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Week 12 Tutorial

ECON90033 - 2023 Semester 2

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Completed on 22 October 2023

Exercise 1

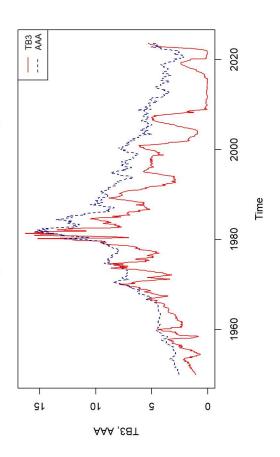
a) Import the data, graph and interpret it.

It's clear both series are non-stationary, but they follow similar time paths and therefore may share a common stochastic trend.

However, the differenced series do not contain trends. Therefore, their means might be stationary.

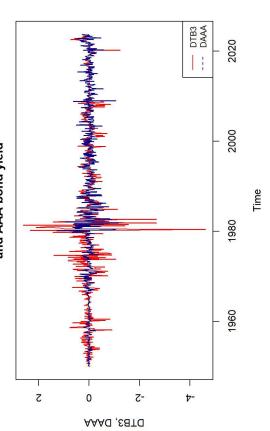
```
e1 <- read_excel("C:/Users/joshc/Documents/2023S2/ECON90033/Tutorials/Week 12/t12e1.xlsx", sheet
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            plot.ts(TB3, ylab = "TB3, AAA", main = "Treasury Bill rate and AAA bond yield",
                                                                                                                                                                                                    ts(start = c(1949, 12), end = c(2023, 9), frequency = 12)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           ts(start = c(1949, 12), end = c(2023,9), frequency = 12)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         legend("topright", legend = c("TB3", "AAA"),
col = c("red", "darkblue"), lty = 1:2, cex = 0.8)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      lines(AAA, col = "darkblue", lty = 2)
                                                                                                                                                                                                                                                                             DTB3 <- diff(TB3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        DAAA <- diff(AAA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                              pull(AAA) %>%
                                                                                                                                                                pull(TB3) %>%
                                                                                                                                                                                                                                                                                                                                                                                                                                              AAA <- e1 %>%
                                                                                                                          TB3 <- e1 %>%
                                            = "Data")
```

Treasury Bill rate and AAA bond yield



```
plot.ts(DTB3, ylab = "DTB3, DAAA", main = "First differences of Treasury Bill rate
                                                                                                                                                  legend("bottomright", legend = c("DTB3", "DAAA"),
col = c("red", "darkblue"), lty = 1:2, cex = 0.8)
                                                      and AAA bond yield", col = "red")
                                                                                                        lines(DAAA, col = "darkblue")
```

First differences of Treasury Bill rate and AAA bond yield



To confirm these observations above, perform the ADF and kPSS tests on the levels and first differences of TB3 and AAA.

Based on the plots, the data generating plots certainly don't have a single linear trend component. It might have a broke nlinear trend components. For this reason you need to conduct tests with both an intercept (Model 2) and both an intercept and a trend (Model 3) for the level series. For the differenced series we use Model 2. We don't discuss the output in detail, but they show that both variables are integrated of order 1 (i.e. I(1)) no matter of the significance level is 10% or 5%.

adf.DTB3 = ur.df(diff(TB3), type = "none", selectlags = "BIC")

