ECOM90024
Forecasting in Economics and Business
Tutorial 4

1.) The updating equations for Holt's multiplicative trend model are given by:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1}b_{t-1})$

Trend Equation: $b_t = \beta \dfrac{l_t}{l_{t-1}} + (1-\beta)b_{t-1}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t b_t^h$

a.) Rewrite the above level and trend equations in their error correction forms and describe the role that the parameters, α and β play in updating the level and trend when new information arrives.

b.) Let $\alpha = 0.2$ and $\beta = 0.4$ and let the initial values of the trend and level be given by $l_0 = 1$ and $b_0 = 0.1$. Then, suppose that you have the following time series data set:

$$\{1,4,9,20,23\}$$

Using Excel, compute the smoothed time series according to the equations provided above.

c.) Using Excel, calculate the h = 4 step ahead point forecasts according to equations provided above equations.

2.) Using the ***rnorm*** command in R, generate and plot Gaussian white noise series that comprises of 200 observations. These will be a set of observations from the following data generating process,

$$Y_t = \varepsilon_t$$

$$\varepsilon_t \sim_{iid} N(0,1)$$

Then, using the ***acf*** and ***pacf*** commands in R, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated series. Do they accord with the properties of the underlying data generating process?

3.) Using the data that you've generated in question 2, generate and plot a series that represents a set of observations from the following MA(1) process,

$$Y_t = \varepsilon_t + 0.9\varepsilon_{t-1} \qquad t = 2, 3, \dots, 200$$

$$\varepsilon_t \sim_{iid} N(0,1)$$

Then, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated MA(1) series. Discuss your findings.

4.) Using the data that you've generated in question 2, generate and plot a series that represents a set of observations from the following AR(1) process,

$$Y_t = 0.9Y_{t-1} + \varepsilon_t \qquad t = 2, 3, \dots, 200$$

$$Y_1 = 1$$

$$\varepsilon_t \sim_{iid} N(0,1)$$

Then, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated AR(1) series. Repeat the exercise using an autocorrelation coefficient of -0.9. Discuss your findings.

ECOM90024
Forecasting in Economics and Business
Tutorial 5

1.) Is the following MA(2) process covariance-stationary?

$$Y_t = (1 + 2.4L + 0.8L^2)\varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

If so, calculate its autocovariances and autocorrelations.

2.) Is the following AR(2) process covariance-stationary?

$$(1 - 1.1L + 0.18L^2)Y_t = \varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

If so, calculate its autocovariances and autocorrelations.

3.) Given the following AR(2) process,

$$Y_t = 0.6Y_{t-1} - 0.08Y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim_{iid} (0,1)$$

Compute the roots of the corresponding lag polynomial and verify that it is indeed a covariance stationary process.

4.) The data file ar2.csv contains 2000 observations that have been simulated from an AR(2) process whose coefficients you do not know.

a.) Using ***R***, compute and plot the sample autocorrelations and partial autocorrelations. Do they accord with the dependence structure of an AR(2) process?

b.) Using your knowledge of the Yule-Walker equations, compute estimates of the AR(2) coefficients.

c.) Using ***R***, verify your computation in part b.) by computing OLS estimates of the AR(2) coefficients.

ECOM90024
Forecasting in Economics and Business
Tutorial 6

1.) Consider the variables Y_t and X_t such that Y_t is described by an AR(1) model,

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

while X_t is described by the following restricted ARMA(4,1) model,

$$X_t = \beta X_{t-4} + u_t + \theta u_{t-1}$$

where both ε_t and u_t are white noise series and $|\phi| < 1$, $|\beta| < 1$ and $|\theta| < 1$ so that the stationarity and invertibility of Y_t and X_t are guaranteed.

Show that the variable $Z_t = Y_t + X_t$ can be described by an ARMA(5,4) model. (*Hint: The lag operator will be useful here*!)

2.) Consider the general MA(∞) representation for a stationary time series Y_t, that is,

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots +$$

and suppose that the parameters $\theta_1, \theta_2, \ldots$ are known.

a.) The 1 step ahead forecast error is defined as:

$$\varepsilon_{t+1|t} = Y_{t+1} - E[Y_{t+1}|\Omega_t]$$

$$\Omega_t = \{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$$

What are the forecast errors for 3 and 4 steps ahead?

b.) What is the covariance between the 3 and 4 step ahead errors?

