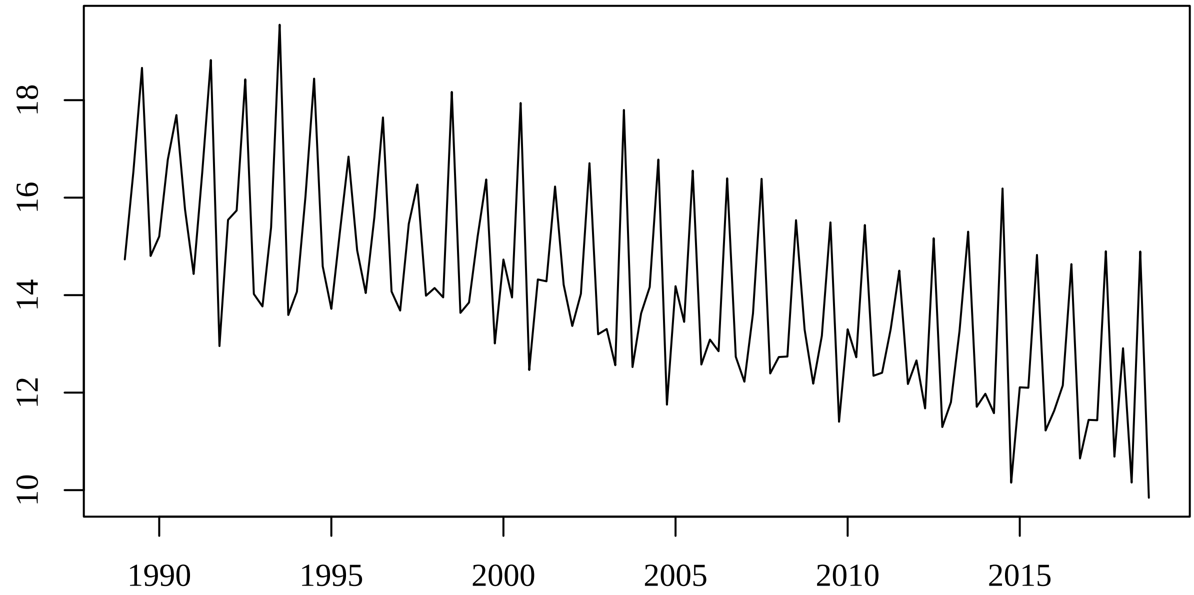
Question 1.

The following plot shows $n = 120$ observations on a quarterly time series from 1989q1 to 2018q4, to be used as the estimation sample. The four observations for 2019q1 to 2019q4 are reserved for forecast evaluation purposes.



(a) Describe the features evident in this plot that would be included in the specification of the deterministic trend equation for the time series.

A model selection search was carried out for this time series. Two deterministic specifications were tried:

- intercept and linear trend (`NoQD`)
- intercept, linear trend and quarterly dummy variables (`QD`)

in combination with ARMA(p,q) models with a variety of p and q. The values of the AICc and Ljung-Box test p-values are tabulated below.

p	q	AICc(NoQD)	LBp(NoQD)	AICc(QD)	LBp(QD)
0	0	462.42	0.000	247.66	0.000
1	0	448.32	0.000	213.64	0.027
2	0	420.09	0.000	208.60	0.065
3	0	258.62	0.000	207.11	0.042
4	0	260.54	0.000	204.76	0.073
5	0	261.23	0.000	206.81	0.064
6	0	263.54	0.000	209.22	0.064
7	0	227.57	0.110	203.39	0.485
8	0	228.99	0.087	204.92	0.467
0	1	373.16	0.000	192.37	0.823
1	1	373.87	0.000	194.12	0.636
2	1	348.77	0.000	193.81	0.517
0	2	368.15	0.000	194.26	0.626
1	2	362.35	0.000	196.89	0.191
2	2	342.13	0.000	195.94	0.311

(b) What is the best model *without* quarterly dummmies? Justify your choice.

(c) What is the best model amongst all of those considered? Justify your choice.

