Forecast error decomposition

Useful for calculating the proportion of movements attributable a variable's own shocks versus other variables.

recall the VMA(∞) representation from previous note:

$$\boxed{x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}} \rightarrow \boxed{E(x) = \mu + \sum_{i=h}^{\infty} \phi_i \varepsilon_{t+h-i}}$$

$$\therefore \boxed{x_{t+h} - E_t(x_{t+h}) = \sum_{i=0}^{h-1} \phi_i \varepsilon_{t+h-i}} \rightarrow$$ the h-period ahead forecast errors

$$\boxed{1 = \underbrace{\frac{\sigma_y^2}{\sigma_y^2(h)} \sum_{i=0}^{h-1} \phi_{11}^2(i)} + \underbrace{\frac{\sigma_z^2}{\sigma_y^2(h)} \sum_{i=0}^{h-1} \phi_{12}^2(i)}}$$

Prop. of $\sigma_y^2(h)$ that is due to shocks in the $\{\varepsilon_{yt}\}$ sequence

Prop. of $\sigma_y^2(h)$ that is due to shocks in the $\{\varepsilon_{zt}\}$ sequence

notes:

- In practice, f/cast error typically driven by own shocks in SR & other variables in LR
- Just like impulse response analysis, variance decomp reqs the estimation of some matrix which requires restriction. We can again rely on the cholesky decomposition.
- If the VAR system is stable, and h is increasing, variance decomps should be approaching some constant.