summary(adf.DTB3)

```
adf.TB3 = ur.df(TB3, type = "drift", selectlags = "BIC")
                                                      summary(adf.TB3)
#Library(urca)
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             ## Residual standard error: 0.3694 on 881 degrees of freedom
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     ## Multiple R-squared: 0.1253, Adjusted R-squared: 0.1233
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ## F-statistic: 63.12 on 2 and 881 DF, p-value: < 2.2e-16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      0.031567 11.029 < 2e-16 ***
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0.004047 -2.808 0.00509 **
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               0.020539 2.391 0.01702 *
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Estimate Std. Error t value Pr(>|t|)
                                                                           ## # Augmented Dickey-Fuller Test Unit Root Test #
                                                                                                                                     ## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            ## Value of test-statistic is: -2.8082 3.9758
                                                                                                                                                                                                                                                                                                                                                                                                             Max
2.6016
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         ## Critical values for test statistics:
                                                                                                                                                                                                                                                                                                                                                                                                           ## Min 1Q Median 3Q
## -3.8227 -0.0830 -0.0155 0.1005
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ## (Intercept) 0.049106
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      0.348167
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ## tau2 -3.43 -2.86 -2.57 ## phil 6.43 4.59 3.78
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        1pct 5pct 10pct
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -0.011365
                                                                                                                                                                                             ## Test regression drift
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  ## Coefficients:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ## z.diff.lag
                                                                                                                                                                                                                                                                                                                                                                              ## Residuals:
                                                                                                                                                                                                                                                                                         ## Call:
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               #
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                             ## Residual standard error: 0.3639 on 881 degrees of freedom
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  ## Multiple R-squared: 0.3543, Adjusted R-squared: 0.3529
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ## F-statistic: 241.7 on 2 and 881 DF, p-value: < 2.2e-16
                                                                                                                                                                                                                                                                                                                                                   0.03787 -20.749 < 2e-16 ***
                                                                                                                                                                                                                                                                                                                                                                        0.03304 5.931 4.33e-09 ***
## # Augmented Dickey-Fuller Test Unit Root Test #
                                          Estimate Std. Error t value Pr(>|t|)
                                                                                                                                                                         ## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
                                                                                                                                                                                                                                       Median 3Q Max
0.0097 0.1082 2.1945
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ## Value of test-statistic is: -20.7494
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                ## Critical values for test statistics:
                                                                                                                                                                                                                                           1Q Median
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            ## tau1 -2.58 -1.95 -1.62
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           1pct 5pct 10pct
                                                                                 ## Test regression none
                                                                                                                                                                                                                                                                                                                                                                          ## z.diff.lag 0.19594
                                                                                                                                                                                                                                                                                                                                                     -0.78587
                                                                                                                                                                                                                                                            ## -3.3414 -0.0808
                                                                                                                                                                                                                                                                                                     ## Coefficients:
                                                                                                                                                                                                                    ## Residuals:
                                                                                                                                                                                                                                                                                                                                                   ## z.lag.1
                                                                                                                                                    ## Call:
                                                                 #
                                                                                                                                                                                             #
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```

```
adf.AAA = ur.df(AAA, type = "drift", selectlags = "BIC")
                                           summary(adf.AAA)
```

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```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              ## Residual standard error: 0.1949 on 881 degrees of freedom
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ## Multiple R-squared: 0.1084, Adjusted R-squared: 0.1064
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ## F-statistic: 53.58 on 2 and 881 DF, p-value: < 2.2e-16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                ## z.diff.lag 0.325805 0.031835 10.234 <2e-16 ***
                                                                                                                                                                                                                                                                                                                                                                                                                                        ## (Intercept) 0.028929 0.016274 1.778 0.0758 .
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0.0712 .
                                                                                                                                                                                                                                                                                                                                                                                                                  Estimate Std. Error t value Pr(>|t|)
## # Augmented Dickey-Fuller Test Unit Root Test #
                                                     ## lm(formula = z.diff \sim z.lag.1 + 1 + z.diff.lag)
                                                                                                                                                                                                                                                                                                    ## Min 1Q Median 3Q Max
## -1.11444 -0.08028 -0.00553 0.07935 1.19397
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    ## Value of test-statistic is: -1.8062 1.6789
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0.002343 -1.806
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            ## Critical values for test statistics:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 ## tau2 -3.43 -2.86 -2.57
## phil 6.43 4.59 3.78
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         1pct 5pct 10pct
                                                                                                      ## Test regression drift
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          -0.004232
                                                                                                                                                                                                                                                                                                                                                                                       ## Coefficients:
                                                                                                                                                                                                                                                                            ## Residuals:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          ## z.lag.1
                                                                                                                                                                                           # Call:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               #
                                                                                  #
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                                                                                                                                                                #
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                                                                                                                                                                                                                                                                                                                                                                                                                  #
```

```
adf.DAAA = ur.df(diff(AAA), type = "none", selectlags = "BIC")
                              summary(adf.DAAA)
```

