Lecture 16: Search model of unemployment

ECON30009/90080 Macroeconomics

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Last Class

- Last class, we looked at measures of the labour market with a particular focus on unemployment
- We showed how we can measure the stock of unemployed, and also how to characterize the flows into and out of unemployment
- Today, we want to look at a search model of unemployment.

Matching function

- Denote market tightness $\theta_t = v_t/u_t$ and given the matching function:

$$M_t = \xi \mathcal{M}(v_t, u_t) = \xi \frac{u_t v_t}{\left(u_t^\alpha + v_t^\alpha\right)^{1/\alpha}}$$

- Derive the job-finding probability in terms of θ_t, $p(\theta_t) = M_t/u_t$:

$$p(\theta_t) = \xi \frac{u_t v_t}{u_t \left(u_t^\alpha \left[1 + \theta_t^\alpha\right]\right)^{/1\alpha}} = \xi \frac{\theta_t}{\left(1 + \theta_t^\alpha\right)^{1/\alpha}}$$

- And the probability the firm fills the vacancy: $q(\theta_t) = M_t/v_t$:

$$q(\theta_t) = \xi \frac{u_t v_t}{v_t \left(u_t^\alpha \left[1 + \theta_t^\alpha\right]\right)^{1/\alpha}} = \xi \frac{1}{\left(1 + \theta_t^\alpha\right)^{1/\alpha}}$$

where $0 < \alpha < 1$

A 2 period search model

Assumptions: household

- Economy lasts for 2 periods. Households live 2 periods. Population $=$ measure 1
- Households get utility from consumption, there is no savings. Households discount the future with factor β, where $0 < \beta < 1$
- If individual is employed, they earn some wage $w(y_t)$ where the wage is some exogenous function of output.
- If non-employed, individual produces at home and has home production equal to h
- The household gets log utility from consumption. Since no savings, consumption is either equal to wage income or to home production.

A 2 period search model

Assumptions: household

- All households start as non-employed in period 1. They can choose whether or not to participate in the labor market

- There exists search frictions in the labour market.

- An unemployed individual finds a job in period t with probability $p(\theta_t)$

- An employed person loses a job in period t with probability s_t. Newly displaced workers have to wait a period before they can search for a job.

- Household choice: choose to participate if the expected benefit from search $>$ than value of staying out of the labour force and producing h with probability 1.

A 2 period search model

Assumptions: firm

- Output is produced using labour and TFP, $y_t = z_t \times 1$. No capital used in production. Assume $y_t > h$ for all t
- All employed workers provide 1 unit of labour. A job is a single firm-worker pair
- A new firm that wants to produce needs a worker and must post a vacancy. Each vacancy costs κ
- In period t, a firm meets an unemployed worker with probability $q(\theta_t)$. Matched firms can produce output.
- If a vacancy goes unfilled at the end of period t, the vacancy expires and no output is produced by that firm

A 2 period search model

Assumptions: firm

- An existing matched firm loses a worker with probability s_t.
- In the absence of a separation shock (probability $1 - s_t$) existing matched firm can continue to produce with their worker
- This implies only new unmatched firms post vacancies. Matched firms do not need to post since they already have a worker.
- New firm's choice: Choose whether or not to create a vacancy

Timing of model

At the start of each period t:

- Observe aggregate productivity z_t
- Firms decide whether or not to post vacancy at unit cost κ
- Separation shocks occur with probability s_t. Newly separated cannot search immediately
- Non-employed household decide whether to search
- Search and matching occurs
- Production (market and home) occurs, households consume

Working backwards: end of period 2

To characterize whether the household wants to participate and how many firms want to enter, we will work backwards and start from the **end** of period 2.

- At the end of period 2, the value of a non-employed household:

$$V_2^U = \ln h$$

- At the end of period 2, the value of an employed household:

$$V_2^E = \ln w[y_2]$$

- At the end of period 2, the value of a filled firm:

$$V_2^F = y_2 - w(y_2)$$

End of period 2: nobody is making any decisions, just producing and consuming.

