Lecture 2
Forecasting foundations and Autoregressive models

Forecasting foundations:

Conditional Expectations

Expected value

Expected value / ``population mean'':

$$E(Y) = \begin{cases} \sum_y y \cdot p(y) & Y \text{ is discrete} \\ \int y \cdot f(y) dy & Y \text{ is continuous} \end{cases}$$

Conditioning

Consider two random variables : Y and X.

- *Conditioning on* $X = x$ implies we take the value of X as known to be x.
- Knowing X might be informative about Y.
- Formally, the "distribution of Y conditional on X" differs from the "distribution of Y".

Conditional expectation and Prediction

- Suppose we know the value of random variable X is x (i.e. condition on $X = x$)
- Using this information, we want to predict the value of Y.
- The “optimal” predictor of Y conditional on $X = x$ is $E(Y|X = x)$.
- “Optimal” : minimum mean squared error

Minimum mean squared error prediction

Let $g(X)$ denote *any* function of X that could be used to predict Y.

The mean squared error (MSE) of $g(X)$ is

$$\mathrm{MSE}(g) = E[\,(\,Y - g(X)\,)^2\,]$$

The expectation is taken over both Y and X.

Can show that $g(X) = E(Y|X)$ minimises $\mathrm{MSE}(g)$.

Minimum mean squared error prediction

- In econometrics, this is why $E(Y|X)$ is used as the "population regression function".

- A functional form is assumed, eg $E(Y|X) = \beta_0 + \beta_1 X$.
- Parameters of $E(Y|X)$ estimated from data.
- The approach will the same for forecasting.

Forecasting

- Suppose we have a time series Y_t for $t = 1, \ldots, n$
- We want to forecast the unknown value of Y_{n+1}.

- The MSE-optimal forecast of Y_{n+1} :

$$E(Y_{n+1} | Y_n, Y_{n-1}, \ldots, Y_1)$$

Forecasting

- The MSE-optimal forecast of Y_{n+1} :

$$E(Y_{n+1}|Y_n, Y_{n-1}, \ldots, Y_1)$$

Steps:

1. Specify a *model* for $E(Y_{n+1}|Y_n, Y_{n-1}, \ldots)$.

2. Estimate model parameters from the data.

3. Evaluate model fit, re-specify as required.

4. Calculate and evaluate forecasts.

Step 1. Time series model specification

We specify a model for

$$E(Y_t | Y_{t-1}, Y_{t-2}, \dots, Y_1), \quad t = 1, 2, \dots, n$$

The model for $t = n + 1$ can forecast Y_{n+1}.

Step 1. Time series model specification

We specify a model for

$$E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_1), \quad t = 1, 2, \ldots, n$$

Example. *Autoregressive model of order one*:

$$E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_1) = \beta_0 + \beta_1 Y_{t-1}$$

Resembles a cross-section regression:

$$E(Y_i|X_i) = \beta_0 + \beta_1 X_i$$

Step 1. Time series model specification

Example. *Autoregressive model of order one*:

$$E(Y_t | Y_{t-1}, Y_{t-2}, \ldots, Y_1) = \beta_0 + \beta_1 Y_{t-1}$$

Assumes

- linear functional form
- only Y_{t-1} is useful for forecasting Y_t
- coefficients are constant over time

Step 2. Model estimation

Example. *Autoregressive model of order one*:

$$E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_1) = \beta_0 + \beta_1 Y_{t-1}$$

Recall the minimum MSE property of the conditional expectations function.

If the model is correct, the true values β_0, β_1 satisfy

$$(\beta_0, \beta_1) = \arg\min_{b_0, b_1} E[\,(\,Y_t - (b_0 + b_1 Y_{t-1})\,)^2\,]$$

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$$(\beta_0, \beta_1) = \arg\min_{b_0, b_1} E\left[\, (\, Y_t - (b_0 + b_1 Y_{t-1})\,)^2 \,\right]$$

The sample analog of this is

$$(\widehat{\beta}_0, \widehat{\beta}_1) = \arg\min_{b_0, b_1} \frac{1}{n-1} \sum_{t=2}^{n} (\, Y_t - (b_0 + b_1 Y_{t-1})\,)^2$$

i.e. least squares estimation!