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```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     ## Residual standard error: 0.1904 on 881 degrees of freedom
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          ## Multiple R-squared: 0.3702, Adjusted R-squared: 0.3688
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 259 on 2 and 881 DF, p-value: < 2.2e-16
                                                                                                                                                                                                                                                                                                                                                         0.03823 -21.606 < 2e-16 ***
                                                                                                                                                                                                                                                                                                                                                                                0.03289 6.765 2.43e-11 ***
## # Augmented Dickey-Fuller Test Unit Root Test #
                                           Estimate Std. Error t value Pr(>|t|)
                                                                                                                                                                            ## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
                                                                                                                                                                                                                                             ## Min 1Q Median 3Q Max
## -1.01082 -0.07914 0.00542 0.08295 1.14681
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               ## Value of test-statistic is: -21.6057
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          ## Critical values for test statistics:
## 1pct 5pct 10pct
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        tau1 -2.58 -1.95 -1.62
                                                                                                                                                                                                                                                                                                                                                                                ## z.diff.lag 0.22250
                                                                                                                                                                                                                                                                                                                                                          -0.82609
                                                                                  ## Test regression none
                                                                                                                                                                                                                                                                                                            ## Coefficients:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              ## F-statistic:
                                                                                                                                                                                                                       ## Residuals:
                                                                                                                                                                                                                                                                                                                                                         ## z.lag.1
                                                                                                                                                       ## Call:
                                                                                                                                                                                                #
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      #
```

kpss.TB3 = ur.kpss(TB3, type = "mu") summary(kpss.TB3)

```
## Critical value for a significance level of:
                                                                                                                                                                                                          10pct 5pct 2.5pct 1pct
                                                                                                                                                                                                                               critical values 0.347 0.463 0.574 0.739
                                                                                                                                    ## Value of test-statistic is: 2.9097
##
                                                                                       ## Test is of type: mu with 6 lags.
## # KPSS Unit Root Test #
                                            #
```

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```
## Critical value for a significance level of:
kpss.DTB3 = ur.kpss(diff(TB3), type = "mu")
                                                                                                                                                                                                                                                                                                                                                                              10pct 5pct 2.5pct 1pct
                                                                                                                                                                                                                                                                                                                                                                                                        ## critical values 0.347 0.463 0.574 0.739
                                                                                                                                                                                                                                                                                              ## Value of test-statistic is: 0.0689
                                                                                                                                                                                                                                   ## Test is of type: mu with 6 lags.
                                                                                                     ## # KPSS Unit Root Test #
                                                                                                                                                                                    ######################################
                              summary(kpss.DTB3)
                                                                                                                                                                                                             #
                                                                                                                                                                                                                                                                                                                         #
```

kpss.AAA = ur.kpss(AAA, type = "mu") summary(kpss.AAA)

```
## Critical value for a significance level of:
                                                                                                                                                                                                                                           10pct 5pct 2.5pct 1pct
                                                                                                                                                                                                                                                                 ## critical values 0.347 0.463 0.574 0.739
                                                                                                                                                                 ## Value of test-statistic is: 2.7442
                                                                                                                 ## Test is of type: mu with 6 lags.
## # KPSS Unit Root Test #
                                                                #
```

```
kpss.DAAA = ur.kpss(diff(AAA), type = "mu")
                               summary(kpss.DAAA)
```

```
## Critical value for a significance level of:
                                                                                                                                                                                                                                         10pct 5pct 2.5pct 1pct
                                                                                                                                                                                                                                                                   ## critical values 0.347 0.463 0.574 0.739
                                                                                                                                                             ## Value of test-statistic is: 0.2718
                                                                                                        ## Test is of type: mu with 6 lags.
```

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Since TB3 and AAA are both I(1) they might be cointegrated. Check this possibility by performing cointegration tests.

