ECON30024
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Solution to Tutorial 5

1. (1) A portfolio of $n \geq 2$ assets is uniquely determined by the expected return on the portfolio. Is this statement true or false?

   (a) True

   (b) False

   Answer: (b)

   A portfolio of $n > 2$ assets cannot be uniquely determined by the expected return on the portfolio. To see this, let $n = 3$ and consider a portfolio $(a_1, a_2, 1-a_1-a_2)$. Given an expected return μ_P, we have

$$\mu_P = a_1\mu_1 + a_2\mu_2 + (1 - a_1 - a_2)\mu_3.$$

   It's clear that a_1 and a_2 cannot be uniquely determined by the equation above. This is way we need to determine the portfolio frontier with $n > 2$ risky assets by solving a minimum variance problem.

   (2) According to the mutual fund theorems, an efficient portfolio of n risky assets and a risk-free asset can be constructed as a portfolio of the risk-free asset and a mutual fund (or composite asset) of the n risky assets. Is this statement true or false?

   (a) True

   (b) False

   Answer: (a)

   (3) Which of the following statements is FALSE?

   (a) The optimum portfolio of an investor is reached at the point at which her efficient frontier is tangent to an indifference curve of her mean-variance objective.

   (b) The portfolio frontier with $n \geq 2$ genuinely different risky assets is a hyperbola.

   (c) The portfolio frontier with three genuinely different risky assets is located to the right of the frontier with any two of the three assets.

   (d) The efficient frontier with many risky assets and a risk free asset is a straight line.

   (e) Risk reduction can be achieved by investing in a portfolio of $n \geq 2$ genuinely different risky assets.

   Answer: (c)

(4) Consider the CAPM setting, suppose there are only two risky assets–two stocks and two investors. Both investors have homogeneous beliefs and mean-variance objectives. Suppose investor 1 invests a total value of \$1000 in risky assets, investor 2 invests \$1500 in risky assets, and the market is in equilibrium. The market portfolio is represented by $(0.4, 0.6)$. The prices of asset 1 and asset 2 are \$10 and \$20 per share, respectively. Which of the following statements is TRUE?

(a) Investor 1 invests \$400 in asset 1.

(b) Investor 2 invests \$900 in asset 2.

(c) The total supply of asset 1 is 100 shares.

(d) The total demand for asset 2 is 75 shares.

(e) All of the rest is true.

Answer: (e)

The risky assets portfolios held by investor 1 and 2 both have the same proportions as the market portfolio, i.e., both represented by $(0.4, 0.6)$. That is, both investors invest 40% of their total value of risky assets holdings in asset 1, and 60% in asset 2.

So the value of asset 1 and asset 2 held by investor 1 is: $(\$400, \$600)$, and held by investor 2: $(\$600, \$900)$.

Therefore, the total demand for asset 1 is (\$400+\$600)/\$10=100 shares, and the total demand for asset 2 is (\$600+\$900)/\$20=75 shares.

As market is in equilibrium, demand equals supply.

(5) Suppose the risk-free rate is 5.5% and the return of the market portfolio has an expected value of 14% and a standard deviation of 10%. The return of stock Z has a standard deviation of 12%, and a correlation coefficient with the market return of 0.2. According to the CAPM, what is the expected rate of return on stock Z?

(a) 6.75%

(b) 10.24%

(c) 7.54%

(d) 9.68%

(e) None of the rest.

Answer: (c)

According to the CAPM, the expected return on stock Z is given by

$$\mu_Z = r_0 + \beta_Z(\mu_M - r_0), \quad \text{where } \beta_Z \equiv \frac{\sigma_{ZM}}{\sigma_M^2}.$$

We first calculate β_Z: Note that

$$\sigma_{ZM} = \rho_{ZM}\sigma_Z\sigma_M,$$

so

$$\beta_Z = \frac{\rho_{ZM}\sigma_Z\sigma_M}{\sigma_M^2} = \rho_{ZM}\frac{\sigma_Z}{\sigma_M} = (0.2)\frac{0.12}{0.1} = 0.24.$$

Hence,

$$\mu_Z = r_0+\beta_Z(\mu_M-r_0) = 0.055+(0.24)(0.14-0.055) = 0.055+0.0204 = 0.0754 = 7.54\%$$

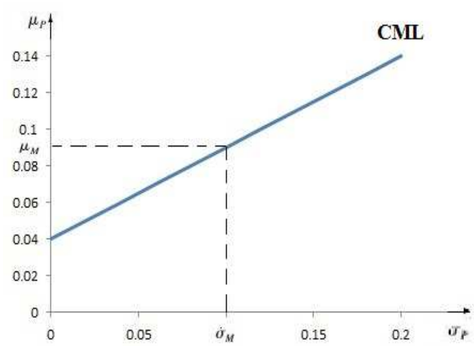
2. (a) The capital market line has an intercept at $r_0 = 0.04$ and its slope equals the Sharpe ratio of the market portfolio which is given by

$$s_M = \frac{\mu_M - r_0}{\sigma_M} = \frac{9\% - 4\%}{0.1} = 0.5.$$

So the capital market line is defined by the following equation

$$\mu_P = r_0 + \frac{\mu_M - r_0}{\sigma_M}\sigma_P = 0.04 + 0.5\sigma_P,$$

as sketched in the following figure:



(b) The Sharpe ratios of Asset 1, Asset 2, and Asset 3 are calculated as

$$s_1 = \frac{\mu_1 - r_0}{\sigma_1} = \frac{7\% - 4\%}{0.3} = 0.1$$
$$s_2 = \frac{\mu_2 - r_0}{\sigma_2} = \frac{8\% - 4\%}{0.2} = 0.2$$
$$s_3 = \frac{\mu_3 - r_0}{\sigma_3} = \frac{10\% - 4\%}{0.15} = 0.4$$

Assets 1-3 do not lie on the capital market line, as their Sharpe ratios are smaller than that of the market portfolio. Note that the market portfolio has the highest Sharpe ratio. This is no surprise, as the market portfolio is an efficient portfolio and it lies on the capital market line. All assets or portfolios of assets lying on the capital market line have the same Sharpe ratio.