Step 3. Evaluate model

Define the (population) forecast error

$$U_t = Y_t - E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

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Define the (population) forecast error

$$U_t = Y_t - E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

Taking conditional expectations of both sides:

$$\begin{aligned} &E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ &= E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ &\quad - E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ &= 0 \end{aligned}$$

Step 3. Evaluate model

Meaning of

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0 \ ?$$

Review: Law of Iterated Expectations

If Z is any random variable then

$$E(Z) = E[\, E(Z|X) \,]$$

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If Z is any random variable then

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Inner expectation averages over Z for each X.

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Inner expectation averages over Z for each X.

Outer expectation then averages over X.

Step 3. Evaluate model

Meaning of

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0 \ ?$$

The LIE implies

$$\begin{aligned} E(U_t) &= E[\, E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \\ &= \qquad 0 \end{aligned}$$

Step 3. Evaluate model

Meaning of

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0 \ ?$$

For any $j = 1, 2, \ldots$:

$$\begin{aligned} &\operatorname{cov}(U_t, U_{t-j}) \\ =&E[\,(U_t - E(U_t))\,(U_{t-j} - E(U_{t-j}))\,] \quad \text{(defn)} \end{aligned}$$

Step 3. Evaluate model

Meaning of

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For any $j = 1, 2, \ldots$:

$$\begin{aligned}
&\operatorname{cov}(U_t, U_{t-j}) \\
=&E[\,(U_t - E(U_t))\,(U_{t-j} - E(U_{t-j}))\,] \quad \text{(defn)} \\
=&E[\,U_t U_{t-j}\,] \quad \text{since } E(U_t) = 0
\end{aligned}$$

Step 3. Evaluate model

Meaning of

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0 \ ?$$

For any $j = 1, 2, \ldots:$

$$\begin{aligned} & E[\, U_t U_{t-j} \,] \\ = & E[\, E(\, U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \quad \textbf{(LIE)} \end{aligned}$$

Step 3. Evaluate model

Meaning of

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For any $j = 1, 2, \ldots$:

$$\begin{aligned} & E[\, U_t U_{t-j} \,] \\ =& E[\, E(\, U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \quad \textbf{(LIE)} \\ =& E[\, U_{t-j} \cdot E(\, U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \quad \text{Why?} \end{aligned}$$

Step 3. Evaluate model

Meaning of

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0 \ ?$$

For any $j = 1, 2, \ldots$:

$$\begin{aligned}
&E[\, U_t U_{t-j} \,] \\
=&E[\, E(\, U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \quad \text{(LIE)} \\
=&E[\, U_{t-j} \cdot E(\, U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \,] \\
=& \; 0.
\end{aligned}$$

Step 3. Evaluate model

Summary :

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0$$

implies

i. $E(U_t) = 0$

ii. $\operatorname{cov}(U_t, U_{t-j}) = 0$ for all $j = 1, 2, \ldots$.

Step 3. Evaluate model

Summary :

$$E(U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0$$

implies

i. $E(U_t) = 0$

ii. $\mathrm{cov}(U_t, U_{t-j}) = 0$ for all $j = 1, 2, \ldots$.

The errors have **no autocorrelation**.

$\Rightarrow$ if a model produces autocorrelated errors, that model must be misspecified.

Why?

$$E(\, U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ = U_{t-j} \cdot E(\, U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

This is a deceptively important step.

Why?

$$E(\,U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ = U_{t-j} \cdot E(\,U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

- $U_{t-j} = Y_{t-j} - E(Y_{t-j} | Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$

Why?

$$E(\,U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$
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- $U_{t-j} = Y_{t-j} - E(Y_{t-j} | Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$
- Y_{t-j} is in the conditioning set.

Why?

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- Y_{t-j} is in the conditioning set.
- $E(Y_{t-j}|Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$ is a function of $Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1$.

Why?

$$E(\,U_t U_{t-j}\mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) \\ = U_{t-j} \cdot E(\,U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

- $U_{t-j} = Y_{t-j} - E(Y_{t-j}|Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$
- Y_{t-j} is in the conditioning set.
- $E(Y_{t-j}|Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$ is a function of $Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1$.
- $Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1$ are in the conditioning set.

