ECON30009/90080 – Tutorial 3

This Version: Semester 2, 2025

Note: these questions are designed to give you some practice solving the life-cycle OLG model but with slightly different assumptions relative to those we discussed in lectures.

Question 1: Solving for equilibrium in the OLG model

The following question asks you to consider a variation of the life-cycle OLG model with a minimal reference level of consumption over the lifetime. That is, we will assume that individuals must consume at least $\overline{c}$ each period and get utility only if their consumption in each period exceeds this minimum amount. Specifically, assume that households in each generation have the following preferences:

$$U(c_t^y, c_{t+1}^o) = \ln\left(c_t^y - \overline{c}\right) + \beta \ln\left(c_{t+1}^o - \overline{c}\right)$$

where $\overline{c} \geq 0$ is a parameter representing the minimum consumption level a household must have. β is a parameter representing the household's discount factor, i.e., the weight they put on consumption when old. Specifically, $0 < \beta < 1$. There are N households in every generation and there is no population growth. The household lives for two periods, and can choose to save in an asset a_{t+1} which gives gross return of $1 + r_{t+1}$ in period $t+1$. Individuals when young supply one unit of labour inelastically and earn a wage w_t per unit of labour supplied. Individuals when old retire and do not have any labour income. They instead consume their savings.

In addition to households in the economy, firms produce output according to a Cobb-Douglas production function $Y_t = zK_t^{\alpha} L_t^{1-\alpha}$ where α is a parameter that takes values between 0 and 1, and z represents TFP which we will assume is exogenous and constant for simplicity. In every period, firms rent capital at rate R_t and hire labour at rate w_t. Capital used in production depreciates at rate $\delta = 1$. This implies $K_{t+1} = I_t$.

a) Set up the household's problem

b) Derive the household's optimality conditions.

c) Set up the firm's problem.

d) Derive the firm's optimality conditions. [Note: you may find it useful at this stage to write things in terms of $k_t = K_t/L_t$]

e) State the markets which clear in equilibrium [Hint: there are 3 different markets that firms and households participate in]

f) Using your equilibrium conditions, derive a transition equation for k_{t+1} in terms of k_t,z, α, β and $\bar{c}$.

g) Assume $\alpha = 0.5$,$z = 2$, $\beta = 0.95$. For the following values of k_t contained in Table 1, use the transition equation you derived in f) to compute the corresponding values of k_{t+1} for $\bar{c} = 0.05$. Plot how k_{t+1} varies with k_t. You can do this part in Excel or any spreadsheet software you prefer.

k_t	k_{t+1}	c_t^y	c_t^o	$y_t = c_t^y + c_t^o + k_{t+1}$	$y_t = zk_t^\alpha$
0.01					
0.02					
0.04					
0.06					
0.08					
0.10					
0.12					
0.14					
0.16					
0.18					
0.20					

Table 1: Equilibrium values of key aggregate outcomes

h) Still assuming $\alpha = 0.5$,$z = 2$, $\beta = 0.95$ and $\bar{c} = 0.05$, compute the values of c_t^y, c_t^o given the values of k_t contained in Table 1. Also compute total expenditure in the economy as given by $y_t = c_t^y + c_t^o + k_{t+1}$.

i) Verify that the goods market clears in equilibrium by computing output supplied as $y_t = zk_t^\alpha$. Is total output supplied equal to output demanded?

j) Finally repeat the exercise in g and compute the values of k_{t+1} but for $\bar{c} = 0.1$. Plot your answer on the same graph where you plotted how k_{t+1} varies with k_t when $\bar{c} = 0.05$. Provide some intuition as to why the two graphs differ.

Question 2: The life-cycle model with population growth

Consider the life-cycle model we discussed in class. In particular, households have preferences

$$U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

And firms have the following production function

$$Y_t = zK_t^\alpha L_t^{1-\alpha}$$

Everything is the same as the example we discussed in Lecture 5 (refer to the lecture slides if you don't remember) ***except*** population grows at a constant rate such that $N_{t+1} = (1+n)N_t$.

a) Derive the household optimality conditions

b) Derive the firm's optimality conditions

c) Derive the transition equation. How does k_{t+1} depend on population growth rate n? Provide some intuition.