FNCE90056: Investment Management

Lecture 1: Risk Aversion and Investment Strategies

A/Prof Andrea Lu and Dr Jun Yu

Department of Finance
Faculty of Business and Economics
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Welcome & Logistics

About myself...

Dr Jun Yu

(Lecturer for Week 1-5)

- Room 11.050 The Spot
- Email: jun.yu1@unimelb.edu.au
- Phone: 03 8344 9881

Academic background

- Ph.D. and M.Sc in Economics, Hong Kong University of Science and Technology
- BA in Electrical Engineering, Wuhan University

Research interests

- Macro-finance, theoretical and empirical asset pricing

Who else is teaching...

A/Prof Andrea Lu

(Subject coordinator & Lecturer for Week 6-12 excluding the MST week)

- Room 12.046 The Spot
- Email: andrea.lu@unimelb.edu.au
- Phone: 03 8344 3326

Andrea will start teaching from Week 6.

She is the subject coordinator for this subject so if you have any logistic issues related to the subject, please contact Andrea directly.

Topics

1. Week 1: Risk aversion and investment strategies
2. Week 2: Modern Portfolio Theory
3. Week 3: Capital Asset Pricing Model (CAPM)
4. Week 4: Empirical evidence on CAPM
5. Week 5: Arbitrage Pricing Theory and multifactor models

6. Week 6: Fixed income valuation
7. Week 7: Coupon bonds
8. Week 9: Term structure

   (Week 9: Midterm test)

9. Week 10: Interest rate risk
10. Week 11: Portfolio performance evaluation

    (Week 12: Revision & Exam information)

Assessments = 10 weekly quizzes + MST + Exam

- **10 weekly multiple-choice quizzes, 1% each.**
  - Assigned on Thursdays at 4pm, due on the following Tuesdays at 8am, any re-take or re-submission after due time will be given a mark of 0
  - Completing each quiz on time with a 50% or above score will give you 1% towards the overall grade for this subject.
  - You may attempt each quiz as many times as you like BEFORE the deadline. However, no marks will be awarded for any submissions (including re-attempts) made after the due date, so be sure to submit on time! And, do not re-take after the deadline. *(I encourage you to discuss the material and work on weekly quizzes in groups, but make sure you submit the quizzes individually.)*
- **Mid-semester test (35%)**:
  - Week 9 during class time: Wednesday 6-9pm and Thursday 1-4pm; Location: to be announced.
- **Final exam (55%)**: Date TBA during assessment period. Closed book. 2 hours. Covers all lectures, but more focus on Topic 6-10.

Resources

- **From the Lecturers**
  - Lectures & lecture slides: news stories, examples, definitions, formulas, theories.
  - Consultation hours on Zoom or in-person:
    - ★ Jun: Thursday 16:30-17:30 via Zoom (Week 2-6), or by appointment
    - ★ Andrea: Friday 9:30-10:30 via Zoom (Week 6-8 and 10-12), or by appointment
  - Try to use Ed Discussion on Canvas to ask questions.
  - But if you have a more personal question, e.g. university asks you to discuss Alternative Exam Arrangements, email subject coordinator: andrea.lu@unimelb.edu.au

  Class attendance is not assessed, but I **strongly** encourage you to attend.

- **Post questions on Ed Discussion on Canvas.**
  Lei Chen (Finance PhD candidate) is the TA and she will join me in answering them.

More Resources

- **Canvas**: announcements, files, discussions.
- **Weekly quizzes** with (video) solutions
- **Textbook**: "Investments 13th Edition". Bodie, Zvi, Alex Kane, and Alan J. Marcus. 2024.
  You are only responsible for material in lectures and problem sets, not the rest of the textbook.
- **Your class-mates**: connect via Ed Discussion on Canvas, and discuss lecture material & work on practice problems together.
- **Subject Guide**
- **Handbook**
- **The news and financial press** (e.g. Wall Street Journal, Financial Times, The Economist)