Why?

$$E(\,U_t U_{t-j}\,|\,Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$
$$= U_{t-j} \cdot E(\,U_t\,|\,Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

- $U_{t-j} = Y_{t-j} - E(Y_{t-j}|Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$
- Y_{t-j} is in the conditioning set.
- $E(Y_{t-j}|Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$ is in the conditioning set.

Why?

$$E(\,U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$
$$= U_{t-j} \cdot E(\,U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

- $U_{t-j} = Y_{t-j} - E(Y_{t-j} | Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$
- Y_{t-j} is in the conditioning set.
- $E(Y_{t-j} | Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$ is in the conditioning set.
- Thus U_{t-j} is in the conditioning set.

Why?

$$E(\,U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$
$$= U_{t-j} \cdot E(\,U_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

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- Thus U_{t-j} is in the conditioning set, hence can come out of the conditional expectation.

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$$E(\,U_t U_{t-j} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$
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- $U_{t-j} = Y_{t-j} - E(Y_{t-j} | Y_{t-j-1}, Y_{t-j-2}, \ldots, Y_1)$
- Thus U_{t-j} is in the conditioning set, hence can come out of the conditional expectation.
- $\mathrm{cov}(U_t, U_{t-j}) = 0$ can be proved because the model conditioning set contains all lags of the dependent and explanatory variables.

Summary of formalities

- A forecasting model is a model for

$$E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

- The "analog principle" implies estimation by least squares.
- The forecast errors

$$U_t = Y_t - E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_1)$$

should not be autocorrelated.

Autoregressive models

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.

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  where X_t contains *deterministic* variables.
- Example: $X_t = 1$ (a constant mean)

Context

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  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- Example: $X_t = (1, \text{Time}_t)'$ (linear time trend)

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- Example: $X_t = (1,\ \text{Time}_t,\ Q_{2,t},\ Q_{3,t},\ Q_{4,t})'$
  (trend+seasonals)

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- Deterministic: values are known without looking at the data.

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- We therefore focus on forecasting Z_t.

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- X_t will include an intercept $\quad \Rightarrow \quad E(Z_t) = 0.$

Context

- We have a time series Y_t for $t = 1, \ldots, n$.
  We want to forecast the unknown value of Y_{n+1}.
- Suppose we have specified a regression:

  $$Y_t = X_t'\beta + Z_t$$

  where X_t contains *deterministic* variables.
- X_t will include an intercept $\quad \Rightarrow \quad E(Z_t) = 0.$
  But $E(Z_t) = 0 \not\Rightarrow E(Z_t | Z_{t-1}, Z_{t-2}, \ldots, Z_1) = 0$

First order autoregressive model

$$E(Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1) = \phi_1 Z_{t-1}$$

First order autoregressive model

$$E(Z_t | Z_{t-1}, Z_{t-2}, \ldots, Z_1) = \phi_1 Z_{t-1}$$

The time series Z_t is regressed on a *lag* of itself.

- *lag* : a value of the time series in the past
- Z_{t-1} : first lag of Z_t
- Z_{t-2} : second lag of Z_t, etc

First order autoregressive model

$$E(Z_t | Z_{t-1}, Z_{t-2}, \ldots, Z_1) = \phi_1 Z_{t-1}$$

It will be convenient to write

$$\mathscr{Z}_{t-1} = \{Z_{t-1}, Z_{t-2}, \ldots, Z_1\}$$

so

$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1}$$

Autoregressive (AR) models

First order autoregressive model:

$$E(Z_t|\mathscr{Z}_{t-1}) = \phi_1 Z_{t-1}$$

Second order autoregressive model:

$$E(Z_t|\mathscr{Z}_{t-1}) = \phi_1 Z_{t-1} + \phi_2 Z_{t-2}$$

Autoregressive (AR) models

First order autoregressive model:

$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1}$$

Second order autoregressive model:

$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1} + \phi_2 Z_{t-2}$$

p^{th} order autoregressive model:

$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1} + \ldots + \phi_p Z_{t-p}$$

AR(p) models

p^{th} order autoregressive model:

$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1} + \ldots + \phi_p Z_{t-p}$$

- Estimation: least squares / maximum likelihood
- Fit evaluation: graphs, autocorrelation testing
- Choice of p: model selection criteria
- Forecasting

AR(1) Forecasting

How does AR(1) forecasting work?

