FORECASTING IN ECONOMICS & BUSINESS ECOM90024

LECTURE 4: EXPONENTIAL SMOOTHING & COVARIANCE STATIONARITY

SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- Given a time series $\{y_t\}_{t=0}^{T}$ and setting $l_0 = y_0$, we can compute a simple exponentially smoothed series $\{l_t\}_{t=0}^{T}$ using the recursive equation:

$$l_t = \alpha y_t + (1 - \alpha) l_{t-1}$$

- We can then use the smoothed series at time t as a one-step ahead forecast by setting:

$$\hat{y}_{t+1|t} = l_t$$

- But what about a two-step ahead forecast?

SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- The two-step ahead forecast is given by:

$$\hat{y}_{t+2|t} = l_{t+1}$$

- Where,

$$l_{t+1} = \alpha y_{t+1} + (1 - \alpha) l_t$$

- However, if we are standing at time period t, we have not observed y_{t+1} yet! So, we must use the next best thing, $\hat{y}_{t+1|t}$. Now, since

$$\hat{y}_{t+1|t} = l_t$$

- We have that

$$l_{t+1} = \alpha l_t + (1 - \alpha) l_t = l_t$$

SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- It follows that this must be true for the h-step horizon forecast:

$$\hat{y}_{t+h|t} = \hat{y}_{t+h-1|t} = \hat{y}_{t+1|t} = l_t$$

- Thus, we have shown that the simple exponential smoothing model produces a flat forecast! In fact, we can think of l_t as the level of the forecast. This is the only thing that is specified in the simple exponential smoothing model.

- The forecast will be updated once we observe new information. To see how this works, let's suppose we now observe y_{t+1}. Then, we rewrite our updating equation as:

$$l_{t+1} = l_t + \alpha(y_{t+1} - l_t) = l_t + \alpha e_{t+1}$$

- Where e_t represents the forecast error $y_{t+1} - l_t$

- This is known as the ***error correction form*** of the updating equation.

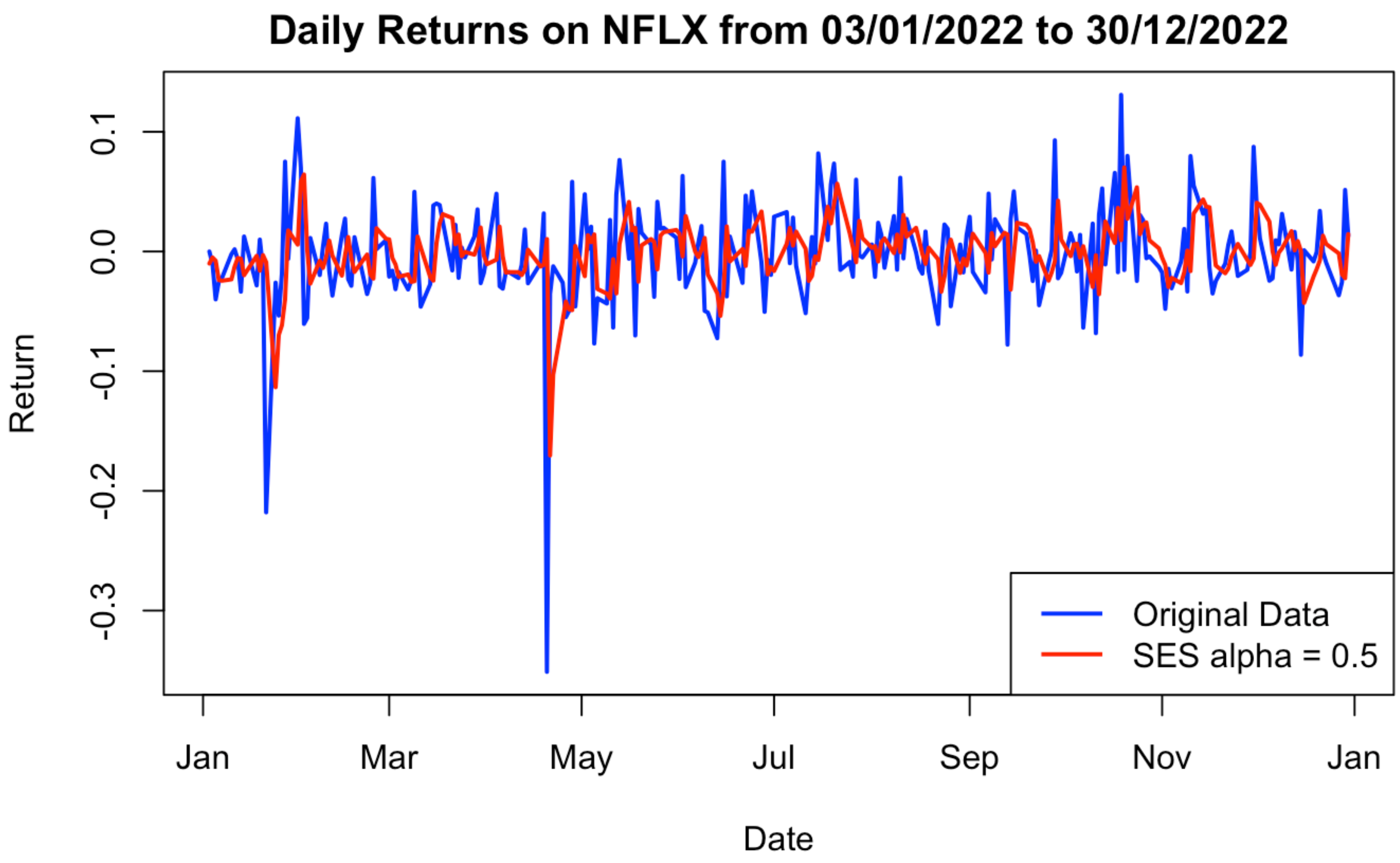
SIMPLE EXPONENTIAL SMOOTHING CONTINUED

$$l_{t+1} = l_t + \alpha(y_{t+1} - l_t) = l_t + \alpha e_{t+1}$$

- Looking at the error correction form, we can see that:
  - If $e_{t+1} < 0$, then the forecast l_t is overestimated (i.e., too high) and thus l_t should be adjusted downwards to get the new value l_{t+1}
  - If $e_{t+1} > 0$, then the forecast l_t is underestimated (i.e., too low) and thus l_t should be adjusted upwards to get the new value l_{t+1}
- We update the level of the forecast as we receive new information.

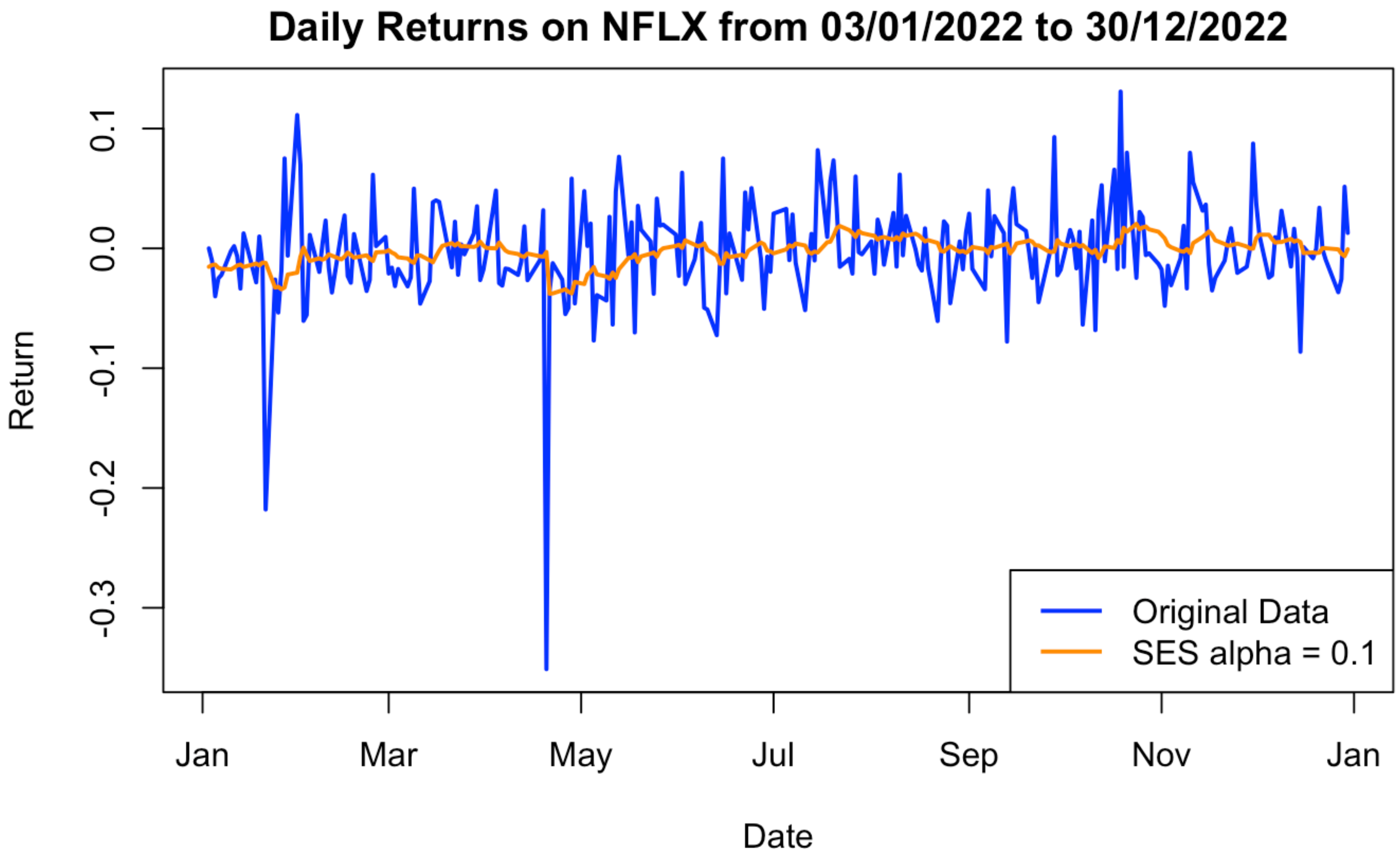
SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- Here we have the daily returns on Netflix Inc.'s stock that have been smoothed using simple exponential smoothing in which we have set the smoothing parameter $\alpha = 0.5$



SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- Notice that when we decrease the value of the smoothing parameter and set it to $\alpha = 0.1$, we obtain a much more smoothed series:



SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- So far, we have arbitrarily chosen our initial value l_0 and smoothing parameter value α.