Our Recommendation on How to Study This Subject

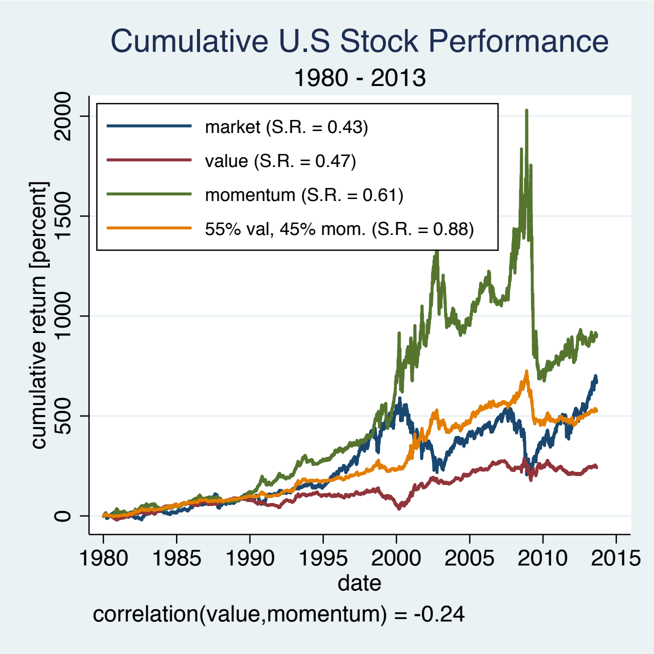
- Before class: take a quick read through the relevant textbook sections.
- During class:
  - Come to class with the lecture notes.
  - Write your own notes on the printed lecture notes.
  - Ask questions!
- After class:
  - Go through lecture notes again
  - Read the relevant textbook sections more closely
  - Work on the problem sets which contains the quiz questions - and link each question to the lecture notes and textbook.
  - Complete the weekly quiz.
  - Review the answers to the weekly quiz (video) posted on Canvas.
  - If you are still hungry for more practice questions - work on the practice questions in the textbook.
  - Ask questions! Ed Discussion or during the office hour.

We are not big fans of excessive practice question drills, so we will not be providing additional practice questions, including mock exams.

Introduction

Agenda

- We'll start to analyze investment strategies today.
- We will keep things simple to start, and then increase the complexity (and realism) as we go.
- We will introduce some statistical concepts and measures that are fundamental in finance.
- Lots of notation today!



- How do we characterize the risk and return properties of these trading strategies?
- That is what we will do in this lecture through the use of statistical measures.

What do we want to be able to do?

For most of you, the central concern will be one of **asset allocation**. This means choosing an optimal combination of risky and safe asset classes. Here are some recommended allocations (from 2001):

	Equities	**Bonds**	**Cash**
Robeco Group	50	50	0
Julius Baer PB	54	39	7
Commerz Int. CM	53	47	0
Credit Suisse PB	43	45	12
Lehman Brothers	60	35	5
Standard Life	60	40	0
Daiwa	50	45	5
Average	**53**	**43**	**4**

Measuring Performance

Returns

- We can define a **net return**, r_1, from time 0 to time 1, in several (equivalent) ways:

$$
\begin{aligned}
r_1 &= \frac{P_1 - P_0}{P_0} && (1)\\
&= \frac{\text{sell price} - \text{buy price}}{\text{buy price}} = \frac{\text{profit}}{\text{investment}} && (2)\\
&= \frac{\text{ending value}}{\text{starting value}} - 1 && (3)
\end{aligned}
$$

(assuming that there is no interim cashflow (i.e. dividends))

- We can also define a **gross return**, from time 0 to time 1

$$
R_1 = 1 + r_1 = \frac{P_1}{P_0} = \frac{\text{sell price}}{\text{buy price}} = \frac{\text{ending value}}{\text{starting value}} \qquad (4)
$$

Excess return and compounded returns

- Simple excess return

$$R^e_{t+1} = R_{t+1} - R^f_t \tag{5}$$

  - R^f_t is the risk-free asset return at time t.
  - The returns on T-bills (T-bill = Treasury bill, a short-term U.S. government bonds) or equivalent bonds are usually considered as risk-free.

- Simple k-period holding return: compounded gross return

$$\begin{aligned} R^{(k)}_{t+1} &= R_{t-k+2} \cdot \dots \cdot R_t R_{t+1} \\ &= \frac{P_{t-k+2}}{P_{t-k+1}} \cdots \frac{P_t}{P_{t-1}} \frac{P_{t+1}}{P_t} = \frac{P_{t+1}}{P_{t-k+1}} \end{aligned} \tag{6}$$

Expected returns

- **Future returns are uncertain**, e.g.
  - Suppose we know that Qantas is about to make an announcement; estimated fuel costs per trip are down by \$0.10 per passenger
  - It's good news: Does this guarantee the share price of Qantas goes up? Unfortunately, no.
- We must make decisions today, before we get to observe the return – **we need to form an expectation of return**
- We usually start by tying investments to the economy
  - What can happen to the state, s, of the economy in the future?
  - What possible return, $r(s)$, will Qantas stock earn in each state, s?
  - How likely is that possible return? Each state, s, has probability $p(s)$.
- The **expected return** is $\mathbb{E}[r] = \sum_s p(s) \times r(s)$

Expected returns: example

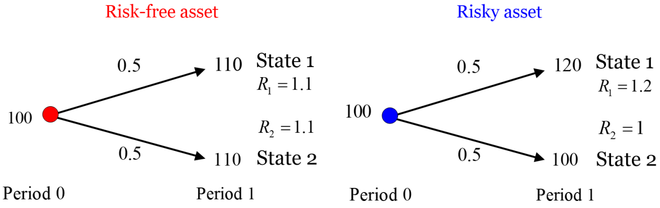
- You think the economy can be either Good or Bad next year. These are the only possible *states*, and so either $s = G$ or $s = B$.
- You make the following forecasts about Qantas stock prices in the good economy and bad economy:

Economy State s	Probability $p(s)$	Qantas Price
$s = G$ (Good)	0.30	\$1.20
$s = B$ (Bad)	0.70	\$0.80

- Suppose the current price of Qantas is \$1.17.
  - Unknown future return r could become either of the following future realised returns depending on which state s occurs:
    1. $r(G) = (1.20 - 1.17)/1.17 = 2.56\%$
    2. $r(B) = (0.80 - 1.17)/1.17 = -31.62\%$
  - Return we're expecting, based on what we know right now: $\mathbb{E}[r] = (0.3 \times 2.56\%) + (0.7 \times (-31.62\%)) = -21.37\%$

Expected Return vs Realized Return

- A simple binary tree model of assets



- Differentiate between ex post and ex ante rate of return
  - ex post return is called realized return, depends on future state
    - ★ for risk-free asset: realized return $R_1 = R_2$, no uncertainty
    - ★ for risky asset: realized return $R_1 \neq R_2$, with uncertainty
  - ex ante return is called expected return, does not depend on future state
    - ★ for risk-free asset: expected return $E(R) = 0.5R_1 + 0.5R_2 = R_1 = R_2$ realized return
    - ★ for risky asset: expected return $E(R) = 0.5R_1 + 0.5R_2 \neq$ realized return

Expected Return vs Realized Return

- The essence of risk is uncertain future outcome
- Risk-free rate $\Leftrightarrow$ no future uncertainty, it is not a random variable
  - but itself varies overtime due to changing economic conditions
- For risky assets, returns realized in the future are **random variables**.
  - it depends on future realized state, we can observe different realizations overtime
  - we can't observe expected return, so need to form expectation
    - we typically use historical average of realized return as a forecast of future expected return
    - we typically measure uncertainty - risk using volatility (standard deviation of realized return)
- Risk premium is the compensation for holding risky asset: higher risk $\Leftrightarrow$ higher expected return $\Leftrightarrow$ lower current price

$$E_t\left(R^e_{t+1}\right) = E_t\left(R_{t+1}\right) - R^f_t \tag{7}$$

- Be aware of the difference between risk premium and excess returns

Standard deviation

How do we measure our uncertainty about returns?

- Let's think about volatility (or standard deviation) of the returns.
- From statistics, the **variance** of returns is:
  $$\sigma^2 = \sum_s p(s)[r(s) - \mathbb{E}[r]]^2$$
- Often, work with the **standard deviation**:
  $$\sigma = \sqrt{\sum_s p(s)[r(s) - \mathbb{E}[r]]^2}$$

Continuing our example:

Economy State s	Probability $p(s)$	Qantas Price	Return $r(s)$
$s = G$ (Good)	0.30	\$1.20	0.0256
$s = B$ (Bad)	0.70	\$0.80	-0.3162
	$\mathbb{E}[r] = (0.3 \times 0.0256) + (0.7 \times (-0.3162)) = -0.2137$		

$$\sigma^2 = p(G)[r(G) - \mathbb{E}[r]]^2 + p(B)[r(B) - \mathbb{E}[r]]^2$$

$$\sigma^2 = 0.3[0.0256 - (-0.2137)]^2 + 0.7[-0.3162 - (-0.2137)]^2 = 0.0245$$

$$\sigma = \sqrt{0.0245} = 0.1567$$

Sharpe Ratio

(How do we measure performance taking into account the risk-return trade off?)

- **Sharpe ratio** $= \dfrac{\mathbb{E}[r] - r_f}{\sigma}$
- Expected excess return per unit of risk.
- One measure of risk-adjusted return.
- More to discuss in depth in future weeks.
- Text book: Reward to variability ratio.

Risk Aversion

Basics

A **risk averse** investor prefers less risk for the same expected return.

- Economists believe that people are typically risk averse (e.g. insurance).
- We see risk aversion through a person's *utility function*.
    - A utility function numerically describes a person's preferences.

- **We will assume that people maximise utility,** U.

Utility

Utility is a way of ranking an individual's preferences, e.g. your friend offers to buy you one of the 10 items on the McDonald's menu. You assign your favourite (fries) 10 utility points, 2nd favourite (milkshake) 9 points,..., hot tea 1 point.

- There are an infinite number of utility functions to represent each person's preferences, e.g. 20 points for your favourite, 18 points for your 2nd favourite, etc. Or 30 points for your favourite, 27 points for your 2nd favourite, etc. Utility is abstract; only the ranking matters.
- Everyone has different preferences, and different utility functions.

