# Lecture 10: Public capital formation and intro to Social Security

ECON30009/90080 Macroeconomics
Semester 2, 2025

Shu Lin Wee
Department of Economics
The University of Melbourne

# Announcements

- Assignment 1 Due. Please upload directly to LMS and not on Gradescope.
- Mid-Semester Test on 11 Sep.
  - ECON30009 Students: Exam is in-class (David Wood Lecture Theatre [Bldg 165]), normal class time
  - ECON90080 Students: Exam is in Rm 315 FBE Bldg, 10am-1130am.
- Practice MST released next week. We will go over practice MST in-class on 8 Sep
- Tutorials on week of 8 Sep will instead be used for consultation hours with tutors (you can attend any tutorial if your tutorial falls on 11 Sep)

# Last Class

- Last class we examined two cases where government spending was either completely financed by:
  - A proportional tax on consumption of the young
  - vs. a proportional tax of consumption spending of the old
  - We saw that the ratio of proportional consumption tax rates $\frac{1+\tau_t^y}{1+\tau_{t+1}^o}$ affected the optimal trade-off between $c_t^y$ and $c_{t+1}^o$
- Suppose $G$ increased in period $t$ from $G$ to $G'$ where $G' > G$. What kind of proportional tax on consumption spending would each generation in period $t$ vote for?

# A mix of tax instruments and debt

- Thus far, we have considered either only a proportional tax on consumption of the young or on the consumption of the old
- In both these examples, the government ran a balanced budget and did not incur any debt.
- Governments, however, regularly issue debt to finance their spending
- We want to consider now when they use both debt and taxes to finance their spending.

# Case 3: Deficit financing

Government Budget Constraint:

$$g_t + (1 + r_t)b_t = \tau_t^y c_t^y + \tau_t^o c_t^o + b_{t+1}$$

- Each period, government spends $G_t = G$. Denote $g_t = G_t/N$.
- Each period, govt issues debt $B_{t+1} = G$ and repays this debt completely with a proportional tax on old consumption
- In per capita terms, this implies

$$\tau_t^y c_t^y = 0, \quad \text{and} \quad \tau_t^o c_t^o = (1 + r_t)b_t \quad \text{and} \quad b_{t+1}, b_t = g$$

- So in per-capita terms, government budget constraint becomes:

$$g + (1 + r_t)b_t = \tau_t^o c_t^o + b_{t+1}$$

# Case 3: Deficit financing

- Firm's problem and optimality conditions are exactly the same as before

# Case 3: Deficit financing

Household budget constraints when $\tau_t^y c_t^y = 0$:

- Budget constraint when young :

$$c_t^y + a_{t+1} + b_{t+1} = w_t + \pi_t$$

# Case 3: Deficit financing

Household budget constraints when $\tau_t^y c_t^y = 0$:

- Budget constraint when young :

$$c_t^y + a_{t+1} + b_{t+1} = w_t + \pi_t$$

- Budget constraint when old

$$(1 + \tau_{t+1}^o)c_{t+1}^o = (1 + r_{t+1})(a_{t+1} + b_{t+1})$$

# Case 3: Deficit financing

Household budget constraints when $\tau_t^y c_t^y = 0$:

- Budget constraint when young :

$$c_t^y + a_{t+1} + b_{t+1} = w_t + \pi_t$$

- Budget constraint when old

$$(1 + \tau_{t+1}^o)c_{t+1}^o = (1 + r_{t+1})(a_{t+1} + b_{t+1})$$

- So LBC is:

$$\boxed{c_t^y + \frac{(1 + \tau_{t+1}^o)c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t}$$

# Case 3: Deficit financing

Household problem when $\tau_t^y c_t^y = 0$:

$$\mathcal{L} = \max \ln c_t^y + \beta \ln c_{t+1}^o + \lambda_t \left[ w_t + \pi_t - c_t^y - \frac{\left(1 + \tau_{t+1}^o\right) c_{t+1}^o}{1 + r_{t+1}} \right]$$

Same problem as Case 1, $\implies$ same form for household optimality conditions!

