Lecture 18: Money in the utility function model

ECON30009/90080 Macroeconomics

Semester 2, 2025

Goals of today's lecture

- We want to introduce money into the model
- Basic building block: RBC model but with money
- What we will find: monetary policy (money supply rule) and nominal variables don't matter for real variables in RBC + money

A 2 period money-in-the-utility-function model

- 2 period economy. Assume measure 1 (continuum) of households in the population, $N = 1$.

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- Households seek to maximize utility

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- which in a way means our households are like . . .

A 2 period money-in-the-utility-function model

- 2 types of assets: you can invest in physical capital (a_t) or hold money (M_t)
- Money is a dominated asset. You can't earn interest keeping money in your pocket
- This is unlike investing in physical capital which can earn you a real return R_t (in nominal terms $P_t R_t$)
- Notation: we will refer to M_t as nominal money balances and $m_t = M_t/P_t$ as real money balances

Household

Household utility function

- Household gets utility from consumption in each period and from holding money as an asset:

$$U\left(c_1, c_2, \frac{M_1}{P_1}, \frac{M_2}{P_2}\right) = \ln c_1 + \gamma \ln\left(\frac{M_1}{P_1}\right) + \beta\left\{\ln c_2 + \gamma \ln \frac{M_2}{P_2}\right\}$$

- Household still gets utility from real variables (quantities)

- No disutility from working. Inelastically supplies 1 unit of labour each period

Household budget constraints

- Budget constraint in period 1 in *nominal* terms:

$$P_1 c_1 + M_1 + P_1 a_2 = P_1 R_1 a_1 + P_1 w_1 + P_1 \pi_1 + \overbrace{P_1 \tau_1}^{\text{nominal transfer from govt}}$$

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- We can make a_2 the subject of the equation:

$$a_2 = \frac{P_2 c_2 + M_2 - [P_2 w_2 + P_2 \pi_2 + P_2 \tau_2 + M_1]}{P_2 R_2}$$

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- and plug this into period 1 budget constraint, to derive the LBC in nominal terms:

$$P_1c_1+M_1+P_1\left\{\frac{c_2 + M_2/P_2}{R_2} - \frac{[w_2 + \pi_2 + \tau_2 + M_1/P_2]}{R_2}\right\} = P_1\left[R_1a_1 + w_1 + \pi_1 + \tau_1\right]$$

Household budget constraints

- Can take the nominal LBC and make it real by dividing by P_1:

$$c_1 + \frac{c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + \tau_1 - \frac{M_1}{P_1} + \frac{\tau_2}{R_2} - \frac{M_2}{P_2}\frac{1}{R_2} + \frac{M_1}{P_1}\frac{P_1}{P_2 R_2}$$

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- Looks similar to the real LBC from our RBC model, but now money can be held.

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- Denote $\Pi_2 = \frac{P_2}{P_1}$, i.e., the gross inflation rate between period 1 and 2. Denote $m_t = M_t/P_t$ as real money balances in period t.

$$c_1 + \frac{c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2}\left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2}\right]\right)$$

Household utility maximization problem

- Household problem is given by:

$$\max_{c_1, c_2, m_1, m_2} \ln c_1 + \gamma \ln m_1 + \beta \{\ln c_2 + \gamma \ln m_2\}$$

s.t.

$$c_1 + \frac{c_2}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2}\left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2}\right]\right)$$

Household utility maximization problem

$$
\begin{aligned}
\mathcal{L} &= \ln c_1 + \gamma \ln m_1 + \beta \left\{ \ln c_2 + \gamma \ln m_2 \right\} \\
&\quad + \lambda \left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2} \left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2} \right] \right) \right] \\
&\quad - \lambda \left[c_1 + \frac{c_2}{R_2} \right]
\end{aligned}
$$

- First order conditions:

$$
\begin{aligned}
(c_1): \quad & \frac{1}{c_1} = \lambda \\
(c_2): \quad & \frac{\beta}{c_2} = \frac{\lambda}{R_2}
\end{aligned}
$$

