ECON30009/90080 – Tutorial 6

This Version: Semester 2, 2025

These questions are designed to give you practice solving a 2 period RBC model.

Q1: RBC model with elastic labour supply

Assume there are N households in an economy. A household born into this economy is endowed with an initial amount of a physical asset, a_1. Each household in the economy lives for two periods and gets utility from consuming in both periods. Further the household observes disutility from supplying labour in each period. Specifically, let the household's preferences be given by

$$U(c_1, c_2, \ell_1, \ell_2) = \ln c_1 - \ell_1 + \beta (\ln c_2 - \ell_2)$$

where $0 < \beta < 1$ is the discount factor (i.e., the factor that the household discounts period 2 utility by). ℓ_t for $t \in \{1, 2\}$ is the amount of labour supplied by the household. Unlike the standard problem we solve in class, the household here can choose the amount of labour they would like to supply to the labour market.

The household is paid w_t for each unit of labour supplied in period t and the household receives rental income, $R_t a_t$, from renting out her physical asset in each period. In addition, the household receives dividend income from the firm. There is no government in this model. The representative firm produces output according to a Cobb-Douglas production function: $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$ where $0 < \alpha < 1$ and z_t is exogenous and can vary between period 1 and period 2. The firm chooses capital and labour to maximize its profits.

a) Set up the household's 2 period utility maximization problem. State what are the choice variables of the household.

b) Set up the Lagrangian to the household's problem and take first order conditions. Derive the Euler equation and LBC. Derive an equation that expresses the consumption-leisure trade-off of the household for each period t. [Hint: to derive the consumption-leisure trade-off you should arrive at an equation that relates the marginal cost of supplying one unit of labour to the marginal benefit of supplying one unit of labour.]

c) Set up the representative firm's problem and solve for the firm's optimality conditions.

d) Write down equations representing the market clearing conditions [For example, $N\ell_1 = L_1$ is an equation that must hold when the labour market clears in period 1. Write down the other market clearing conditions.]

e) Using information from the household euler equation, the household LBC and the firm's optimality conditions. Derive an expression for aggregate consumption $C_1 = Nc_1$ in terms of pre-determined K_1, choice variables L_1, K_2, exogenous variable z_1 and parameters of the model.

f) Using the goods market clearing condition, derive an expression for K_2 in terms of pre-determined K_1, choice variable L_1, exogenous variable z_1 and parameters of the model.

g) We have one last optimality condition we have not utilized yet, the consumption-leisure trade-off in period 1. Use the consumption leisure trade-off, your answer in parts e) and f) to solve for ℓ_1 in terms of pre-determined K_1, exogenous z_1 and parameters of the model.

h) Solve for C_1 and K_2 in terms of pre-determined K_1, exogenous variables and parameters of the model. Show that a rise in z_1 generates positive co-movement in C_1 and K_2 with GDP= Y_1. Show that even information about higher z_2 cannot drive an expansion in period $t = 1$,

Q2: RBC model with government investment

Consider the RBC model we discussed in class. Suppose there is a positive exogenous government spending shock in period 1 only. Each unit of government spending $G_1 > 0$ goes towards public capital formation, i.e., building capital for period 2, K_2^G. The government runs a balanced budget and finances this government spending G_1 by collecting a lump-sum tax, τ_1, from each household in period 1 only. There are N households.

Households live 2 periods and have the following preferences: $U(c_1, c_2) = \ln c_1 + \beta \ln c_2$ where $0 < \beta < 1$. They inelastically supply 1 unit of labour each period and receive wage income w_t for their labour where $t \in \{1, 2\}$. Households are born with initial capital a_1 and receive rental income from renting out their capital. The rental rate is given by R_t for $t \in \{1, 2\}$. Households also receive dividend income from firms.

The representative firm in this model produces according to a Cobb-Douglas production function where $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$. The firm rents capital at rate R_t and hires labour at wage rate w_t.

a) Write down the government budget constraint

b) Set up the firm's profit maximization problem and characterize the firm's optimality conditions

c) Set up the household's utility maximization problem and characterize the household's optimality conditions

d) Solve for k_2 and c_1 in terms of exogenous variables, parameters of the model and pre-determined k_1. You may denote $g_1 = G_1/N$.

e) Show that a rise in government investment is not a candidate for driving business cycles. In particular, explain how k_2 and c_1 change with an increase in government investment.