# ECON30009/90080 – Assignment 1

## This Version: Semester 2, 2025

(Due no later than August 28, Thursday, 4pm)

**Assignment Overview**

- This assignment is graded out of 80 points. While the assignment is scored out of 80 points, it is worth 8% of your total grade for this course.

- Please type the solutions to your assignment (e.g., in Word or LaTeX) and convert them to a PDF file for online submission through Gradescope on the LMS. Handwritten submissions can also be scanned and submitted through Gradescope as a PDF, but they must be legible for marking.

- **Please note this is a group assignment with up to 3 students per group. You can form your groups directly in Gradescope.** You don't need to be in the same tutorial to form a group and you can submit your assignment individually if you prefer.

- All students within the same group will receive the same mark and no two groups may submit the same assignment. You can collaborate with members of your own group (and all group members must provide input), but not with other groups. **Please list all members of your group clearly on the first page of your submission** and make sure you keep draft copies of your own working (for each member of the group).

- **This assignment should reflect your own work and ideas** (see also the section: **Artificial Intelligence Software in the Preparation of Material for Assessment** in the Subject Guide). AI assistance tools are not required for this assignment, but if you do use them it should be for **editing and proofing of your work only**. Any use of AI tools for editing purposes needs to be clearly acknowledged at the start of your assignment. Further information on the acceptable use of these tools can be found here.

## Question 1: Using Logarithms (10 points)

In this class, we use a lot of logarithms, in particular the natural log. When calculating GDP growth rates, economists like to use the difference in the natural log of GDP as an approximation for an economy's growth.

*Some background info:* As a review, let's recall what happens with compound interest rates. Suppose you invest $X_1$ for one year at interest rate $r = 2\%$. At the end of the year, you get $X_2 = (1+r)X_1$, i.e. $X_2 = (1.02)X_1$. Interest can be compounded, for example, you might get a return every quarter. Suppose the annual interest rate is still $r = 2$, then compounded quarterly, your final return at the end of the year is $X_2 = (1 + \frac{r}{4})^4 X_1$ or $X_2 \approx 1.0202$. As the frequency of compounding becomes high, the formula $(1 + \frac{r}{t})^t$ converges to :

$$\lim_{t\to\infty}(1 + \frac{r}{t})^t = e^r$$

This implies that if your money gets compounded continuously, at the end of the year you would have $X_2 = e^r X_1$.
To get back the interest rate or growth rate of your investment, we can do a very simple transformation of the above equation by taking the natural logarithm on both sides

$$\ln\frac{X_2}{X_1} = \ln X_2 - \ln X_1 = r$$

We can now do a very simple subtraction of two log variables to get out the growth rate!

a Download quarterly data on Real GDP for the Australian economy from FRED (you may use this link: here) for the period covering 1959q4-2025q1. Calculate the growth rate of Real GDP for each quarter by using the formula: $g_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$. Find the average growth rate across all quarters. [Note in your answer, your growth rate series would start from 1960q1 onwards]

b Now calculate the growth rate for each quarter as: $\hat{g}_t = \ln(GDP_t) - \ln(GDP_{t-1})$. Again find the average growth rate across all quarters. Is there a large discrepancy between your answer in part (a) and part (b)? Plot the two calculated growth rate series, $g_t$ and $\hat{g}_t$ together on the same plot.

c Calculate the correlation between the growth rate $g_t$ and $\hat{g}_t$ you found in part (a) and part (b)

*Final notes on logarithms*: One more useful aspect about using logs is in terms of making comparisons. Suppose $GDP_t = 100$. Saying that GDP rose by 50% in period $t+1$ but then fell by 50% in $t+2$ does not give us back 100 in period $t+2$. However, if we said that GDP rose 50 log points in $t+1$ and dropped 50 log points in $t+2$, we would be back at the same level of GDP in period $t+2$ as in period $t$ You can see this clearly from the formula $\ln GDP_t = \ln GDP_{t-1} + g_t$.

