A Short Answer Questions: Total 30 points (Answer all 5 questions in Section A.)

Write your answer in the space provided. Each question is worth 6 points.

1. Suppose job-seekers and vacancies come together via a Cobb-Douglas matching function, i.e., $M = \xi u_t^{\alpha} v_t^{1-\alpha}$. Show that the job-finding rate is increasing in labour market tightness $\theta_t = v_t/u_t$

   Answer:

   Denote the job-finding rate as $p(\theta)$.

$$p(\theta) = \frac{M}{u} = \frac{\xi u^{\alpha} v^{1-\alpha}}{u} = \xi u^{\alpha-1} v^{1-\alpha} = \xi \theta^{1-\alpha}$$

   where the last equality applies because $\theta = v/u$. Clearly, the job-finding rate is increasing in labour market tightness. One can show:

$$\frac{dp(\theta)}{d\theta} = (1-\alpha)\xi\theta^{-\alpha}$$

2. Suppose households do **not** derive any utility from holding real money balances, State whether any households would hold money in the RBC model with money. Provide a brief intuition for your answer.

   Answer:

   No, households would not hold money in the RBC model if there is no reason to hold money. This is because money is a dominated asset and one could earn a higher return by investing in a physical asset. In the RBC model, households would only hold money if they derive utility from it, or if they required money for transaction purposes.

3. The classical dichotomy exists in the New Keynesian model with sticky nominal wages. State whether this statement is true, and provide some brief intuition to support your answer.

   Answer:

   The statement is false. The classical dichotomy does not exist in the New Keynesian model with sticky nominal wages. Real variables cannot be analyzed separately from nominal variables. In this model, the wage rate does not adjust to clear the labour market. Firm's labour demand is affected by the price level through the real wage.

4. Suppose household preferences are given by $U(c_1, c_2, m_1, m_2) = \ln c_1 + \gamma \ln m_1 + \beta \{\ln c_2 + \gamma \ln m_2\}$: and the household also has the following life-time budget constraint.

$$c_1 + \frac{c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2}\left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2}\right]\right)$$

   where R_t is the rate of return to a physical asset, w_t is the real wage rate, π_t is dividend income and τ_t represents a transfer. Further, m_t represents real money balances while c_t represent consumption and Π_t is the gross inflation between period $t-1$ and t. Given this information, derive an IS curve.

   Answer:

   The IS curve is derived from the household's Euler equation. We can write down the Lagrangian of this problem:

$$\begin{aligned}\mathcal{L} &= \ln c_1 + \gamma \ln m_1 + \beta\{\ln c_2 + \gamma \ln m_2\} \\ &\quad + \lambda\left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2}\left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2}\right]\right) - c_1 - \frac{c_2}{R_2}\right]\end{aligned}$$

   Taking FOCs wrt c_1 and c_2, we have:

$$\frac{1}{c_1} = \lambda \quad \text{and} \quad \frac{\beta}{c_2} = \frac{\lambda}{R_2}$$

   Combining these two equations, we have the household euler equation:

$$\frac{1}{c_2} = \frac{\beta R_2}{c_2}$$

   Taking logs and re-arranging, and using the Fisher equation, we have:

$$\Delta \ln c = \ln \beta + \ln R_2 = \ln \beta + \ln(1 + i_2) - \ln \Pi_2 \approx \ln \beta + i_2 - \pi_2^e$$

   where π_2^e is the net inflation rate and i_2 is the net nominal interest rate. The equation above shows how consumption growth is affected by β and the nominal interest rate less inflation. Since consumption growth is a function of output growth, the above equation traces out how output growth is affected by variations in i_2 and thus represents an IS curve.

5. Consider the static New Keynesian model with sticky nominal wages. Explain how an increase in government spending financed by passive monetary policy affects labour, output and consumption.
Answer:

An increase in government spending in the static New Keynesian model financed by passive monetary policy requires more money to be supplied/created to finance the extra government spending. Holding all else constant, there is now more output being demanded at the same price level with the increase in government spending. To clear the goods market, the price level rises and this in turn lowers the real wage and raises the equilibrium amount of labour used in production. With more labour used in production, output increases. The increase in income leads households to increase both their real money balances and consumption.

also acceptable if you use graphs or equations to answer:
The government budget constraint is given by

$$PG = M^s \implies G = \frac{M^s}{P}$$

In equilibrium money supply is equal to money demand, and we know from the household's problem that optimal consumption is positively related to real money balances. Thus an increase in government spending must mean a rise in consumption demand as well through the expansion of money supply. Output also rises because labour demand and labour used in production increases when the price level rises with the increase in demand, causing a decline in the real wage.