Consider the AR(1) model

$$Y_t = X_t'\beta + Z_t$$
$$E(Z_t | \mathscr{Z}_{t-1}) = \phi_1 Z_{t-1}$$

How does AR(1) forecasting work?

At $t = n + 1$:

$$
\begin{aligned}
Y_{n+1} &= X'_{n+1}\beta + Z_{n+1} \\
E(Z_{n+1}|\mathscr{Z}_n) &= \phi_1 Z_n
\end{aligned}
$$

The optimal forecast for Y_{n+1} given data to time n is

$$
E(Y_{n+1}|\mathscr{Y}_n) = E(X_{n+1}|\mathscr{Y}_n)'\beta + E(Z_{n+1}|\mathscr{Y}_n)
$$

How does AR(1) forecasting work?

At $t = n + 1$:

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The optimal forecast for Y_{n+1} given data to time n is

$$E(Y_{n+1}|\mathscr{Y}_n) = E(X_{n+1}|\mathscr{Y}_n)'\beta + E(Z_{n+1}|\mathscr{Y}_n)$$

$$\uparrow$$

$$\mathscr{Y}_n = \{Y_n, Y_{n-1}, \ldots, Y_1\}$$

How does AR(1) forecasting work?

At $t = n + 1$:

$$Y_{n+1} = X'_{n+1}\beta + Z_{n+1}$$
$$E(Z_{n+1}|\mathscr{Z}_n) = \phi_1 Z_n$$

The optimal forecast for Y_{n+1} given data to time n is

$$E(Y_{n+1}|\mathscr{Y}_n) = E(X_{n+1}|\mathscr{Y}_n)'\beta + E(Z_{n+1}|\mathscr{Y}_n)$$

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At $t = n + 1$:

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Y_{n+1} &= X'_{n+1}\beta + Z_{n+1} \\
E(Z_{n+1}|\mathscr{Z}_n) &= \phi_1 Z_n
\end{aligned}
$$

The optimal forecast for Y_{n+1} given data to time n is

$$
\begin{aligned}
E(Y_{n+1}|\mathscr{Y}_n) &= E(X_{n+1}|\mathscr{Y}_n)'\beta + E(Z_{n+1}|\mathscr{Y}_n) \\
&= X_{n+1}{}'\beta + E(Z_{n+1}|\mathscr{Z}_n)
\end{aligned}
$$

because X_t is deterministic.

How does AR(1) forecasting work?

At $t = n + 1$:

$$Y_{n+1} = X'_{n+1}\beta + Z_{n+1}$$
$$E(Z_{n+1}|\mathscr{Z}_n) = \phi_1 Z_n$$

The optimal forecast for Y_{n+1} given data to time n is

$$\begin{aligned} E(Y_{n+1}|\mathscr{Y}_n) &= E(X_{n+1}|\mathscr{Y}_n)'\beta + E(Z_{n+1}|\mathscr{Y}_n) \\ &= \quad '\beta + E(Z_{n+1}|\mathscr{Z}_n) \\ &= \quad '\beta + \phi_1 Z_n \end{aligned}$$

How does AR(1) forecasting work?

At $t = n + 1$:

$$Y_{n+1} = X'_{n+1}\beta + Z_{n+1}$$
$$E(Z_{n+1}|\mathscr{Z}_n) = \phi_1 Z_n$$

The optimal forecast for Y_{n+1} given data to time n is

$$E(Y_{n+1}|\mathscr{Y}_n) = X'_{n+1}\beta + \phi_1 Z_n$$

Lecture 2 Summary

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- MSE optimal forecasting is formalised using *conditional expectations*.
- For one-step-ahead forecasts, a correct model for the conditional expectations must have errors **without autocorrelation**.
- An autoregressive model is a regression of Y_t on its "lags" $Y_{t-1}, \ldots, Y_{t-p}$.