For this class, **as investors, we get utility from portfolio returns: we prefer returns which have higher expected values, and less risk.** One measure of risk is the standard deviation (or volatility) of returns.

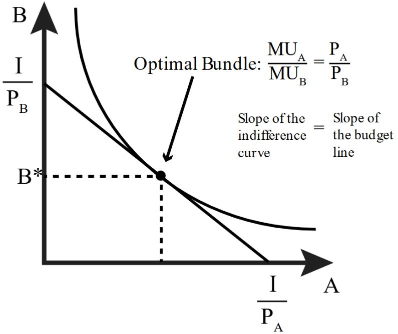
ECON 101 : Consumers' Optimal Choice of Two Goods

- How do we analyze consumers' optimal choice of two goods?
- Mathematically, we assume that consumers maximize utility subject to a budget constraint

$$\max_{C_A, C_B} U(C_A, C_B)$$

$$P_A C_A + P_B C_B = I$$

- Graphically, we use indifference curves to illustrate the optimal choices



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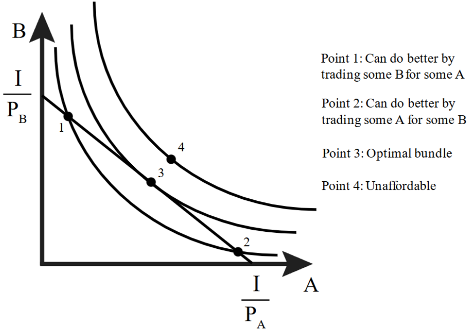


Utility

- There are an endless number of possible utility functions, we use a similar approach to analyze investors' optimal investment choices
- E.g. $U_i = \dfrac{W_i^{1-\gamma}}{1-\gamma}$ is a CRRA (Constant Relative Risk Aversion) utility function.
- We'll use a simplification for now. For an investment i:

$$U_i = \underbrace{\mathbb{E}\left[r_i\right]}_{\substack{\text{higher expected} \\ \text{returns increase} \\ \text{utility}}} - \underbrace{\frac{1}{2}\gamma\,\sigma_i^2}_{\substack{\text{higher volatility} \\ \text{decreases} \\ \text{utility}}} \tag{8}$$

- γ is a constant called the "coefficient of risk aversion".
  - Someone who does not care about risk is **risk neutral**, and $\gamma = 0$. This is uncommon but a reasonable model in some situations.
  - Someone who does not like risk is **risk averse**, and $\gamma > 0$. This is typical.

A working model for decisions, $\gamma = 2$

Look at the following set of investments i:

Investment (i)	$\mathbb{E}[r_i]$	σ_i^2	Utility U_i
A	0.05	0	
B	0.05	0.0225	
C	0.05	0	
D	0.0625	0.0264	
E	0.05	0	
F	4.05	24.5025	

1. First example: compare A to B. Which is better?
   - $U_A = 0.05, U_B = 0.0275$
2. Second example: compare C to D. Which is better?
   - $U_C = 0.05, U_D = 0.0361$
3. Third example: compare E to F. Which is better?
   - $U_E = 0.05, U_F = -20.4525$

A working model for decisions, $\gamma = \frac{1}{2}$

Would our assessment of this set of investments change?

Investment (i)	$\mathbb{E}[r_i]$	σ_i^2	Utility U_i
A	0.05	0	
B	0.05	0.0225	
C	0.05	0	
D	0.0625	0.0264	
E	0.05	0	
F	4.05	24.5025	

1. First example: compare A to B. Which is better?
   - $U_A = 0.05, U_B = 0.0444$
2. Second example: compare C to D. Which is better?
   - $U_C = 0.05, U_D = 0.0559$
3. Third example: compare E to F. Which is better?
   - $U_E = 0.05, U_F = -2.076$

Optimal Allocation

What is portfolio?

Instead of holding individual assets, often times we hold a portfolio of different assets.

A **portfolio** is any collection of investments

- A share of Qantas is a (simple) portfolio
- 12 shares of Qantas, 15 shares of Microsoft, and 8 Government bonds is a portfolio
- A house, an education, 11 shares of Qantas, a lottery ticket, and some US currency is also a portfolio

Some formulas to keep in mind:

- $Var[X+Y] = Var[X] + Var[Y] + 2Cov[X,Y]$, $Var[cX] = c^2 Var[X]$, and $Cov[cX, dY] = cdCov[X,Y]$.
- Risk-free rate is not a random variable, therefore has 0 correlation with any risky asset returns

One risky, one safe asset case

- We compared individual assets, but we could choose some combination.
- Consider two assets, R_1 (risky with σ_1) and $R_2 = R_f$(risk-free with $\sigma_2 = 0$). We have

$$\begin{aligned} \bar{R}_p - R_f &= w_1 \bar{R}_1 + (1 - w_1) R_f - R_f = w_1 \left(\bar{R}_1 - R_f \right), \\ \sigma_p^2 &= w_1^2 \sigma_1^2 \Longrightarrow \sigma_p = w_1 \sigma_1. \end{aligned} \tag{9}$$