B Longer Analytical Questions: 70 points. (Answer all parts of all 2 questions in Section B.)

Question 1: 35 points

Consider the two period search model of unemployment. There is no savings by households and no capital is used in production by firms. Households get utility from consuming their income at the end of each period. We will assume log utility from consumption. Conditional on being employed, the household inelastically supplies one unit of labour to the firm and receives an exogenous wage equal to $\bar{w}$. This wage is constant across periods. If non-employed, the household produces home goods worth h. There is no disutility to working. Households discount the future with discount factor β where $0 < \beta < 1$. There is a measure 1 of households in the population.

A job is a single firm-worker pair. A firm needs a worker to produce. Specifically, output is given by $y_t = z_t \times 1 = z_t$ where z_t is TFP. We will assume the lowest possible value of z_t, i.e., $\underline{z} > \bar{w}$ implying that $y_t > \bar{w}$. Further assume that $\bar{w} > h$. An unmatched firm must pay a vacancy posting cost of κ to post a vacancy. Search is random and new firms in period t fill their vacancies with probability $q(\theta_t)$ and households find jobs with probability $p(\theta_t)$ where $\theta_t = v_t/u_t$ where v_t denotes vacancies and u_t denotes unemployed job-seekers. Let the matching function in this economy be given by

$$M_t = \xi \frac{u_t v_t}{(u_t^\alpha + v_t^\alpha)^{1/\alpha}}$$

where $0 < \alpha < 1$. At the start of period 1, η fraction of the individuals in the economy are employed while $1 - \eta$ fraction of individuals are non-employed. Individuals separate from their jobs with probability $s(z_t)$ where $\frac{ds}{dz_t} < 0$, that is, individuals are less likely to lose their jobs when the economy is booming and TFP z_t is high. Individuals who lose their jobs in period 1 cannot search the labour market immediately and must wait until period 2 before they can search for a job.

(a) (6 points) Write down the values of an employed household, non-employed household and of the matched firm at the end of period 2.

(b) (6 points) Write down the value of a vacancy for an unmatched firm at the start of period 2. Suppose there is free entry of firms. Solve for θ_2.

(c) (6 points) Now write down the values of an employed household, non-employed household and the value of a matched firm at the end of period 1.

(d) (6 points) Write down the value of a vacancy at the start of period 1. Suppose there is free entry of firms. Solve for θ_1.

(e) (6 points) Suppose a recession occurs in period 1 and z_1 falls. State what happens to θ_1. Show what happens to the unemployment rate in period 1. Does it increase or decrease? In your answer, explain what are the forces contributing to the increase or decrease in the unemployment rate.

(f) (5 points) Suppose the recession occurs instead in period 2 and z_2 falls. State whether knowledge of tomorrow's TFP affects θ_1 and the unemployment rate in period 1. Provide some intuition to accompany your answer.

(a) (6 points) Write down the values of an employed household, non-employed household and of the matched firm at the end of period 2.

Answer

At the end of period 2, the value of an employed household is given by:

$$V_2^E = \ln \bar{w}$$

The value of a non-employed household at the end of period 2 is given by:

$$V_2^U = \ln h$$

The value of a matched firm at the end of period 2 is given by:

$$V_2^F = y_2 - \bar{w}$$

(b) (6 points) Write down the value of a vacancy for an unmatched firm at the start of period 2. Suppose there is free entry of firms. Solve for θ_2.

Answer

The value of a vacancy at the start of period 2 is given by:

$$\widetilde{V}_2^V = -\kappa + q(\theta_2)V_2^F$$

Under free entry, the value of a vacancy is driven to zero. This implies:

$$\kappa = q(\theta_2)V_2^F$$

Given the matching function, we know that

$$q(\theta_t) = \xi \frac{u_t v_t}{v_t(u_t^\alpha + v_t^\alpha)^{1/\alpha}} = \xi \frac{1}{(1+\theta_t^\alpha)^{1/\alpha}}$$

Using the form of $q(\theta_t)$ and the form of V_2^F, we can plug this into the free entry condition:

$$\kappa = \frac{\xi(y_2 - \bar{w})}{(1+\theta_2^\alpha)^{1/\alpha}}$$

which implies

$$\theta_2 = \left(\left[\frac{\xi(y_2 - \bar{w})}{\kappa}\right]^\alpha - 1\right)^{1/\alpha}$$

(c) (6 points) Now write down the values of an employed household, non-employed household and the value of a matched firm at the end of period 1.

