Lecture 2: A Two Period Consumption-Savings Problem

ECON30009/90080 Macroeconomics

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Consumption-Savings

Why do we care about consumption?

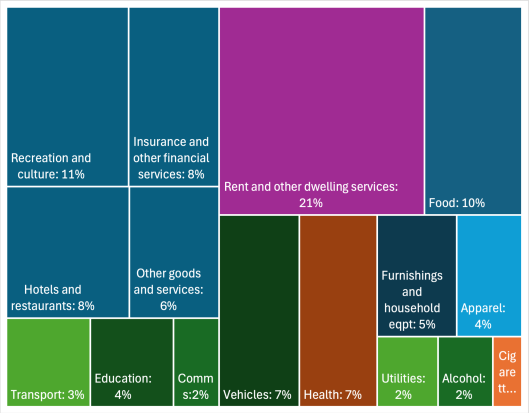
Consumption spending constitutes a large part of GDP

- Consumption spending makes about 52% of Australia's GDP

$$Y = C + I + G + X - M$$

- In the US, consumption spending makes up about 70% of GDP

Consumption Spending



Source: ABS, March 2025 National Accounts

Consumption as a composite good

- Although households decide on many items to spend on, for macro, we will treat consumption as a *composite* good
- We will treat C as aggregate household final consumption expenditure
- and c as individual household final consumption expenditure
- If there are N individuals, and if all households are identical:

$$C = \sum_{i=1}^{N} c_i = Nc$$

Building a Consumption-Savings Model

The Household's Consumption-Savings Problem

- Goal: write down a simple problem where households choose how much to consume and save

- For simplicity: consider a household who lives 2 periods.
  - Why don't we consider a household who only lives 1 period?

- For simplicity: consider a household who does not work and gets exogenous income (endowment)

The Household's Consumption-Savings Problem

- Household cares about consumption today when young (c^y) and tomorrow when old (c^o)
- Households can save in an asset a which pays $1 + r$ tomorrow
- Endogenous choice variables: Can choose c^y, c^o and a
- Exogenous variables: Income y^y today when young and y^o tomorrow when old
  - We will relax this assumption that income is exogenous later when we think about workers who have to earn their income

Relative price of consumption today when young

- r the interest rate is a relative price.
- Assume that all households are identical (representative households)
  - All households are identical, each household is too small to individually affect the market for assets (No Warren Buffets).
  - which means implicitly we are assuming that while a household can choose how much to save in a, they cannot individually control r
- In other words, we are considering the **individual** problem of a household who takes prices as given.

Decision-making in Economics

The Economics Approach

- Agents have an objective they want to achieve
- Agents can face constraints that affect their decision-making
  - Given scarcity of resources, how should we allocate $x, y, z \ldots$
- To achieve their objective, agents weigh the **marginal benefit** of an action against its **marginal cost**

The household problem

To build a consumption-savings model, we need to:

- identify what the household's objective is: objective function
- identify what constraints the household faces
- identify what trade-off she/he faces at the margin when making a choice

Household Constraints

Household budget constraints

- Let price of consumption $P = 1$, i.e., consumption is a numeraire good.
- Budget constraint is in real terms: how many consumption goods can you buy
- Budget constraint today when young:

$$c^y + a = y^y$$

- Budget constraint tomorrow when old:

$$c^o = y^o + (1+r)a$$

  - Why doesn't the household save in assets when old?

Household budget constraints

- a appears in both time periods' budget constraints
- Make a subject of 2nd period budget constraint

$$a = \frac{c^o}{1+r} - \frac{y^o}{1+r}$$

- And plug in above into 1st period budget constraint $c^y + a = y^y$:

$$c^y + \frac{c^o}{1+r} - \frac{y^o}{1+r} = y^y$$

- Re-arrange and get household lifetime budget constraint (LBC):

$$c^y + \frac{c^o}{1+r} = y^y + \frac{y^o}{1+r}$$

- LHS of above is present value of consumption, RHS is present value of income

Draw the household lifetime budget constraint

- We can re-arrange to the LBC to make c^o:

$$c^o = y^o + (1+r)y^y - (1+r)c^y$$

- Draw the budget constraint in (c^y, c^o) space.