- An alternative approach would be to use an objective function to choose these values:

$$\underset{\alpha, l_0}{\arg\min} \sum_{t=1}^{T} e_t^2$$

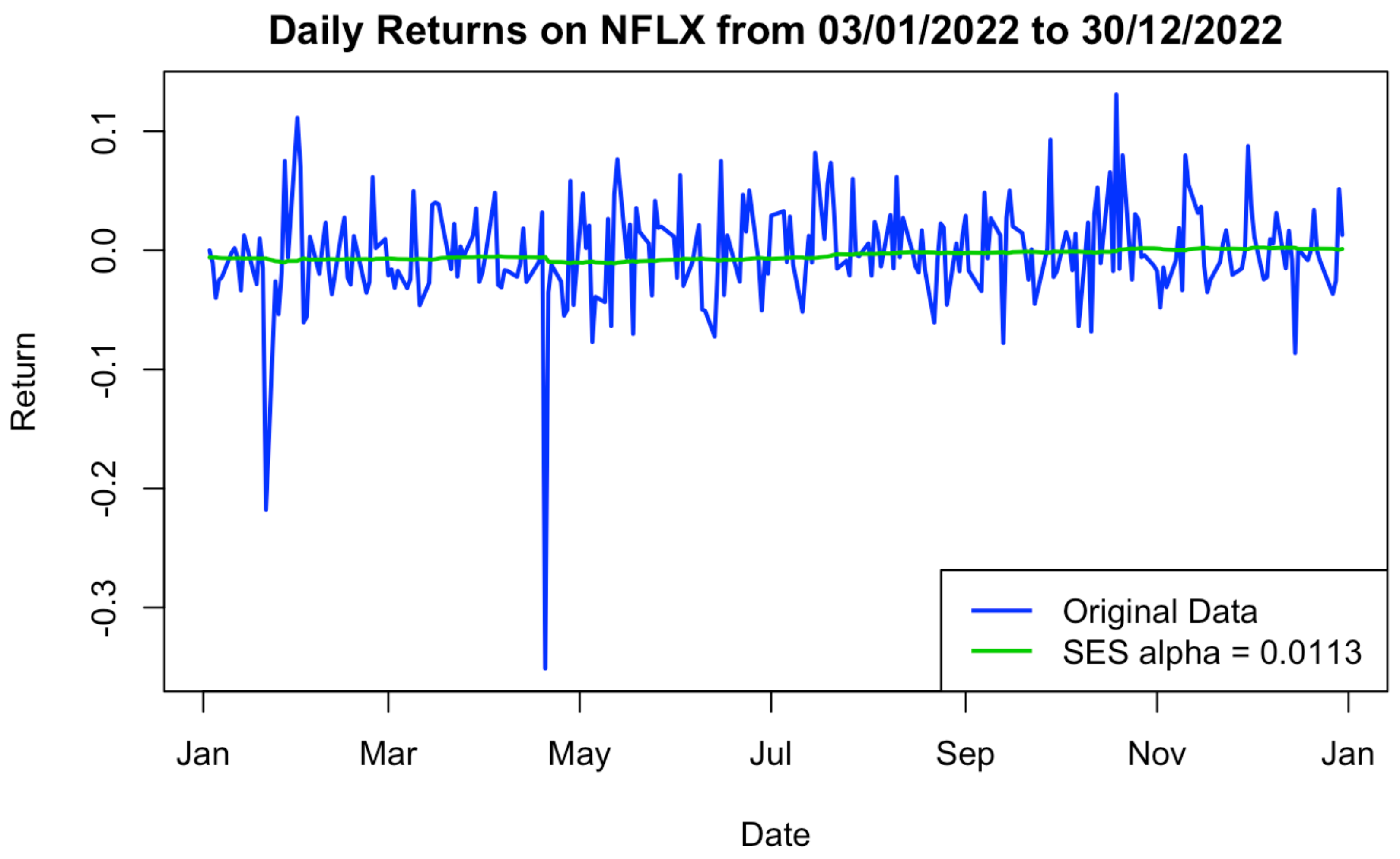
- Where e_t represents the forecast error $y_{t+1} - l_t$

- Note that in most cases, the choice of initial value won’t matter very much. We would only need to care about the initial value of we had an extremely small number of time series observations.

SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- The estimated smoothing parameter $\alpha = 0.0113$ minimizes the sum of squared forecast errors.

- This is unsurprising as the level of the time series is extremely stable.

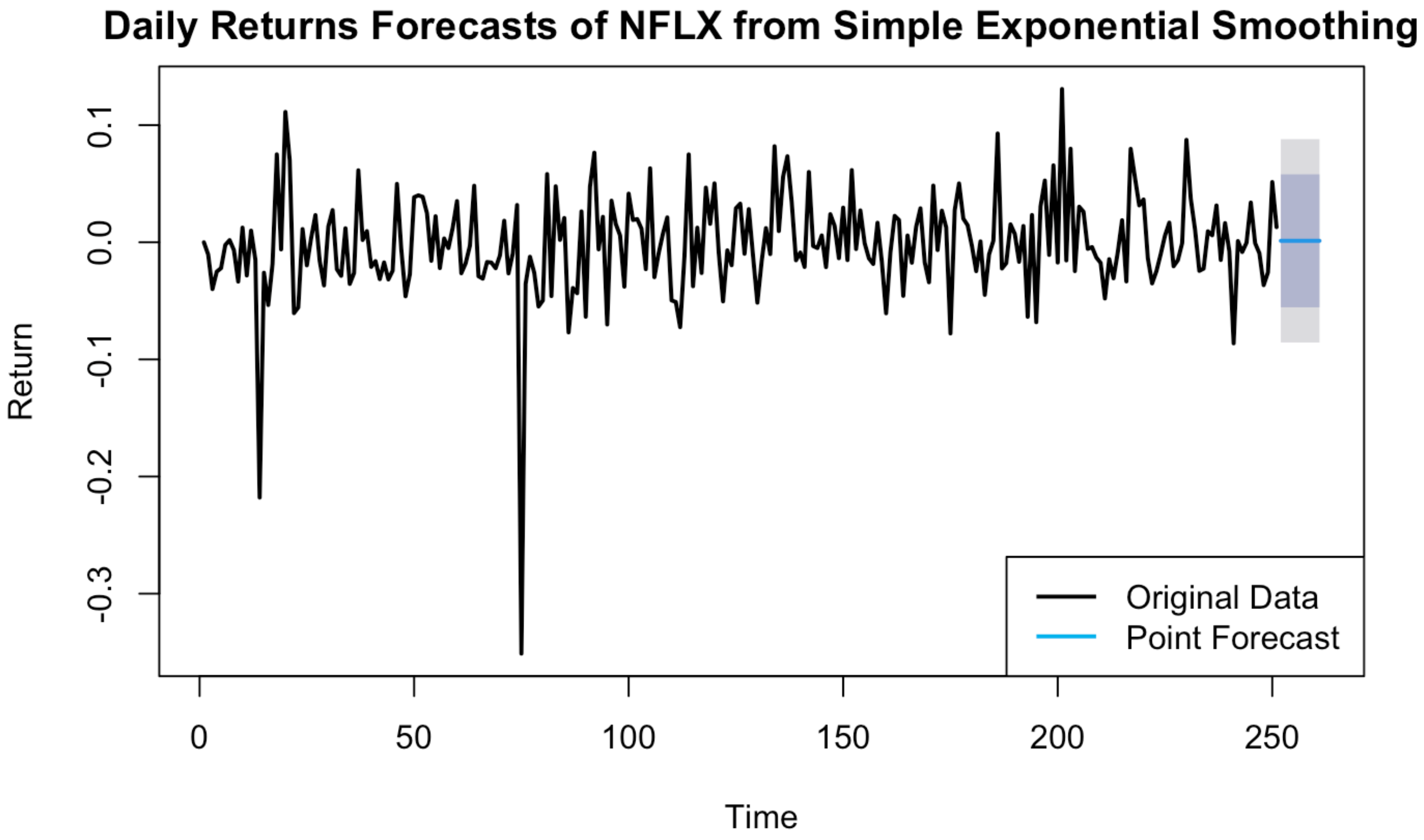


SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- The forecasts generated from the simple exponential smoothing model are flat by construction:

$$\hat{y}_{t+h|t} = \hat{y}_{t+h-1|t} = \hat{y}_{t+1|t} = l_t$$

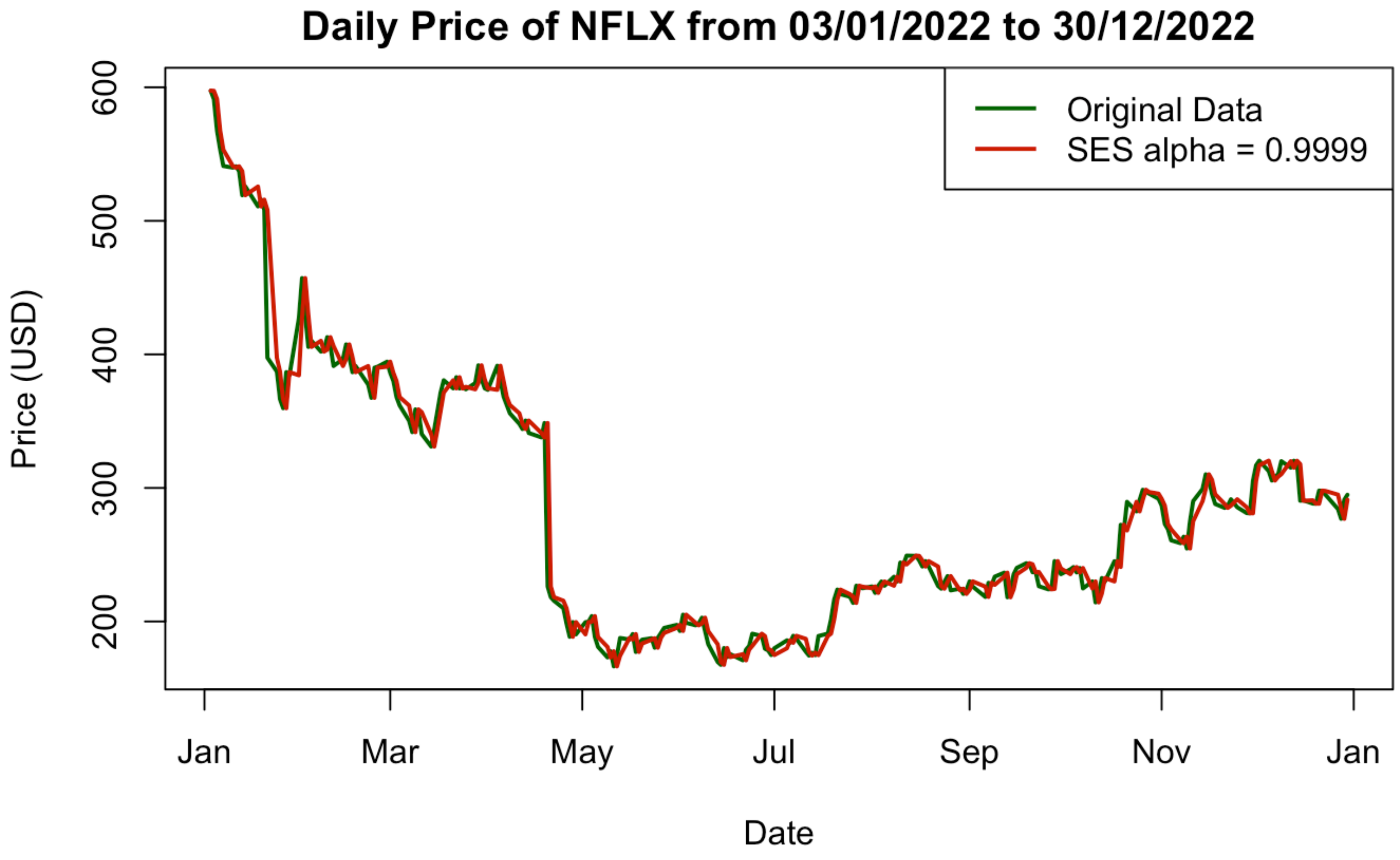
- Note that the intervals generated from the ses() function are 80% and 95% prediction intervals and are depicted as shaded regions.