Answer

At the end of period 1, the value of an employed household is given by:

$$V_1^E = \ln \bar{w} + \beta \left\{ [1 - s(z_2)] V_2^E + s(z_2) V_2^U \right\}$$

The employed household at the end of period 1 gets current utility from consuming her wages, and with some probability $s(z_2)$, the employed household is separated from her job and gets future value βV_2^U. With some probability $1 - s(z_2)$, the household retains her job and gets future value βV_2^E.

Since we are told $\bar{w} > h$, being employed is a better state than being non-employed and staying out of the labor force. The non-employed person will always choose to search in period 2. The value of a non-employed household at the end of period 1 is given by:

$$V_1^U = \ln h + \beta \left\{ [1 - p(\theta_2)] V_2^U + p(\theta_2) V_2^E \right\}$$

At the end of period 1, the non-employed individual enjoys current utility from home production. With probability $p(\theta_2)$ the non-employed individual finds a job and gets future value βV_2^E. With probability $1 - p(\theta_2)$, the individual remains unemployed and enjoyes future value βV_2^U.

Finally, the value of the matched firm at the end of period 1 is given by:

$$V_1^F = y_1 - \bar{w} + \beta [1 - s(z_2)] V_2^F$$

At the end of period 1, the matched firm enjoys current profits $y_1 - \bar{w}$. With probability $1 - s(z_2)$, the firm retains the worker next period and gets future value βV_2^F. With probability $s(z_2)$, the firm and worker separate, and the firm gets zero future value.

(d) (6 points) Write down the value of a vacancy at the start of period 1. Suppose there is free entry of firms. Solve for θ_1.

The value of a vacancy is given by:

$$\widetilde{V}_1^V = -\kappa + q(\theta_1) V_1^F$$

Under free entry, the value of a vacancy is driven to zero:

$$\kappa = q(\theta_1) V_1^F$$

Substituting in the form of $q(\theta)$ and V_1^F, we can solve for θ_1:

$$\theta_1 = \left(\left[\frac{\xi \left\{ y_1 - \bar{w} + \beta [1 - s(z_2)](y_2 - \bar{w}) \right\}}{\kappa} \right]^{\alpha} - 1 \right)^{1/\alpha}$$

(e) (6 points) Suppose a recession occurs in period 1 and z_1 falls. State what happens to θ_1. Show what happens to the unemployment rate in period 1. Does it increase or decrease? In your answer, explain what are the forces contributing to the increase or decrease in the unemployment rate.

Answer

We know that $y_1 = z_1$. So if a recession occurs in period 1 and TFP falls, output produced by a matched firm-worker pair is lower. This in turn reduces current profits of a matched firm, and diminishes the incentive of new firms to create a vacancy in period 1.

Since fewer firms create vacancies, the ratio of vacancies to unemployed job-seekers falls, i.e., labour market tightness, θ_1, falls. We can see this from the expression of θ_1

$$\theta_1 = \left(\left[\frac{\xi\left\{y_1 - \bar{w} + \beta[1 - s(z_2)](y_2 - \bar{w})\right\}}{\kappa}\right]^{\alpha} - 1\right)^{1/\alpha}$$

θ_1 is positively related to $y_1 = z_1$. Thus, if output and thus current profits of a matched firm falls, firms create fewer vacancies and θ_1 falls.

A fall in θ_1 reduces the job-finding rate, since there are fewer job opportunities per job-seeker. A decline in $p(\theta_1)$ reduces the rate at which the unemployed leave unemployment, raising the unemployment rate. Further, a decline in z_1 raises the separation rate since we are told the separation rate is declining in TFP. Since more people lose their jobs in a recession, the unemployment rate rises further.

(f) (5 points) Suppose the recession occurs instead in period 2 and z_2 falls. State whether knowledge of tomorrow's TFP affects θ_1 and the unemployment rate in period 1. Provide some intuition to accompany your answer.