- Euler equation

$$\frac{1}{c_t^y} = \frac{\beta(1 + r_{t+1})}{c_{t+1}^o \left(1 + \tau_{t+1}^o\right)}$$

- LBC

$$c_t^y + \frac{\left(1 + \tau_{t+1}^o\right) c_{t+1}^o}{1 + r_{t+1}} = w_t + \pi_t$$

# Case 3: Deficit financing

In equilibrium

- Substitute Euler into LBC:

$$c_t^y = \frac{1}{1+\beta}\left[w_t + \pi_t\right]$$

- From budget constraint of young:

$$a_{t+1} = w_t + \pi_t - c_t^y - b_{t+1}$$

- In eqm, we know from market clearing, firm's optimality and govt budget constraint:

$$\boxed{k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^{\alpha} - g}$$

# Case 3: Deficit financing

Growth path affected by $g$

$$\boxed{k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^{\alpha} - g}$$

- Under deficit financing, the growth path of $k_{t+1}$ is lower with higher levels of $g$
- In this case, public debt crowds out private investment in physical capital
- This in turn means that the growth in output per person is lower

# Case 3: Deficit financing

Consumption affected via lower growth path of $k_t$

- From budget constraint of old

$$(1+\tau_t^o)c_t^o = (1+r_t)(k_t+b_t)$$

and we know

$$\tau_t^o c_t^o = (1+r_t)b_t$$

# Case 3: Deficit financing

Consumption affected via lower growth path of $k_t$

- From budget constraint of old

$$(1+\tau_t^o)c_t^o = (1+r_t)(k_t+b_t)$$

and we know

$$\tau_t^o c_t^o = (1+r_t)b_t$$

- Then from firm optimality:

$$c_t^o = R_t k_t = \alpha z k_t^\alpha$$

# Case 3: Deficit financing

Consumption affected via lower growth path of $k_t$

- From budget constraint of old

$$(1+\tau_t^o)c_t^o = (1+r_t)(k_t+b_t)$$

and we know

$$\tau_t^o c_t^o = (1+r_t)b_t$$

- Then from firm optimality:

$$c_t^o = R_t k_t = \alpha z k_t^\alpha$$

- And we earlier had:

$$c_t^y = \frac{1}{1+\beta}\left[w_t + \pi_t\right] = \frac{1}{1+\beta}(1-\alpha)z k_t^\alpha$$

# Crowding out private investment

- In this example with deficit financing, households are still making the same choices regarding how much of their income to set aside for consumption and savings.
- The difference is they now face two instruments that they can save in
- Specifically, they can choose to save and invest in capital, $k_{t+1}$, or save by buying government debt, $b_{t+1}$
- Here the issuance of public debt crowds out private savings in capital.
- This in turn leads the economy on a lower growth path of $k_t$ and $y_t$.

# Crowding out private investment

- In this example with deficit financing, households are still making the same choices regarding how much of their income to set aside for consumption and savings.
- Why are consumption choices unchanged in this example if there is a proportional tax on consumption of the old?

# Crowding out private investment

- To answer this, we need to ask ourselves what is happening to the rate of return on savings.
- Investment was lower with government spending paid with deficit financing

$$\boxed{k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^{\alpha} - g}$$

- This has implications for $R_{t+1} = 1 + r_{t+1}$.
- We know $MPk \uparrow$ when $k \downarrow$ because of diminishing marginal product.
- So even though there's a tax on consumption of old, the returns to savings are now higher and offset the cost coming from a higher tax

# Crowding out private investment

- From Household Euler equation:

$$\frac{1}{c_t^y} = \frac{\beta(1 + r_{t+1})}{c_{t+1}^o \left(1 + \tau_{t+1}^o\right)}$$

