Lecture 11: The Effects of Social Security in a Life-cycle Model

ECON30009/90080 Macroeconomics

Semester 2, 2025

Outline

- Last class: introduction to social security.
- Compare fully funded social security vs. PAYG in economy with a constant population

Adding Fully-Funded Social Security to the Model

- Suppose in period 1 the economy is in the steady state with no government activity.
- Then in period 2 the government introduces a Fully-Funded social security policy.
  - In every period $t \geq 2$ the each working person is taxed (lump-sum) s and the proceeds are used to invest in capital
  - Then in $t+1$, when the same person retires, they receive $(1+r_{t+1})s$ as their social security benefit.
- How does this fiscal policy affect individuals' decisions, welfare, and growth?

Household budget constraints

- Budget constraint of the young:

$$c_t^y + a_{t+1} = w_t + \pi_t - s$$

- Budget constraint of old:

$$c_{t+1}^o = (1 + r_{t+1})(a_{t+1} + s)$$

- Which implies the following lifetime budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t$$

LBC same as if $s = 0$

Household and firm

No change to household's and firm's optimality conditions:

- Household still chooses c_t^y, c_{t+1}^o to maximize lifetime utility subject to lifetime budget constraint

- Firm still chooses how much capital and labour to rent and hire in order to maximize profits

Government

- No government spending, $G_t = 0$.
- Consider generation t. Govt levies a lump-sum tax s on each young household of generation t
- Govt invests the s to return $(1 + r_{t+1})s$ to generation t when they are old in period $t + 1$
- Essentially, government is simply saving on behalf of the working generation

Equilibrium

- Since no change to household's problem (same as if there were no government)

$$\max U(c_t^y, c_{t+1}^o)$$

s.t.

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t$$

Equilibrium

- Since no change to household's problem (same as if there were no government)
- And no change to the firm's problem

$$\max F(z_t, K_t, L_t) - w_t L_t - r_t K_t$$

Equilibrium

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  Let's see this with the example we've been working with in class

An example

- As before assume log utility:

$$U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

- Output given by Cobb-Douglas production function:

$$F(z_t, K_t, L_t) = z_t K_t^\alpha L_t^{1-\alpha}$$

- Full depreciation, $\delta = 1$, and zero population growth

An example

- From household optimality, we had:
  - Euler equation:
$$\frac{1}{c_t^y} = \frac{\beta(1+r_{t+1})}{c_{t+1}^o}$$
  - LBC
$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t + \pi_t$$
- As before, using firm optimality and market clearing:
$$c_t^y = \frac{1}{1+\beta}(1-\alpha)zk_t^\alpha$$
- Form of c_t^y same as in our market economy when there was no govt

An example

- From capital market clearing, we have:

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- growth path of k_t, and thus y_t unaffected!

Fully funded social security eqm achieves market economy eqm

- Fully funded social security policy does not alter the transition equation.
- Which means the time paths of k_t, w_t, r_t and y_t are unchanged.
- Individual consumption (c_t^y, c_{t+1}^o) and welfare are also not affected.
- Same equilibrium as market economy without government

Fiscal Neutrality of Fully-funded Social Security

- In other words, Fully-funded social security is **neutral**.
- Intuition:
  - The government is simply saving on behalf of the working generation.
  - However, since **social security pays the same rate of return as private saving**, individuals are indifferent between the two.
  - Households $\downarrow$ their private saving in capital by exactly the same amount as the saving the government undertakes on their behalf
  - . . . leaving overall capital formation unchanged!

Fully-funded Social Security

- Fully-funded social security system achieves the same equilibrium as a market economy without government

- which means in steady state, the fully-funded social security system gives us:

$$\bar{k} = \left[\frac{\beta}{1+\beta}(1-\alpha)z\right]^{1/(1-\alpha)}$$

- However, we already saw (in Lecture 7), that the market economy does not necessarily give us the pareto-optimal outcome: $\bar{k}^{SP} = [\alpha z]^{1/(1-\alpha)}$

- Since the fully-funded social security system gives us back the outcomes from a market economy without a govt $\implies$ such a system will not be pareto-improving.

