statement of the relevant test statistic and its distribution under the null: $nR^2 \overset{H_0}{\underset{a}{\sim}} \chi_2^2$, where n is the sample size and R^2 is the coefficient of determination from the auxiliary regression of the disturbances from the restricted model against all of the explanatory variables in the unrestricted model. The restricted model here is

$$LOGSAL_i = \beta_0 + \beta_1 EDUC_i + \beta_2 GENDER_i + u_i,$$

where $i = 1, 2, \ldots, 474$.

(iii) A decision rule: Cannot accept H_0 if $LM_1 = nR^2 > \chi_{2,1-\alpha}^2 = 5.99$, where $\chi_{2,1-\alpha}^2$ is the critical value that cuts off an $100\alpha\%$ critical value from a χ_2^2 distribution. In this case we have chosen $\alpha = 0.05$.

(iv) Use data to compute test statistic: $LM_1 = 474 * 0.5684 \approx 208.75$
Note that, in principal, all the other stuff should be arrnaged before ever looking at the data or other results based on the same data to ensure that you test procedures have the sampling distributions and sizes that you think they have. Contamination by prior use of the data will jeopardize this.

(v) Draw a conclusion: $LM1 > 5.99$ and so the evidence does not support the null hypothesis.

(c) Use $LM2$ to test $H_0 : \beta_3 + \beta_4 = 0$.

Solution:

See code in file `Ex10Q2_Rcode.R`.

Here $H_0 : \beta_3 + \beta_4 = 0$ versus $H_1 : \beta_3 + \beta_4 \neq 0$. Under the null hypothesis, $LM_2 = nR^2 \underset{a}{\sim} \chi^2_1$, where R^2 comes from the restricted regression. In order to obtain the restricted regression we shall first re-write the unrestricted model by adding and subtracting $\beta_4 MINORITY_i$ to obtain

$$\begin{aligned}
LOGSAL_i &= \beta_0 + \beta_1 EDUC_i + \beta_2 GENDER_i + \beta_3 MINORITY_i \\
&\quad + \beta_4 JOBCAT_i + \beta_4 MINORITY_i - \beta_4 MINORITY_i + u_i \\
&= \beta_0 + \beta_1 EDUC_i + \beta_2 GENDER_i + (\beta_3 + \beta_4) MINORITY_i + \beta_4 Z_i + u_i, \\
&= \beta_0 + \beta_1 EDUC_i + \beta_2 GENDER_i + \delta MINORITY_i + \beta_4 Z_i + u_i,
\end{aligned}$$

where $Z_i = JOBCAT_i - MINORITY_i$. (Equally, we could have added and subtracted $\beta_3 JOBCAT_i$ to get a similar outcome, but that would have results in Z_i replacing the variable $MINORITY_i$ in the equations, with a coefficient of $-\beta_3$. The coefficient of $JOBCAT_i$ would then be $\delta = \beta_3 + \beta_4$.) We could estimate this model by ordinary least squares/maximum likelihood and obtain an estimate of β_3 by subtracting $\hat{\beta}_4$ from $\hat{\delta}$. Of course, variances would be more difficult because you would need to solve

$$\operatorname{Var}\left[\hat{\beta}_3\right] = \operatorname{Var}\left[\hat{\delta} - \hat{\beta}_4\right] = \operatorname{Var}\left[\hat{\delta}\right] + \operatorname{Var}\left[\hat{\beta}_4\right] - 2\operatorname{Cov}\left[\hat{\delta}, \hat{\beta}_4\right].$$

The restricted equation is obtained by imposing $\delta = \beta_3 + \beta_4 = 0$ during estimation, which is done by simply omitting the variable $MINORITY_i$ from the estimated equation:

$$LOGSAL_i = \beta_0 + \beta_1 EDUC_i + \beta_2 GENDER_i + \beta_4 Z_i + u_i$$

So to fit the restricted model we need to create the new variable Z_i and omit $MINORITY_i$ from the fitted equation. Thus, in the notation of the preamble, we see that $x_1 = MINORITY$ and $X_2 = [\mathbf{1}, EDUC, GENDER, Z]$. LM_2 is then nR^2, where R^2 is the coefficient of determination from the auxiliary regression of the residuals from the restricted equation on the residuals from a regression of $MINORITY$ on $[\mathbf{1}, EDUC, GENDER, Z]$. Any value of $LM_2 > \chi^2_{1,0.95} = 3.84$ will be deemed evidence against H_0. We see that $LM_2 = 474{*}0.0787 = 37.30097 > 3.84$, which we count as evidence against the null hypothesis.

