Lecture 14: The 2 period RBC model and Drivers of Business Cycles

ECON30009/90080 Macroeconomics

Semester 2, 2025

An example with log utility and cobb-douglas production

- The firm's problem:

$$\pi_t = \max_{K_t, L_t} z_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t$$

- Solving the firm's problem, we get two optimality conditions:
  - Optimal labour demand :
  $$(1-\alpha) z_t k_t^{\alpha} = w_t$$
  - and optimal capital demand:
  $$\alpha z k_t^{-(1-\alpha)} = R_t$$

  for $t = \{1, 2\}$ and $k_t = K_t / L_t$.

- Under perfect competition, $\pi_t = 0$

An example with log utility and cobb-douglas production

- The household's problem:

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} - c_1 - \frac{c_2}{R_2} \right]$$

- Solving the household's problem, we get two optimality conditions:
  - Euler equation:

$$\frac{1}{c_1} = \frac{\beta R_2}{c_2}$$

  - And the LBC:

$$R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} - c_1 - \frac{c_2}{R_2} = 0$$

- Plugging the Euler equation into the LBC, we get a decision rule for c_1:

$$c_1 = \frac{1}{1+\beta} \left[R_1 k_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} \right]$$

An example with log utility and cobb-douglas production

- In equilibrium, prices adjust to make all markets clear. This means $L_t = N$, $K_t = Na_t$ and $C_t + I_t = Y_t$ for $t = \{1, 2\}$

- Since a_1 pre-determined (born with initial endowment), this means k_1 is also predetermined.

- So from firm's optimality conditions and market clearing, we know prices w_t, R_t

- And so c_1 becomes:

$$c_1 = \frac{1}{1+\beta}\left[z_1 k_1^{\alpha} + \frac{1-\alpha}{\alpha}k_2\right]$$

An example with log utility and cobb-douglas production

- k_2 is endogenous and affected by households' savings decision in period 1

$$K_2 = Na_2 \implies k_2 = a_2 = R_1 k_1 + w_1 + \pi_1 - c_1$$

- We know w_1, R_1 and the form of c_1, so we can solve for k_2:

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^{\alpha}$$

- And we can again use this form of k_2 and get c_1 entirely in terms of pre-determined variables, exogenous TFP and parameters:

$$c_1 = \frac{1}{1+\alpha\beta} z_1 k_1^{\alpha}$$

An example with log utility and cobb-douglas production

- Can the RBC get co-movement right?

$$y_1 = z_1 k_1^{\alpha}$$

$$c_1 = \frac{1}{1+\alpha\beta} z_1 k_1^{\alpha}$$

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^{\alpha}$$

- To get a boom in the RBC model and have output,consumption and investment all increase, you need a rise in TFP z_1 !

An example with log utility and cobb-douglas production

- Can the RBC get co-movement right?

$$
\begin{aligned}
y_1 &= z_1 k_1^{\alpha} \\
c_1 &= \frac{1}{1+\alpha\beta} z_1 k_1^{\alpha} \\
k_2 &= \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^{\alpha}
\end{aligned}
$$

- Surprisingly, *even good news about tomorrow's productivity doesn't matter* for today's outcomes. z_2 doesn't show up in any of the equations

- RBC prediction: fluctuations in TFP (not news about it) drive the business cycle

- An increase in z_t in t causes an expansion in economic activity in t. A decline in z_t causes a contraction in t

What about government spending shocks?

Government spending shock in period 1

- Suppose we introduce a government spending shock in this model.
- Government spends G_1 and finances it fully by collecting a lump-sum tax T_1 levied on all households in period 1 .

$$G_1 = T_1 \implies g_1 = \tau_1 \text{ in per capita terms}$$

- No other government spending in any other period
- Assume this government spending is just government consumption (doesn't go towards production nor a public good)

Government spending shock in period 1

- Using same example as before, differences show up in household budget constraints:
  - budget constraint in period 1:

$$R_1 a_1 + w_1 + \pi_1 = c_1 + a_2 + \tau_1$$

  - budget constraint in period 2:

$$R_2 a_2 + w_2 + \pi_2 = c_2$$

- and so lifetime budget constraint is:

$$R_1 a_1 + w_1 + \frac{w_2}{R_2} + \pi_1 + \frac{\pi_2}{R_2} = c_1 + \frac{c_2}{R_2} + \tau_1$$

Government spending shock in period 1

- Firm optimality conditions same as before

- Household Euler equation same as before

- Plug Euler into LBC

$$c_1 = \frac{1}{1+\beta}\left[R_1 a_1 + w_1 + \frac{w_2}{R_2} - \tau_1\right]$$

- We know w_1, w_2, R_1, R_2 from firm's optimality and market clearing. And we know $\tau_1 = g_1$ from government budget constraint

$$c_1 = \frac{1}{1+\beta}\left[z_1 k_1^{\alpha} + \frac{(1-\alpha)}{\alpha} k_2 - g_1\right]$$

