ECON90080 – Assignment 1

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Question 1

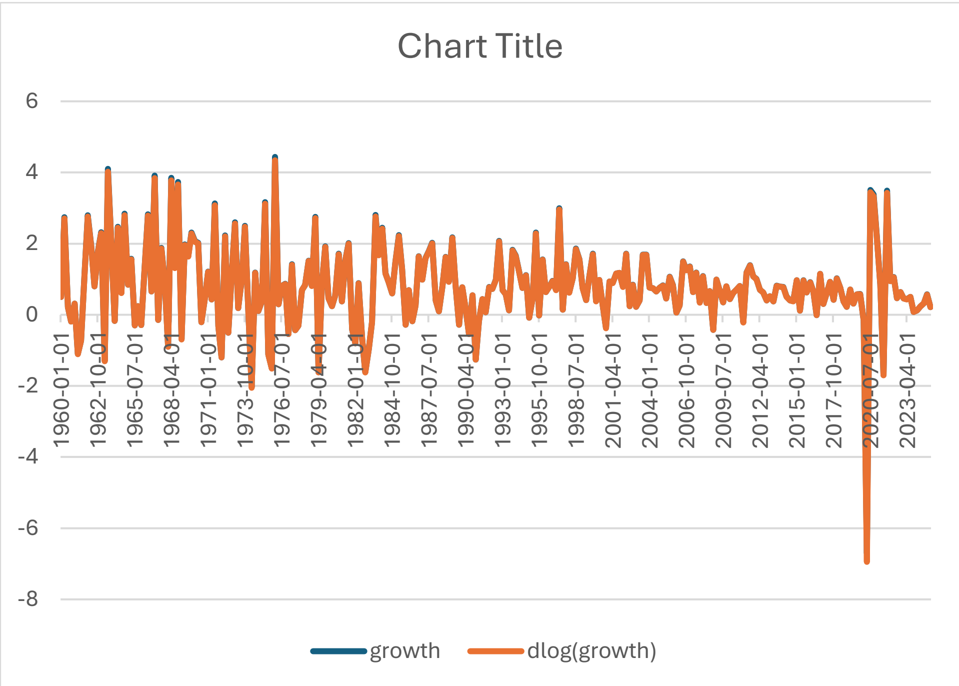
a. **Calculate the growth rate of real GDP for each quarter from 1960q1 onwards using the explicit growth formula.**

After converting to percentages, the average quarterly real GDP growth rate is 0.8211%.

b. **Repeat the exercise above, but use the difference in logs growth rate formula.**

After converting to percentages, the average quarterly real GDP growth rate is 0.8112%. Figure 1 compares the growth rate profiles between calculation methods. There are very minimal differences between them and is only just observable when growth is extremely high (i.e. during the large spikes during the 1960s and 1970s and COVID-19 periods).

Figure 1: Real GDP quarterly growth by calculation method



c. **Calculate the correlation in growth rates produced in part (a) and (b).**

The correlation between the two growth rate time series is 0.9985.

Question 2

a. **Compute the ratio of consumption spending to after-tax income for each reference age.**

The fourth column of Table 1indicates the ratio of consumption after spending to after-tax income for each reference age group.

Table 1: Consumption spending, after-tax income and their ratio across age

Age	Annual Consumption Expenditure	After-tax Income	Ratio
< 25 years	49560	51278	0.97
25-34 years	71867	84939	0.85
35-44 years	90939	109075	0.83
45-54 years	97319	115653	0.84
55-64 years	83379	97276	0.86
65-74	65149	65461	1.00
≥ 75 years	53031	49981	1.06

b. Provide intuition for why the ratio of consumption spending to after-tax income is less than one during the prime-age (25-54) working years. Also provide some intuition for the observed consumption spending to after-tax income ratio for individuals aged 65 and older.

The ratio of consumption spending to after-tax income is less than one during prime-age because these households are saving for the future when they can no longer work. If this ratio was greater than 1 whilst working, they risk not having enough savings to retire with. At this point, households are earning more than they are consuming.