Question 3

All R code for this question can be found in the file `Ex10Q3_Rcode.R`.

(a) Determine the log-likelihood for the case of Cauchy disturbances. Show that the maximum likelihood estimates for α and β are obtained from the two conditions

$$\sum_{i=1}^{n} u_i \left(1+u_i^2\right)^{-1} = 0$$
$$\sum_{i=1}^{n} u_i x_i \left(1+u_i^2\right)^{-1} = 0.$$

Solution:
The likelihood is the joint density of the sample. As the data are assumed to be iid we see that

$$f(y_1,\ldots,y_n;\alpha,\beta) = \prod_{i=1}^{n} f(y_i-\alpha-\beta x_i) = \prod_{i=1}^{n}\left(\pi\left(1+(y_i-\alpha-\beta x_i)^2\right)\right)^{-1}$$

As $f(y_1,\ldots,y_n;\alpha,\beta) = \mathcal{L}(\alpha,\beta;y_1,\ldots,y_n)$, the log-likelihood is

$$\ln\mathcal{L}(\alpha,\beta) = -\sum_{i=1}^{n}\ln\left(\pi\left(1+(y_i-\alpha-\beta x_i)^2\right)\right) = -n\ln\pi - \sum_{i=1}^{n}\ln\left(1+(y_i-\alpha-\beta x_i)^2\right).$$

The score for α and β are, respectively,

$$\begin{aligned}
\frac{\partial\ln\mathcal{L}(\alpha,\beta)}{\partial\alpha} &= -\sum_{i=1}^{n}\frac{\partial\ln\left(1+(y_i-\alpha-\beta x_i)^2\right)}{\partial\left(1+(y_i-\alpha-\beta x_i)^2\right)}\frac{\partial\left(1+(y_i-\alpha-\beta x_i)^2\right)}{\partial\alpha}\\
&= 2\sum_{i=1}^{n}\frac{(y_i-\alpha-\beta x_i)}{(1+(y_i-\alpha-\beta x_i)^2)},\\
\frac{\partial\ln\mathcal{L}(\alpha,\beta)}{\partial\beta} &= -\sum_{i=1}^{n}\frac{\partial\ln\left(1+(y_i-\alpha-\beta x_i)^2\right)}{\partial\left(1+(y_i-\alpha-\beta x_i)^2\right)}\frac{\partial\left(1+(y_i-\alpha-\beta x_i)^2\right)}{\partial\beta}\\
&= 2\sum_{i=1}^{n}\frac{(y_i-\alpha-\beta x_i)x_i}{(1+(y_i-\alpha-\beta x_i)^2)}.
\end{aligned}$$

Consequently, the first-order conditions are

$$\frac{\partial\ln\mathcal{L}(\widehat{\alpha},\hat{\beta})}{\partial\alpha} = 0 \implies 2\sum_{i=1}^{n}\frac{(y_i-\widehat{\alpha}-\hat{\beta}x_i)}{\left(1+(y_i-\widehat{\alpha}-\hat{\beta}x_i)^2\right)} = 0 \implies \sum_{i=1}^{n}\frac{e_i}{(1+e_i^2)} = 0,$$
$$\frac{\partial\ln\mathcal{L}(\widehat{\alpha},\hat{\beta})}{\partial\beta} = 0 \implies 2\sum_{i=1}^{n}\frac{(y_i-\widehat{\alpha}-\hat{\beta}x_i)x_i}{\left(1+(y_i-\widehat{\alpha}-\hat{\beta}x_i)^2\right)} = 0 \implies \sum_{i=1}^{n}\frac{x_i e_i}{(1+e_i^2)} = 0,$$

as required. It is worth noting, as we shall use it below, that defining X to have i-th row $[1,x_i]$ and ϵ to have i-th element $e_i/(1+e_i)^2$ allows us to re-write the scores in the form$X'\epsilon$.

(b) Use the R package `nleqslv` to obtain mles for α and β based on the Cauchy distribution. Determine also the (asymptotic) standard errors of these estimates.

Hint: You probably want to use an opg form of the information matrix here.