Household lifetime budget constraint (LBC)

- The slope of LBC $\implies$ opportunity cost of consumption today
$$\frac{\partial c^o}{\partial c^y} = -(1+r)$$
- By giving up 1 unit of c^y and saving it, can consume $(1+r)$ units of c^o tomorrow

- The LBC represents the feasible set of consumption choices the household can make over his/her lifetime.

Household Objective

The Household: Assumptions

Assume households are:

- Representative: All the households have the same preferences
  - thinking of a typical decision maker to represent all households.
- Optimizing: they maximize their preferences given some constraints
  - which means they choose some optimal combination of (c^y, c^o)

The Decision of The Household

The household (HH) has an objective: be happy!

- The household gets happiness from consuming c^y and c^o
- The utility function U represents the happiness of the HH:
  - $U(c^y, c^o)$ represents the level of utility associated with a bundle (c^y, c^o)
  - We say that (c_1^y, c_1^o) is **preferred** to (c_2^y, c_2^o) if and only if

$$U(c_1^y, c_1^o) > U(c_2^y, c_2^o)$$

Properties of the Utility Function

Some assumptions we will make about the utility function:

- The household prefers more to less

$$\frac{\partial U(c^y, c^o)}{\partial c^y} > 0; \qquad \frac{\partial U(c^y, c^o)}{\partial c^o} > 0$$

- Each additional unit of consumption provides a smaller increase in utility (or less gain in happiness)

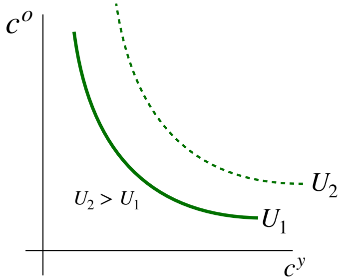
$$\frac{\partial^2 U(c^y, c^o)}{\partial (c^y)^2} < 0; \qquad \frac{\partial^2 U(c^y, c^o)}{\partial (c^o)^2} < 0$$

  - Assume that HH likes diversity (1 apple + 1 orange is better than 2 oranges)

Indifference Curves

- Easy to plot with 1 good an increasing utility function with diminishing marginal utility [How would you draw this?]

- With 2 goods, we represent the household's preferences with an **Indifference Curve**



An indifference curve is a curve connecting all the combinations of (c^y, c^o) for which the consumer is indifferent (provides the same utility).

The Marginal Rate of Substitution

- The negative slope of an indifference curve at a particular point is known as the **marginal rate of substitution (MRS)** at that point.

- It tells us the rate at which the household is willing to substitute consumption when young for consumption when old while maintaining the same level of utility.

- It is equal to the ratio of marginal utilities:

$$MRS_{c^y,c^o} = \frac{\partial U(c^y, c^o)/\partial c^y}{\partial U(c^y, c^o)/\partial c^o} = \frac{\partial c^o}{\partial c^y}$$

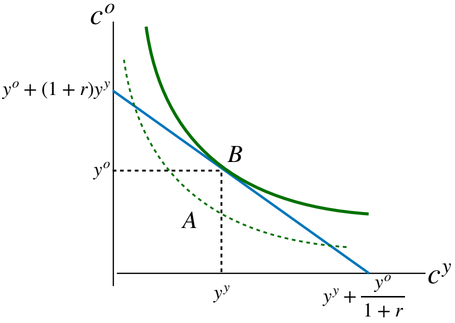
Consumer Maximization

Consumer Maximization

A consumption bundle is:

- Affordable if it lies on or within the budget set.
- Optimal if it is affordable and is on the highest indifference curve.

Optimal and Sub-optimal Bundles



- Only B is optimal
- Optimality required bundle to be on budget constraint
- Slope of indifference curve $=$ slope of budget constraint

Solving the household problem

The household problem:

$$\max_{c^y, c^o} U(c^o, c^y)$$

$$\text{s.t.} \qquad c^y + \frac{c^o}{1+r} = y^y + \frac{y^o}{1+r}$$

- This is a constrained optimization problem
  - A few ways to solve:
    - Substitution method
    - Lagrangian method (This is what we will focus on today!)