SIMPLE EXPONENTIAL SMOOTHING CONTINUED

- Let's try applying the simple exponential smoothing to the daily price of NFLX.

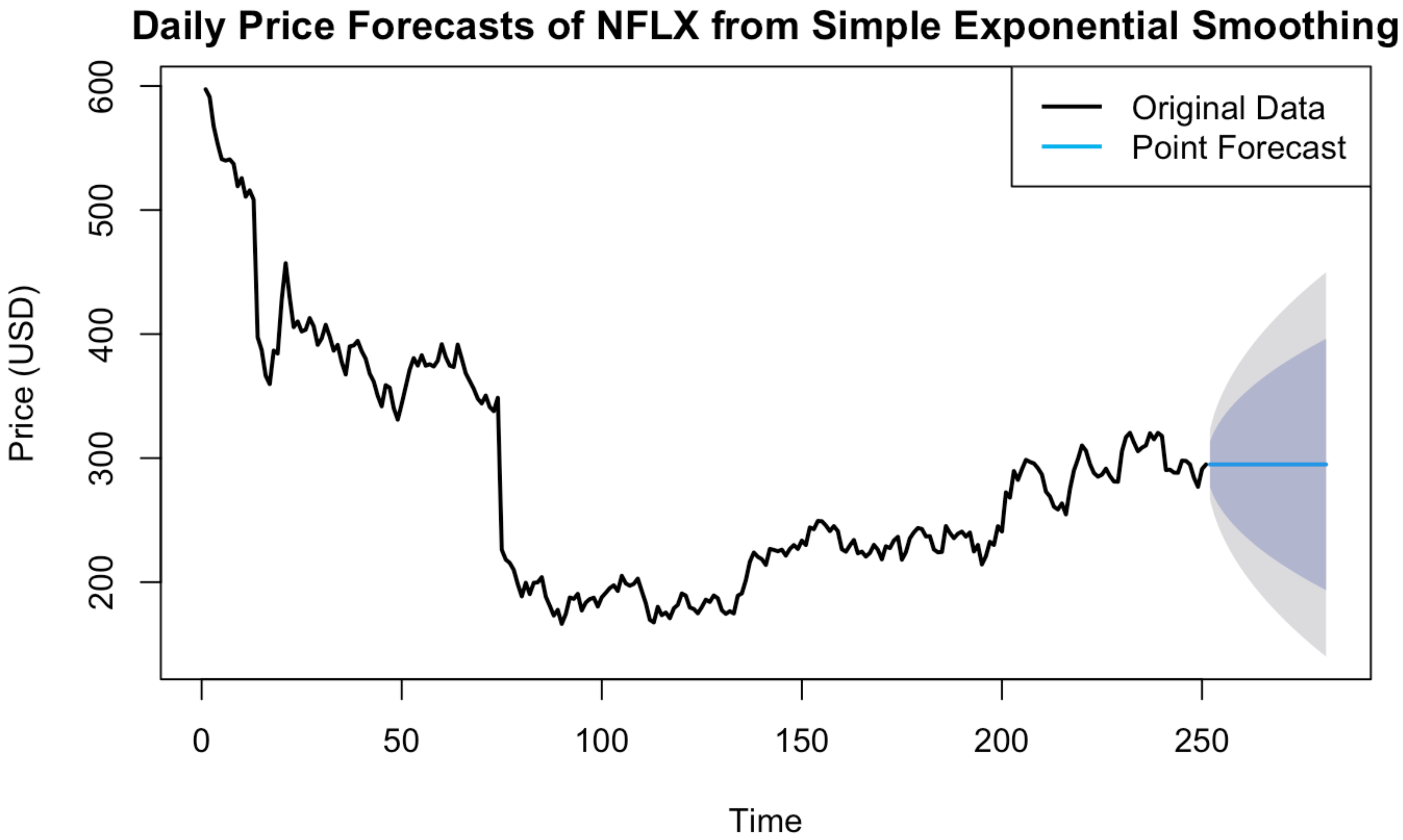
- Compared to the daily returns, we can see that the level of the price series is changing dramatically as we move along the sample. Hence new observations contain lots of information about the level of the series and are thus given a great deal of weight!



SIMPLE EXPONENTIAL SMOOTHING

- Simple exponential smoothing is not appropriate for time series with trends. It only specifies the level of the time series and thus can only generate flat forecasts.

- A flat forecast for this price series doesn't seem very plausible!



EXPONENTIAL SMOOTHING: HOLT'S LINEAR TREND

- In order to accommodate a time trend, we will have to augment the updating equations to incorporate a trend component b_t:

$$\text{Level Equation:} \qquad l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$$

$$\text{Trend Equation:} \qquad b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$

$$Forecasting\ Equation: \qquad \hat{y}_{t+h|t} = l_t + hb_t$$

- Where again, h denotes the forecast horizon.

- b_t is a weighted average of the change in the level $l_t - l_{t-1}$ and the estimated trend for time $t-1$

EXPONENTIAL SMOOTHING: HOLT'S LINEAR TREND

- We can also write Holt's linear trend model in an error correction form. First, we rearrange the level equation:

$$l_t = l_{t-1} + b_{t-1} + \alpha\big(y_t - (l_{t-1} + b_{t-1})\big)$$

- Since $\hat{y}_{t+1|t} = l_t + b_t$ and $e_t = y_t - \hat{y}_{t|t-1} = y_t - (l_{t-1} + b_{t-1})$

$$l_t = \hat{y}_{t|t-1} + \alpha e_t$$

- So that the level is adjusted by the forecast error with weight α

EXPONENTIAL SMOOTHING: HOLT'S LINEAR TREND

- For the trend updating equation, we would similarly write

$$b_t = b_{t-1} + \beta(l_t - (l_{t-1} + b_{t-1}))$$

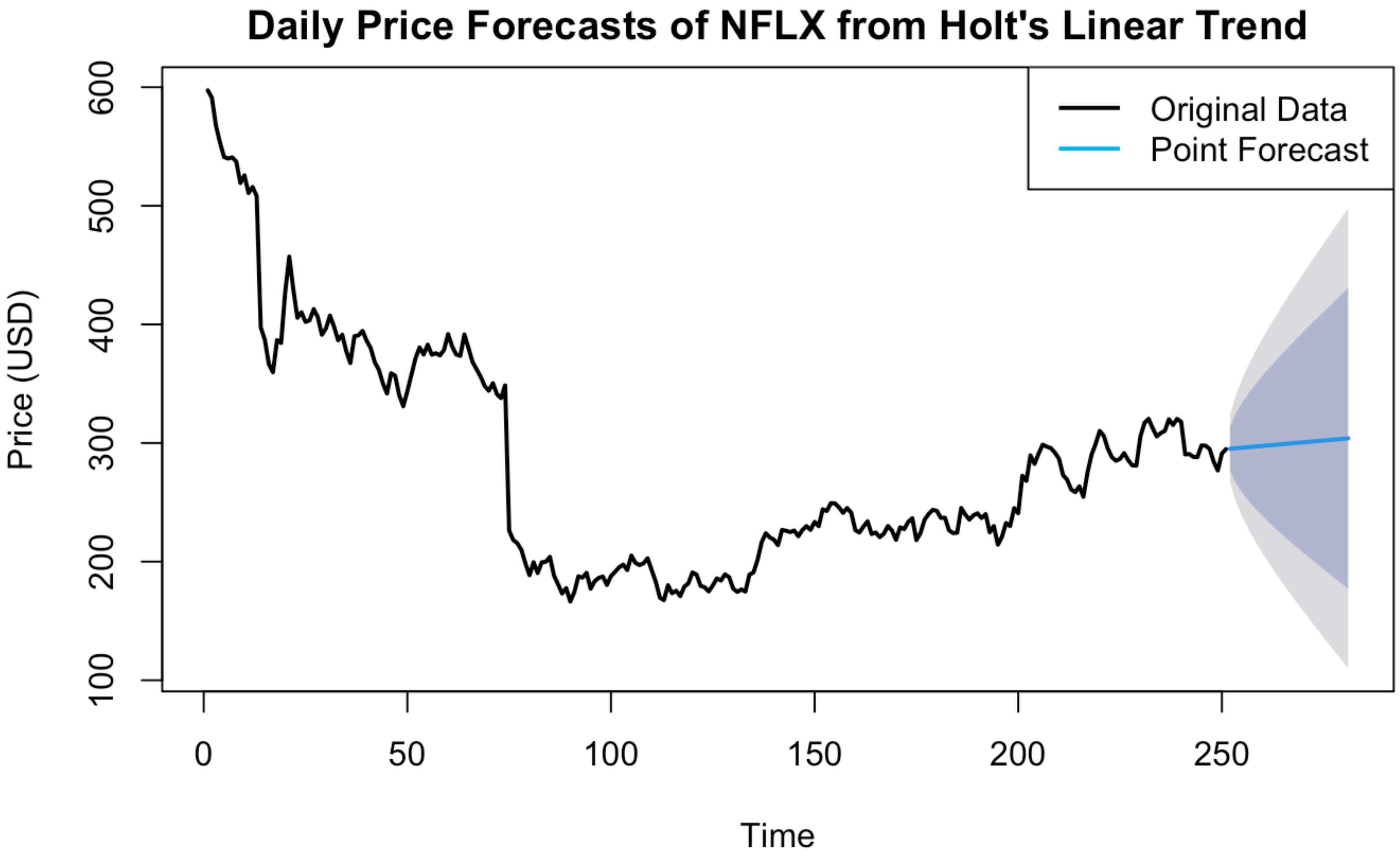
- Since $(l_t - (l_{t-1} + b_{t-1}) = l_t - \hat{y}_{t|t-1} = \alpha e_t$, we have that

$$b_t = b_{t-1} + \alpha\beta e_t$$

- Thus, the trend component is adjusted by the forecast error with weight $\alpha\beta$

EXPONENTIAL SMOOTHING: HOLT'S LINEAR TREND

```
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
##  holt(y = nflx.p$adjusted, h = 30, initial = "optimal")
##
##   Smoothing parameters:
##     alpha = 0.9999
##     beta  = 0.0168
##
##   Initial states:
##     l = 596.0382
##     b = -3.9072
##
##   sigma:  14.4239
```