- With $w_1 = \dfrac{\sigma_p}{\sigma_1}$ we have

$$\bar{R}_p - R_f = w_1 \left(\bar{R}_1 - R_f \right) = \frac{\sigma_p}{\sigma_1} \left(\bar{R}_1 - R_f \right)$$

- Rearrange terms to obtain

$$\bar{R}_p = \sigma_p \frac{\bar{R}_1 - R_f}{\sigma_1} + R_f$$

One risky, one safe asset case

- This defines a straight line, called the capital allocation line (CAL), on a mean-standard deviation $\left(\bar{R}_p, \sigma_p\right)$ diagram.
  - $\bar{R}_p$ is linear in $\sigma_p \Rightarrow$ a straight line connecting asset 1 and 2

$$\underbrace{\bar{R}_p}_{y} = \underbrace{R_f}_{a} + \underbrace{\frac{\bar{R}_1 - R_f}{\sigma_1}}_{b} \underbrace{\sigma_p}_{x}$$

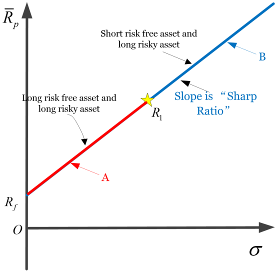
  - If $\sigma_p = \sigma_2 = 0$, $\bar{R}_p = \bar{R}_2 = R_f$, $w_1 = \dfrac{\sigma_p}{\sigma_1} = 0$.
  - If $\sigma_p = \sigma_1$, $\bar{R}_p = \bar{R}_1$, $w_1 = \dfrac{\sigma_p}{\sigma_1} = 1$.
- The slope is called the Sharpe ratio of the risky asset:

$$S_1 = \frac{\bar{R}_1 - R_f}{\sigma_1} \tag{10}$$

- Any portfolio that combines a single risky asset with the risk-free asset has the same Sharpe ratio as the risky asset itself.

Capital allocation line

- If we plot all the combination of $\bar{R}_p, \sigma$ in graph, it will be



- For any portfolio point A on the red part of the line
  - $\bar{R}_A \leq \bar{R}_1$, $\sigma_A = w_1\sigma_1 \leq \sigma_1$: positive $0 < w_1 < 1$ holding of both risk-free asset and risky asset R_1

    $\Rightarrow$ long position in both risk-free asset and risky asset
- For any portfolio point B on the blue part of the line
  - $\bar{R}_B > \bar{R}_1$, $\sigma_B = w_1\sigma_1 > \sigma_1$: negative $(1 - w_1 < 0)$ (positive $w_1 > 1$) holding of risk-free asset (risky asset)

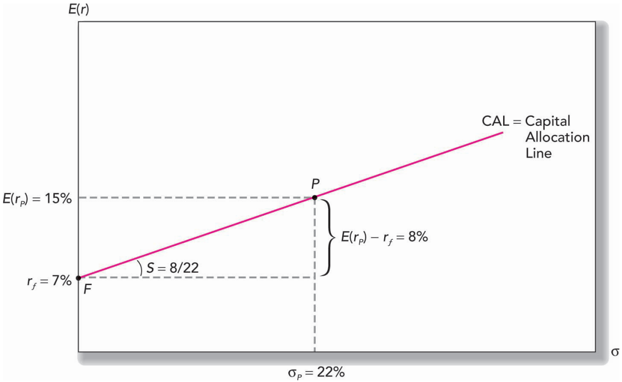
    $\Rightarrow$ short position in risk-free asset, long position in risky asset

Short selling

- Short selling (or shorting or going short) is selling something you don't own.
- Corresponds to negative portfolio weight.
- Buying a security is a way of betting its price will go up. Selling and short selling are ways of betting the price go down.
- Mechanics of trade:
    1. Borrow the security from counterparty X.
    2. Sell the security to counterparty Y at the current market price.
    3. Hope the price falls.
    4. Eventually buy back the security, and return it to the owner X.
- The potential losses of short selling are unlimited.

A concrete example:

Investment (i)	$\mathbb{E}[r_i]$	σ_i^2	Utility U_i
Risk-free Bond	0.07	0	
Stock	0.15	0.0484	
A portfolio			
50% Stock, 50% Bond	0.11	0.0121	



Optimal allocation

If we can choose any combination, which one is best?
$\longrightarrow$ The one that maximises utility.