Optimal trade-off between $c_t^y$ and $c_{t+1}^o$ still the same when increase in $\tau_{t+1}^o$ nullified by increase in $r_{t+1}$

# Assumptions on government spending

- Importantly, we had assumed government spending was wasteful.
- And so government spending here took away resources which could otherwise have gone towards private investment
- In reality, not all government spending is wasteful

# China breaks more records with surge in solar and wind power

**Between January and May, China added 198 GW of solar and 46 GW of wind, enough to generate as much electricity as Indonesia or Turkey**

Workers of Huaneng Yili Xexin Photovoltaic Power Plant inspect equipment on a barren slope in Yili, Xinjiang province, China, on 30 November, 2024. Photograph: Costfoto/NurPhoto/REX/Shutterstock

China's installations of wind and solar in May are enough to generate as much electricity as Poland, as the world's second-biggest economy breaks further records with its rapid buildup of renewable energy infrastructure.

- Large scale clean energy investment led by government

# Public capital formation

- Governments often undertake investment in public infrastructure
- They can invest in roads, public buildings, ICT networks, etc., all of which form part of the public capital stock
- This public capital stock can be used in the production of goods and services

# Deficit financing for public capital goods

- Let's reuse the set-up in our previous example (Case 3) but now assume the government's spending goes towards building the capital stock.
- Each unit of $G$ spent goes towards building $K_{t+1}^G$ units of capital.
- Assume that investment in private and public capital are perfect substitutes, which implies that capital stock in $t+1$ will be:

$$K_{t+1} = Na_{t+1} + K_{t+1}^G$$

and in per-capita terms when there is zero population growth:

$$k_{t+1} = a_{t+1} + k_{t+1}^g$$

# Deficit financing and public capital formation

Government Budget Constraint:

- As before, we will assume in each period, government issues debt $b_{t+1} = g$ and repays this debt completely with a proportional tax on old consumption

- This implies – in per-capita terms– a government budget constraint of the form:

$$g + (1 + r_t)b_t = \tau_t^o c_t^o + b_{t+1}$$

where each unit of $g$ goes towards $k_{t+1}^g$

# Deficit financing and public capital formation

Firm and households:

- No changes here. Same as what we solved earlier!

# Deficit financing with public capital formation

In equilibrium

- We observed private investment of

$$a_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^{\alpha} - g$$

- Capital now evolves according to:

$$k_{t+1} = a_{t+1} + k_{t+1}^{g}$$

- Plugging in for $a_{t+1}$, this gives us:

$$\boxed{k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)zk_t^{\alpha}}$$

- Because private and public investment in capital are perfect substitutes, the growth path of $k_t$ and $y_t$ are unaffected.

# Thinking critically about government policy

- **Modeling assumptions matter!**
  - It's not trivial to measure what government spending multipliers are (recall PIH vs. Keynesian consumption function)
  - But in assessing how government spending affects the economy, we also need to take into account the type of government spending
  - And how it's financed

# Social Security

# Redistributive role of government

- So far, we have been talking about how government spending can affect the economy, and how it depends on our modeling assumptions
- However, governments also typically have a redistributive role
- In particular, the government can conduct transfer payments (provide subsidies to businesses, unemployment insurance, welfare checks, etc)
- Transfer payments are **not** counted as government expenditure in GDP accounting because no final good or service is produced.
- Instead transfer payments involve a reallocation of resources

# Introduction to Social Security

- One transfer payment we will focus on is Social Security
- Social security was first introduced in Germany in the late 1800.
- Put forward by German Chancellor Otto von Bismarck, Bismarck argued: *"... those who are disabled from work by age and invalidity have a well-grounded claim to care from the state."*
- Today, social security viewed as a form of social insurance
  - alleviate poverty among elderly, can provide disability and health insurance

# Introduction to Social Security

- In some instances, social security also seen as a way to require individuals to save in advance for their retirement
- One of our goals will be to investigate if the provision of social security helps us get closer to the socially/pareto optimal choice of savings and investment
- Recall (Lecture 7!): the market economy can fail to give us pareto-optimal long run $\bar{k}$

# Introduction to Social Security

- Social security policies can affect national saving, investment, and output according to the life-cycle model.
- There are two main ways to fund a social security program: **Fully-Funded** and **Pay-As-You-Go**.