Pay-as-you-go (PAYG) Social security

Adding PAYG Social Security to the Model

- Now suppose instead of a Fully-Funded system, in period 2 the government introduces PAYG social security.
  - The working generation in t is taxed (lump-sum) s and the same amount s is immediately transferred to each member of the <u>retired generation in period t</u>
- Note that generation 1 clearly benefits from this policy: they receive a windfall of s when retired without having had to pay s when they worked.

Government

- No government spending, $G_t = 0$
- Levies lump-sum tax on each young household in period t: s
- And immediate transfers s to retired household in period t
- Under lump-sum taxes, PAYG social security is a pure transfer system
- Under zero population growth, PAYG is revenue neutral (doesn't affect govt budget constraint)

Household budget constraints

- Budget constraint of the young:

$$c_t^y + a_{t+1} + s = w_t + \pi_t$$

- Budget constraint of old:

$$c_{t+1}^o = (1 + r_{t+1})(a_{t+1}) + s$$

- Which implies the following lifetime budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t - s + \frac{s}{1 + r_{t+1}}$$

Household and firms

- While the firm's problem and thus optimality conditions are unchanged
- The household's problem is different
- Further, we already know one of the household's optimality conditions (LBC) is different from the market economy without govt!

Differential savings

- Notably, s is not invested in capital formation but is a mere transfer between generations within period
- The transfer s gives a lower "return" (of 1) relative to the return on private savings a_{t+1}
- The PAYG social security shifts resources from the working generation to the retired generation
- However, shifting resources away from the working generation means they have less resources to consume *and save*
- Let's see this with the example we've been using in class.

An example

- Household's problem is now:

$$\max \ln c_t^y + \beta \ln c_{t+1}^o$$

s.t.

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t - s + \frac{s}{1 + r_{t+1}}$$

- Household optimality conditions:
  - Euler:

$$\frac{1}{c_t^y} = \frac{\beta(1 + r_{t+1})}{c_{t+1}^o}$$

  - LBC:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t - s + \frac{s}{1 + r_{t+1}}$$

An example

- plugging in Euler into LBC, we have:

$$c_t^y = \frac{1}{1+\beta}\left(w_t + \pi_t - s + \frac{s}{1+r_{t+1}}\right)$$

- Further from young budget constraint:

$$a_{t+1} = w_t + \pi_t - s - c_t^y$$

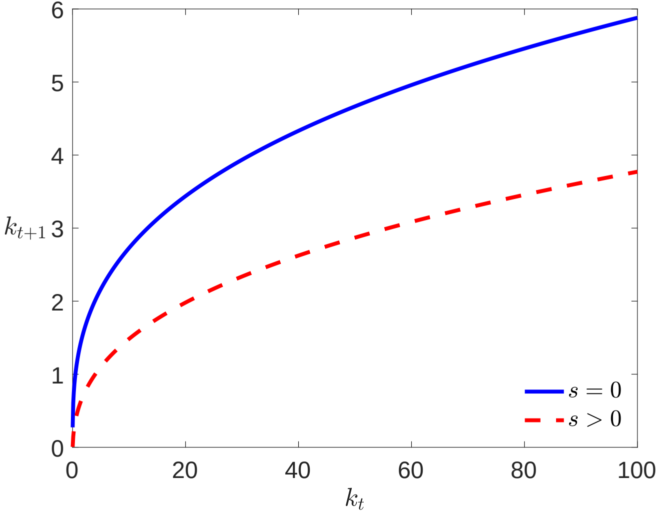
- In equilibrium (using firm optimality and capital market clearing):

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^\alpha - \frac{s}{1+\beta}\left[\frac{\beta(1+r_{t+1})+1}{1+r_{t+1}}\right] < \frac{\beta}{1+\beta}(1-\alpha)zk_t^\alpha$$

- Can't solve this analytically: r_{t+1} also depends non-linearly on k_{t+1}, but we can numerically

An example

Suppose $z = 1$, $\beta = 0.95$, $\alpha = 1/3$



PAYG Redistribution

- The PAYG social security shifts the transition curve down.
- Hence the time paths and steady state levels will be lower for k_t, y_t
- The PAYG social security shifts resources from the working generation to the retired generation, so it reduces capital formation and output.
- Is the introduction of PAYG social security then necessarily worse for households?
    - It depends, as we shall see in next class.

Wrapping up

- Today: Social Security in the model with constant population
- Next class: relaxing no population growth assumption