Household utility maximization problem

$$\begin{aligned}\mathcal{L} &= \ln c_1 + \gamma \ln m_1 + \beta \left\{ \ln c_2 + \gamma \ln m_2 \right\} \\ &\quad + \lambda \left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \frac{1}{R_2}\left(\tau_2 - \left[m_2 - \frac{m_1}{\Pi_2} \right]\right)\right] \\ &\quad - \lambda \left[c_1 + \frac{c_2}{R_2} \right]\end{aligned}$$

- First order conditions cont'd:

$$(m_1): \quad \frac{\gamma}{m_1} = \lambda \left(1 - \frac{1}{R_2 \Pi_2} \right)$$

$$(m_2): \quad \frac{\beta\gamma}{m_2} = \frac{\lambda}{R_2}$$

and FOC wrt λ gives back LBC

Household optimality conditions

- Combine FOC wrt c_2 and m_2 to get optimal money demand in $t = 2$

$$\frac{\gamma}{m_2} = \frac{1}{c_2}$$

  m_2 is like a "consumption good". Get utility from it, can't bring it into next period, because economy ends after $t = 2$

- Hence optimal money demand comes from the optimal trade-off between m_2 and c_2

Household optimality conditions

- Combine FOC wrt m_1 and c_1 to get optimal money demand in $t = 1$

$$\frac{\gamma}{m_1} = \frac{1}{c_1}\left(1 - \underbrace{\frac{1}{R_2 \Pi_2}}_{\text{due to store of value role}}\right)$$

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$$\frac{\gamma}{m_1} = \frac{1}{c_1}\left(1 - \underbrace{\frac{1}{R_2\Pi_2}}_{\text{due to store of value role}}\right)$$

- Fisher equation:

$$\underbrace{1 + i_t}_{\text{Gross nominal interest rate}} = \Pi_t R_t$$

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- Substituting the Fisher equation into the above and re-arranging:

$$m_1 = \gamma c_1 \frac{1 + i_2}{i_2} = \gamma c_1 \left(\frac{1}{i_2} + 1\right)$$

Household optimality conditions

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- Substituting the Fisher equation into the above and re-arranging:

$$m_1 = \gamma c_1 \frac{1 + i_2}{i_2} = \gamma c_1 \left(\frac{1}{i_2} + 1\right)$$

- Holding all else constant, m_1 declining in i_2. Money is a dominated asset, opportunity cost to holding money is the nominal interest rate

Household optimality conditions

- Combine FOC wrt c_1 and c_2 to get Euler equation (optimal intertemporal trade-off in consumption today vs tomorrow):

$$\frac{1}{c_1} = \frac{\beta R_2}{c_2}$$

- Finally, also have LBC.

An IS curve

- From the household's Euler equation, actually have an IS curve:

$$c_2 = \beta R_2 c_1$$

Taking logs:

$$\Delta \ln c = \ln \beta + \ln R_2 = \ln \beta + i_2 - \pi_2^e$$

where $\ln(1 + i_2) \approx i_2$ and $\ln \Pi_2 \approx \pi_2^e$

  - $\Delta \ln c$ is consumption growth (in eqm related to output growth)
  - $\ln \beta$ acts like a demand shock. Lower β, more impatient household, wants to consume more today.
  - i_2 net nominal interest rate
  - π_2^e expected net inflation rate in period 2.

An IS curve

- From postulated relationship to optimizing behaviour $\implies$ we can actually derive an IS curve from our household's optimality condition
  - Since consumption growth is a function of output growth, we can trace out an IS-curve that shows all the combinations of (i, Y).
  - Our model gives us a way to formalize why this occurs.
  - The IS curve we derived came from the household's optimal trade-off of how much to consume today vs. how much to invest so as to consume for tomorrow.
  - How much you want to invest and save depends on the interest rate

Firm

Firms' profit maximization problem

- Firm's profit maximization problem is:

$$\max_{K_t, L_t} P_t z_t K_t^{\alpha} L_t^{1-\alpha} - P_t R_t K_t - P_t w_t L_t$$

which is equivalent to:

$$\max_{K_t, L_t} P_t \{ z_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t \}$$

P_t just scales the firm's problem. So choices of the firm are the same as derived in the RBC model.