3.) Suppose that the time series Y_t is governed by the following process,

$$Y_t = Y_{t-1} + \varepsilon_t$$

Where ε_t is a white noise series with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t^2] = \sigma^2$ for all t. Also suppose that that Y_t is observed every six months, but that it is aggregated to annually observed time series X_T by taking the sum of the two observations of Y in year T. Show that X_T can be described by

$$X_T = X_{T-1} + u_T$$

Where u_T is an MA(1) process with first order autocorrelation equal to ${}^{1}/_{6}$. (*Hint: Let periods t and $t-1$ be in year T*)

4.) For each of the following stationary time series processes

a.) $Y_t = \mu + \beta Y_{t-1} + u_t$

b.) $Y_t = \mu + u_t + 0.6u_{t-1} + 0.2u_{t-2}$

Where u_t is a white noise process with $E[u_t] = 0$ and $E[u_t^2] = \sigma^2$

i.) Derive the unconditional mean $E[Y_t]$
ii.) Derive the unconditional variance $Var(Y_t)$
iii.) Derive the first-order autocovariance $Cov(Y_t, Y_{t-1})$

ECOM90024
Forecasting in Economics and Business
Tutorial 7

---

1. Let $x_1, x_2, \dots, x_n$ be a set of realizations of from an i.i.d. sequence $X_1, X_2, \dots, X_n$ in which each X_i is characterized by a Poisson distribution function:

$$P(X = x;\ \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

   a.) Using words, provide an explanation of the parameter λ and the role it plays in determining the shape of the distribution function.

   b.) Write down the likelihood and log-likelihood function associated with the set of realizations $x_1, x_2, \dots, x_n$.

   c.) Given the realizations $x_1, x_2, \dots, x_n$, what is the maximum likelihood estimate of the parameter λ?

   d.) Using the rpois() function in ***R***, generate a set of 500 independent realizations from a Poisson random variable where $\lambda = 2$. Using these realizations, compute the maximum likelihood estimate of the parameter λ. Does your estimate conform to expectations?

   e.) Repeat part d an additional 499 times. You will have 500 samples of 500 observations from which you will obtain 500 estimates of the parameter λ. Plot a histogram of the estimates and discuss its shape. (***Hint: Try writing a loop in R:*** https://www.r-bloggers.com/how-to-write-the-first-for-loop-in-r/)

2. Suppose that you are analyzing a time series model that behaves according to an ARMA(1,1) process,

$$Y_t = \phi Y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

$$\varepsilon_t \sim_{i.i.d.} (0, \sigma^2)$$

a.) Given the information set $\Omega_t = \{Y_t, Y_{t-1}, \dots, \varepsilon_t, \varepsilon_{t-1}, \dots\}$, write down the expressions for the one-step, two-step and h-step ahead forecasts.

b.) Write down the expressions for the one-step, two-step and h-step ahead forecast errors and their associated variances. What happens to the forecast error variance when $h \to \infty$

c.) Using the data contained in *tute7.csv*, estimate an ARMA(1,1) model in ***R*** using the *mle()* function and compare your estimates with those produced by the *Arima()*

function. Then, use the estimates and the formulas that you derived in part b) to compute h-step ahead 95% interval forecasts for $h = 1,2, \ldots ,10$. Compare your intervals with those produced by the *forecast()* function and describe what happens to the interval forecasts as h increases.

ECOM90024
Forecasting in Economics and Business
Tutorial 8

---

1. Consider the following AR(4) process,

$$Y_t = \mu + \delta t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \varepsilon_t$$

   a.) Given the above process, derive a regression model that will allow you to test whether the process possesses a unit root. In doing so, make sure to derive expressions of the regression coefficients in terms of the parameters of the above process.

   b.) Having derived the regression equation, explain how it can be used to test for the presence of a unit root. Make sure to write down the elements of the test (i.e., null and alternative hypotheses, test statistic and decision rule).

2. Using the R statistical environment, simulate 1000 observations from an AR(4) (with no mean or deterministic trend) using a set of coefficients generated from the generateAR() function from the DREGAR package (see R-Week5.pdf)

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \varepsilon_t$$

   a.) Install and load the urca package. The function ur.df() performs an Augmented Dickey Fuller Test using the following syntax:

```
ur.df(y, type = c("none", "drift", "trend"), lags = k)
```

   Where `y` is the time series data, `type` is the form of the ADF regression where

   `"none"` = $\Delta Y_t = \rho Y_{t-1} + \sum_{j=2}^{k} \beta_j \Delta Y_{t-j+1} + \varepsilon_t$

   `"drift"` = $\Delta Y_t = \mu + \rho Y_{t-1} + \sum_{j=2}^{4} \beta_j \Delta Y_{t-j+1} + \varepsilon_t$

   `"trend"` = $\Delta Y_t = \mu + \delta t + \rho Y_{t-1} + \sum_{j=2}^{4} \beta_j \Delta Y_{t-j+1} + \varepsilon_t$

   and `lags` is the number of lags to include in the ADF regression. Use the ur.df() function to perform an ADF test on your simulated data. Make sure to explain the lag that you've chosen to specify in the function. Do your results conform to your expectations?

   b.) Using the data that you have simulated, construct a new time series Z_t that is defined as:

$$Z_t = \sum_{s=0}^{t} Y_s$$

   (Hint: you can use the cumsum() function).

Generate a plot of Z_t and describe what you see. Use the ur.df() function to perform an ADF test on Z_t. You can use the same lag specification used in part (a). Do your results conform to your expectations?