Working backwards: start of period 2

Consider now the **start** of period 2.

- At the start of period 2, the firm decides whether or not to create a vacancy.
- The value of a vacancy is given by:

$$\widetilde{V}_2^V = -\kappa + q(\theta_2)V_2^F$$

- Note each firm is too small to influence θ_2, they take θ_2 as given.
- Under free entry, new firms will enter the labour market until the value of a vacancy is **driven to zero**

Working backwards: start of period 2

Free Entry

- Under free entry, new firms will enter the labour market until the value of a vacancy is **driven to zero**
- If $\kappa > q(\theta_2)V_2^F$: cost is greater than expected benefit of creating a job, firms would want to exit the labour market.
- If $\kappa < q(\theta_2)V_2^F$: cost is less than the expected benefit of creating a job, more firms would want to enter the labour market
- In equilibrium, cost of posting vacancy $=$ expected benefit of creating job. No new firms want to either enter or exit the market at this point

Working backwards: start of period 2

Free Entry

- Under free entry, new firms will enter the labour market until the value of a vacancy is **driven to zero**

- This implies:

$$q(\theta_2)V_2^F = \kappa$$

  and we can solve for θ_2 using the form of $q(\theta)$ and V_2^F:

$$\theta_2 = \left(\left[\frac{\xi \left(y_2 - w[y_2] \right)}{\kappa} \right]^{\alpha} - 1 \right)^{1/\alpha}$$

- Labor market tightness is determined in equilibrium from the free entry condition

Working backwards: start of period 2

Participation

- At start of period 2, non-employed household decides on whether to participate
- Expected value of search at start of period 2:

$$\widetilde{V}_2^S = p(\theta_2)V_2^E + (1 - p(\theta_2))V_2^U$$

- Participate as long as expected benefit of search $>$ producing at home with probability 1.

  Participate as long as $\widetilde{V}_2^S > V_2^U$

- Clearly, participate so long as $w(y_2) > h \implies V_2^E > V_2^U \implies \widetilde{V}_2^s > V_2^U$

Working backwards: period 2

- We just solved for period 2.
- We know θ_2 from the free entry condition
- We know all households participate in the labor force as long as $w(y_2) > h$.

Working backwards: end of period 1

- At the **end of period 1**, the value of a non-employed household:

$$V_1^U = \ln h + \beta \left\{ p(\theta_2) V_2^E + [1 - p(\theta_2)] V_2^U \right\}$$

- At the end of period 1, the value of an employed household:

$$V_1^E = \ln w(y_1) + \beta \left\{ s V_2^U + (1 - s) V_2^E \right\}$$

- At the end of period 1, the value of the matched firm:

$$V_1^F = y_1 - w(y_1) + \beta (1 - s) V_2^F$$

Note if firm loses worker in second period, equivalent to firm shutting down

Working backwards: start of period 1

Free Entry

- At the **start of period 1**, firms decide whether or not to create a vacancy:

$$\widetilde{V}_1^V = -\kappa + q(\theta_1) V_1^F$$

- Under free entry, firms enter until the cost of posting a vacancy is equal to its expected benefit:

$$\theta_1 = \left(\left[\frac{\xi \left\{ y_1 - w(y_1) + \beta(1-s)[y_2 - w(y_2)] \right\}}{\kappa} \right]^{\alpha} - 1 \right)^{1/\alpha}$$

- θ_1 determined in equilibrium from free entry condition.

Working backwards: start of period 1

Participation

- At the **start of period 1**, expected benefit from participating:

$$\widetilde{V}_1^S = p(\theta_1)V_1^E + (1 - p(\theta_1))V_1^U$$

- Then as long as $V_1^E > V_1^U$, household will always participate.

Unemployment and vacancies

- Now that we have characterized the choices, we can show how unemployment evolves in this economy

- Since all individuals start out non-employed and since all participate in the labour force so long as $V_1^E > V_1^U$, we know that total unemployed at end of period 1 is:

$$u_1 = 1 - p(\theta_1)$$

- Since total job-seekers at start of period 1 is equal to labour force $=$ population $=$ 1, this means vacancies $v_1 = \theta_1$.