Answer

If z_2 is known to fall tomorrow, $s(z_2)$ increases and y_2 falls. Both a rise in $s(z_2)$ and a fall in y_2 lowers expected future profits. This is because the firm has a higher likelihood of losing his worker in period 2 and conditional on retaining his worker, future profits are also lower. Since expected future profits are lower, the expected benefit of creating a vacancy is lower and firms have less incentive to create a vacancy in period 1. Consequently, θ_1 falls if firms know that z_2 is lower.

A decline in θ_1 lowers the job-finding rate $p(\theta_1)$. Unemployed job-seekers find it harder to find a job because there are fewer job opportunities available. Consequently, the unemployment rate rises in period 1 because fewer unemployed individuals find a job and leave unemployment. Notably, in this case when z_2 falls, the separation rate in period 1 $s(z_1)$ is unchanged. So the rise in the unemployment rate only stems from a lower job-finding rate.

Question 2: 35 points

Consider the two period RBC model. Households have utility given by $U(c_1, c_2) = (1-\beta)\ln c_1 + \beta \ln c_2$ where $0 < \beta < 1$. Households are born with physical asset a_1 and inelastically supply 1 unit of labour to firms each period. Households receive a gross rate of return R_t for each unit of their asset that they rent to firms and they receive a wage rate w_t for each unit of labour that they supply to firms. In addition, households receive dividend income from firms. There are N households in the economy.

Firms produce according to a Cobb-Douglas production function, $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$, and rent capital at rate R_t and hire labour at rate w_t in each period t. Capital depreciates fully after use in production after 1 period.

There exists a government that spends exogenous amount G_t. Government spending is wasteful. The government balances its budget and completely finances its spending within a period by collecting a proportional tax, τ_t, on consumption expenditure in that period.

(a) (3 points) Write down the government budget constraint in period t.

(b) (5 points) Set up the firm's problem and solve for the firm's optimality conditions.

(c) (6 points) Set up the household's problem and derive the optimality conditions of the household.

(d) (6 points) Suppose $G_1 = G_2 = 0$ and $\tau_1 = \tau_2 = 0$. Denote $k_2 = K_2/L_2$. Solve for k_2 in terms of parameters of the model, exogenous variables and predetermined k_1.

(e) (6 points) Suppose that z_2 increases. Show that consumption, investment and output do not respond to good news about TFP in period 2.

(f) (6 points) Now suppose that government spending is not zero in all periods. The government decides to spend only in the first period, $G_1 > 0, G_2 = 0$. The government finances this spending by only collecting a proportional tax on consumption expenditure in period 1. Show how an increase in G_1 affects consumption and investment

(g) (3 points) Given your answer in (e), provide some intuition as to whether government spending shocks can drive business cycles in the economy.

(a) (3 points) Write down the government budget constraint in period t.

Answer

$$G_t = \tau_t c_t$$

(b) (5 points) Set up the firm's problem and solve for the firm's optimality conditions.

Answer

The firm's profit maximization problem is given by:

$$\max_{K_t, L_t} z_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t$$

Denoting $k_t = K_t/L_t$ and taking FOCs:

$$R_t = \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} = \alpha z_t k_t^{\alpha-1}$$

$$w_t = (1-\alpha) z_t K_t^{\alpha} L_t^{-\alpha} = (1-\alpha) z_t k_t^{\alpha}$$

(c) (6 points) Set up the household's problem and derive the optimality conditions of the household.

Answer

The household's utility maximization problem is given by:

$$\max_{c_1, c_2} (1-\beta) \ln c_1 + \beta \ln c_2$$

s.t.

$$(1+\tau_1) c_1 + \frac{(1+\tau_2) c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2}$$

We can write down the Lagrangian of this problem:

$$\mathcal{L} = (1-\beta) \ln c_1 + \beta \ln c_2 + \lambda \left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} - (1+\tau_1) c_1 - \frac{(1+\tau_2) c_2}{R_2} \right]$$

Taking FOCs:

$$(c_1): \quad \frac{1-\beta}{c_1} = \lambda (1+\tau_1)$$

$$(c_2): \quad \frac{\beta}{c_2} = \lambda \frac{1+\tau_2}{R_2}$$

$$(\lambda): \quad (1+\tau_1) c_1 + \frac{(1+\tau_2) c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2}$$

Combining the FOCs wrt c_1 and c_2, we get the Euler equation:

$$\frac{\beta R_2}{c_2} \frac{1+\tau_1}{1+\tau_2} = \frac{1-\beta}{c_1}$$

and the FOC wrt λ gives us back the LBC which is the other household optimality condition.

