ECOM40006/ECOM90013 Econometrics 3
Department of Economics
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Week 4 Tutorial Exercise

Semester 1, 2025

1. Take the opportunity to ask any questions that you may have about the lecture material.

2. Consider the simple linear regression model

$$y_i = x_i + u_i, \qquad , i = 1, \ldots, n, \tag{1}$$

so that $\beta = 1$ is the true parameter value. Also assume that the classical assumptions about the disturbance term are satisfied to that $\mathrm{E}[u_i \mid x_i] = 0$ and that the u_i given x_i are independent and identically distributed, with $0 < \mathrm{Var}[u_i \mid x_i] = \sigma^2 < \infty$. The OLS estimator for β is

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}. \tag{2}$$

Given these assumptions show that:

(a) If $x_i = i$ then $\mathrm{plim}_{n\to\infty} \hat{\beta} = \beta = 1$.

Hint: $\displaystyle\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$.

(b) If $x_i = i^{-1}$ then $\mathrm{plim}_{n\to\infty} \hat{\beta} \neq \beta = 1$.

Hint: $\displaystyle\sum_{i=1}^{n} i^{-2} = \frac{\pi^2}{6}$.

What do these two results suggest about consistency and the accumulation of knowledge?