ECON30024
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Solution to Tutorial 11

1. (1) You have just purchased a 10-year zero-coupon bond with a yield to maturity of 10% and a face value of \$1,000. What would be your rate of return if you sell the bond at the end of the year? Assume that the yield to maturity on the bond is 11% at the time you sell.

   (a) 1.4%   (b) 10%   (c) 11%   (d) 5.5%   (e) None of the rest.

   Answer: (a)

   The rate of return from holding the zero-coupon bond for a year or the one-year holding period yield, is given by

$$\frac{p_{9,t+1}}{p_{10,t}} - 1,$$

   where $p_{10,t}$ denotes the price of the bond at present date t, when the bond's time to maturity is 10 years, and $p_{9,t+1}$ denotes the price of the bond one year later, when the bond's time to maturity is 9 years.

   $p_{10,t}$ can be calculated using the bond's face value and yield to maturity at t, $y_{10,t} = 10\%$:

$$p_{10,t} = \frac{1000}{(1 + y_{10,t})^{10}} = \frac{1000}{(1 + 10\%)^{10}} = 385.5433.$$

   $p_{9,t+1}$ can be calculated using the bond's face value and yield to maturity at $t+1$, $y_{9,t+1} = 11\%$:

$$p_{9,t+1} = \frac{1000}{(1 + y_{9,t+1})^{9}} = \frac{1000}{(1 + 11\%)^{9}} = 390.9248.$$

   So the one-year holding period yield is calculated as

$$\frac{p_{9,t+1}}{p_{10,t}} - 1 = \frac{390.9248}{385.5433} = 0.014 = 1.4\%.$$

   (2) A coupon-paying bond that pays interest of \$100 annually has a face value of \$1,000, matures in 5 years, and is selling today at a \$72 discount from the face value. The yield to maturity on this bond is

   (a) 6.00%   (b) 8.33%   (c) 10.39%   (d) 12.00%   (e) 60.00%

   Answer: (d)

The yield to maturity on a coupon-paying bond is determined by equation (2) on Topic 10 slides:

$$p = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \cdots + \frac{c+m}{(1+y)^n}$$

In this quesiton, the present price $p = 1000 - 72$, $n = 5$, $c = 100$, and $m = 1000$, so the yield to maturity y satisfies the following equation:

$$928 = \frac{100}{1+y} + \frac{100}{(1+y)^2} + \frac{100}{(1+y)^3} + \frac{100}{(1+y)^4} + \frac{100+1000}{(1+y)^5}$$

Let y take each of the values in (a)-(e), we can see that $y = 12\%$ makes the equation above approximately hold. So the yield to maturity on this bond is about 12%.

(3) Which of the following statements regarding yield to maturity is FALSE?

(a) Given present price, a bond has a certain rate of return if it is held to maturity.

(b) The yield to maturity on a zero-coupon bond only depends on the bond's time to maturity.

(c) If you buy a zero-coupon today and hold it to maturity, then you will earn a risk-free annual rate of return that is equal to the bond's yield to maturity today.

(d) Under the arbitrage principle, at a given date, all zero-coupon bonds with the same time to maturity have the same spot yields.

Answer: (a)

(4) Which of the following statements regarding the yield curve is TRUE?

(a) The yield curve is a graphical representation of the term structure of interest rates.

(b) The yield curve plots the yields to maturity on bonds against their time to maturity.

(c) The yield curve can be upward sloping or downward sloping.

(d) The shape of the yield curve can vary over time.

(e) All of the rest.

Answer: (e)

(5) According to the liquidity preference theory, an yield curve should be

(a) flat.

(b) upward sloping.

(c) downward sloping.

(d) inverted U shape.

(e) None of the rest.