So let's imagine **investing fraction** w **in the risky stock, and** $1 - w$ **in the riskless bond.**

- Utility is $U_i = \mathbb{E}[r_i] - \frac{1}{2}\gamma\,\sigma_i^2$.
- $\mathbb{E}[r] = w\,\mathbb{E}[r_{\text{stock}}] + (1-w)\,r_f$
- $\sigma^2 = Var\left[w \cdot r_{\text{stock}} + (1-w)r_f\right] = w^2\sigma^2_{stock}$
- Thus, $U_i = r_f + [\mathbb{E}[r_{\text{stock}}] - r_f]w - \frac{1}{2}\gamma w^2 \sigma^2_{\text{stock}}$
- What is w^*, the w that maximises U_i? Set first derivative equal to zero:

$$U' = [\mathbb{E}[r_{\text{stock}}] - r_f] - \gamma\,\sigma^2_{\text{stock}}\,w$$

$$U' = 0 \implies w^* = \frac{\mathbb{E}[r_{\text{stock}}] - r_f}{\gamma\,\sigma^2_{\text{stock}}}$$

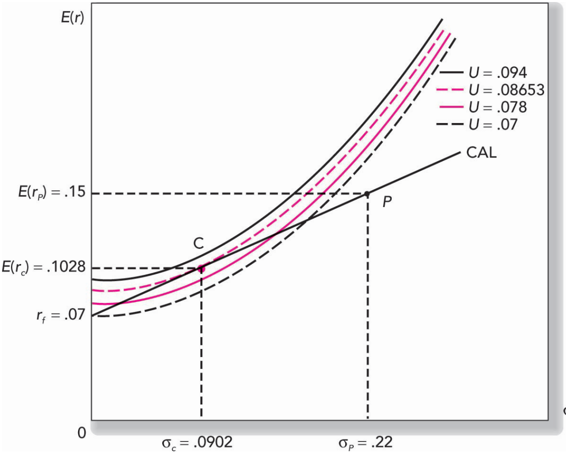
Suppose our stock and bond are as before, but with a **greater risk aversion** $\gamma = 4$.

$$U = r_f + \left[\mathbb{E}\left[r_{\text{stock}}\right] - r_f\right] w - \frac{1}{2}\gamma\sigma^2_{\text{stock}} w^2$$

$$w^* = \frac{\mathbb{E}\left[r_{\text{stock}}\right] - r_f}{\gamma\sigma^2_{stock}} = \frac{[0.15 - 0.07]}{4 * (0.22)^2} = 0.41$$

$$\mathbb{E}[r] = (0.41 \times 0.15) + ((1 - 0.41) \times 0.07) = 0.103$$

$$\sigma = 0.41 \times 0.22 = 0.09$$



Suppose our stock and bond are as before, but with a **greater risk aversion** $\gamma = 4$.

Indifference curve:

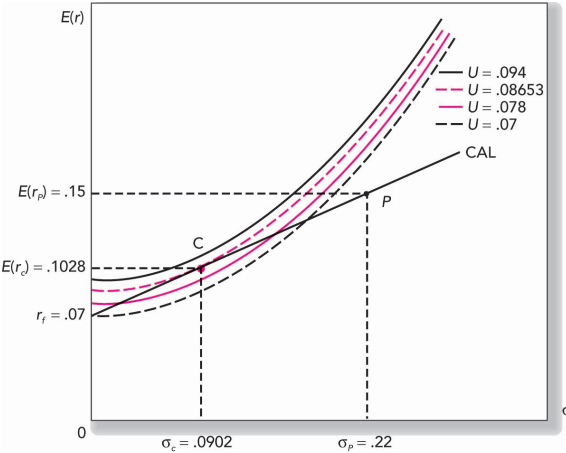
$$\bar{U} = E(r_p) - \frac{1}{2}\gamma\sigma_p^2$$

To get the same $\bar{U}$, if $E(r_p)$ increases, σ_p also needs to increase.

Fixing $E(r_p)$, larger σ_p decreases utility, curve moves downwards

Fixing σ_p, larger $E(r_p)$ increases utility, curve shifts upwards

If γ changes, how will the curve change?



Modelling Returns: Normality

Example: stock market returns

(What if there are infinite number of state, and return follows a distribution...)

- The stock market has an average annual return of 7.41% and a return standard deviation of 18.90%.
- **Assuming that the stock market return is normally distributed**, we want to answer the following questions:
  1. What is the probability that the stock market return will be positive?
  2. What is the 95% confidence interval for the stock market return?

To answer these, let's have a refresher on the Normal distribution...

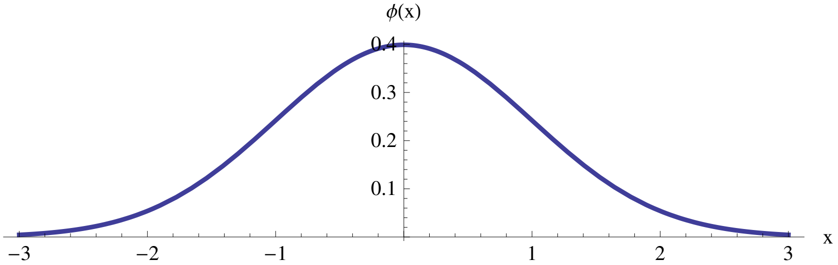
Normal distribution

Modeling r_t as a normal distribution with mean μ and volatility σ has several benefits:

1. Very tractable: symmetric, stable, & only two parameters (μ and σ).
2. Captures asset return dynamics well in most applications.
3. With returns assumed to be normally distributed, we can compute probabilities, confidence intervals, and perform significance tests.