(c) The beta-coefficient for asset j is defined by

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2} = \frac{\rho_{jM}\sigma_j\sigma_M}{\sigma_M^2} = \rho_{jM}\frac{\sigma_j}{\sigma_M}.$$

Using this definition, we can calculate the beta-coefficients for Asset 1 to 3 as follows:

$$\beta_1 = \rho_{1M}\frac{\sigma_1}{\sigma_M} = (0.2)\frac{0.3}{0.1} = 0.6$$
$$\beta_2 = \rho_{2M}\frac{\sigma_2}{\sigma_M} = (0.4)\frac{0.2}{0.1} = 0.8$$
$$\beta_3 = \rho_{3M}\frac{\sigma_3}{\sigma_M} = (0.8)\frac{0.15}{0.1} = 1.2.$$

Recall that according the the CAPM, an asset's beta-coefficient indicates its market or systematic risk. So in terms of systematic risk, Asset 1 is least risky, followed by Asset 2, with Asset 3 being most risky.

3. (a) In the context of the CAPM, the security market line expresses the relationship between the expected rates of return μ_j on assets, or portfolios of assets, and their beta coefficients, β_j, which is represented by the following equation

$$\mu_j = r_0 + (\mu_M - r_0)\beta_j.$$

Using the information provided:

$$\begin{aligned} \text{Asset 1}: \quad & 0.12 = r_0 + (\mu_M - r_0)(1.8), \\ \text{Asset 2}: \quad & 0.07 = r_0 + (\mu_M - r_0)(0.8), \end{aligned}$$

we can solve for r_0 and $\mu_M - r_0$:

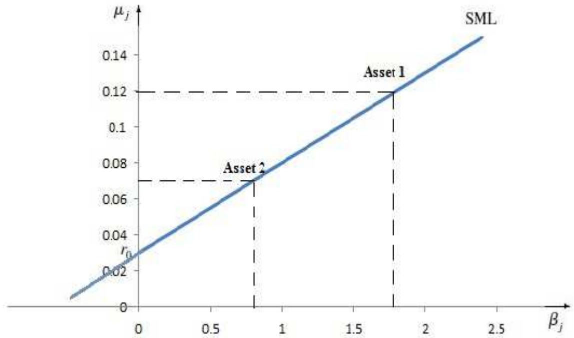
$$r_0 = 0.03, \quad \mu_M - r_0 = 0.05.$$

So the SML is represented by

$$\mu_j = 0.03 + 0.05\beta_j,$$

and it is plotted in the following figure. It's easy to verify that Asset 3 also lies on the SML.

The security market line:



(b) Applying the SML, the predicted rate of return

$$\mu_4 = 0.03 + (0.05)(2.0) = 13\% < 16\%.$$

That is, the average rate of return on the asset is higher than predicted by the CAPM. Therefore according to the CAPM, this asset is underpriced; its price should go up to yield a lower expected return than observed. Alternatively, this may be evidence against the CAPM.

4. (a) Here are some important differences between the CML and SML in the context of the CAPM.

   - They are different concepts: The CML is the portfolio frontier, also the efficient frontier, for all investors in equilibrium. The SML refers to the linear relationship between an asset's expected rate of return and its beta.
   - All assets or portfolios located on the CML are (mean-variance) efficient. However, not all assets or portfolios located on the SML are efficient.
   - If the CAPM is true, all assets should locate on the SML in equilibrium, i.e., their expected returns should have the linear relationship with their betas as predicted by the CAPM.

   (b) Here is a list of the assumptions for the CAPM and how they are used in the derivation.

   - All investors behave according to a single-period investment horizon: the portfolio selection problem of an individual investor is a single-period decision problem.
   - All investors select their portfolios according to a mean-variance objective: the objective function in individual investor's portfolio selection problem is a mean-variance objective.
   - All investors have homogeneous beliefs, i.e., μ_j's, σ_j's and σ_{ij}'s are the same for all investors: all investors have the same portfolio frontier and efficient frontier, and hence hold the same proportions of risky assets in their risky asset portfolio (i.e., the same target portfolio Z)

- Markets are in equilibrium: market clearing for all risky assets imply that the risky asset portfolio Z is the market portfolio M in which the proportions of each asset j is its market share in the risky assets market.
- Frictionless markets: the portfolio selection problem is only subject to the budget constraint, no need to consider any other constraint associated with market frictions.
- Unlimited risk-free borrowing and lending: the portfolio frontier with n risky assets and a risk free asset is a straight line such that the CML is a straight line.
- Competitive asset markets: individual investors take asset prices and beliefs about asset returns as given when making their own portfolio decisions; they do not view their own portfolio choice would affect asset prices or returns.