As regards normality, we stick to it this time because the results based on the *t* distribution are worse. We are going to focus instead on the possibility of a unit root in the conditional variance and on the leverage effect.

c) Estimate an *IGARCH*(1,1) model with a constant mean equation for *DLNDAX*.

```
spec_v2 = ugarchspec(mean.model = list(armaOrder = c(0,0), include.mean = TRUE),
                     variance.model = list(model = "iGARCH", garchOrder = c(1,1)),
                     distribution.model = "norm")
fit_v2 = ugarchfit(spec = spec_v2, data = DLNDAX)
print(fit_v2)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000722    0.000111   6.4894 0.000000
omega   0.000003    0.000001   3.0288 0.002455
alpha1  0.117372    0.011137  10.5393 0.000000
beta1   0.882628          NA       NA       NA
```

$$\widehat{DLNDAX}_t = 0.000722 + e_t \quad , \quad e_t \sim N(0, \hat{h}_t)$$

$$\hat{h}_t = 0.000003 + 0.117372 e_{t-1}^2 + 0.882628 \hat{h}_{t-1}$$

$\beta_1 = 1 - \alpha_1$, so it is not estimated.

Without presenting the details, the rest of the printout suggests that

i. The robust standard errors make *omega* and *alpha1* insignificant.
ii. As one should expect, all four model specification criteria favour the *GARCH*(1,1) model over this restricted model.
iii. The weighted *LB* tests do not detect autocorrelation.
iv. The weighted *ARCH LM* tests do not detect any remaining *ARCH* effect.
v. The Nyblom stability tests reject stability for *omega*.
vi. The sign bias tests detect some leverage effect.
vii. The adjusted Pearson tests reject normality.

As an additional check, it is useful to test H_0: $\alpha_1 + \beta_1 = 1$ on the *GARCH* model.

This linear restriction can be tested with the likelihood ratio (*LR*), Wald or Lagrange multiplier (*LM*) tests based on the unrestricted *GARCH* model and the restricted *IGARCH* model.

In the *LR* test, for example, the test statistic is

$$\lambda = 2(\ln L_{ur} - \ln L_r)$$

where L_{ur} and L_r are the likelihood values of the unrestricted and restricted models, respectively,

Under H_0 it follows a chi-square distribution with degrees of freedom equal to the number of restrictions.

There is not a specific *R* function to perform the *LR* test on a *GARCH* model, but we can do it step-by-step.

Likelihood values:

```
url = likelihood(fit_v1)
print(round(url,2))
```
27009.85

```
rl = likelihood(fit_v2)
print(round(rl,2))
```
26988.86

Test statistic:

```
lambda = 2*(log(url) - log(rl))
print(round(lambda,5))
```
0.00155

p-value:

```
pvalue = 1 - pchisq(q = lambda, df = 1)
print(round(pvalue,4))
```
0.9685

H_0: $\alpha_1 + \beta_1 = 1$ is maintained, supporting the *IGARCH* model.

d) Estimate a *TGARCH*(1,1) model with a constant mean equation for *DLNDAX*.

```
spec_v3 = ugarchspec(mean.model = list(armaOrder = c(0,0), include.mean = TRUE),
                     variance.model = list(model="fGARCH", submodel="TGARCH",
                                           garchOrder = c(1,1)),
                     distribution.model = "norm")
fit_v3 = ugarchfit(spec = spec_v3, data = DLNDAX)print(fit_v3)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : fGARCH(1,1)
fGARCH Sub-Model        : TGARCH
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000307    0.000113   2.7264 0.006402
omega   0.000344    0.000044   7.8660 0.000000
alpha1  0.074508    0.007240  10.2905 0.000000
beta1   0.915468    0.008538 107.2171 0.000000
eta11   0.734277    0.057214  12.8339 0.000000
```

$$\widehat{DLNDAX}_t = 0.000307 + e_t \quad , \quad e_t \sim N(0, \hat{h}_t)$$

$$\hat{h}_t = 0.000344 + 0.074508 e_{t-1}^2 + 0.734277 d_{t-1} e_{t-1}^2 + 0.915468 \hat{h}_{t-1}$$

The estimate of the conditional variance for $e_{t-i} < 0$, $d = 1$ is

$$\hat{h}_t = 0.000344 + (0.074508 + 0.734277) e_{t-1}^2 + 0.915468 \hat{h}_{t-1}$$
$$= 0.000344 + 0.808785 e_{t-1}^2 + 0.915468 \hat{h}_{t-1}$$

while for $e_{t-i} > 0$, $d = 0$ it is

$$\hat{h}_t = 0.000344 + 0.074508 e_{t-1}^2 + 0.915468 \hat{h}_{t-1}$$

e) Estimate an *EGARCH*(1,1) model with a constant mean equation for *DLNDAX*.

```
spec_v4 = ugarchspec(mean.model = list(armaOrder = c(0,0), include.mean = TRUE),
                     variance.model = list(model="eGARCH", garchOrder = c(1,1)),
                     distribution.model = "norm")
fit_v4 = ugarchfit(spec = spec_v4, data = DLNDAX)
print(fit_v4)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : eGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.000329    0.000093    3.5438 0.000394
omega  -0.203867    0.000674 -302.2640 0.000000
alpha1 -0.091653    0.004462  -20.5415 0.000000
beta1   0.976486    0.000155 6280.4241 0.000000
gamma1  0.121289    0.001190  101.9641 0.000000
```

$$\widehat{DLNDAX}_t = 0.000329 + e_t \quad , \quad e_t \sim N(0, \hat{h}_t)$$

$$\hat{h}_t = -0.203867 - 0.091653 \frac{e_{t-1}}{\sqrt{\hat{h}_{t-1}}} + 0.121289 \frac{|e_{t-1}|}{\sqrt{\hat{h}_{t-1}}} + 0.976486 \ln \hat{h}_{t-1}$$

The point estimates are all significant and they satisfy $\alpha_1 + \gamma_1 = 0.02963 > 0$, $\alpha_1 = -0.091653 < 0$, and $\gamma_1 = 0.121289 > 0$.

f) Estimate a *GARCH-M*(1,1) model with a constant mean equation for *DLNDAX*.

```
spec_v5 = ugarchspec(mean.model = list(armaOrder = c(0,0),
                                        include.mean = TRUE, archm = TRUE, archpow = 2),
                     variance.model = list(model="sGARCH", garchOrder = c(1,1)),
                     distribution.model = "norm")
fit_v5 = ugarchfit(spec = spec_v5,  data = DLNDAX)
print(fit_v5)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000375    0.000177   2.1136 0.034554
archm   2.903712    1.158441   2.5066 0.012191
omega   0.000004    0.000001   4.4390 0.000009
alpha1  0.099579    0.006274  15.8707 0.000000
beta1   0.879628    0.007120 123.5366 0.000000
```

$$\widehat{DLNDAX}_t = 0.000375 + 2.903712\hat{h}_t + e_t$$

$$e_t \sim N(0, \hat{h}_t)$$

$$\hat{h}_t = 0.000004 + 0.099579 e_t^2 + 0.879628\hat{h}_{t-1}$$

The point estimates are all significant at the 1.3% level.