Government spending shock in period 1

- From capital market clearing and using 1st period household budget constraint:

$$\begin{aligned} k_2 = a_2 &= R_1 k_1 + w_1 - c_1 - \tau_1 \\ &= \frac{\alpha\beta}{1+\alpha\beta}\left(z_1 k_1^{\alpha} - g_1\right) \end{aligned}$$

- which implies

$$c_1 = \frac{1}{1+\alpha\beta}\left(z_1 k_1^{\alpha} - g_1\right)$$

- And thus we have:

$$y_1 = c_1 + k_2 + g_1$$

  Total output supplied = total output demanded

Government spending shock in period 1

- We have:

$$y_1 = c_1 + k_2 + g_1 \implies \frac{dy_1}{dg_1} > 0$$

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta}\left(z_1 k_1^{\alpha} - g_1\right) \implies \frac{dk_2}{dg_1} < 0$$

$$c_1 = \frac{1}{1+\alpha\beta}\left(z_1 k_1^{\alpha} - g_1\right) \implies \frac{dc_1}{dg_1} < 0$$

- Govt. spending increase crowds out consumption and investment. Cannot predict observed co-movement in data.

- This prediction goes against the idea of govt intervening to stimulate the economy during a recession

Only TFP shocks can generate observed co-movement in data

- Key takeaway: only random fluctuations in TFP can drive business cycles
- Govt. spending shocks can't generate the correct co-movement.
- In fact, the implications of the RBC model are so stark that it says the market economy can replicate the social planner's outcomes (achieve pareto efficiency).

Pareto Optimality

The social planner's problem

- Consider the 2 period social planner problem. The planner wants to maximize household lifetime utility subject to the resources available in the economy

- Note that at the end of period 2, the economy ends. So this implies

$$c_2 = z_2 k_2^\alpha$$

- Plug in for c_2 in the utility function and the social planner's problem becomes:

$$\max_{c_1, k_2} \ln c_1 + \beta \ln \left(z_2 k_2^\alpha\right)$$

s.t.

$$c_1 + k_2 = z_1 k_1^\alpha$$

Note TFP is exogenous so the planner also doesn't get to choose z_1, z_2.

The social planner's problem

$$\mathcal{L} = \ln c_1 + \beta \ln \left(z_2 k_2^{\alpha} \right) + \lambda \left[z_1 k_1^{\alpha} - c_1 - k_2 \right].$$

- Taking FOCs:

$$(c_1): \quad \frac{1}{c_1} = \lambda$$

$$(k_2): \quad \frac{\beta}{z_2 k_2^{\alpha}} \left[\alpha z_2 k_2^{\alpha - 1} \right] = \lambda$$

$$(\lambda): \quad z_1 k_1^{\alpha} - c_1 - k_2 = 0$$

The social planner's problem

- Combining FOC wrt c_1 and k_2 to get the planner's optimal trade-off between consumption and investment:

$$\frac{1}{c_1} = \frac{\alpha\beta}{k_2} \implies k_2 = \alpha\beta c_1$$

- Combine above with resource constraint:

$$c_1 = \frac{1}{1+\alpha\beta} z_1 k_1^{\alpha}$$

and this implies

$$k_2 = \frac{\alpha\beta}{1+\alpha\beta} z_1 k_1^{\alpha}$$

Same results as market economy without government spending! The market economy makes the same choices as the social planner.

The RBC model

- The RBC model predicts that the market economy is pareto efficient!
  - This is different from the OLG model where there was a dynamic inefficiency
  - There the dynamic inefficiency arose because we had a "missing market": the young in generation $t-1$ could not trade/contract with the young of generation t.
- In the RBC model, markets are complete, there is perfect competition and all agents are identical
- ... which brings us back to the (neo) classical view that government shouldn't intervene and markets are efficient ...

Criticisms of RBC

- The reason for why the economy experiences recessions in the RBC model is just *weird*: technological regress?
  - Recessions are efficient responses to lower exogenous TFP (no govt intervention required!)
  - No other shock (even good news or bad news about tomorrow!) introduced in this model can get co-movement right.
- More generally, we measure a Solow **residual** in data. To interpret this as "productivity shocks" may be inappropriate
  - Solow residual captures everything we did not measure. Accounting for capacity utilization, you actually find measured TFP moves opposite to booms and recessions
- The model is silent about unemployment

What's the point of RBC?

- Today, few economists actually believe that short-run fluctuations in economic activity are efficient responses to changes in TFP
- But the RBC model makes one thought-provoking critique: just because you observe fluctuations doesn't necessarily make those fluctuations inefficient.
- The efficiency prediction of the RBC model forces us to think more carefully about the conditions under which there should be policy intervention
    - For e.g., when do markets fail, and/or what frictions, externalities exist in reality?
    - How should we incorporate these frictions into the model and what's the policy to address that exact friction?

The road ahead

- This class: driver of RBC is TFP shocks
- Also RBC predicts market economy is pareto efficient
- Next class: introducing search frictions (you have to look for a job and not everyone gets employed)