Conversely, this ratio is greater than one after prime-age because this is generally when households retire, implying they need to use the savings they have spent their working life accumulating to fund their consumption decisions. At this point, households are consuming more than they are earning.

Question 3

a. Set up the household problem and derive the household's optimality conditions.

$$\max_{c_t^y,\, c_{t+1}^o \geq 0} \quad U(c_t^y, c_{t+1}^o) = (c_t^y)^\beta (c_{t+1}^o)^{1-\beta}$$

$$s.t. \quad c_t^y + \frac{c_{t+1}^o}{R_{t+1}} = w_t$$

To derive optimality conditions, we must solve the Lagrangian for all its first order conditions (FOCs):

$$\mathcal{L} = (c_t^y)^\beta (c_{t+1}^o)^{1-\beta} + \lambda\left(w_t - c_t^y - \frac{c_{t+1}^o}{R_{t+1}}\right)$$

$$\frac{\partial \mathcal{L}}{\partial c_t^y} = \beta (c_t^y)^{\beta-1} (c_{t+1}^o)^{1-\beta} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}^o} = (1-\beta)(c_t^y)^\beta \, (c_{t+1}^o)^{-\beta} - \frac{\lambda}{R_{t+1}} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = w_t - c_t^y - \frac{c_{t+1}^o}{R_{t+1}} = 0$$

Then, to derive the Euler our equation, we must divide the first two FOCs by each other.

$$\frac{\frac{\partial\mathcal{L}}{\partial c_t^y}}{\frac{\partial\mathcal{L}}{\partial c_{t+1}^o}} = \frac{\beta\left(c_t^y\right)^{\beta-1}(c_{t+1}^o)^{1-\beta}}{(1-\beta)\left(c_t^y\right)^{\beta}(c_{t+1}^o)^{-\beta}} = \frac{\lambda}{\frac{\lambda}{R_{t+1}}}$$

$$\frac{\beta}{1-\beta} * \frac{c_{t+1}^o}{c_t^y} = R_{t+1}$$

This final expression is our Euler equation and one of our optimality conditions. The other is the lifetime budget constraints, noted above as the constraint for the household's optimisation problem.

b. Set up the firm's problem and derive the firm's optimality conditions.

The representative firm has the following Cobb-Douglas production function:

$$y_t = zK^{\alpha}L^{1-\alpha}\,, \quad 0 < \alpha < 1$$

This firm is also profit-maximising and has the following optimisation problem:

$$\max_{K_t, L_t} \quad \pi_t = zK^{\alpha}L^{1-\alpha} - R_tK_t - w_tL_t$$

For the firm, the optimality conditions are the marginal product of labour (MPL) and marginal product of capital (MPK). Practically, this means equating the marginal products to the wage rate (w) and the capital rental rate (R) respectively. If this wasn't the case, the firm would change its productions inputs.

For these expressions we express them in per capita terms $\left(\frac{K}{L} \equiv k_t\right)$:

$$MPL = \frac{\partial\pi_t}{\partial L} = (1-\alpha)zk_t^{\alpha} = w$$

$$MPK = \frac{\partial\pi_t}{\partial K} = \alpha zk_t^{\alpha-1} = R$$

c. Using your equilibrium conditions, derive an equation the expresses c_t^y in terms of the predetermined variable, k_t parameters α, β, and exogenous variable $z_t = z$.

Doing this takes a few steps. First, we need to insert the Euler equation into the lifetime budget constraint:

$$c_t^y + \frac{1}{R_{t+1}}\left(\frac{1-\beta}{\beta}R_{t+1}c_t^y\right) = w_t$$

$$c_t^y + \left(\frac{1-\beta}{\beta}c_t^y\right) = w_t$$

$$\left[1 + \frac{1-\beta}{\beta}\right] c_t^y = w_t$$

$$\frac{1}{\beta} c_t^y = w_t$$

$$c_t^y = \beta w_t$$

d. Derive the transition equation, i.e., an equation that shows how k_{t+1} evolves as a function of k_t, parameters of the model and exogenous variable $z_t = z$. Explain in one or two sentences why knowing this transition equation is sufficient to describe how the key aggregate macroeconomic variables evolve over time in this model economy.