Normal distribution

The **probability density function (PDF)** is the probability of the random variable being different outcomes:



$$f(r_t) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} \exp\left(-\frac{(r_t-\mu)^2}{2\sigma^2}\right).$$

Normal distribution

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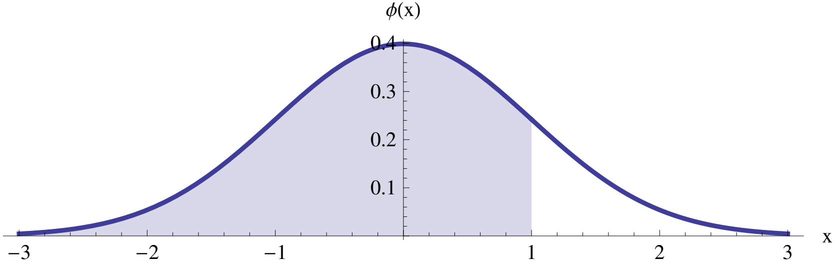
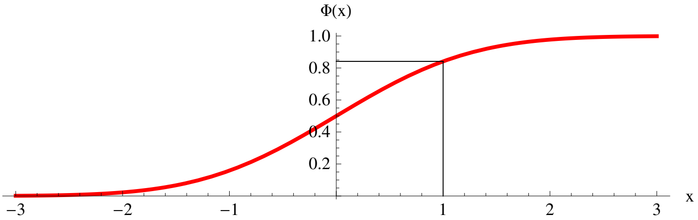


$$f(r_t) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} \exp\left(-\frac{(r_t-\mu)^2}{2\sigma^2}\right).$$

Normal distribution

The **cumulative distribution function (CDF)** is the probability of the random variable being less than or equal to some number c:



$$Pr(r_t \leq c) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} \int_{-\infty}^{c} \exp\left(-\frac{(r_t - \mu)^2}{2\sigma^2}\right) \mathrm{d}r_t.$$

How do we compute $Pr(r_t \leq c)$ in Excel? $\longrightarrow$ NORMDIST(c, μ, σ, 1).

Example: stock market returns

- The stock market has an average annual return of 7.41% and a return standard deviation of 18.90%.
- Assuming that the stock market return is normally distributed:

1. **Q: What is the probability that the stock market return will be positive?**
   A: Excel: = 1 - NORMDIST(0, 0.0741, 0.1890, 1)
   = 65%

2. **Q: What is the 95% confidence interval for the stock market return?**
   A: Excel: = 0.0741 ± 0.189*NORMSINV(0.975)
   C.I. lower bound: -29.63%, C.I. upper bound: 44.45%

Value at Risk (VaR)

- The stock market return follows normal distribution with an average return of 7.41% and a return standard deviation of 18.90%.
- **Q: What is the probability that the stock market return will be less than 7.41%?**
  A: 50%
- **X% VaR** is the value (point) below which X% of returns lie in the distribution.
- 50% VaR is the value (point) below which 50% of returns lie in the distribution.
  7.41%
- Risk management objective: e.g. 1% VaR.

Deviations From Normality

Are returns normally distributed?

Fama/French Forum

Fama and French answer topical and timeless questions.

Hosted by Dimensional

JAN 30, 2012

Q & A

Q&A: Are Stock Returns Normally Distributed?

What is the best way to describe the distribution of stock returns—a normal distribution, lognormal, or something else? What should investors do with this information?

EFF/KRF: Distributions of daily and monthly stock returns are rather symmetric about their means, but the tails are fatter (i.e., there are more outliers) than would be expected with normal distributions. (This topic takes up half of Gene's 1964 PhD thesis.) In the old literature on this issue, the popular alternatives to the normal distributions were non-normal symmetric stable distributions (which are fat-tailed relative to the normal) and t-distributions with low degrees of freedom (which are also fat-tailed). The message for investors is: expect extreme returns, negative as well as positive.

FAMA AND FRENCH ON OTHER TOPICS

ABOUT FAMA AND FRENCH

Eugene F. Fama
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Kenneth R. French
The Carl E. and Catherine M. Heidt Professor of Finance at the Tuck School of Business at Dartmouth College

SECTIONS

Questions & Answers

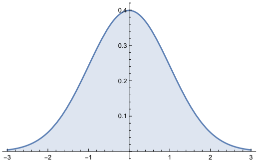
fat tails: the probability of extreme outcomes is higher than implied by the normal distribution.