EXPONENTIAL SMOOTHING: EXPONENTIAL TREND

- We can also specify the model to have a multiplicative form if we want to model an exponential trend:

$$\text{Level Equation:} \qquad l_t = \alpha y_t + (1-\alpha)(l_{t-1}b_{t-1})$$

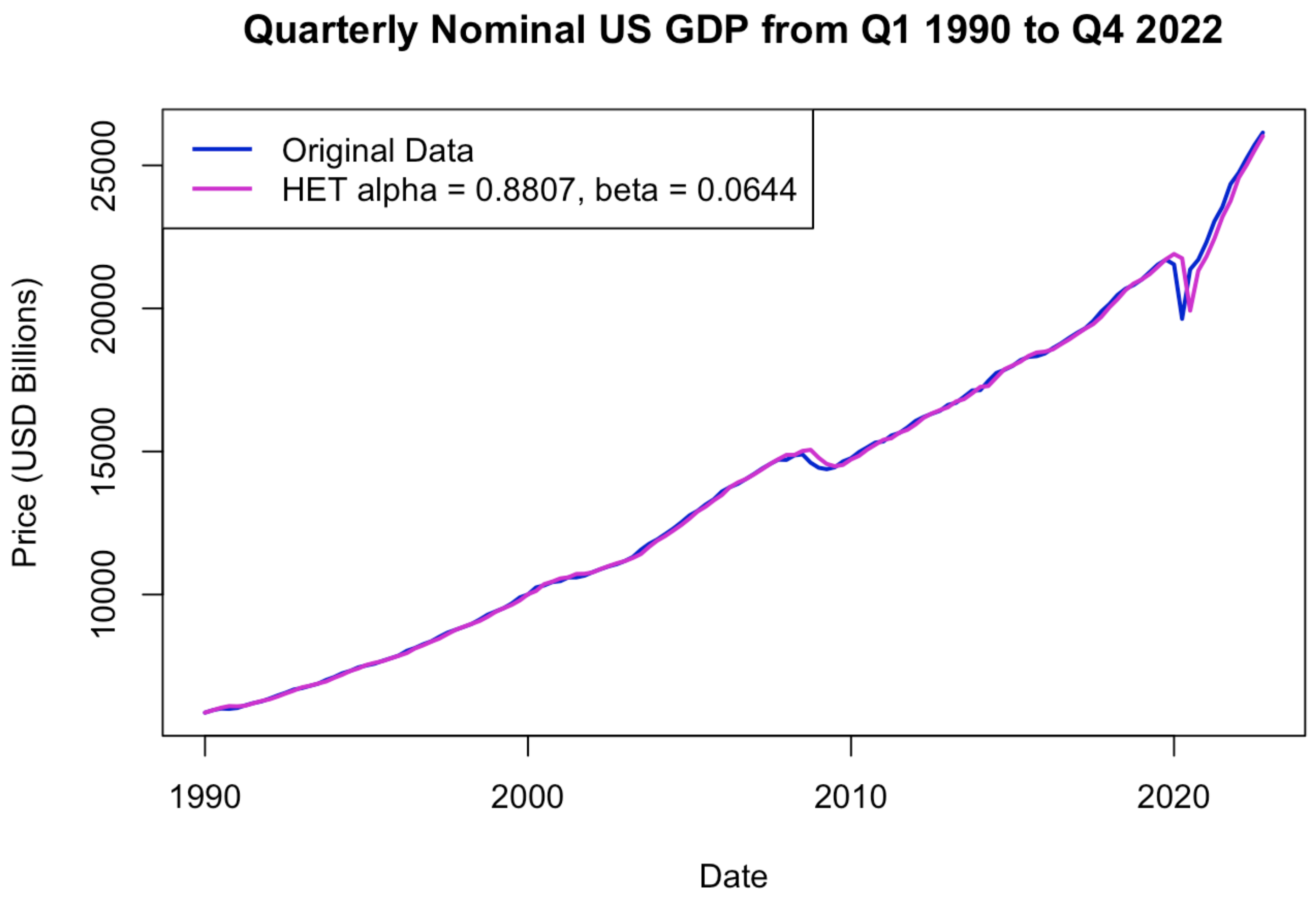
$$\text{Trend Equation:} \qquad b_t = \beta \frac{l_t}{l_{t-1}} + (1-\beta)b_{t-1}$$

$$Forecasting\ Equation: \qquad \hat{y}_{t+h|t} = l_t b_t^h$$

- Where b_t is now the growth rate of the trend.

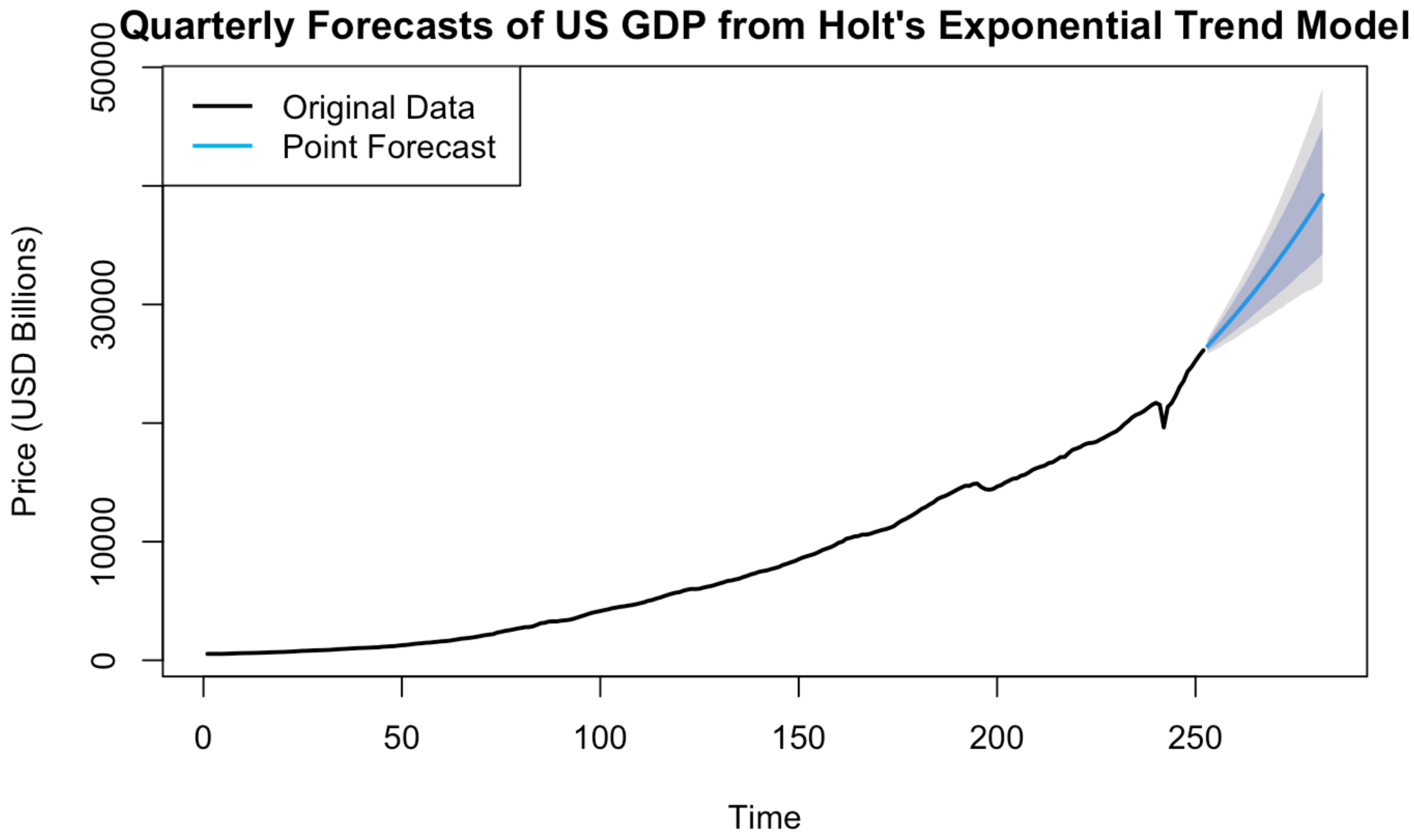
EXPONENTIAL SMOOTHING: EXPONENTIAL TREND

```
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
##  holt(y = us.gdp$price, h = 20, initial = "optimal")
##
##   Smoothing parameters:
##     alpha = 0.8807
##     beta  = 0.0644
##
##   Initial states:
##     l = 5791.2645
##     b = 84.1193
##
##   sigma:  264.8207
```



EXPONENTIAL SMOOTHING: EXPONENTIAL TREND

- Here we can see that our point forecasts reflect the multiplicative specification of our model:



EXPONENTIAL SMOOTHING: DAMPED TREND

- The linear trend model assumes that the trend is increasing/decreasing by a constant amount as we move from t to $t+1$

- The exponential trend model assumes that the trend is growing by a constant rate as we move from t to $t+1$

- We can add greater flexibility to our specification by allowing the trend to die out over time. It may also be a more plausible/reasonable model for the phenomenon that we are studying (i.e., most things do not increase or grow at the same rate forever!)

EXPONENTIAL SMOOTHING: DAMPED TREND

- The additive damped trend model is given by:

Level Equation: $l_t = \alpha y_t + (1 - \alpha)(l_{t-1} + \phi b_{t-1})$

Trend Equation: $b_t = \beta(l_t - l_{t-1}) + (1 - \beta)\phi b_{t-1}$

$Forecasting\ Equation$: $\hat{y}_{t+h|t} = l_t + \left(\phi + \phi^2 + \cdots + \phi^h\right)b_t$

- Where $0 < \phi < 1$

EXPONENTIAL SMOOTHING: DAMPED TREND

- The multiplicative damped trend is given by:

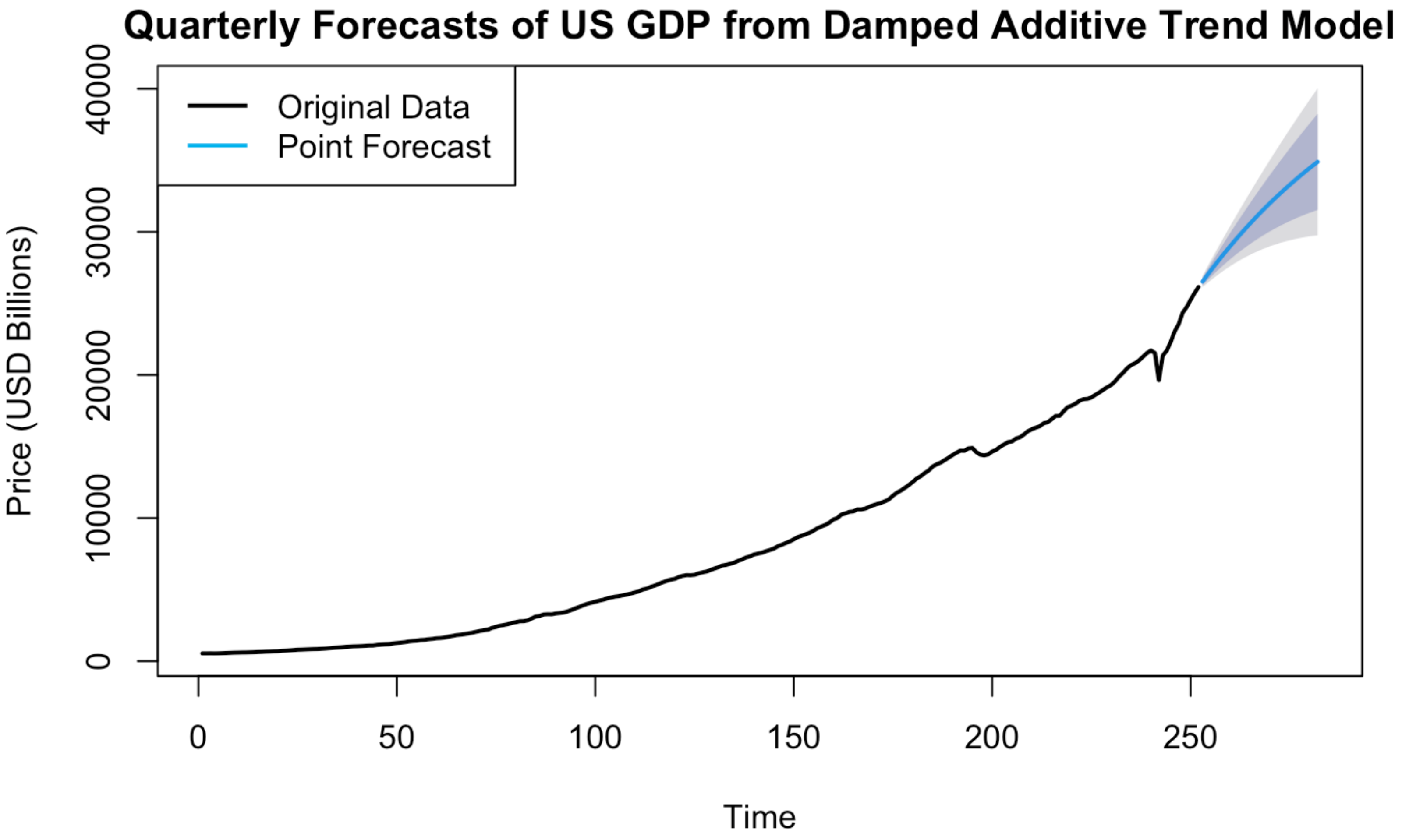
$$\text{Level Equation:} \qquad l_t = \alpha y_t + (1-\alpha)(l_{t-1} b_{t-1}^{\phi})$$

$$\text{Trend Equation:} \qquad b_t = \beta \frac{l_t}{l_{t-1}} + (1-\beta) b_{t-1}^{\phi}$$

$$Forecasting\ Equation\text{:}\, \hat{y}_{t+h|t} = l_t b_t^{(\phi+\phi^2+\cdots+\phi^h)}$$