Answer: (b)

2. Key points:

    - The equity premium puzzle is a quantitative puzzle regarding the gap between the observed average equity premium (stock returns over risk-free bond returns) and the equity premium implied by a dynamic asset pricing model.
    - Describe the magnitude of the equity premium in the data.
    - Briefly describe the model (infinitely-lived identical consumers, save through holding a risk-less bond and a risky equity, aim to maximise expected lifetime utility, flow utility function is CRRA)
    - Briefly summarise the model's quantitative prediction on the equity premium under reasonable risk aversion measures for the representative consumer (The model predicts that the equity premium is a function of the risk aversion parameter and the covariance between equity return and consumption growth rate. Given average covariance between equity return and consumption growth rate in the data, the predicted equity premium for reasonable risk aversion parameter is much lower than the observed equity premium).

3. (a) The coupon-paying bonds is equivalent to a portfolio of the following 4 zero-coupon bonds:

    - zero-coupon bond 1: paying \$20 after 1 year, i.e., its time to maturity is 1 year, face value is \$20
    - zero-coupon bond 2: paying \$20 after 2 year, i.e., its time to maturity is 2 year, face value is \$20
    - zero-coupon bond 3: paying \$20 after 3 year, i.e., its time to maturity is 3 year, face value is \$20
    - zero-coupon bond 4: paying \$100 after 4 year, i.e., its time to maturity is 4 year, face value is \$100

   (b) Denote the yield to maturity on these 4 zero-coupon bonds as y_1, y_2, y_3, y_4, respectively. The arbitrage principle under certainty implies that the yields on maturity on all zero-coupon bonds with the same time to maturity are equal. Therefore, y_j ($j = 1, 2, 3, 4$) equals the yield to maturity on the zero-coupon bond listed in the table which has a face value of \$100 and time to maturity j years.

Therefore, y_1, y_2, y_3, and y_4 are calculated as:

$$y_1 = \frac{100}{96} - 1 = 0.0417$$
$$y_2 = \left(\frac{100}{90}\right)^{1/2} - 1 = 0.0541$$
$$y_3 = \left(\frac{100}{84}\right)^{1/3} - 1 = 0.0598$$
$$y_4 = \left(\frac{100}{80}\right)^{1/4} - 1 = 0.0574$$

(c) The 'fair value' of the coupon-paying bond is defined to be the price at which the bond would be traded in a frictionless market and in the absence of arbitrage opportunities.

Because the coupon-paying bond is equivalent to the 4 zero-coupon bonds, its fair value should equal the sum of the prices of the 4 zero-coupon bonds. The price of each zero-coupon bond can be calculated using its yield on maturity obtained in (b):

$$p_1^{ZC} = \frac{20}{1+y_1},\ p_2^{ZC} = \frac{20}{(1+y_2)^2},\ p_3^{ZC} = \frac{20}{(1+y_3)^3},\ p_4^{ZC} = \frac{100}{(1+y_4)^4}.$$

So the fair value of the coupon-paying bond is given by

$$p = p_1^{ZC} + p_2^{ZC} + p_3^{ZC} + p_4^{ZC} = \frac{20}{1+y_1} + \frac{20}{(1+y_2)^2} + \frac{20}{(1+y_3)^3} + \frac{100}{(1+y_4)^4} = \$134.$$

4. First note that the US term spread is positive most of the time since 1982, implying that the yield on 10-year treasury bond is higher than the yield on 3-month treasury bill. This is consistent with the liquidity theory of the term structure; bonds with longer maturity needs to pay a term premium for risk-averse investors to be willing to hold them.

   Second, the negative term spread appeared only several times, each time followed by a recession within 18 months. A negative term spread implies an inverted yield curve. The fact that inverted yield curves often precede recessions lead economists to believe inverted yield curves can serve as a useful indicator for upcoming recessions. As discussed in Topic 10, the idea is that if a recession is expected to come, the market would expect the central bank to lower short-term interest rate in the future, which would lower the spot yield on long-term bonds to the extent to invert the current yield curve.

   Note that the term spread has been negative most of the time since October 2022. If history repeats itself, we would probably expect a recession of the US economy not far in the future.