ECOM40006/ECOM90013 Econometrics 3
Department of Economics
University of Melbourne

Assignment 1

Semester 1, 2025

1. In this question we will assume that $x \sim \mathrm{N}(\mu, \Sigma)$ is a p-vector, as is μ, and that the $p \times p$ matrix $\Sigma > 0$.

   (a) If v is any fixed p-vector, show that

   $$g = \frac{v'(x-\mu)}{\sqrt{v'\Sigma v}} \sim \mathrm{N}(0,1). \qquad \text{(1 mark)}$$

   (b) If v is now a random vector independent of x for which $P(v'\Sigma v = 0) = 0$, show that $g \sim \mathrm{N}(0,1)$ and is independent of v. Why have we assumed $P(v'\Sigma v = 0) = 0$? Provide an equivalent statement of this assumption not involving Σ. (4 marks)

   (c) Hence show that if $y = [y_1, y_2, y_3]' \sim \mathrm{N}(0, I_3)$ then

   $$h = \frac{y_1 e^{y_3} + y_2 \log|y_3|}{[e^{2y_3} + (\log|y_3|)^2]^{1/2}} \sim \mathrm{N}(0,1). \qquad \text{(2 marks)}$$

2. Suppose that $x \sim \mathrm{N}(\mu, \Sigma)$, where the $p \times p$ matrix $\Sigma > 0$, and that v is a fixed p-vector. If r_i, the ith element of the vector r, is the correlation between x_i and $v'x$, show that $r = (cD)^{-1/2}\Sigma v$, where $c = v'\Sigma v$ and $D = \mathrm{diag}(\Sigma)$. (3 marks)

   Bonus question: When does $r = \Sigma v$? (1 mark)

Your answers to the Assignment should be submitted via Canvas no later than 4:30pm, Thursday 27 March.

No late assignments will be accepted but an incomplete exercise is better than nothing.

Your mark for this assignment may contribute up to 10% towards your final mark in the subject.