There are several tests to consider

- Engle-Granger (EG)
- Basically an ADF-type test on the residuals from a simple linear regression of TB3 on AAA (or vice versa) but with different critical values
- EG tests do this and test under the null hypothesis of two or more series being not cointegrated.
- Since the results can be sensitive to normalisation (i.e. choice of Y), best practice is to run the test
- Interpreting EG test output:
- Each printout has three panels, corresponding to the three possibl especifications of the ADF test regressions: Model 1, Model 2 and Model 3. We are using model 1 in this instance.
- significantly between printouts which also leads to different p-values. However, in both cases the null Comparing the models to eachother you can see the normalistaion does matter as the t-value vary hypothesis of no cointegration can be rejected at the 2% significance level, implying that they are

```
coint.test(TB3,AAA)
                                   # Library(aTSA)
# EG test
```

```
## Note: p.value = 0.01 means p.value <= 0.01
                                                                                                                                                                                                                                                                                         : p.value = 0.10 means p.value >= 0.10
                                                                ## Engle-Granger Cointegration Test
                                                                                 ## alternative: cointegrated
                                                                                                                                                                                                                      Type 3: quadratic trend
                                                                                                                                      0.01
                                                                                                                                                                                                                                     EG p.value
                                                                                                                         EG p.value
                                                                                                                                                                             EG p.value
                                                                                                                                                                                            -0.633 0.100
                                                                                                                                                                                                                                                6.0000 0.0773 0.1000
                                                                                                                                                                Type 2: linear trend
                          ## Number of inputs: 1
                                                                                                          ## Type 1: no trend
## lag EG p
                                                                                                                                      -3.88
                                         ## Model: y \sim X + 1
## Response: TB3
             ## Input: AAA
                                                                                                                                      6.00
                                                                                                                                                                                            6.000
                                                                                                                                                                               lag
                                                                                                                                                                                                                                      lag
                                                                                                                                                                                                                    # #
                                                                                                                                                                                                                                                #
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                                                                                                                                                                             #
                                                                                                                                                                                                                                                              #
```

file:///C:/Users/joshc/Documents/2023S2/ECON90033/Tutorials/Week 12/Week-12-script.html

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```
## Note: p.value = 0.01 means p.value <= 0.01 ## note: p.value = 0.10 means p.value >= 0.10
                                                                                                                ## Engle-Granger Cointegration Test
                                                                                                                                                                                                                       ## Type 2: linear trend
## lag EG p.value
## 6.0 0.3 0.1
## -----
## Type 3: quadratic trend
                                                                                                                               ## alternative: cointegrated
                                                                                                                                                                                                                                                                                                                  0.10
                                                                                                                                                                              EG p.value
                                                                                                                                                                                                                                                                                                      EG p.value
                                                                                                                                                                                           6.000 -3.337 0.017
                                                                     ## Number of inputs: 1
                                                                                                                                                            ## Type 1: no trend
                                                                                    ## Model: y ~ X + 1
                                                                                                                                                                                                                                                                                                                   1.14
coint.test(AAA,TB3)
                                       ## Response: AAA
                                                                                                    ----- ##
                                                      ## Input: TB3
                                                                                                                                                                              lag
                                                                                                                                                                                                                                                                                                      lag
                                                                                                                                                                                                                                                                                                                   6.00
                                                                                                                                                                             ##
                                                                                                                                                                                                          #
                                                                                                                                                                                                                                                                                                   ##
```

- Johansen tests:
- deterministic term outside the EC), or "trend" (a trend variables but no constant in EC and a constant The tricky part of using Johansen tests is knowing which deterministic assumptions to apply: "none" (no deterministic term in EC but a constant outside EC), "const" (a constant in the EC, but no outside EC)
- Use the following approach for choosing the right term:
- "none" is appropriate for (linearly) trending series, granted all trends are stochastic.
- · "const" is proper only if none of the variables appear to have a sustained tendency to increase or decrease
- "trend" is reasomable when there is some long-run linear growth that the cointegrating relation does not account for.

Because neither TB3 and AAA are trending linearly, but seem to have some broken trend, "const" is a reasonable

For lag length in this model, we use the VARselect() function. SC selects lag length 3, whereas all other measures select lag length 10.

value of this test is large (0.18), therefore we can maintain the null hypothesis of no autocorrelation of orders 1-3 To determine what's correct, estimate VAR(3) and perform the BG test for autocorrelation of orders 1-3. The p-Therefore, we use K = 3.

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```
VARselect(data, type = "const")
                                                                                       data = cbind(AAA, TB3)
                                          # library(urca)
                                                                                                                                 #library(vars)
# J tests
```

```
## AIC(n) -5.33838552 -5.532230868 -5.610549434 -5.610157777 -5.606011398
                                                                                                                                                                                                   -5.32587465 -5.511379421 -5.581357408 -5.572625171 -5.560138214
                                                                                                                                                                                                                                  -5.30567753 -5.47717557 -5.534230798 -5.512033816 -5.486082113
                                                                                                                                                                                                                                                                                      ## 6 7 8 9 10
## AIC(n) -5.601631527 -5.658245229 -5.652268434 -5.670675971 -5.68893683
                                                                                                                                                                                                                                                                                                                                                   -5.547417763 -5.595690886 -5.581373512 -5.591440471 -5.60136075
                                                                                                                                                                                                                                                                                                                                                                        -5.459896917 -5.494705294 -5.466923174 -5.463525388 -5.45998092
                                                                                                                                                                                                                                                                                                                                                                                                           0.003691852 0.003488657 0.003509581 0.003445582 0.00338325
                                                                                                                                                                                                                                                            ## FPE(n) 0.00480362 0.003957152 0.003659061 0.003660497 0.003675711
                         SC(n) FPE(n)
3 10
                            ## AIC(n) HQ(n)
## $selection
                                                                                                              ## $criteria
                                                                                                                                                                                                   ## HQ(n)
## SC(n)
                                                                                                                                                                                                                                                                                                                                                HQ(n)
SC(n)
                                                        10
                                                      # #
                                                                                                                                           #
                                                                                                                                                                                                                                                                                                                                                ##
```

```
serial.test(var3, lags.bg = 3, type = "BG")
var3 = VAR(data, p = 3, type = "const")
                                                                                                                                            ## Breusch-Godfrey LM test
                                                                                                                                                                             #
```