Bigger Hint: The R package `nleqslv` needs you to give it a function to find the zero of. Below is the code that I used to define such a function. It may save you some time.

```
##
# Create a function to construct the score.
# This is needed by nleqslv.
# - theta are values for the parameters
# - x and y are your regression variables
# (in an obvious notation)
##
cauchy.score <- function(theta,x,y){
   alpha <- theta[1]
   beta <- theta[2]
   n <- length(y)
   e <- rep(0,n)
   for(i in 1:n)
      {
         temp=y[i]-alpha-beta*x[i]
         e[i]=temp/(1+temp*temp)
      }
   score.alpha <- sum(e)
   score.beta <- t(x) %*% e
   score=c(score.alpha,score.beta)
   return(score)
}
```

Solution:
First, we know that the asymptotic distribution of mles is $\hat{\theta} \underset{a}{\sim} \mathrm{N}(\theta, (I_n(\theta))^{-1})$, where $I_n(\theta)$ is Fisher's information for the sample. So, if we can evaluate the expectations then the best way to estimate $I_n(\theta)$ is simply to plug $\hat{\theta}$ into your expression for it. That is, $\hat{I}_n(\theta) = I_n(\hat{\theta})$. Typically, we can't and so we have to choose between an estimator based on the hessian:

$$\hat{I}_n^H(\theta) = -\left.\frac{\partial^2 \ln \mathcal{L}(\theta)}{\partial\theta\partial\theta'}\right|_{\theta=\hat{\theta}} = -\frac{\partial^2 \ln \mathcal{L}(\hat{\theta})}{\partial\theta\partial\theta'} = -\sum_{i=1}^{n} \frac{\partial^2 \ln \mathcal{L}(\hat{\theta}; y_i)}{\partial\theta\partial\theta'}$$

and one based on the variance of the scores, the so-called *outer product of gradient* (opg) form

$$\hat{I}_n^{opg}(\theta) = \left.\frac{\partial \ln \mathcal{L}(\theta)}{\partial\theta}\frac{\partial \ln \mathcal{L}(\theta)}{\partial\theta'}\right|_{\theta=\hat{\theta}} = \sum_{i=1}^{n} \frac{\partial \ln \mathcal{L}(\hat{\theta}; y_i)}{\partial\theta}\frac{\partial \ln \mathcal{L}(\hat{\theta}; y_i)}{\partial\theta'}.$$

The advantage of the opg form is that we have had to find these derivatives already in order to construct the first-order conditions, so it doesn't require any extra differentiation. That is, it is attractive because it is easy (hence the well-meaning, if somewhat mis-guided tip). This is how I calculate $\hat{I}_n(\theta)$ in the code in `Ex10Q3_Rcode.R`. To yield the standard errors we still need to invert it and then take square roots of the elements on the leading diagonal. All up we find that

$$y = \underset{(0.1829)}{0.1364} + \underset{(0.0206)}{0.9146}\, x.$$

The values in parentheses are estimated standard errors.

(c) Estimate α and β by maximum likelihood, based on the Normal distribution. Compute also the standard errors of these estimates. Compare the results with those obtained by OLS.

Solution:
The log-likelihood function for α and β is given by

$$\ln \mathcal{L}(\alpha, \beta; y) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \alpha - \beta x_i)^2.$$

Consequently, the first order conditions are:

$$\frac{\partial \ln \mathcal{L}}{\partial \alpha} = 0 \implies \sum_{i=1}^{n} e_i = 0,$$
$$\frac{\partial \ln \mathcal{L}}{\partial \beta} = 0 \implies \sum_{i=1}^{n} x_i e_i = 0,$$
$$\frac{\partial \ln \mathcal{L}}{\partial \sigma^2} = 0 \implies \hat{\sigma}_n^2 = \frac{e'e}{n},$$

where $e = y - \widehat{\alpha}_n - \hat{\beta}_n x$. We see that the FOCs for α and β (i) are identical to those for OLS and (ii) that they don't depend upon $\hat{\sigma}^2$, so it matters not if variance estimates differ. Now OLS doesn't offer a variance estimate but we use $s_n^2 = (n-2)^{-1}e'e$. So the only difference between the two procedures will be in our estimated standard errors. Just to confirm, we can check the second-order conditions for the mles. Writing $\theta = [\alpha, \beta, \sigma^2]'$, and recalling that $u_i = y_i - \alpha - \beta x_i$'

