Week 3

Semester 1, 2025

Tutorial Questions

Given that it is a while since we had a lecture, and the previous lecture was mostly about defining concepts (which doesn't generate much by way of tutorial questions), I thought that this would be a good time to do a little bit of matrix revision. (I say revision because, if nothing else, much of it will have been covered in Daniel's Math Camp). Failing that, take a look at the (incomplete) Matrices handout that is now in the Handouts folder on the LMS. Obviously, if you have any questions about the lecture material, then the tutorial is a good place to ask.

The following questions will be based around the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 2 & 2 & 2 \\ -1 & 4 & 5 & 3 \end{bmatrix}$$

Using these matrices, answer the following questions. Show all workings (which means that you can't just get a computer to tell you the final answer).

1. Find the sum $\mathbf{A} + \mathbf{B}$ and the difference $\mathbf{A} - \mathbf{B}$.
2. Find the determinant of both $\mathbf{A}$ and $\mathbf{B}$.
3. Use elementary row and column operations to reduce each of $\mathbf{A}$ and $\mathbf{B}$ to equivalent canonical form $\mathbf{C}$. (See Section 8.1 of the Matrices handout.) Determine the matrices $\mathbf{P}$ and $\mathbf{Q}$ required to achieve the final outcome.
4. Using your answer to 3, determine the rank of both $\mathbf{A}$ and $\mathbf{B}$.
5. Using your answer to 3, determine the inverse of $\mathbf{A}$.
6. Find a full rank factorization for $\mathbf{B}$ and use it to construct a Moore-Penrose generalized inverse for $\mathbf{B}$. Show that the conditions of a Moore-Penrose generalized inverse are actually satisfied.
7. Find eigenvalues and eigenvectors for $\mathbf{A}$ and $\mathbf{B}$.
8. Confirm the spectral decomposition for $\mathbf{B}$.
9. Use elementary matrix operations to find the inverse of $\mathbf{C}$. Show all workings. (You may wish to check your answer via computer but you need to do the calculations by hand.)
10. Use elementary matrix operations to construct the equivalent canonical form of $\mathbf{D}$ and thereby determine its rank. Construct a Moore-Penrose inverse of $\mathbf{D}$ based on a full rank decomposition.