First we look at the "trace" J-test. There are four parts to the printout.

Chi-squared = 16.225, df = 12, p-value = 0.1811

data: Residuals of VAR object var3

#

- The first shows three estimated eigenvalues in decreasing order
- The second part shows the trade statistics for the null hypotheses of r_0 <= 1 and r_0 = 0. These need to be evaluated in reverse order, starting with r_0 = 0.
- The r_0 = 0 test statistics is very large (38.27), much larger than the 1% critical value (24.60)
- However, the r_0 <= 1 test statistic is smaller than the correponding 10% value (7.52)
- Therefore, at the 1%, 5% and 10% critical values alike r_0 = 0 is rejects but r_0 <= is maintained
- This implies r = 1.
- The third part shows the estimated corintegration relations that correspond to the three eigenvalues in the
- Since we concluded r = 1, we consider only the first eigenvalue and cointegrating relation

file:///C:/Users/joshc/Documents/2023S2/ECON90033/Tutorials/Week 12/Week-12-script.htm

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- Therefore, the estimated EC term, normalised with respect to the first endogenous variable is: AAA -2.2585 - 1.0265TB3 = Epsilon
- The fourth part of the printout shows the estimated speed of adjustment coefficients. Again, only the first column is relevant (i.e. alpha = [-0.0191, 0.0107])

```
Test type: trace statistic , without linear trend and constant in cointegration
                    j.trace = ca.jo(data, type = "trace", K = 3, ecdet = "const")
                                                                                                                                                                                                                                                                                                                                             Values of teststatistic and critical values of test:
                                                                                                                                                                                                                                                                                              ## [1] 3.781984e-02 4.780644e-03 -2.930396e-19
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                ## AAA.d -0.01913395 -0.002857251 7.630392e-19
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             constant
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          ## TB3.d 0.01069988 -0.008111077 -7.637752e-19
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Eigenvectors, normalised to first column:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -1.026532 0.0557922 -0.2276885
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            ## constant -2.258470 -7.0850332 7.3511208
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     ## (These are the cointegration relations)
##
                                                                                                                                                                                                                                                                                                                                                                                                             ## r <= 1 | 4.23 7.52 9.24 12.97
## r = 0 | 38.27 17.85 19.96 24.60
                                                                                                                                                                                                                                                                                                                                                                                          test 10pct 5pct 1pct
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             TB3.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        TB3.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             1.000000 1.0000000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              ## (This is the loading matrix)
                                                                                                                                                    ## # Johansen-Procedure #
                                                                                                                                 #######################
                                                                                                                                                                             ######################################
                                                                                                                                                                                                                                                                     ## Eigenvalues (lambda):
 TB3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      AAA.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             AAA.13
data = cbind(AAA,
                                             summary(j.trace)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ## Weights W:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             ## AAA.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ## TB3.13
                                                                                                           ##
                                                                                                                                                                                                                      #
                                                                                                                                                                                                                                                                                                                                       ##
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 #
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 #
                                                                                                                                                                                                                                                                                                                  #
                                                                                                                                                                                                                                                                                                                                                                                    #
                                                                                                                                                                                                                                                                                                                                                                                                                                                              #
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    #
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    #
```

