10 Impulse response analysis

Objective is to trace the effects of structural innovations or shocks ($\varepsilon_{\cdot t}$) on the entire time path of LHS variables.

↳ To do this we use the VMA representation of the VAR model. → VAR generates fcasts, VMA for studying dyn. props of system & calculate forecast errors.

$$\boxed{\begin{aligned} y_t &= a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + u_{1t} \\ z_t &= a_{20} + a_{21} y_t + a_{22} z_{t-1} + u_{2t} \end{aligned}} \rightarrow \boxed{\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \sum_{i=0}^{\infty} (A_1 L)^i \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \sum_{i=0}^{\infty} (A_1 L)^i \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}$$

with $\boxed{\mu = \begin{bmatrix} \mu_y \\ \mu_z \end{bmatrix}}$ and $\boxed{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \frac{1}{1 - b_{12} b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}}$

$$\boxed{\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \mu \\ \mu_z \end{bmatrix} + \frac{1}{1 - b_{12} b_{21}} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} = \begin{bmatrix} \varphi_{11}(i) & \varphi_{12}(i) \\ \varphi_{12}(i) & \varphi_{22}(i) \end{bmatrix}}$$

∴ We get the final form of the VMA(∞) rep. of VAR(1) system.

$$x = \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_z \end{bmatrix} + \sum_{i=0}^{\infty} \underbrace{\begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix}}_{\phi_i} \begin{bmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{bmatrix} = \mu + \sum_{i=0}^{\infty} \phi \varepsilon_{t-i}$$

The elements of the $\phi_i (i = 0, 1, 2, \ldots)$ matrices measure the effect of the ε_{zt} shocks on current & future values of $\{y_t\}$ & $\{z_t\}$
↳ $\{\varphi_{11}(i)\}, \{\varphi_{12}(i)\}, \{\varphi_{21}(i)\}, \{\varphi_{22}(i)\}$ $(i = 0, 1, 2, \ldots)$ sets of coefficients are the impulse response functions.

The first element of ϕ_0 are called the impact multipliers whereas the sum of n elements are the intermediate multipliers

Before doing this you need to make sure the VAR system is stationary:

e.g. $\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \rightarrow I - A_1 L = \begin{bmatrix} 1-0.8L & -0.2L \\ -0.4L & 1-0.1L \end{bmatrix}$

Determine inverse characteristic equation:

$$|I - A_1 L| = (1-0.8L)(1-0.1L) - (0.02L)(-0.4L) = 1 - 0.9L = 0$$

$\therefore$ single root, $L = {}^{10}/_{9}$, is outside unit circle. $\therefore$ VAR(1) system is stable & $\{y_t\}$, $\{z_t\}$ processes are stationary.