Using the expression k_t defined above, we express the MPL in per capita terms again:

$$MPL = w_t = (1-\alpha) z k_t^\alpha$$

We also know households consume a fraction β of their income and save the rest:

$$c_t^y = \beta w_t$$

$$a_{t+1} = (1-\beta) w_t$$

And with full depreciation ($\delta = 1$), tomorrow's capital stock equal's today's savings:

$$k_{t+1} = a_{t+1}$$

Therefore, we can define k_{t+1} in terms of savings and wages:

$$k_{t+1} = (1-\beta) w_t$$

$$k_{t+1} = (1-\beta)(1-\alpha) z k_t^\alpha$$

Today's capital stock determins wages, which determine savings, which determins next period's capital. Thus, all other macroeconomic variables are a function of k_t: it determines everything else in the system and if you know it's form you know everything there is to know about this economy.

e. Write down what the long-run steady state capital per person $\bar{k}$ is in this economy.

At the steady state, capital per worker doesn't change. Therefore:

$$k_{t+1} = k_t = \bar{k}$$

Hence, we can plug this $\bar{k}$ term into both sides of our transition equation and solve for it:

$$\bar{k} = (1-\beta)(1-\alpha) z \bar{k}^\alpha$$

$$\bar{k}^{1-\alpha} = (1-\beta)(1-\alpha) z$$

$$\bar{k} = [(1-\beta)(1-\alpha)z]^{\frac{1}{1-\alpha}}$$

f. **In class, we've largely assumed $z_t = z$ for simplicity. Now suppose that there is a production externality and productivity is instead endogenous and affected by the level of capital stock in the economy (you can think of this as the more capital is produced and used in production, the more productive and adept we become at using this capital). Each household and firm, however, thinks that they are individually too small to affect the capital stock and thus they take z_t as given. Let the production externality take that form the z_t has an increasing relationship with K_t, that is:**

$$\boldsymbol{z_t = \bar{z}K_t^{1-\alpha}}$$

Are the firm's and households' optimality conditions any different? What about the transition equation? If yes, derive the new transition equation. Finally, is there a steady state in this economy?

Optimality conditions remain unchanged under this positive production externality as the representative agents in this model are still atomistic and hence too small to impact TFP by themselves. Therefore, when optimising, firms still equate their MPL and MPK to the wage and capital rental rates respectively.

However, this positive production externality does change output's function form as TFP now depend on total capital:

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$$

$$Y_t = \bar{z}K_t^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}$$

$$Y_t = \bar{z}K_t L_t^{1-\alpha}$$

Expressing this in per-capita terms gives us:

$$y_t = \frac{Y_t}{L_t} = \bar{z}N^{1-\alpha}k_t$$

Production is now linear in k_t and no longer concave: the externality removes diminishing returns to capital in aggregate. This has implications for the transition equation, is it means savings are now also linear in k_t/do not suffer diminishing returns.

$$w_t = (1-\alpha)\,\bar{z}K_t L_t^{1-\alpha}$$

$$k_{t+1} = (1-\beta)(1-\alpha)\bar{z}N^{1-\alpha}k_t$$

This also means there are three difference steady state possibilities, are the removal of diminishing returns means $\bar{k}$ doesn't always settle as some pre-determined level:

- If the slope coefficient of k_t ($(1-\beta)(1-\alpha)\bar{z}N^{1-\alpha}$) us less than one, capital per worker will shrink overtime.
- If it is exactly equal to 1, any value of k_t will be indefinitely stable.
- If greater than 1, growth in the capital stock will become explosive.