ECOM40006/ECOM90013 Econometrics 3
Department of Economics
University of Melbourne

Assignment 2

Semester 1, 2025

1. Let $Y_1, Y_2, \ldots, Y_n$ denote a simple random sample of size n from a Normal population with mean μ and variance 1. Consider the first observation Y_1 as an estimator for μ.

   (a) Show that Y_1 is an unbiased estimator for μ. (1 mark)

   (b) Find $\Pr(|Y_1 - \mu| \leq 1)$. (2 marks)

   (c) Based on your answer to 1(b), is Y_1 a consistent estimator for μ? Explain your answer both theoretically and intuitively. (2 marks)

2. The Constant Elasticity of Substitution (CES) production function is of the form

$$Q = A\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho},$$

   where K and L are the factor inputs, capital and labour say, and the parameters of the function are $A > 0$, $0 < \delta < 1$, and $-1 < \rho \neq 0$. In this model the elasticity of substitution can be shown to be $\varepsilon = 1/(1-\rho)$. Suppose that you have an estimator for the parameters of the CES production function with joint limiting distribution of the form

$$\sqrt{n}\left(\begin{bmatrix}\hat{A}\\ \hat{\delta}\\ \hat{\rho}\end{bmatrix} - \begin{bmatrix}A\\ \delta\\ \rho\end{bmatrix}\right) \xrightarrow{d} \mathrm{N}(0, \Sigma), \qquad \Sigma = \begin{bmatrix}\sigma_A^2 & \sigma_{A\delta} & \sigma_{A\rho}\\ \sigma_{A\delta} & \sigma_\delta^2 & \sigma_{\delta\rho}\\ \sigma_{A\rho} & \sigma_{\delta\rho} & \sigma_\rho^2\end{bmatrix}.$$

   (a) What is the marginal limiting distribution of $\hat{\rho}$? (1 mark)

   (b) If $\hat{\Sigma}$ denotes a consistent estimator for Σ, derive an operational 95% confidence interval for ε, where

$$\hat{\Sigma} = \begin{bmatrix}\hat{\sigma}_A^2 & \hat{\sigma}_{A\delta} & \hat{\sigma}_{A\rho}\\ \hat{\sigma}_{A\delta} & \hat{\sigma}_\delta^2 & \hat{\sigma}_{\delta\rho}\\ \hat{\sigma}_{A\rho} & \hat{\sigma}_{\delta\rho} & \hat{\sigma}_\rho^2\end{bmatrix}.$$ (4 marks)

   By ‘operational’ is meant that your answer cannot depend upon any unknown parameters. Be sure to include all steps of your derivation. (4 marks)

Your answers to the Assignment should be submitted via the LMS no later than 4:30pm, Thursday 17 April.

No late assignments will be accepted and an incomplete exercise is better than nothing.

Your mark for this assignment may contribute up to 10% towards your final mark in the subject.