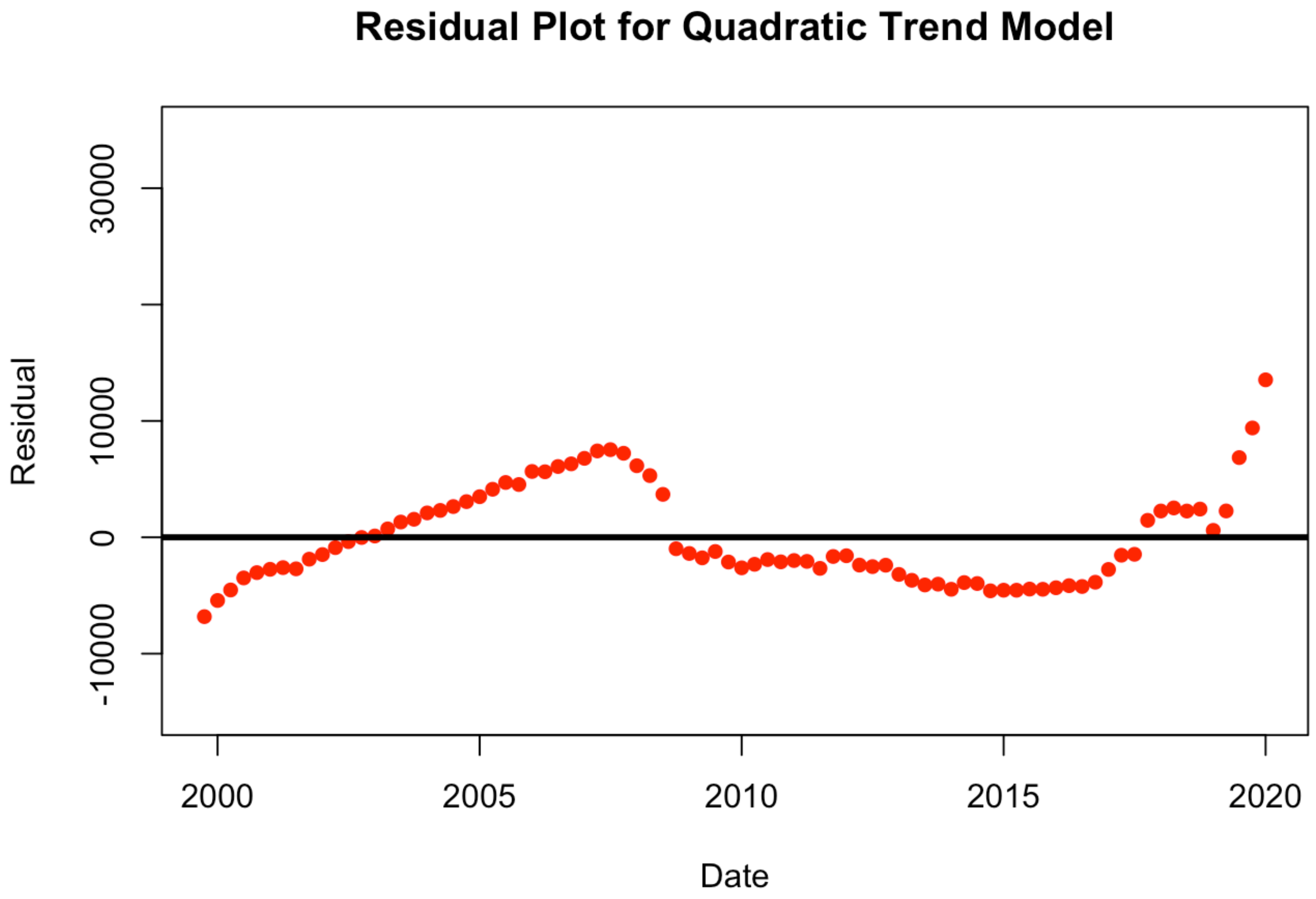
EXPONENTIAL SMOOTHING: DAMPED TREND

- Here are the forecasts for quarterly US gdp generated from an additive damped trend model:



CHARACTERISING CYCLES

- The trend and seasonal components of a time series are typically determined ***ex-ante*** to the estimation and analysis.
- Looking back at the residual plots of our trend and seasonal component models, we can clearly see that there are cyclical components that we have yet to account for.
- Cycles can be thought of broadly as the stable, mean-reverting dynamics that are not captured by trends or seasonal factors.
- While we can capture cycles through smoothing, we would like a way to model them explicitly.



COVARIANCE STATIONARY TIME SERIES

- A realization of a time series is an ordered set,

$$\{\dots, y_{-2}, y_{-1}, y_0, y_1, y_2, \dots\}$$

- In theory, a time series realization begins in the infinite past and continues into the infinite future.

- In practice however, the data we observe is just a finite subset of a realization called the ***sample path***

$$\{y_1, y_2, \dots, y_T\}$$

COVARIANCE STATIONARY TIME SERIES

- If we are to use historical information to learn about the future, we need the future to be like the past.

- That is, in order to forecast the cyclical component of a time series, we need the mathematical properties that govern the future evolution of a time series to be the same as those that govern its history.

- Specifically, we require that:

  1. The mean of the time series be stable over time.
  2. The autocovariance structure (i.e. the covariances between current and past values) to be stable over time.
  3. The variance of the time series to be finite.

- A time series that possesses these three properties is said to be ***covariance stationary***.

COVARIANCE STATIONARY TIME SERIES

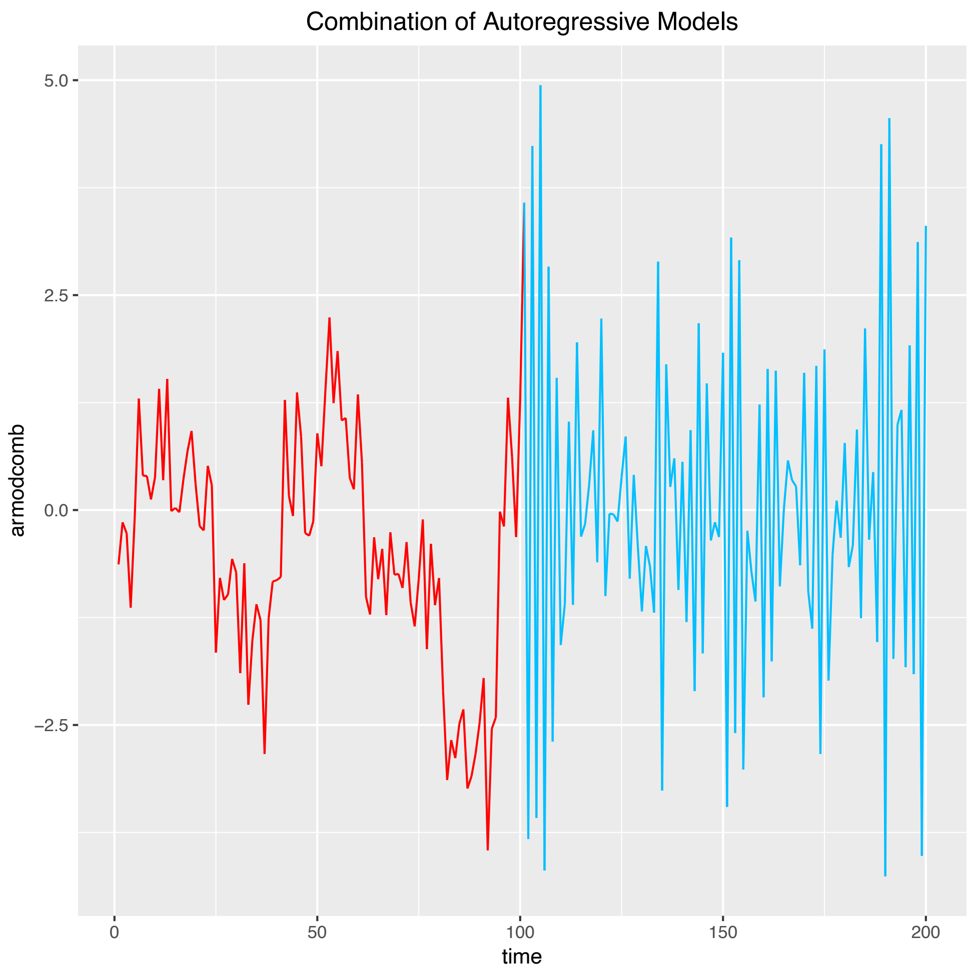
- Here we have a time series in which the first 100 observation is generated from

$$Y_t = 0.9Y_{t-1} + \varepsilon_t$$

And the following 100 observations is generated from

$$Y_t = -0.9Y_{t-1} + \varepsilon_t$$

Let's suppose the data generating process switches between these two models. The autocovariance structure is clearly not stable! We would not be able to consistently estimate the underlying parameters with a single regression let alone compute forecasts!



COVARIANCE STATIONARY TIME SERIES

- We say that the mean of a time series is stable over time if

$$E[Y_t] = \mu \text{ for all } t$$

- The ***autocovariance function*** of a time series is defined as

$$\gamma(t, \tau) = cov(Y_t, Y_{t-\tau}) = E[(Y_t - \mu)(Y_{t-\tau} - \mu)]$$

- The autocovariance function evaluated at τ measures the strength of association between two observations separated by τ time periods. It provides a basic summary of the ***cyclical dynamics of a time series***.
- We say that a time series has a ***time-invariant autocovariance structure*** if,

$$\gamma(t, \tau) = \gamma(\tau) = \gamma(-\tau)$$

COVARIANCE STATIONARY TIME SERIES

- Autocovariances are hard to interpret and compare across different time series due to the fact that they scaled in terms of the cross product of the variables.
- In practice, we work with the autocorrelation function since it is scale free and bounded between -1 and 1. Recall that

$$corr(X,Y) = \frac{cov(X,Y)}{\sigma_x \sigma_y}$$

- Therefore the ***autocorrelation function*** is obtained by dividing the autocovariance function by the variance

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)}$$

- Where $\gamma(0) = var(Y_t)$

COVARIANCE STATIONARY TIME SERIES

- The autocorrelation function measures the simple correlation between Y_t and $Y_{t-\tau}$.