Now we conduct the same process but using the maximum eigenvalue test

- Only the second part of the new prinout differs from the previous, the rest are the same.
- · However, the statistical conclusions do not change: we reejct the first null hypothesis but maintain the second, implying r = 1.

```
j.trace = ca.jo(data, type = "eigen", K = 3, ecdet = "const")
data = cbind(AAA, TB3)
                                                                                 summary(j.trace)
```

file:///C:/Users/joshc/Documents/2023S2/ECON90033/Tutorials/Week 12/Week-12-script.html

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Then you need to derive the second equation of the VECM...

```
# Step 1
EC.1 <- lm(AAA ~ TB3)
Summary(EC.1)
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                                                                                                                                                                                                                                                                                                                                                                                                 ## Residual standard error: 1.359 on 884 degrees of freedom
                                                                                                                                                                                                                                                                                                                                                                                                                          Multiple R-squared: 0.7649, Adjusted R-squared: 0.7646
                                                                                                                                                                                                                                                                                                                                                                                                                                                  ## F-statistic: 2876 on 1 and 884 DF, p-value: < 2.2e-16
                                                                                                                                                                                                                                                              41.58 <2e-16 ***
                                                                                                                                                                                                                                                                                      <2e-16 ***
                                                                                                                                                                                                                                    Estimate Std. Error t value Pr(>|t|)
                                                                                                                                                                                                                                                                                        53.63
                                                                                                                              ## -2.6549 -1.2472 -0.0918 1.2109 3.6640
                                                                                                                                                                                                                                                        ## (Intercept) 3.13472 0.07539
                                                                                                                                                                                                                                                                                          0.01485
                                               ## lm(formula = AAA ~ TB3)
                                                                                                                                                                                                                                                                                        0.79623
                                                                                                                                                                                                         ## Coefficients:
                                                                                                    ## Residuals:
##
## Call:
                                                                                                                                                                                                                                                                                      ## TB3
                                                                                                                                                                                                                                  #
```

```
# Step 2
e.1 = ts(EC.1$residuals, start = c(1950,1), end = c (2023,9), frequency = 12)

# Step 3

DAAA = window(diff(AAA), start = c(1950,2), end = c (2023,9),frequency = 12)

# Step 4

le.1 = window(stats::lag(e.1, -1), start = c(1950,2), end = c (2023,9), frequency = 12)

# Step 5

ec.11 <- lm(DAAA ~ le.1)

summary(ec.11)
```

```
\#\# Test type: maximal eigenvalue statistic (lambda max) , without linear trend and constant in c
                                                                                                                                                                                                                  ## Values of teststatistic and critical values of test:
                                                                                                                                                                      ## [1] 3.781984e-02 4.780644e-03 -2.930396e-19
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     ## Eigenvectors, normalised to first column:
                                                                                                                                                                                                                                                                                                                                                                                                                   constant
                                                                                                                                                                                                                                                                                                                                                                                                                                       1.000000 1.0000000 1.0000000
                                                                                                                                                                                                                                                                                                                                                                                                                                                          -1.026532 0.0557922 -0.2276885
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 ## constant -2.258470 -7.0850332 7.3511208
                                                                                                                                                                                                                                                                                                                                                                    ## (These are the cointegration relations)
##
                                                                                                                                                                                                                                                                                 ## r <= 1 | 4.23 7.52 9.24 12.97
                                                                                                                                                                                                                                                                                                   ## r = 0 | 34.04 13.75 15.67 20.20
                                                                                                                                                                                                                                                                test 10pct 5pct 1pct
                                                                                                                                                                                                                                                                                                                                                                                                                  TB3.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            ## (This is the loading matrix)
                     ## # Johansen-Procedure #
###############################
                                          ###############################
                                                                                                                                                   ## Eigenvalues (lambda):
                                                                                                                                                                                                                                                                                                                                                                                                                  AAA.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           ## Weights W:
                                                                                                           ointegration
                                                                                                                                                                                                                                                                                                                                                                                                                                     ## AAA.13
                                                                                                                                                                                                                                                                                                                                                                                                                                                        ## TB3.13
                                                                                                                                #
                                                                                                                                                                                                                                                                                                                           #
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  #
                                                                                                                                                                                           #
                                                                                                                                                                                                                                     # #
                                                                                                                                                                                                                                                                                                                                                                                                            #
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     #
```