$$\frac{\partial^2 \ln \mathcal{L}}{\partial \theta \partial \theta'} = \begin{bmatrix} -\frac{n}{\sigma^2} & -\frac{1}{\sigma^2}\sum_{i=1}^{n} x_i & -\frac{1}{\sigma^4}\sum_{i=1}^{n} u_i \\ -\frac{1}{\sigma^2}\sum_{i=1}^{n} x_i & -\frac{1}{\sigma^2}\sum_{i=1}^{n} x_i^2 & -\frac{1}{\sigma^4}\sum_{i=1}^{n} x_i u_i \\ -\frac{1}{\sigma^4}\sum_{i=1}^{n} u_i & -\frac{1}{\sigma^4}\sum_{i=1}^{n} x_i u_i & \frac{n}{2\sigma^4} - \frac{1}{\sigma^6}\sum_{i=1}^{n} x_i u_i^2 \end{bmatrix}$$

and so

$$
\begin{aligned}
I(\theta) &= -\operatorname{E}_X\left[\operatorname{E}\left[\frac{\partial^2 \ln \mathcal{L}}{\partial\theta\partial\theta'} \mid X\right]\right] \\
&= \operatorname{E}\begin{bmatrix} \frac{n}{\sigma^2} & \frac{1}{\sigma^2}\sum_{i=1}^n x_i & \frac{1}{\sigma^4}\sum_{i=1}^n \operatorname{E}[u_i \mid x_i] \\ \frac{1}{\sigma^2}\sum_{i=1}^n x_i & \frac{1}{\sigma^2}\sum_{i=1}^n x_i^2 & \frac{1}{\sigma^4}\sum_{i=1}^n x_i \operatorname{E}[u_i \mid x_i] \\ \frac{1}{\sigma^4}\sum_{i=1}^n \operatorname{E}[u_i \mid x_i] & \frac{1}{\sigma^4}\sum_{i=1}^n x_i \operatorname{E}[u_i \mid x_i] & -\frac{n}{2\sigma^4} + \frac{1}{\sigma^6}\sum_{i=1}^n x_i \operatorname{E}[u_i^2 \mid x_i] \end{bmatrix} \\
&= \begin{bmatrix} \frac{n}{\sigma^2} & \frac{1}{\sigma^2}\sum_{i=1}^n x_i & 0 \\ \frac{1}{\sigma^2}\sum_{i=1}^n x_i & \frac{1}{\sigma^2}\sum_{i=1}^n x_i^2 & 0 \\ 0 & 0 & \frac{n}{2\sigma^4} \end{bmatrix}
\end{aligned}
$$

because $\operatorname{E}[u_i \mid x_i] = 0$ and $\operatorname{E}[u_i^2 \mid x_i] = \sigma^2$. If we let the i-th row of $X = [1, x_i]$ then we see that

$$I(\theta) = \begin{bmatrix} \sigma^{-2}X'X & 0 \\ 0 & \frac{n}{2\sigma^4} \end{bmatrix}$$

and, hence, the covariance matrix of the mle is

$$(I(\theta))^{-1} = \begin{bmatrix} \sigma^2(X'X)^{-1} & 0 \\ 0 & \frac{2\sigma^4}{n} \end{bmatrix},$$

which, for the coefficient estimates is identical to that for OLS and so, in practice, they will only differ in estimator for σ^2. A litle bit of fiddly maths and we can show that

$$(X'X)^{-1} = \frac{1}{n\sum_{i=1}^n (x_i - \overline{x})^2}\begin{bmatrix} \sum_{i=1}^n x_i^2 & -\sum_{i=1}^n x_i \\ -\sum_{i=1}^n x_i & n \end{bmatrix}.$$

Therefore, $\operatorname{Var}[\widehat{\alpha}] = \sigma^2\sum_{i=1}^n x_i^2/[n\sum_{i=1}^n (x_i - \overline{x})^2$ and $\operatorname{Var}\left[\hat{\beta}\right] = \sigma^2/\sum_{i=1}^n (x_i - \overline{x})^2$, with the only difference between the estimated standard errors of the mle and OLS being the estimator for σ^2. We note that $s_n^2 = n\hat{\sigma}^2/(n-2) > \hat{\sigma}^2$. So, the standard errors of OLS will be slightly larger than those of the mle but the t-ratios, which are of the form $t_{\hat{\theta}} = \hat{\theta}/\sqrt{\operatorname{Var}\left[\hat{\beta}\right]}$ will be smaller for OLS. Which means that if, at a given size, one can reject the null hypothesis that a regression coefficient is equal to zero based on the OLS results, then so too will the case for the mle. The final results are

$$\begin{array}{rccc} y = & 0.1979 & + & 0.9315 \quad x \\ & (0.1878) & & (0.0390) \end{array}$$

for maximum likelihood and

$$\begin{array}{rccc} y = & 0.1979 & + & 0.9315 \quad x \\ & (0.1886) & & (0.0392) \end{array}$$

for ols. Again, the values reported in parentheses for both equations are estimated standard errors. We see that this sample size is such that the consistency of all estimators is really kicking in and there is very little between them (and then only in the estimated standard errors).