# Fully-Funded Social Security

- **Fully-Funded** social security system (also known as defined-contributions):

# Fully-Funded Social Security

- **Fully-Funded** social security system (also known as defined-contributions):
  - The govt. taxes the working generation and invests the proceeds. When working generation retires, the accumulated savings are paid out as social security benefits.

# Fully-Funded Social Security

- **Fully-Funded** social security system (also known as defined-contributions):
  - The govt. taxes the working generation and invests the proceeds. When working generation retires, the accumulated savings are paid out as social security benefits.
  - Works similar to private pensions offered by firms – individuals contribute to their accounts and receive payments from their own accounts when eligible for benefits.

# Fully-Funded Social Security

- **Fully-Funded** social security system (also known as defined-contributions):
  - The govt. taxes the working generation and invests the proceeds. When working generation retires, the accumulated savings are paid out as social security benefits.
  - Works similar to private pensions offered by firms – individuals contribute to their accounts and receive payments from their own accounts when eligible for benefits.
- Institutional details can very across countries (e.g., how funds are invested and managed, how payouts are distributed, etc.)

# Fully-Funded Social Security

- **Fully-Funded** social security system (also known as defined-contributions):
  - The govt. taxes the working generation and invests the proceeds. When working generation retires, the accumulated savings are paid out as social security benefits.
  - Works similar to private pensions offered by firms – individuals contribute to their accounts and receive payments from their own accounts when eligible for benefits.
- Institutional details can very across countries (e.g., how funds are invested and managed, how payouts are distributed, etc.)
- Commonality: **value of the benefit paid in retirement linked to contributions made while working**. higher contribution $\implies$ higher payout in retirement.

# PAYG social security system

- **Pay-As-You-Go (PAYG)** social security system:

# PAYG social security system

- **Pay-As-You-Go (PAYG)** social security system:
  - The govt. taxes the working generation and then immediately uses the proceeds to pay social benefits to the current retired generation.

# PAYG social security system

- **Pay-As-You-Go (PAYG)** social security system:
  - The govt. taxes the working generation and then immediately uses the proceeds to pay social benefits to the current retired generation.
  - In a *balanced* system, the amount paid in by the employed over a given time frame is equal to the amount paid out to those receiving benefits over the same time frame.

# PAYG social security system

- **Pay-As-You-Go (PAYG)** social security system:
  - The govt. taxes the working generation and then immediately uses the proceeds to pay social benefits to the current retired generation.
  - In a *balanced* system, the amount paid in by the employed over a given time frame is equal to the amount paid out to those receiving benefits over the same time frame.
  - Most OECD countries are primarily PAYG systems. However, budget pressures from population ageing is leading to reform and change to these systems.

# PAYG social security system

- **Pay-As-You-Go (PAYG)** social security system:
  - The govt. taxes the working generation and then immediately uses the proceeds to pay social benefits to the current retired generation.
  - In a *balanced* system, the amount paid in by the employed over a given time frame is equal to the amount paid out to those receiving benefits over the same time frame.
  - Most OECD countries are primarily PAYG systems. However, budget pressures from population ageing is leading to reform and change to these systems.
- Under PAYG, it is the tax levied on the **future working generation** that determines your benefit, not necessarily your own contribution while working.