d) Estimate a VECM

Like testing for cointegration, a VECM can be estimated based on the EG or J approaches.

In the former, we have to estimate the long-run equilibrium relationship between the two variables, then estimated the VECM eqns one-by-one with the lagged residuals for the equilibrium used as the EC term.

- Step 1: estimate the long-run equilibrium between the two variables.
- This prinout show the LR equilibrium relationship is: AAA = 3.1347 + 0.7962(TB3)
- There is a significantly positive relationship between AAA and TB3.
- Step 2: save the residuals from this regression
- Step 3: take the first difference of the dependent variable (AAA)
- Step 4: take the lag of the residuals from LR equilibrium eqn
- Step 5: regress the former (first difference of the dependent variable) on the latter (the lag of the residuals from the LR equilibrium eqn)

Then you have the printout of the first VECM equation: DAAA = 0.0030 - 0,0263(ec_t-1).

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```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                                                                                                                                                                                                                                                                                                                             Adjusted R-squared: 0.02879
                                                                                                                                                                                                                                                                                                    ## Residual standard error: 0.2032 on 882 degrees of freedom
                                                                                                                                                                                                                                                                                                                                              ## F-statistic: 27.17 on 1 and 882 DF, p-value: 2.319e-07
                                                                                                                                                                                        Estimate Std. Error t value Pr(>|t|)
                                                                                                        -1.16592 -0.08071 -0.01575 0.07993 1.23484
                                                                                     30
                                                                                                                                                                                                                                                                                                                           ## Multiple R-squared: 0.02989,
                                                                                     Median
                    ## lm(formula = DAAA ~ le.1)
                                                                                     10
                                                                                                                                                  ## Coefficients:
                                                                ## Residuals:
                                                                                     Min
## Call:
                                                                                 #
                                                                                                          #
                                                                                                                                                                                                                                       #
                                                                                                                           #
                                                                                                                                                                       #
                                                                                                                                                                                                                                                                                #
```

To derive the second equation of the VECM

You need to regress the relevant series on the lagged residuals from the LR equation.