Government/Monetary Authority

Govt/Monetary authority

- Monetary authority/Govt sets the money supply.
- For period 1, this is as simple as setting:

$$M_1^s = \underbrace{\bar{M}}_{\text{exogenous money supply target}}$$

- For period 2, this money supply rule follows:

$$M_2^s = \theta M_1^s$$

Note that if $\theta = 1$, money supply is constant. If $\theta > 1$, money supply is growing.

Govt/Monetary authority

- Govt/monetary authority earns revenue from printing (creating) money $\implies$ seigniorage revenue

- No govt spending, so revenue earned from printing money is transferred to households:

$$P_1\tau_1 = M_1^s$$

$$P_2\tau_2 = M_2^s - M_1^s$$

Equilibrium

Equilibrium

Equilibrium is a set of allocations and prices such that:

- Households choose c_1, c_2, m_1, m_2 to maximize their lifetime utility
- Firms choose K_t, L_t to maximize their profits each period
- Govt/monetary authority balances the government budget constraint each period
- Prices adjust such that all markets (goods, labour, asset, money) clears

Solving for equilibrium

- Starting from the household problem: plug the Euler equation into the LBC:

$$c_2 = \beta R_2 c_1 \quad \text{from Euler eqn}$$

and LBC becomes:

$$c_1 + \frac{\beta R_2 c_1}{R_2} = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \left(\frac{\tau_2}{R_2} - \frac{1}{R_2} \left[m_2 - m_1 \frac{1}{\Pi_2} \right] \right)$$

Solving for equilibrium

- Thus far, we have:

$$(1+\beta)c_1 = R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} + (\tau_1 - m_1) + \left(\frac{\tau_2}{R_2} - \frac{1}{R_2}\left[m_2 - m_1 \frac{1}{\Pi_2}\right]\right)$$

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- In equilibrium, all markets clear: $L_t = N$, $K_t = a_t N$, $M_t^s = N M_t$, and $N = 1$. Substitute in firm optimality conditions and govt budget constraints into LBC:

$$(1+\beta)\,c_1 = z_1 k_1^{\alpha} + \frac{1-\alpha}{\alpha} k_2 + \underbrace{\cancel{(\tau_1 - m_1)} + \cancel{\left(\frac{\tau_2}{R_2} - \frac{1}{R_2}\left[m_2 - m_1 \frac{1}{\Pi_2}\right]\right)}}_{\text{from govt budget constraints}}$$

Solving for equilibrium

- We have from plugging the Euler into LBC and imposing equilibrium:

$$c_1 = \frac{1}{1+\beta}\left(z_1 k_1^\alpha + \frac{1-\alpha}{\alpha} k_2\right)$$

- k_2 is endogenous, so haven't solved for c_1 yet. But we know from goods market clearing:

$$k_2 = z_1 k_1^\alpha - c_1$$

- So using goods market clearing and plugging in for c_1, we have:

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^\alpha$$

and therefore

$$c_1 = \frac{1}{1+\alpha\beta} z_1 k_1^\alpha$$

Solving for equilibrium

- Looking at the key real variables in the economy:

$$y_1 = z_1 k_1^\alpha$$

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^\alpha$$

$$c_1 = \frac{1}{1+\alpha\beta} z_1 k_1^\alpha$$

- Real output, investment and consumption (since $N = 1$) do not depend on M_1^s / P_1.

- So what does money supply do?

To be continued next class

- So what does money supply do?
- Hint: we built on the RBC model and added money to it. Is the prediction surprising that monetary policy (via money supply rules) do not affect real variables?
- Next class: what happens to nominal variables in the money-in-the-utility function model?
- To come: having a proper role for monetary policy $\Longrightarrow$ moving to New Keynesian model