3.) Download the data contained in the file sp500.csv into your R environment.

a.) Generate a plot of the data and briefly describe the primary visual characteristics of the data?

b.) Generate sample ACF and PACF plots. What are they telling us about the dependence structure of the data?

c.) Perform an ADF test using the ur.df() function in which `type = "drift"`. Set the lag length using the method of Ng & Perron discussed in class. In the output, the relevant test statistic is the first number that is reported. Compare this test statistic with the critical value labelled tau2. Do your results conform with your results in parts (a) and (b)?

d.) Compute the continuously compounded return on the S&P 500 index using the following formula:

$$r_t = 100 \times (\ln(P_t) - \ln(P_{t-1}))$$

Plot the returns and repeat part (c) using the continuously compounded returns that you've just generated and report your results.

ECOM90024
Forecasting in Economics and Business
Tutorial 8

1.) Let ε_t be a sequence of innovations that behaves according to an ARCH(1) process,

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

$$\upsilon_t \sim_{i.i.d.} (0,1)$$

$$\alpha_0 > 0, \alpha_1 \geq 0$$

a.) Show that ε_t is serially uncorrelated. That is, verify that all of its autocorrelations (apart from its zero-th autocorrelation) are zero.

Suppose now that ε_t behaves according to an ARCH(m) process. Will ε_t be still serially uncorrelated?

b.) Suppose that ε_t, in addition to behaving according to an ARCH(m) process, also represents the innovations to an ARMA(p, q) process,

$$Y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \alpha + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

What are the forecasting implications of incorporating an ARCH(m) structure into the time series behaviour of Y_t?

With a plain ARMA process, the forecast intervals depends only on the forecast horizon.

When we incorporate an ARCH or GARCH structure to our innovations, then our forecast horizons will also depend on the information.

2.) The file "btc.csv" contains observations of the daily closing price of Bitcoin from 08/05/2016 to 08/05/2019. Using ***R*** you are required to do the following:

a.) Generate the daily returns on Bitcoin as the log difference of the daily price.

b.) Verify using the sample ACF and PACF, as well as appropriately specified Box tests that the daily returns are serially uncorrelated.

c.) Using the methods described in the lecture, test for the presence of ARCH effects in the daily Bitcoin returns.

d.) Estimate an appropriate ARCH model for the returns. Verify that your ARCH specification is adequate by plotting the standardized residuals from your ARCH estimation.

ECOM90024
Forecasting in Economics and Business
Tutorial 9

1.) Let ε_t be a sequence of innovations that behaves according to a GARCH(2,1) process,

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2$$

$$\upsilon_t \sim i.i.d.\ N(0,1)$$

$$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \beta_2 \geq 0$$

a.) Show that the GARCH(2,1) model can be rewritten as an ARMA(2,2) process for the squared innovations ε_t^2.

b.) Derive the unconditional variance of ε_t and explain why it is different to the conditional variance.

c.) Explain why the GARCH parameters are restricted to the values,

$$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \beta_2 \geq 0$$

Are these the only restrictions that must be imposed on these parameters?

d.) Given the information set $\Omega_t = \{\varepsilon_t, \varepsilon_{t-1}, \dots\}$, derive expressions for the 1-step and 2-step ahead forecasts of the conditional variance in terms of the conditioning variables.

2.) Let R_t represent the return on a financial asset from period $t-1$ to t and suppose that it is governed by the following GARCH specification for $t = 1,2, \dots, T$.

$$R_t = \mu + \beta R_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (\varepsilon_{t-1} - \theta \sigma_{t-1})^2 + \beta_1 \sigma_{t-1}^2$$

$$\upsilon_t \sim_{i.i.d} N(0,1)$$

a.) Given the above specification, derive the unconditional variance of ε_t and the set of conditions on the parameters $\alpha_0, \alpha_1, \theta, \beta_1$ that guarantee the non-negativity and finiteness of the conditional and unconditional variance. You may assume that the process ε_t is covariance stationary.

b.) Explain how the leverage effect is captured by the above specification. Why would this specification be useful in the analysis of financial returns?

3.) The file “tsla.csv” contains observations of the daily closing price of Tesla stock from 16/05/2016 to 16/05/2019. Using ***R*** you are required to do the following:

a.) Generate the daily returns on Tesla’s stock as the log difference of the daily price.

b.) Verify using the sample ACF and PACF, as well as appropriately specified Box tests that the daily returns are serially uncorrelated.

c.) Estimate a GARCH(1,1) model for the returns. Verify that the specification is adequate by showing that the squared standardized residuals from the GARCH estimation are serially uncorrelated.

d.) The values of the conditional volatility $\hat{\sigma}_t$ can be computed by applying the predict() function to the object in which you have stored your GARCH output (Note: the predict function will produce two columns of output, make sure to only use the first column!). Using the parameter estimates that you obtained in part c, compute the h-step ahead forecast of the conditional variance for $h = 1,2, \dots ,10$. (Hint: you can use a loop to compute these values.)