ECOM90024
Forecasting in Economics and Business
Tutorial 9

1.) Let ε_t be a sequence of innovations that behaves according to a GARCH(2,1) process,

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2$$

$$\upsilon_t \sim i.i.d.\ N(0,1)$$

$$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \beta_2 \geq 0$$

a.) Show that the GARCH(2,1) model can be rewritten as an ARMA(2,2) process for the squared innovations ε_t^2.

b.) Derive the unconditional variance of ε_t and explain why it is different to the conditional variance.

c.) Explain why the GARCH parameters are restricted to the values,

$$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \beta_2 \geq 0$$

Are these the only restrictions that must be imposed on these parameters?

d.) Given the information set $\Omega_t = \{\varepsilon_t, \varepsilon_{t-1}, \dots\}$, derive expressions for the 1-step and 2-step ahead forecasts of the conditional variance in terms of the conditioning variables.

2.) Let R_t represent the return on a financial asset from period $t-1$ to t and suppose that it is governed by the following GARCH specification for $t = 1,2, \dots, T$.

$$R_t = \mu + \beta R_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (\varepsilon_{t-1} - \theta \sigma_{t-1})^2 + \beta_1 \sigma_{t-1}^2$$

$$\upsilon_t \sim_{i.i.d} N(0,1)$$

a.) Given the above specification, derive the unconditional variance of ε_t and the set of conditions on the parameters $\alpha_0, \alpha_1, \theta, \beta_1$ that guarantee the non-negativity and finiteness of the conditional and unconditional variance. You may assume that the process ε_t is covariance stationary.

b.) Explain how the leverage effect is captured by the above specification. Why would this specification be useful in the analysis of financial returns?

3.) The file “tsla.csv” contains observations of the daily closing price of Tesla stock from 16/05/2016 to 16/05/2019. Using ***R*** you are required to do the following:

a.) Generate the daily returns on Tesla’s stock as the log difference of the daily price.

b.) Verify using the sample ACF and PACF, as well as appropriately specified Box tests that the daily returns are serially uncorrelated.

c.) Estimate a GARCH(1,1) model for the returns. Verify that the specification is adequate by showing that the squared standardized residuals from the GARCH estimation are serially uncorrelated.

d.) The values of the conditional volatility $\hat{\sigma}_t$ can be computed by applying the predict() function to the object in which you have stored your GARCH output (Note: the predict function will produce two columns of output, make sure to only use the first column!). Using the parameter estimates that you obtained in part c, compute the h-step ahead forecast of the conditional variance for $h = 1,2, \dots ,10$. (Hint: you can use a loop to compute these values.)