Unemployment and vacancies

- At the start of period 2, only the unemployed search for jobs. The total unemployed at the start of period 2 is given by u_1

- This means we know the number of vacancies posted at the start of period 2: $v_2 = \theta_2 u_1$

- And we know the total unemployed at the end of period 2:

$$u_2 = [1 - p(\theta_2)]u_1 + s(1 - u_1)$$

- At this point, we have solved for all labor market variables and choices in a 2 period search model of unemployment (Done!)

What happens in a recession?

A decline in $y_1 = z_1$

- Suppose z_1 falls and thus y_1 falls
- Further suppose that $w(y) = \bar{w}y^{\gamma}$ where $0 \leq \gamma < 1$ and $0 < \bar{w} < 1$
- Note that γ represents the elasticity of wages with respect to labour productivity (labour productivity=output per labour)
- The higher γ is, the more elastic the wage rate is to changes in labour productivity
- A lower γ implies the wage is more sticky and less responsive to changes in y.
- For $\gamma = 0$, wage rate is perfectly rigid and fixed at $\bar{w}$

A decline in $y_1 = z_1$

- Suppose z_1 falls and thus y_1 falls

- For any $0 \leq \gamma < 1$, firm's current profits fall when y_1 falls because wage rates fall by a less than proportionate amount

- From free entry condition in period 1, this means fewer firms post vacancies:

$$\theta_1 = \left(\left[\frac{\xi \left\{ y_1 - \bar{w} y_1^{\gamma} + \beta(1-s)[y_2 - \bar{w} y_2^{\gamma}] \right\}}{\kappa} \right]^{\alpha} - 1 \right)^{1/\alpha}$$

- Fewer vacancies posted means tightness θ_1 falls in a recession.

A decline in $y_1 = z_1$

- As long as $\bar{w}y_1 > h$, all households will want to search for a job in period 1
- A decline in y_1 (recession) causes unemployment at the end of period 1 to rise

$$u_1 = (1 - p(\theta_1))$$

- Since firms post fewer vacancies when current profits are lower, θ_1 falls and probability of finding a job falls.
- Harder to find a job, unemployment rate rises
- Aggregate output lower: $Y_1 = u_1 h + (1 - u_1) y_1$
- Aggregate consumption lower: $C_1 = u_1 h + (1 - u_1)\bar{w}y_1^{\gamma}$

A different implication from RBC

- The search model of unemployment introduced a search friction in the market: while individuals willing to supply labour, they are not always matched to jobs
- Interestingly, in this model, fluctuations in TFP today is not the only driver of business cycles.
- Observe that good news or bad news about tomorrow's y_2 can affect outcomes in period 1

A different implication from RBC

- Observe that good news or bad news about tomorrow's y_2 can affect outcomes in period 1

- From free entry condition in period 1:

$$\theta_1 = \left(\left[\frac{\xi \left\{ y_1 - \bar{w} y_1^{\gamma} + \beta(1-s)[y_2 - \bar{w} y_2^{\gamma}] \right\}}{\kappa} \right]^{\alpha} - 1 \right)^{1/\alpha}$$

- Future profits also matter for firm's expected benefit of creating a job.

- Good news about tomorrow can boost vacancy creation today, $y_2 \uparrow \implies \theta_1 \uparrow$

A different implication from RBC

- Good news about tomorrow can boost vacancy creation today, $y_2 \uparrow \implies \theta_1 \uparrow$
- This implies fewer unemployed at the end of period 1:

$$u_1 = (1 - p(\theta_1))$$

- And higher aggregate output due to fewer unemployed: $Y_1 = (1 - u_1)y_1 + u_1 h$
- And higher aggregate consumption since u_1 lower: $C_1 = (1 - u_1)\bar{w}y^\gamma + u_1 h$

Wrapping up

- Today: a look at a search model of unemployment

- Introduction of search frictions can give rise to a role for news to drive business cycles

- Next class: bringing in money