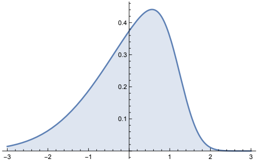
Skewness measures the asymmetry of a distribution.

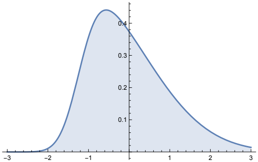
- Negative skewness: the data are skewed to the left. Downside risk is underestimated by standard deviation.
- Positive skewness: the data are skewed to the right. Downside risk is overestimated by standard deviation.
- Skewness is related to the third power of a random variable:

$$\text{skewness} \equiv \frac{\mathbb{E}\left[(r - \mathbb{E}[r])^3\right]}{\sigma^3}.$$

- Skewness for a normal distribution is zero.







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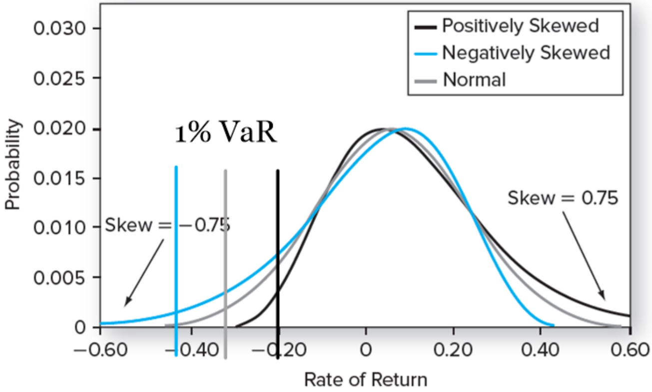
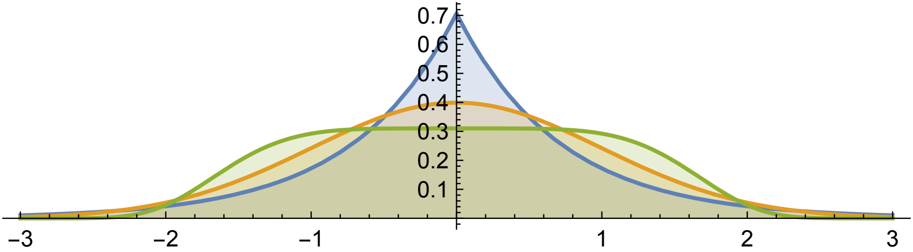


Kurtosis

Kurtosis is a measure of the tails (and pointiness) of a distribution.

- Positive kurtosis: the distribution is peaked (pointier) and more data are in the tails of the distribution relative to a normal distribution.
- Negative kurtosis: the distribution is flatter and fewer data are in the tails of the distribution relative to a normal distribution.
- Excess Kurtosis, or Kurt, is related to the fourth power of random variable, and is zero for a normal distribution:

$$\text{Kurt} = \frac{\mathbb{E}\left[(r - \mathbb{E}(r))^4\right]}{\sigma^4} - 3$$



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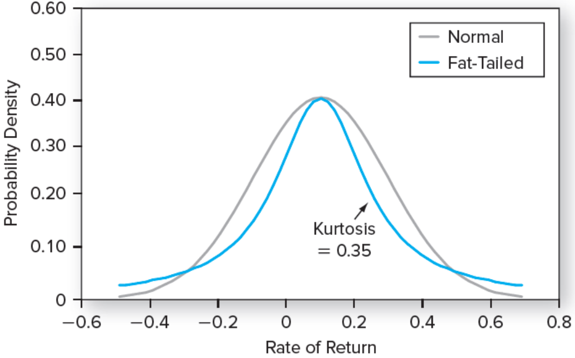

Additional issues with normal assumption

- Is the normal distribution a reasonable assumption?

Certainly not!

- Unrealistic values: Financial assets usually exhibit limited liability, so the smallest possible net return is -100%, or smallest gross return is 0.
    - ★ But the normal distribution's support is $(-\infty, \infty)$.
    - ★ Hence, realizations of net returns below -1 (or gross return below 0) will always have a positive probability.
- Hard to deal with compounded returns: the product of normal distributed random variables (compounded returns) is not normal distributed.
    - ★ If R_t and R_{t+1} are both normal distributed gross returns, compounded return $R_t R_{t+1}$ is not normal distributed.

Conclusion

Summary

1. Returns are uncertain. We can think of the trade-off between risk and reward through volatility and expected return.
2. Risk aversion matters for the level of utility.
3. Compute optimal allocations using mean, volatility, and risk aversion.
4. Modelling returns with normal distribution allows us to compute probabilities and confidence intervals of returns using just mean and volatility.
5. Deviations from normality happen. Care must be taken when returns exhibit either extreme skewness or extreme kurtosis.