(d) Test the hypothesis that $\alpha = 0$ using the results in 3(b). Test this result also using the results in 3(c). Use 5% significance level.

Solution:
For OLS, everyone will be happy just using a t-statistics compared to standard Normal critical values. Here the obvious thing to do is to use a Wald test, as we have all the unrestriced estimates already and to note that, as we are only testing a single restriction, the Wald test reduces to something of the form

$$W = t^2 \underset{a}{\overset{H_0}{\sim}} \chi^2_1,$$

where $t = \widehat{\alpha}/\hat{\sigma}_\alpha$, a t-ratio of estimate divided by its estimated standard error. (You should make sure that you can do the maths of this for yourself.) In the absence of any information about sign, we will test our null hypothesis, $H_0 : \alpha = 0$, against a two-sided alternative, that is $H_1 : \alpha \neq 0$. We could use W but we get axactly the same partition of the sample space into acceptance and rejection regions if we use

$$t \underset{a}{\overset{H_0}{\sim}} \mathrm{N}(0, 1)$$

and treat large values of $\text{abs}\,(t)$ as evidence against H_0. In order to test at the 5% level, this will mean a critical value cutting of a 2.5% upper tail. We see that this is approximately 1.96. So our rejection region for all of our tests is all values of the t-statistic that are greater than 1.96 in absolute value. Our results are given in Table 1. As predicted, $t_{normal} > t_{ols}$, but none of the t-ratios are very different, with all concluding that the sample data supports the null hypothesis that $\alpha = 0$.

Table 1: Tests of $H_0 : \alpha = 0$ against $H_1 : \alpha \neq 0$

Model*	t-statistic	Decision (Reject H_0 if $\text{abs}\,(t) > 1.96$)
Cauchy	$t_{caichy} \approx \frac{0.1364}{0.0914} \approx 1.4916$	Data supports H_0 as $1.4916 < 1.96$.
Normal	$t_{normal} \approx \frac{0.1979}{0.1878} \approx 1.0534$	Data supports H_0 as $1.0534 < 1.96$.
OLS	$t_{ols} \approx \frac{0.1979}{0.1886} \approx 1.0490$	Data supports H_0 as $1.0490 < 1.96$.

*The Cauchy and Normal models are estimated by maximum likelihood, as described above, whereas OLS is unencumbered by any distributional assumption.

(e) Construct histograms of the residuals from the models estimated in 3(b) and 3(c). On the basis of this information, which of the two models do you prefer. Justify your answer.

Solution:
In Figure 1 we see that the histograms lie almost perfectly one atop the other. Consequently there is nothing in terms of model fit to choose between them. On that basis, I would go with the model that is easiest to work with, which is the Normal/OLS model.

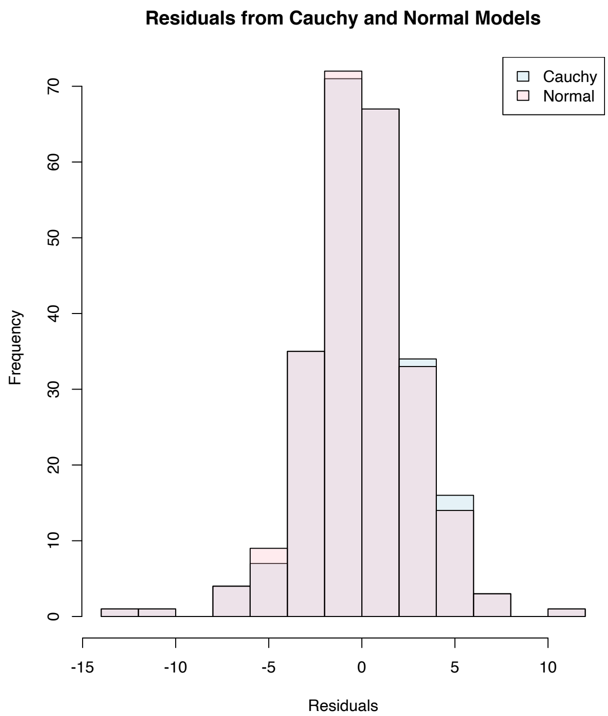

Figure 1: Figure for Question 5