- We can also compute a quantity that measures the correlation between Y_t and $Y_{t-\tau}$ after ***controlling*** for the effects of Y_{t-1} through to $Y_{t-\tau+1}$. This is known as the ***partial correlation*** between Y_t and $Y_{t-\tau}$.

- The partial autocorrelation is determined from a regression of Y_t on $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-\tau}$ and is given by the regression coefficient multiplying $Y_{t-\tau}$.

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \cdots + \alpha_\tau Y_{t-\tau} + \varepsilon_t$$

COVARIANCE STATIONARY TIME SERIES

- The ***partial autocorrelation function*** (PACF) $p(\tau)$ can therefore computed by a series of autoregressions:

$$Y_t = \alpha_0 + \alpha_1\, Y_{t-1} + \varepsilon_t$$
$$Y_t = \alpha_0 + \alpha_1\, Y_{t-1} + \alpha_2\, Y_{t-2} + \varepsilon_t$$
$$\vdots$$
$$Y_t = \alpha_0 + \alpha_1\, Y_{t-1} + \alpha_2\, Y_{t-2} + \cdots + \alpha_\tau\, Y_{t-\tau} + \varepsilon_t$$

- Where

$$p(\tau) = \alpha_\tau$$

WHITE NOISE PROCESS

- Suppose that we had a time series that possesses the following data generating process:

$$Y_t = \varepsilon_t$$

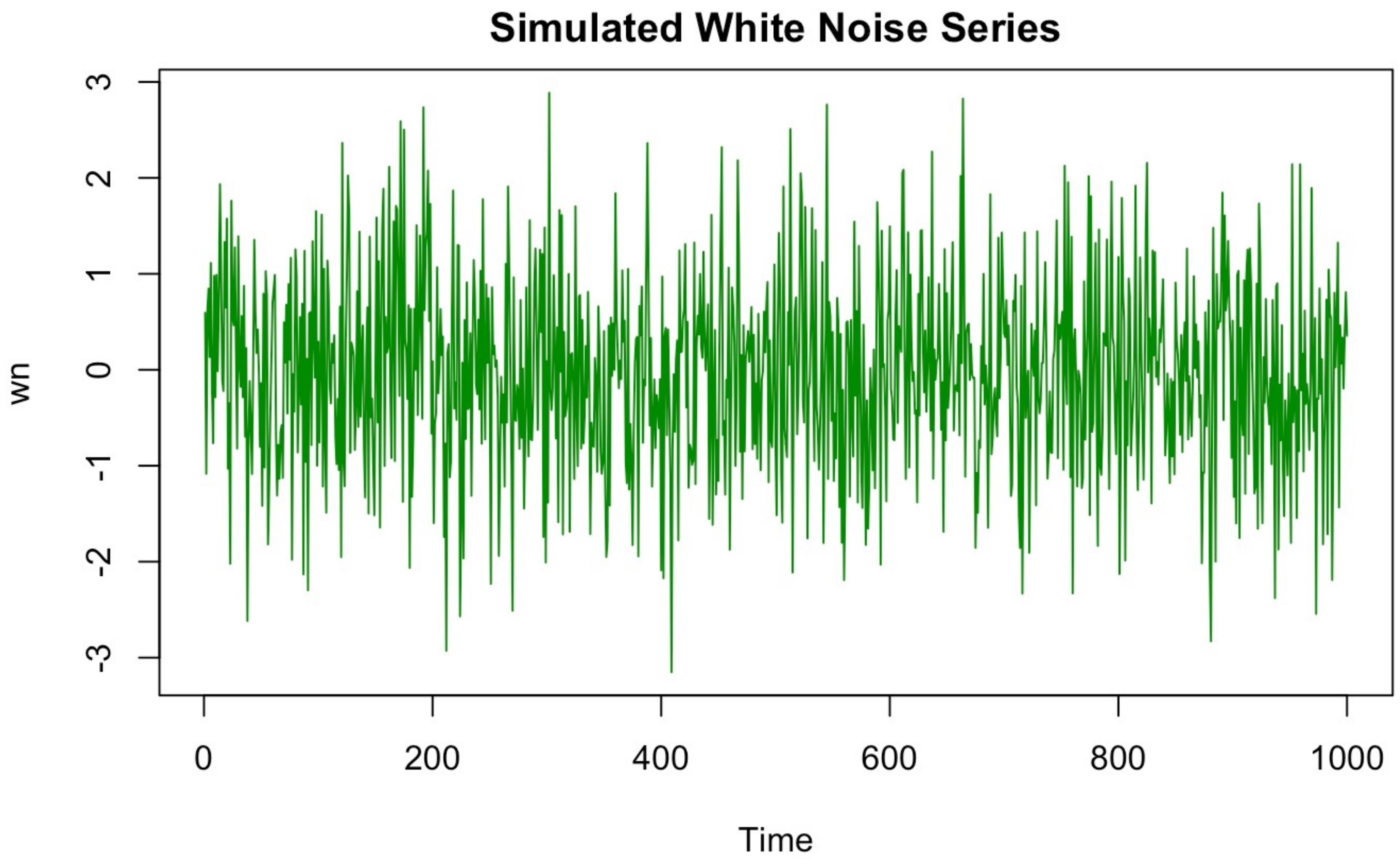
$$\varepsilon_t \sim_{iid} N(0, \sigma^2)$$

- A white noise process will have the following ***stochastic properties***:

$$E[Y_t] = 0$$

$$var(Y_t) = \sigma^2$$

$$\gamma(\tau) = \begin{cases} \sigma^2, & \tau = 0 \\ 0, & \tau \geq 1 \end{cases}$$

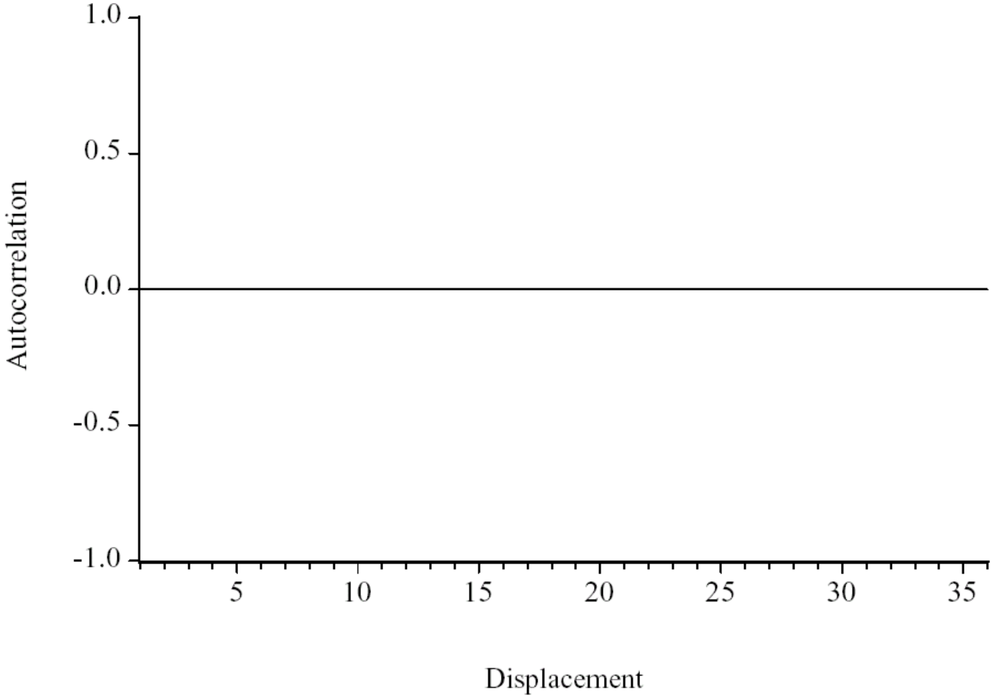


WHITE NOISE PROCESS

- The theoretical autocorrelation (ACF) and partial autocorrelation (PACF) functions will be given by:

$$\rho(\tau) = \begin{cases} 1, & \tau = 0 \\ 0, & \tau \geq 1 \end{cases}$$

$$p(\tau) = \begin{cases} 1, & \tau = 0 \\ 0, & \tau \geq 1 \end{cases}$$



- Clearly, a white noise process is covariance stationary.

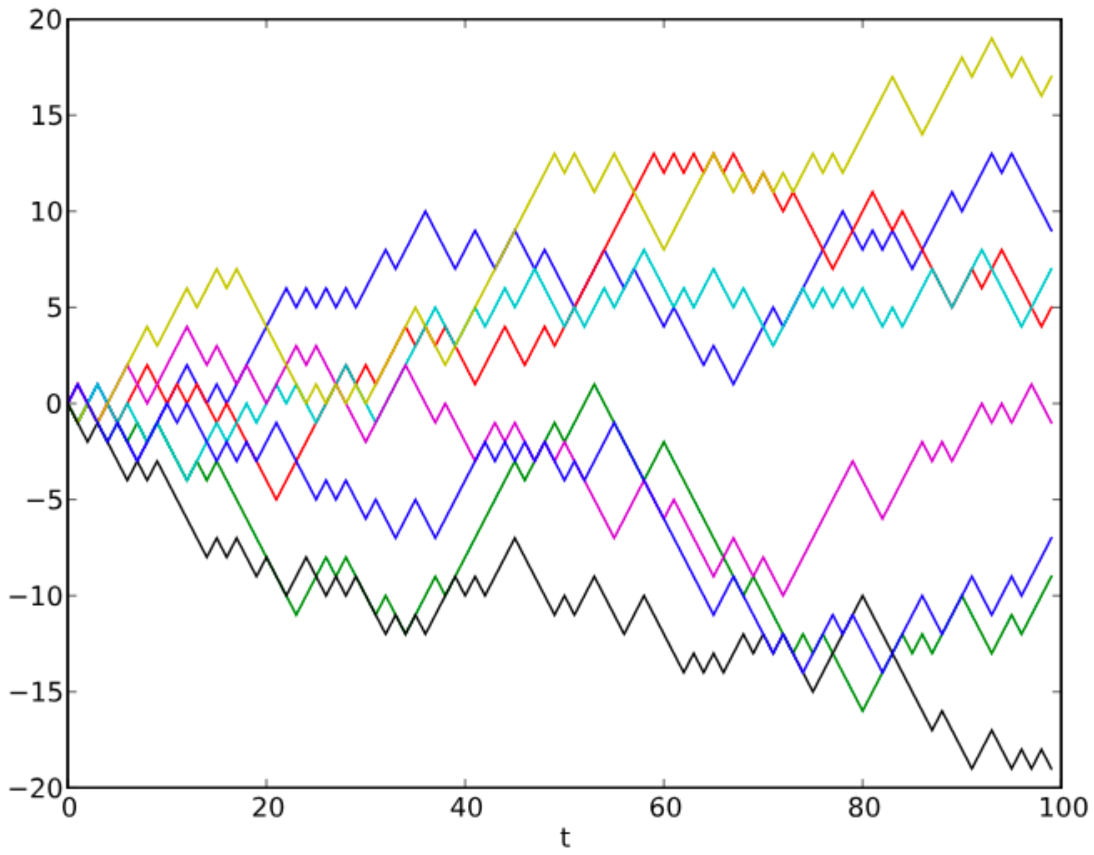
RANDOM WALK

- Let's consider a time series process that is characterized as the sum of a sequence of iid errors.

$$Y_t = \sum_{i=1}^{t} \varepsilon_i$$

$$\varepsilon_t \sim_{iid} N(0, \sigma^2)$$

- This process is known as a random walk and has many applications in finance, engineering and the natural sciences.



