ECOM40006/ECOM90013 Econometrics 3
Department of Economics
University of Melbourne

Week 8 Tutorial Exercise

Semester 1, 2025

1. Ask any questions that you may have about the lecture materials, etc. If there is still time then attempt the following questions.
2. Read, and make sure that you understand, the handouts "Estimation is Constructive" and "Generalized Least Squares Re-Visited" (both of which can be found on the LMS.
3. Consider a simple random sample of size n from a population with Bernoulli distribution with parameter $0 < \pi < 1$, so that

$$\mathcal{L}(\pi) = \prod_{i=1}^{n} (1-\pi)^{1-y_i} \pi^{y_i}.$$

   Find the maximum likelihood estimator of π. Verify that the second order condition for a maximum is satisfied.
4. Consider a simple random sample of size n from a population with exponential density function

$$f(y;\lambda) = \lambda \exp\{-\lambda y\}, \qquad \lambda > 0, y \geq 0.$$

   Find the maximum likelihood estimator of λ. Verify that the second order condition for a maximum is satisfied.
5. Consider a simple random sample of size n from a Poisson population with probability mass function

$$f(y;\lambda) = \frac{\exp\{-\lambda\}\lambda^y}{y!}, \qquad \lambda > 0, y = 0, 1, 2, 3, \ldots$$

   Find the maximum likelihood estimator of λ. Verify that the second order condition for a maximum is satisfied.
6. Consider a simple CAPM model of the form

$$y_i = \alpha + \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

   where y_i and x_i are the excess returns in a particular sector and the whole market, respectively. Assume that the model satisfies the assumptions of a classical linear regression model and, specifically that, given the x's, the disturbances have zero mean, are homoskedastic, and are independent of one another. Now assume that the disturbances have a t_5 distribution, scaled to have unit variance, so that the density of ϵ_i is of the form

$$f(\epsilon_i) = \frac{c_5}{\sigma}\left(1 + \frac{\epsilon_i^2}{5\sigma^2}\right)^{-3}$$

where c_5 is a scaling constant (that does not depend on σ) so that $\int f(\epsilon_i)\,\mathrm{d}\epsilon_i = 1$. The log=likelihood is given by

$$\ln \mathcal{L}_n(\alpha, \beta, \sigma^2) = \sum_{i=1}^{n} \ln f(\epsilon_i) = n \ln c_5 - \frac{n}{2} \ln \sigma^2 - 3 \sum_{i=1}^{n} \ln\left(1 + \frac{(y_i - \alpha - \beta x_i)^2}{5\sigma^2}\right).$$

Derive the various scores that will require solution in order to estimate this model.

7. Stan the Statistician wishes to test the null hypothesis that the mean of some population is zero against the alternative that it is three. If the population is $\mathrm{N}(\mu, 1)$ and Stan uses the statistic $z = \sqrt{n}(\overline{y} - \mu_0)$, using data from a simple random sample of size n, as his test statistic, find the size and power of his test if he uses as his acceptance region $\{z : z \leq 2\}$.