# The Effects of Population Ageing on the PAYG Model

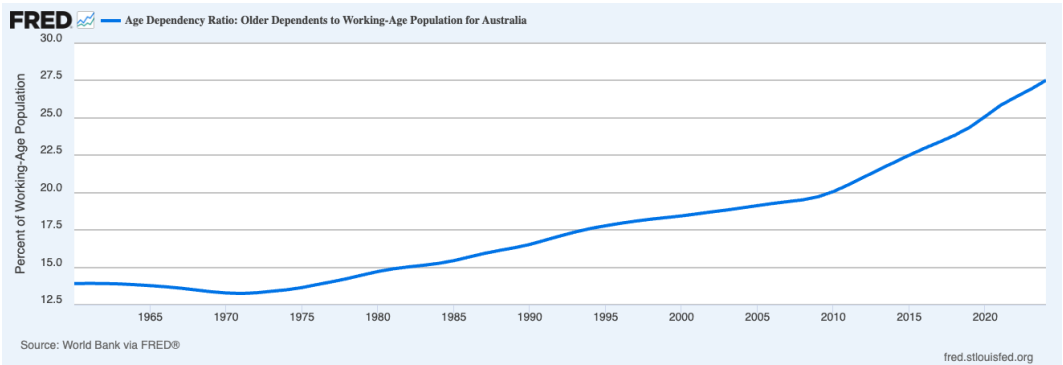
- With a constant ratio of working to retired individuals PAYG, and a constant benefit ratio, PAYG balance can be sustained.
- If working population grows faster than the retired: easier to fund the same benefit-level with each new generation, holding fixed contributions per worker.
- However, slower growth in the working population relative to the retired population can be problematic.
  - In this case: contribution base is falling in relative size compared to the benefit base.

# Population Ageing: A Challenge to the PAYG Model?

- Absent changes in other fiscal parameters (e.g., more borrowing or reducing government expenditure) the only options to address an increasing age dependency ratio on PAYG sustainability are to:
  - require higher contributions from the working age population; and/or
  - reduce the benefits paid to the retired population.
- How prevalent is this challenge?

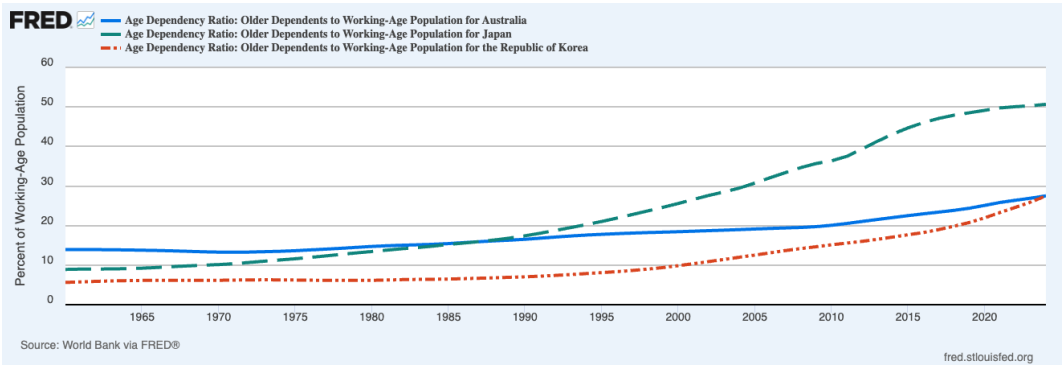
# Age dependency ratio rising

Age dependency ratio: the ratio of older dependents ($> 64$ years) to the working-age population ($15 - 64$ years).



# Age dependency ratio rising

Many developed economies observe a rising age-dependency ratio



# Implications of a Rising Age Dependency Ratio

- If benefits per person remain constant in real terms, the rise in old age dependency ratio $\implies$ real contributions per worker must $\uparrow$ absent changes in fiscal policy.
- However, the constant real benefits assumption is possibly conservative (e.g., in advances in medical technology and the demand for health)?
- This has led to social security reform in a number of countries such as changing retirement ages, eligibility for benefits etc.
- This in turn has implications for individuals incentives to save and work.

# Wrapping up

- This class: finishing up on government spending and introduction to social security
- Next class: social security in the OLG model