RANDOM WALK

Recall that:

$Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z)$

$Cov(aX,Y) = aCov(X,Y)$ for constant a

If X and Y are independent then $Cov(X,Y) = 0$

- The stochastic properties of a random walk are:

$$E[Y_t] = 0$$

$$Var(Y_t) = Var\left(\sum_{i=1}^{t} \varepsilon_i\right) = t\sigma^2$$

$$\gamma(t+\tau,t) = Cov(Y_{t+\tau},Y_t) = Cov\left(Y_t + \sum_{s=1}^{\tau} \varepsilon_s \,, Y_t\right)$$

$$= Cov\,(Y_{t+\tau},Y_t) = t\sigma^2$$

- Since the autocovariance function of a random walk is dependent on t, we have violated the condition for covariance stationarity.

RANDOM WALK

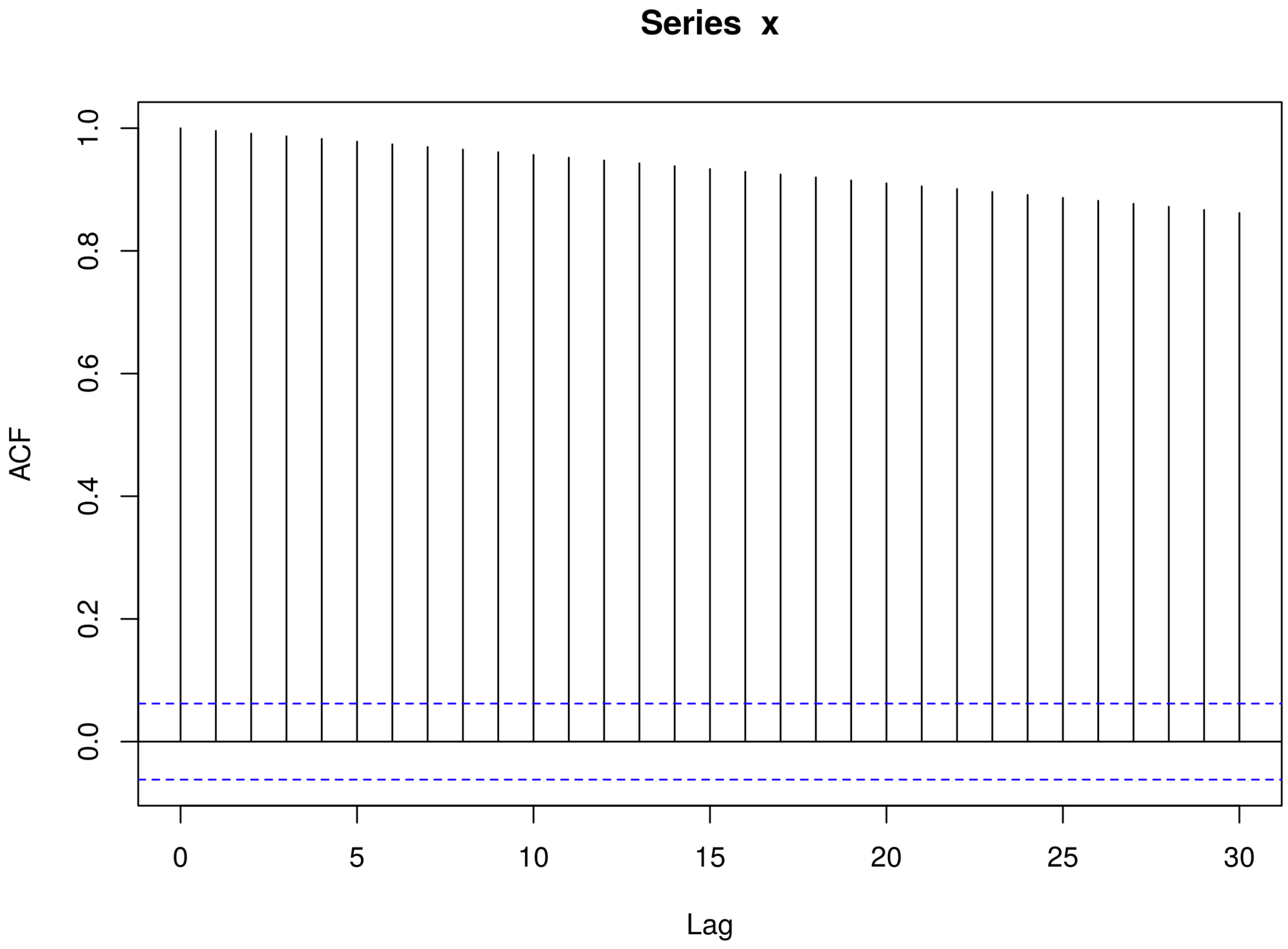
- Since the autocovariance function of a random walk is a function of t, then it also must be true for the autocorrelation function:

$$\rho(t,\tau) = \frac{Cov(Y_{t+\tau}, Y_t)}{\sqrt{Var(Y_{t+\tau})Var(Y_t)}} = \frac{t\sigma^2}{\sqrt{t\sigma^2(t+\tau)\sigma^2}} = \frac{1}{\sqrt{1 + {}^{\tau}/_{t}}}$$

- Not only is the autocorrelation function dependent upon t, it also decays very slowly as τ increases.

RANDOM WALK

- Visually,



AR(1) MODEL

- Let's now consider something in between a random walk and a white noise series,

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim_{iid} N(0, \sigma^2)$$

- Since $\phi = 1$ gives us the random walk model, we will assume that $|\phi| < 1$. This is known as a ***first order autoregressive*** or an AR(1) process.

- Unlike a random walk, the errors do not accumulate but rather die out over time.

AR(1) MODEL

- To see this, note that we can always rewrite an AR(1) model as the following:

$$Y_t = \phi Y_{t-1} + \varepsilon_t = \phi(\phi Y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

- So that

$$Y_t = \varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \phi^3\varepsilon_{t-3} + \cdots$$

- This is known as an ***infinite moving average*** or MA(∞) process. From this we can easily see that

$$E[Y_t] = E[\varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \phi^3\varepsilon_{t-3} + \cdots] = 0$$

AR(1) MODEL

- To compute the variance, we have that

$$\gamma(0) = E[(Y_t - \mu)^2] = E[(\varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \phi^3\varepsilon_{t-3} + \cdots)^2]$$

- Since the errors are all $i.i.d.$, the expectation of their cross products will be zero (e.g., $E[\varepsilon_t\varepsilon_{t-1}] = 0$). Therefore,

$$\gamma(0) = E[\varepsilon_t^2 + \phi^2\varepsilon_{t-1}^2 + \phi^4\varepsilon_{t-2}^2 + \phi^6\varepsilon_{t-3}^2 + \cdots]$$

- So that

$$\gamma(0) = \sigma^2 + \phi^2\sigma^2 + \phi^4\sigma^2 + \phi^6\sigma^2 + \cdots$$

AR(1) MODEL

- Therefore the variance of an AR(1) model is an infinite geometric sum,

$$\gamma(0) = \sigma^2(1 + \phi^2 + \phi^4 + \phi^6 + \cdots)$$

- We know from high school mathematics that for an infinite geometric sum where $|r| < 1$,

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1 - r}$$

- Therefore, the variance of an AR(1) model is given by

$$\gamma(0) = \frac{\sigma^2}{1 - \phi^2}$$

AR(1) MODEL

- To compute the autocovariance function, we write,

$$\gamma(\tau) = E[(Y_t - \mu)(Y_{t-\tau} - \mu)]$$

$$\gamma(\tau) = E[(\varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \cdots)\times(\varepsilon_{t-\tau} + \phi\varepsilon_{t-\tau-1} + \phi^2\varepsilon_{t-\tau-2} + \cdots)]$$

- Again, the cross products will be equal to zero, leaving,

$$\gamma(\tau) = E[\phi^\tau\varepsilon_{t-\tau}^2 + \phi^{\tau+2}\varepsilon_{t-\tau-1}^2 + \phi^{\tau+4}\varepsilon_{t-\tau-2}^2 + \cdots]$$

- Which yields,

$$\gamma(\tau) = \sigma^2(\phi^\tau + \phi^{\tau+2} + \phi^{\tau+4} + \cdots) = \frac{\sigma^2\phi^\tau}{1-\phi^2}$$

- Since the autocovariance function of an AR(1) process is only a function of τ, we can conclude that it is a covariance stationary process.

AR(1) MODEL

- The autocorrelation function is straightforward to compute,

$$\rho(t,\tau) = \frac{Cov(Y_{t+\tau}, Y_t)}{\sqrt{Var(Y_{t+\tau})Var(Y_t)}} = \frac{\gamma(\tau)}{\gamma(0)} = \frac{\frac{\sigma^2}{1-\phi^2}}{\frac{\sigma^2\phi^\tau}{1-\phi^2}} = \phi^\tau$$

- Hence the autocorrelation function of an AR(1) process will show a geometric decay.

SAMPLE AUTOCORRELATIONS

- To compute sample autocorrelations from the data, we know that the autocorrelation at displacement τ for the covariance stationary series Y is

$$\rho(t,\tau) = \frac{Cov(Y_{t+\tau}, Y_t)}{\sqrt{Var(Y_{t+\tau})Var(Y_t)}} = \frac{Cov(Y_{t+\tau}, Y_t)}{Var(Y_t)}$$

- Therefore the sample autocorrelations are simply the sample analogues of these components

$$\hat{\rho}(\tau) = \frac{\frac{1}{T-1}\sum_{t=1}^{T}(y_t - \bar{y})(y_{t-\tau} - \bar{y})}{\frac{1}{T-1}\sum_{t=1}^{T}(y_t - \bar{y})^2} = \frac{\sum_{t=1}^{T}(y_t - \bar{y})(y_{t-\tau} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

SAMPLE AUTOCORRELATIONS

- We can use the sample autocorrelation function to test whether an observed time series is generated from a white noise process.
- It can be shown that if the true data generating process is white noise, then the distribution of sample autocorrelations in large samples for all τ is,