(d) (6 points) Suppose $G_1 = G_2 = 0$ and $\tau_1 = \tau_2 = 0$. Denote $k_2 = K_2/L_2$. Solve for k_2 in terms of parameters of the model, exogenous variables and predetermined k_1.

Answer

Note that the Euler equation can be re-arranged as:

$$(1+\tau_2)c_2 = \frac{\beta}{1-\beta}R_2(1+\tau_1)c_1$$

Substituting the above into the LBC:

$$(1+\tau_1)c_1 = (1-\beta)\left[R_1a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2}\right]$$

Since $G_1 = G_2 = 0$ and $\tau_1 = \tau_2 = 0$, in equilibrium, we have:

$$c_1 = (1-\beta)\left[z_1k_1^\alpha + \frac{1-\alpha}{\alpha}k_2\right]$$

From goods market clearing, we know:

$$k_2 = z_1k_1^\alpha - c_1$$

which implies

$$k_2 = z_1k_1^\alpha - (1-\beta)\left[z_1k_1^\alpha + \frac{1-\alpha}{\alpha}k_2\right]$$

and therefore:

$$k_2 = \frac{\alpha\beta}{[1-\beta(1-\alpha)]}z_1k_1^\alpha$$

(e) (6 points) Suppose that z_2 increases. Show that consumption, investment and output in period 1 do not respond to good news about TFP in period 2.

Answer

We had solved for $k_2 = \frac{\alpha\beta}{[1-\beta(1-\alpha)]}z_1k_1^\alpha$. Note that consumption in period 1 is given by:

$$c_1 = (1-\beta)\left[z_1k_1^\alpha + \frac{1-\alpha}{\alpha}k_2\right] = \frac{1-\beta}{[1-\beta(1-\alpha)]}z_1k_1^\alpha$$

and $y_1 = z_1k_1^\alpha$. Since k_2, c_1 and y_1 are independent of z_2, they do not respond to good news about TFP in period 2.

(f) (6 points) Now suppose that government spending is not zero in all periods. The government decides to spend only in the first period, $G_1 > 0, G_2 = 0$. The government finances this spending by only collecting a proportional tax on consumption expenditure in period 1. Show how an increase in G_1 affects consumption and investment

Answer

Note that from substituting the Euler equation into the LBC, we had:

$$(1+\tau_1)c_1 = (1-\beta)\left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2}\right]$$

Denote $g_1 = G_1/N$. We know that the government runs a balanced budget and $\tau_1 c_1 = g_1$. In equilibrium, we have:

$$c_1 = (1-\beta)\left[z_1 k_1^\alpha + \frac{1-\alpha}{\alpha}k_2\right] - g_1$$

From goods market clearing, we have:

$$k_2 = z_1 k_1^\alpha - c_1 - g_1$$

which is equivalent to:

$$k_2 = z_1 k_1^\alpha - \left\{(1-\beta)\left[z_1 k_1^\alpha + \frac{1-\alpha}{\alpha}k_2\right] - g_1\right\} - g_1 = \frac{\alpha\beta}{[1-\beta(1-\alpha)]} z_1 k_1^\alpha$$

which implies that c_1 is given by:

$$c_1 = \frac{1-\beta}{[1-\beta(1-\alpha)]} z_1 k_1^\alpha - g_1$$

From the above equations, we can see that an increase in G_1 (and thus g_1) leads to no change in investment k_2. Conversely, a rise in G_1 (and thus a rise in g_1) lowers consumption in period 1, c_1 falls. Intuitively, government spending is financed with a proportional tax on consumption expenditure. The more government spending is increased, the higher the proportional tax. Since consumption in period 1 is more expensive when government spending is higher, households optimally choose to lower their consumption in period 1.

(g) (3 points) Given your answer in (e), provide some intuition as to whether government spending shocks can drive business cycles in the economy.

Answer

We just showed that investment spending does not respond to changes in G_1 but consumption spending falls. This suggests that government spending shocks cannot drive business cycle movements in economy as it cannot predict the correct co-movement across consumption, investment and GDP.

END OF EXAM.