The Lagrangian Method

- The Lagrangian method transfers a constrained optimization problem into a
  - unconstrained optimization problem
  - with a pricing problem
- The new function to be optimized is called a Lagrangian
- Each constraint has a shadow price, called a Lagrange Multiplier (denoted by λ)
- In the new unconstrained household optimization problem, λ prices the added value the household gets from one more unit of income

Consumer Maximization: Lagrangian Method

Write the lagrangian:

$$\max_{c^y, c^o, \lambda} \mathcal{L}(c^y, c^o, \lambda) = U(c^y, c^o) + \lambda \left[y^y + \frac{y^o}{1+r} - c^y - \frac{c^o}{1+r} \right]$$

where λ is our lagrange multiplier. First order conditions (FOC):

$$(c^y): \qquad \frac{\partial U(c^y, c^o)}{\partial c^y} - \lambda = 0$$

$$(c^o): \qquad \frac{\partial U(c^y, c^o)}{\partial c^o} - \frac{\lambda}{1+r} = 0$$

$$(\lambda): \qquad y^y + \frac{y^o}{1+r} - c^y - \frac{c^o}{1+r} = 0$$

- The shadow value, λ, of having one more unit of consumption when young given by the marginal utility of c^y

Consumer Maximization

From the FOCs, we can derive the consumption Euler equation (also known as the household intertemporal condition) :

- Combine FOC wrt c^y and c^o

$$(c^y): \qquad \frac{\partial U(c^y, c^o)}{\partial c^y} = \lambda$$

$$(c^o): \qquad \frac{\partial U(c^y, c^o)}{\partial c^o}(1+r) = \lambda$$

Consumer Maximization

From the FOCs, we can derive the consumption Euler equation (also known as the household intertemporal condition) :

$$\boxed{\frac{\partial U(c^y, c^o)}{\partial c^y} = (1+r)\frac{\partial U(c^y, c^o)}{\partial c^o}}$$

- LHS of above is marginal benefit of 1 more unit of c^y
- RHS of above is marginal cost of 1 more unit of c^y (if had instead saved, could buy $(1+r)$ units of c^o which you value at the marginal utility of c^o)

Our household makes decisions by balancing marginal benefit vs marginal cost!

Optimality

Let's re-arrange the Euler equation:

$$\frac{\partial U(c^y, c^o)/\partial c^y}{\partial U(c^y, c^o)/\partial c^o} = (1 + r)$$

- Observe LHS is the ratio of marginal utilities. RHS is the relative price of c^y
- From our FOCs, we have that a consumption bundle is optimal when:
  - Is on the budget line

$$\text{(FOC wrt } \lambda\text{)} : \qquad y^y + \frac{y^o}{1+r} - c^y - \frac{c^o}{1+r} = 0$$

  - Slope of indifference curve (MRS) = slope of budget line

Exactly like our graphical solution!

Optimality

- Solving our household problem gave us two optimality conditions:

  - Euler equation: household tells you how she/he would optimally trade off consumption today (young) vs. tomorrow (old)

$$\boxed{\frac{\partial U(c^y, c^o)}{\partial c^y} = (1+r)\frac{\partial U(c^y, c^o)}{\partial c^o}}$$

  - Lifetime budget constraint (LBC): household's choice must be affordable

$$\boxed{c^y + \frac{c^o}{1+r} = y^y + \frac{y^o}{1+r}}$$

An example for next class

- Suppose preferences are given by:

$$U(c^y, c^o) = \ln c^y + \beta \ln c^o$$

where $\beta \in (0, 1)$ is a parameter representing the discount factor, i.e., the weight that households put on consumption tomorrow when old.

- Then problem becomes:

$$\max_{c^y, c^o} \ln c^y + \beta \ln c^o$$

$$\text{s.t.} \qquad c^y + \frac{c^o}{1+r} = y^y + \frac{y^o}{1+r}$$

Come prepared for next class: solve for c^y in terms of r, y^y, y^o and β.

Roadmap

- Today: a first look at a consumption-savings problem
- Next week: Permanent income hypothesis and introduction to firm's problem