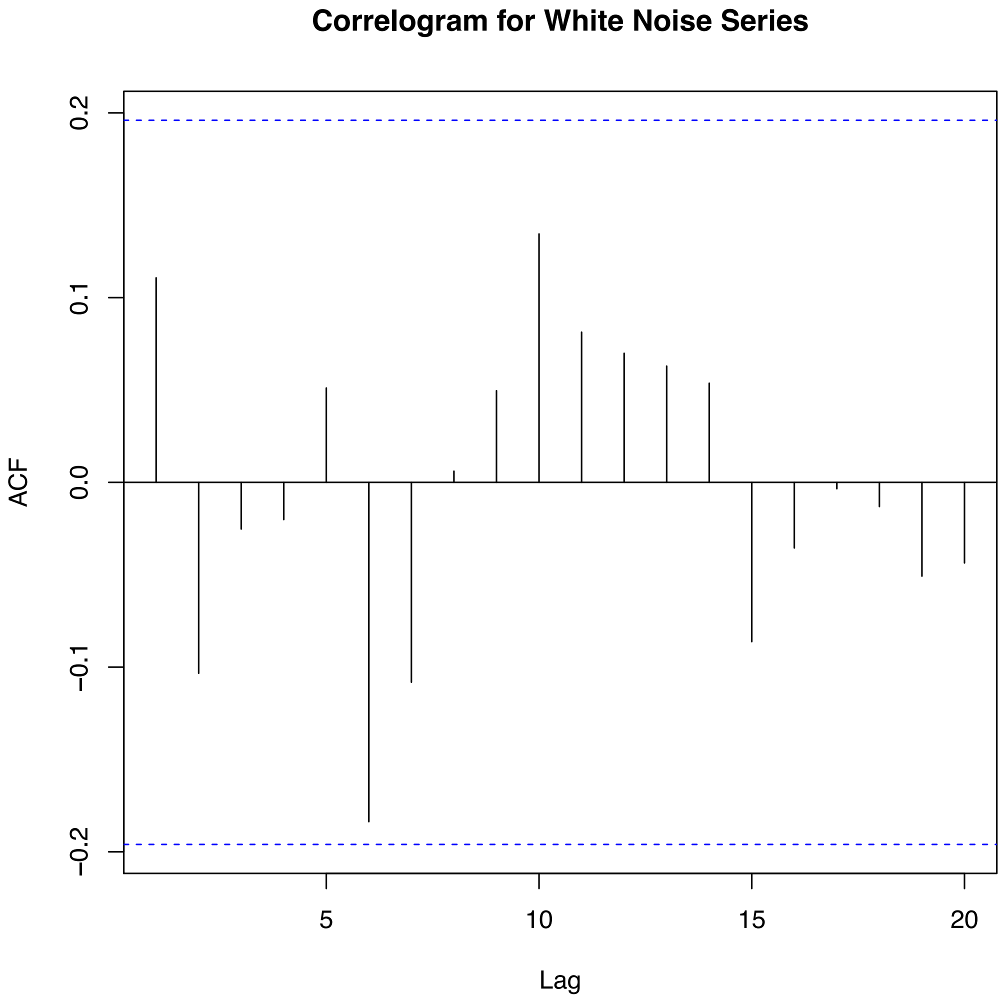
$$\hat{\rho}(\tau) \sim N\left(0, \frac{1}{T}\right)$$

- Thus, if a series is white noise then approximately 95% of sample autocorrelations should fall in the interval

$$0 \pm \frac{2}{\sqrt{T}}$$

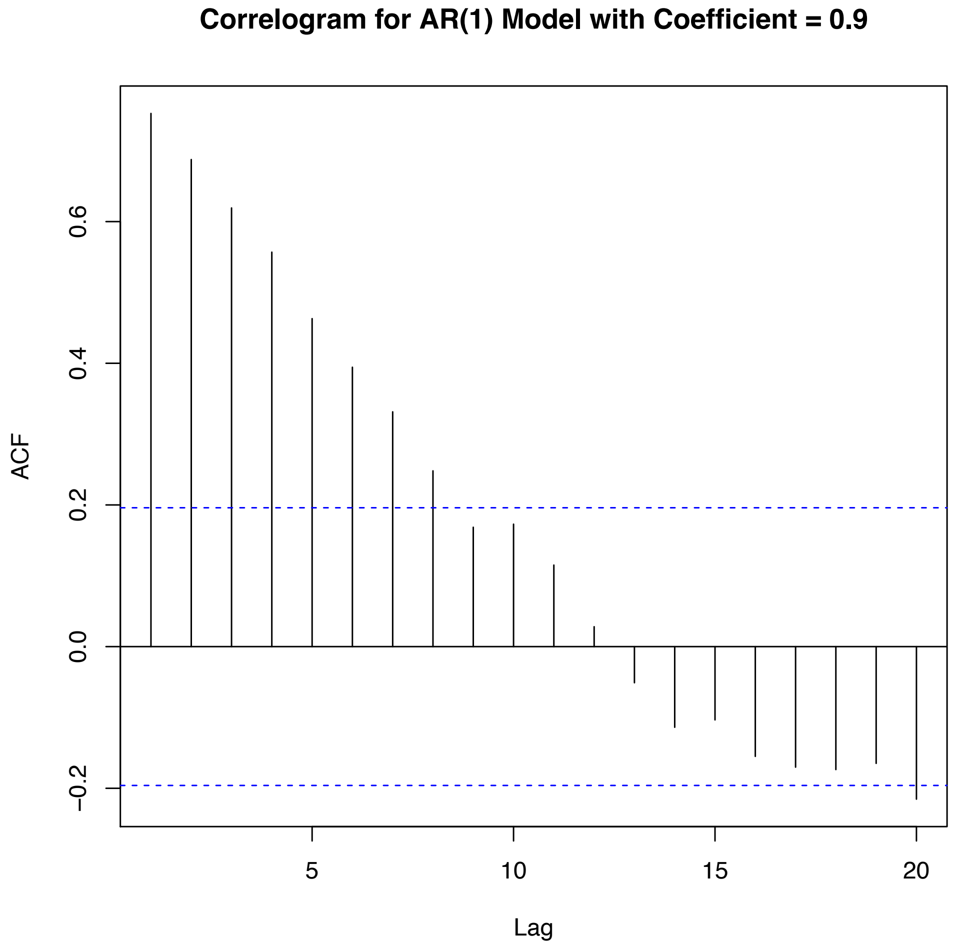
SAMPLE AUTOCORRELATIONS

- Visually,



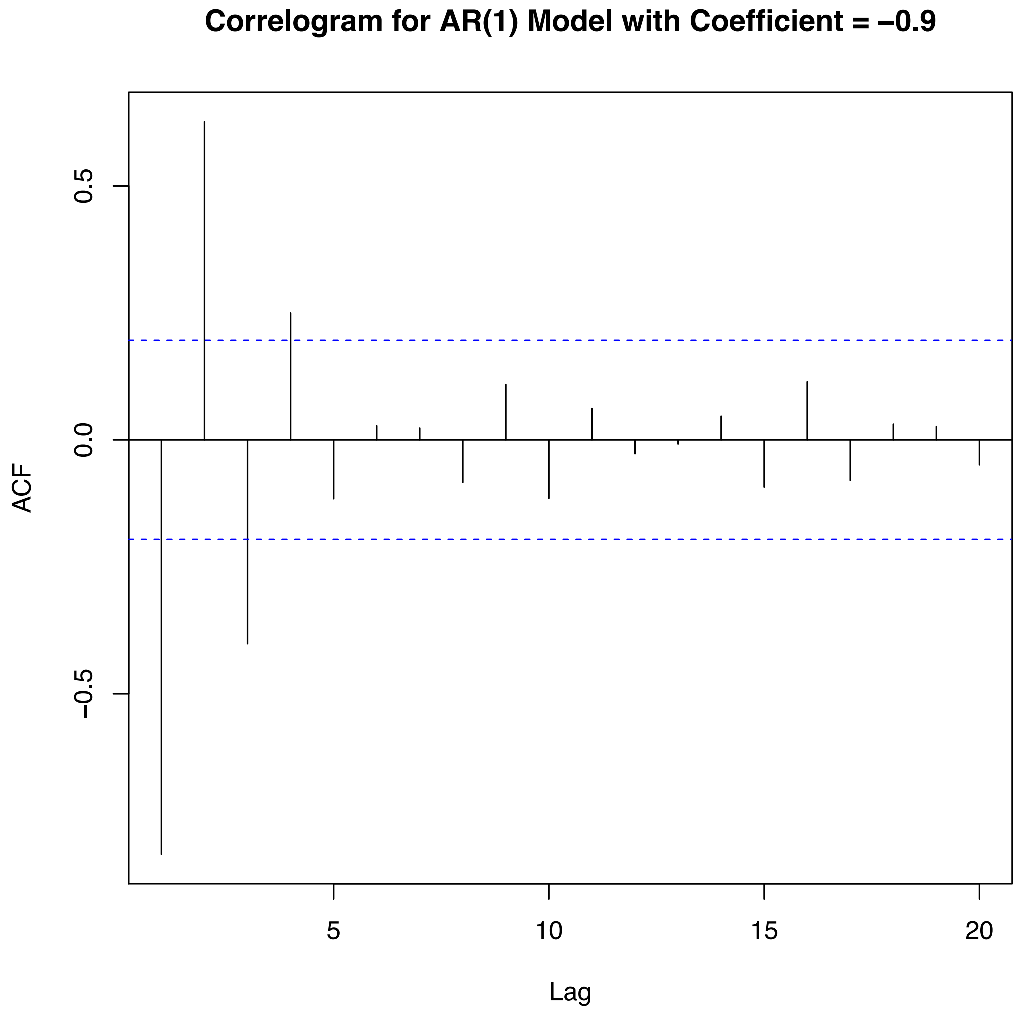
SAMPLE AUTOCORRELATIONS

- Compared to,



SAMPLE AUTOCORRELATIONS

- As well as,



SAMPLE AUTOCORRELATIONS

- Correlograms are useful because we can get an immediate visual impression of the covariance structure of a time series.
- However, we cannot formally infer whether a series is white noise by simply looking at the correlogram. This is because the two standard error bands only provide 95% bounds for the sample autocorrelations taken one at the time.
- In order to determine whether a series is white noise, we need to test the null hypothesis that all of its correlations are ***jointly*** zero.

SAMPLE AUTOCORRELATIONS

- To construct a joint test, we first rewrite the expression

$$\hat{\rho}(\tau) \sim N\left(0, \frac{1}{T}\right)$$

- As

$$\sqrt{T}\hat{\rho}(\tau) \sim N(0,1)$$

- We know from our introductory statistics course that the square of a normal random variable is a chi-squared random variable with one degree of freedom. Therefore,

$$T\hat{\rho}^2 \sim \chi_1^2$$

SAMPLE AUTOCORRELATIONS

- Additionally, it is also the case that the sum of independent χ^2 variables is also a χ^2 with degrees of freedom equal to the sum of the degrees of freedom of the variables summed. Therefore, we have that

$$Q_{BP} = T \sum_{\tau=1}^{m} \hat{\rho}^2(\tau) \sim \chi_m^2 \quad \text{(Box-Pierce Q-Statistic)}$$

- Alternatively,

$$Q_{LB} = T(T+2) \sum_{\tau=1}^{m} \left(\frac{1}{T-\tau}\right) \hat{\rho}^2(\tau) \sim \chi_m^2 \quad \text{(Ljung-Box Q-Statistic)}$$

- For moderate and large T, these two statistics are essentially the same.

SAMPLE AUTOCORRELATIONS

- The testing procedure for whether a series is white noise is as follows:

1. Compute the first m autocorrelations. As a general rule, choose $m \approx \sqrt{T}$
2. Compute the Box-Pierce or Ljung-Box Q-Statistic (i.e. Q_{BP} or Q_{LB} or both!).
3. Compare the computed test statistics to the critical value of χ_m^2 associated with a significance level of your choice, $\alpha\%$. (This is a right tailed test)!
4. If the test statistic lies in the rejection region, reject the null hypothesis that the data generating process is white noise.

SAMPLE AUTOCORRELATIONS

- Visually,

H_0: *The data generating process is white noise*

H_A: *The data generating process has serial correlation*