```
DTB3 = window(diff(TB3), start = c(1950,2), end = c(2023,9).
                                                                                                                                                                       Max
                                                                                                                                                                                    2.6071
                                                                                                                                                                      Median 3Q
0.0029 0.1135
                                                                                                                      ## lm(formula = DTB3 ~ le.1)
                            ec.12 = lm(DTB3 \sim le.1)
                                                                                                                                                                                     -4.6153 -0.0866
                                                                                                                                                                       10
               frequency = 12)
                                              summary(ec.12)
                                                                                                                                                    ## Residuals:
## Min
                                                                                         ##
## Call:
                                                                                                                                                                                    #
                                                                                                                                        #
```

Reflections on this model

-0.0009282

Adjusted R-squared:

0.671 0.719

Estimate Std. Error t value Pr(>|t|)

0.360 0.426

0.013276 0.009805

(Intercept) 0.004786

Coefficients:

#

#

0.004173

le.1

Residual standard error: 0.3947 on 882 degrees of freedom

F-statistic: 0.1811 on 1 and 882 DF, p-value: 0.6705

Multiple R-squared: 0.0002053,

- The first regression is strongly significant although R_squared is very low.
- The second regression is insignificant, as its its estimate of the speed of adjustment coefficient. This means the system of AAA and TB3 adjusts to deviations from the LR equilibrium through changes in AAA and the development of TB3 are not directly linked to the equilibrium error.

file:///C.;/Users/joshc/Documents/2023S2/ECON90033/Tutorials/Week 12/Week-12-script.html

Week 12 Tutorial 03/11/2023, 18:05 · We did not check if these VECM equations are free of autocorrelation, and indeed they are not. However, if you augmented these equations with the first lags of the first differences of AAA and TB3, it is possible to eliminate autocorrelation.

residuals from the new equilibrium regresison and estimate the two equations of the VECM. Do this to prep The results of the EG two-step procedure can be sensitive to normalisation. Therefore, in practice it would be important to re-estimate the LR equilibrium between the two variables normalised to TB3, using the

Now we move onto the Johansen procedure, starting with the trace method.

- comparing this to the previous J-trace printout, you can see that:
- i. the coefficients of the EC term (ect1) arethe same estimated speed of adjustment coefficients term as in the fourth part of the J trade test printout.
- ii. the \$beta vector is the estimated cointegration realtion and it is the same as teh first column vector in the third part of teh J-trace test printout.
- From this printout we get all the terms for the VECM(2):
- DAAA(hat) = -0.0191(ect_t-1) + 0.3407(DAAA_t-1) 0.2765(DAAA_t-2) + 0.0407(DTB3) + 0.0298(DTB3_t-2)
- DTB3(hat) = 0.0107(ect_t-1) + 0.2963(DAAA_t-1) 0.2491(DAAA_t-2) + 0.3405(DTB3_t-1) -0.1355(DTB3_t-2)
- The EC term is:
- $ect = AAA 2.2585 1.0265(TB3_t)$

```
vecm.j = cajorls(j.trace, r = 1)
                          print(vecm.j)
```

```
## lm(formula = substitute(form1), data = data.mat)
                                                                                                        -0.24907
                                                                          0.01070
                                                                                     0.29626
                                                                                               0.34051
                                                                                                                     -0.13554
                                                                                                                                                                                                -2.258470
                                                                                                                                                                            1.000000
                                                                                                                                                                                      -1.026532
                                                                                     0.34070
                                                                                               0.04067
                                                                                                         -0.27653
                                                                                                                     0.02975
                                                                          -0.01913
                                                                AAA.d
                                                   ## Coefficients:
                                                                                                                                                                                                ## constant
                                                                                    ## AAA.dll
                                                                                               ## TB3.dl1
                                                                                                         ## AAA.dl2
                                                                                                                    ## TB3.dl2
                                                                                                                                                                          ## AAA.13
                                                                                                                                                                                  ## TB3.13
                                                                                                                                                   ## $beta
                    ## Call:
                                                                        ## ect1
## $rlm
                                          #
                                                               #
                                                                                                                              ##
                                                                                                                                                               #
```

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e) Obtain the VAR(3) representaion of the estimated VECM(2) model.

Any VECM can be transformed into an equivalent VAR in levels with the vec2var() function.

From this, we get the following VAR(3) model:

```
• AAA(hat) = 0.0432 + 1.3407(AAA_t-1) - 0.6172(AAA_t-2) + 0.2574(AAA_t-3) + 0.0407(TB3_t-1) - 0.0109(TB3_t-2) - 0.0101(TB3_t-3)
```

• TB3(hat) = $0.0242 + 0.2963(AAA_t) - 0.5453(AAA_t-2) + 0.2598(AAA_t-3) + 1.3405(TB3_t-1) - 0.4761(TB3_t-2) + 0.1246(TB3_t-3)$

```
U.4/61(1B3_F-2) + U.1246(1B3_F-3)

var.j = vec2var(j.trace, r = 1)

print(var.j)
```

```
## Coefficient matrix of lagged endogenous variables:
## AAA.11 TB3.11
## AAA 1.3406951 0.04066789
## TB3 0.2962563 1.34051385
## AAA -0.6172566 -0.040602015
## TA3 -0.6476354 -0.47605245
## AAA -0.617256 -0.01002015
## AAA 0.617256 -0.01002015
## AAA 0.617256 -0.01016613
## AAA 0.2573975 -0.01016613
## AAA 0.2573975 -0.01016613
## Coefficient matrix of deterministic regressor(s).
## AAA 0.0432345
## AAA 0.0432345
## AAA 0.0432345
```