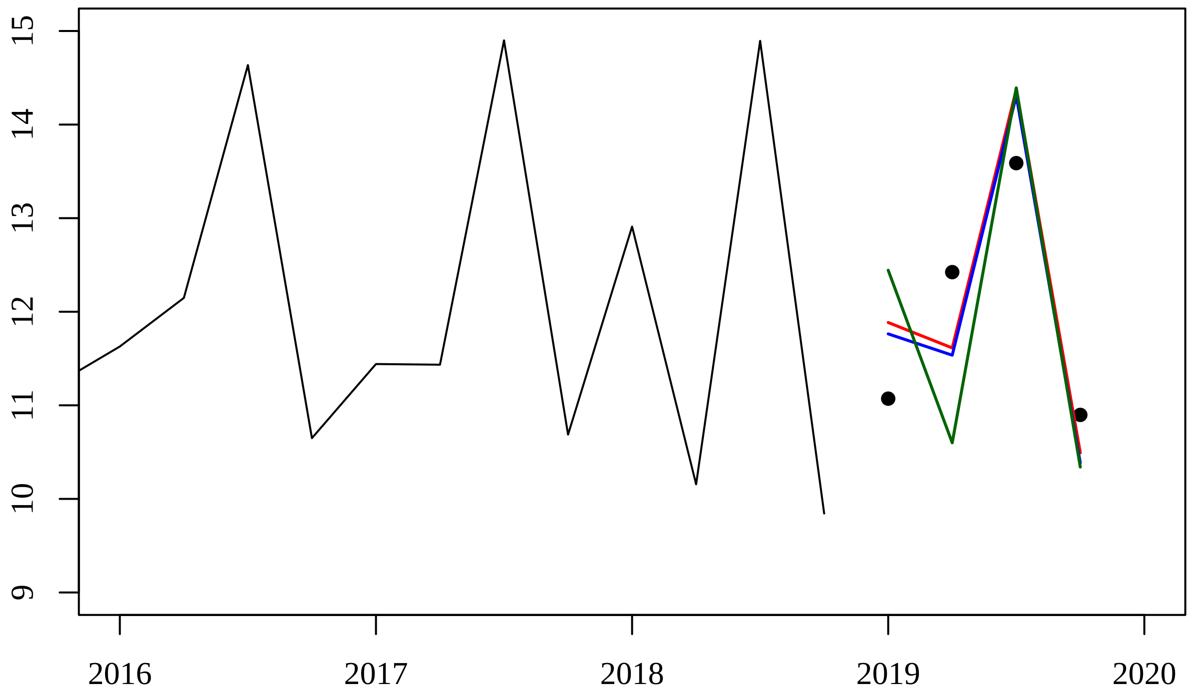
Forecasts and RMSE for the four quarters of 2019 were computed from three models

A: ARMA(0,1) with quarterly dummies, RMSE= 0.725

B: ARMA(7,0) with quarterly dummies, RMSE= 0.716

C: ARMA(7,0) *without* quarterly dummies, RMSE= 1.242

and plotted below, along with the actual values for this period.



(d) Discuss the accuracy of the forecasts from these models, and also how they relate to the in-sample AICc values.

The coefficient estimates for Model A are as follows:

ma1	intercept	time	Q1	Q2	Q3
-0.855	14.980	-4.487	0.552	1.048	3.846

(e) Write out the estimated model in equation form.

(f) According to Model A, in which quarter does Y_t have its highest unconditional mean value? In which quarter does Y_t have its lowest unconditional mean value?

(g) On the basis of the information shown so far, give an explanation for why Models A and B can give such similar forecasts despite their quite different functional forms.

(h) When we construct a 95% prediction interval, what does the probability / proportion 95% refer to?

The following table shows the normal 95% prediction intervals for models B and C, and their width (respectively `B:Lower`, `B:Upper`, `B:Width` for model B, and `C:Lower`, `C:Upper`, `C:Width` for model C). Column `Actual` shows the actual values of Y_t in the forecast period.

	Actual	B:Lower	B:Upper	B:Width	C:Lower	C:Upper	C:Width
h=1	11.071	10.74	12.79	2.05	11.32	13.57	2.25
h=2	12.423	10.26	12.81	2.56	9.10	12.10	3.00
h=3	13.588	13.03	15.59	2.56	12.89	15.90	3.01
h=4	10.898	9.10	11.67	2.57	8.83	11.85	3.01

(i) Compare the quality of the prediction intervals from models B and C on the basis of their inclusion / coverage of the actual value of Y_t.

(j) Compare the quality of the prediction intervals from models B and C on the basis of their width.

(k) Compare the overall forecast accuracy and prediction interval quality of Models B and C. To what do you attribute the difference in performance between the two models?