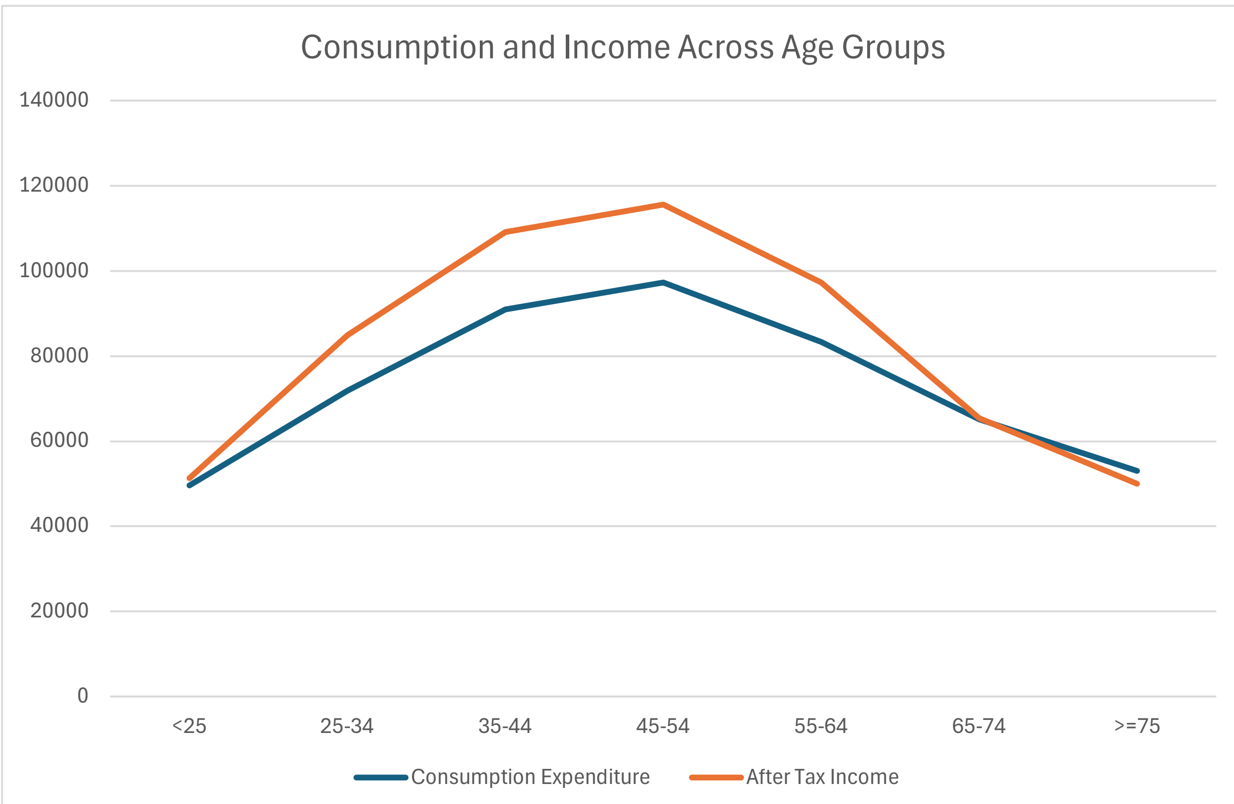
## Question 2: Applying the consumption-savings model to data (10 points)

Figure 1 uses 2023 data from the US Bureau of Labor Statistics Consumer Expenditure Survey (for those interested in this dataset, you can find the data series here: BLS CEX Survey.) Figure 1 plots consumption spending and after tax income by reference age group. Associated values are recorded in Table 1.

| Age | Annual Consumption Expenditure | After-tax Income |
|---|---|---|
| <25 years | 49560 | 51278 |
| 25-34 years | 71867 | 84939 |
| 35-44 years | 90939 | 109075 |
| 45-54 years | 97319 | 115653 |
| 55-64 years | 83379 | 97276 |
| 65-74 years | 65149 | 65461 |
| $\geq 75$ | 53031 | 49981 |

Table 1: Consumption spending and after-tax income across age

a) Compute the ratio of consumption spending to after-tax income for each reference age group




Source: BLS CEX data 2023

Figure 1: Consumption spending and after-tax income across age groups

b) In your own words and from the lens of the consumption-saving model, provide some intuition for why the ratio of consumption spending to after-tax income is less than one during the prime-age (25-54) working years. Also provide some intuition for the observed consumption spending to after-tax income ratio for individuals aged 65 and older.

# Question 3: Equilibrium in the OLG model (60 points)

Consider the OLG model we studied in class. Suppose there are only households and firms in the economy, i.e., there is no government. In each period, there are always $N$ new households born into the economy. In other words, there are always exactly $N$ households in each generation. Assume that households of generation $t$ have the following preferences:

$$U(c_t^y, c_{t+1}^o) = (c_t^y)^\beta (c_{t+1}^o)^{1-\beta}$$

Each household works and inelastically supplies one unit of labour when they are young, earning them wage income $w_t$ in period $t$. When they are old, each household retires. Young households in period $t$ can save in physical capital, $a_{t+1}$, which pays a gross return of $R_{t+1} = 1 + r_{t+1}$.

Firms produce output via a Cobb-Douglas production function $Y_t = z_t K_t^\alpha L_t^{1-\alpha}$. For the first part of this question, you may assume $z_t = z$ for all $t$. Firms rent capital at rental rate $R_t$ and hire labour at wage rate $w_t$. Capital evolves according to:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where $\delta = 1$, i.e, there is **full** depreciation.

a Set up the household problem and derive the household's optimality conditions

b Set up the firm's problem and derive the firm's optimality conditions c Using your equilibrium conditions, derive an equation that expresses $c_t^y$ in terms of predetermined variable, $k_t$ parameters $\alpha, \beta$, and exogenous variable $z_t = z$.

d Derive the transition equation, i.e., an equation that shows how $k_{t+1}$ evolves as a function of $k_t$, parameters of the model and exogenous variable $z_t = z$. Explain in one or two sentences why knowing this transition equation is sufficient to describe how the key aggregate macroeconomic variables evolves over time in this model economy.

e Write down what the long-run steady state capital per person $\bar{k}$ is in this economy.

f In class, we've largely assumed $z_t = z$ for simplicity. Now suppose that there is a production externality and productivity is instead endogenous and affected by the level of capital stock in the economy (you can think of this as the more capital is produced and used in production, the more productive and adept we become at using this capital). Each household and firm, however, thinks that they are individually too small to affect the capital stock and thus they take $z_t$ as given.

Let the production externality take the form that $z_t$ has an increasing relationship with $K_t$, that is:

$$z_t = \bar{z} K_t^{1-\alpha}$$

Are the firms' and households' optimality conditions any different? What about the transition equation? If yes, derive the new transition equation. Finally, is there a steady state in this economy?