# Quantitative Analysis of Finance I

## ECON90033

**WEEK 9**

***VECTOR AUTOREGRESSION***

***GRANGER (PREDICTIVE) CAUSALITY***

Reference:
HMPY: § 4.5-4.6

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# VECTOR AUTOREGRESSION (*VAR*)

- Single-equation univariate time-series methods based on traditional smoothing methods and the Box-Jenkins methodology, are often sufficient to capture the dynamic path of some variable of interest.

  There are, however, situations when these techniques are insufficient, and we need to study the relationships between several variables using single equation structural multivariate models.

  For example, intervention analysis serves to trace the effect of a deterministic independent variable (intervention) on a stationary dependent variable by formally testing whether there is a significant change in the level or trend of a stationary variable at some known break date.

  Transfer function models represent a natural extension of this analysis. In this case the purely deterministic intervention function is replaced with a lag polynomial (transfer function) which shows how some change in an independent variable is transferred to the dependent variable.

- Although we do not have time for single-equation structural multivariate models, it is important to mention that in general they have at least two potential shortcomings.

  i. Single-equation multivariate models are based on the implicit assumption of some one-way relationship running from the set of independent variables to the dependent variable.

     $\longrightarrow$ These models do not allow for any feedback from the dependent variable to the independent variables, and also exclude correlation between the independent variables and the error term at all leads and lags.

     Improper model specifications or/and correlation between the independent variable and the error term, however, can make the estimators biased.

  ii. Single-equation multivariate econometric models are not fully compatible with reality where, at least in principle, everything depends on everything, either directly or indirectly.

  Simultaneous equation models (see the handout of the same title on the subject website) can provide some remedy to these shortcomings, but they might also create new ones.

- Christopher Sims proposed an alternative modelling and estimation strategy based on vector autoregressions (*VAR*).

  Although Sims advocated unrestricted reduced form *VAR* systems, it is useful to start with the

  Structural vector autoregressive (*SVAR*) system:

  A set of autoregressive structural equations.

  ⟵ The time path of each left-hand side (endogenous) variable is determined by its own history and by current and past realizations of other left-hand side variables.

  For example, in a first-order bivariate *SVAR* system of *Y* and *Z*

$$\boxed{\begin{aligned} y_t &= b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt} \\ z_t &= b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt} \end{aligned}}$$

  each variable might have some contemporaneous and one-period delayed effect on the other variable.

  Namely, $-b_{12}$ and $-b_{21}$ measure the contemporaneous effects, i.e., the impact of a unit increase of $z_t$ on $y_t$, and that of $y_t$ on $z_t$, respectively; while $\gamma_{12}$ and $\gamma_{21}$ measure the delayed effects, i.e., the impact of a unit increase of $z_{t-1}$ on $y_t$, and that of $y_{t-1}$ on $z_t$, respectively.

Being a structural system, *SVAR* is subject to the usual identification problem of structural simultaneous-equation models and, if possible, it must be recovered from the corresponding reduced-form system.

Returning to the *SVAR*(1) system, it can be shown that if $b_{12}\, b_{21} \neq 1$, it can be re-written as

$$\boxed{\begin{aligned} y_t &= a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + u_{1t} \\ z_t &= a_{20} + a_{21} y_{t-1} + a_{22} z_{t-1} + u_{2t} \end{aligned}}$$

which is a multivariate generalization of *AR* models,

a first-order bivariate *VAR* in standard or reduced form, *VAR*(1) in brief.

- There are two major differences between this standard *VAR* model and the *SVAR* model (sometimes referred to as the primitive system):

  i. The $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ error terms in *SVAR* are uncorrelated with each other, but the $\{u_{1t}\}$ and $\{u_{2t}\}$ error terms in *VAR* might be correlated contemporaneously.

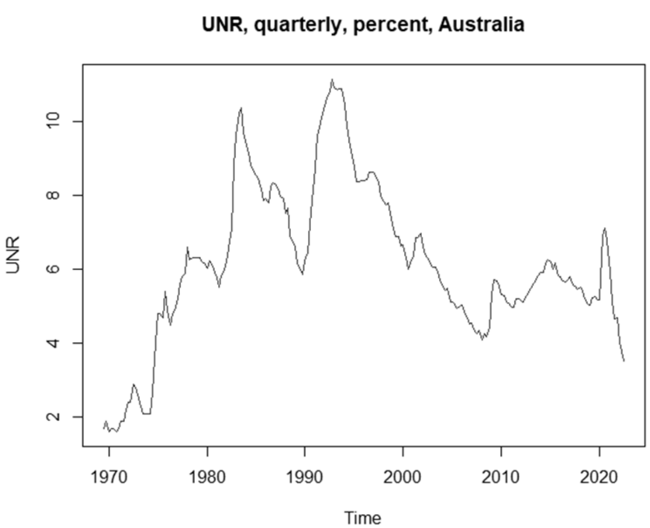
  ii. A *VAR* model can be estimated equation-by-equation by OLS because each equation has the same predetermined variables on the right side.

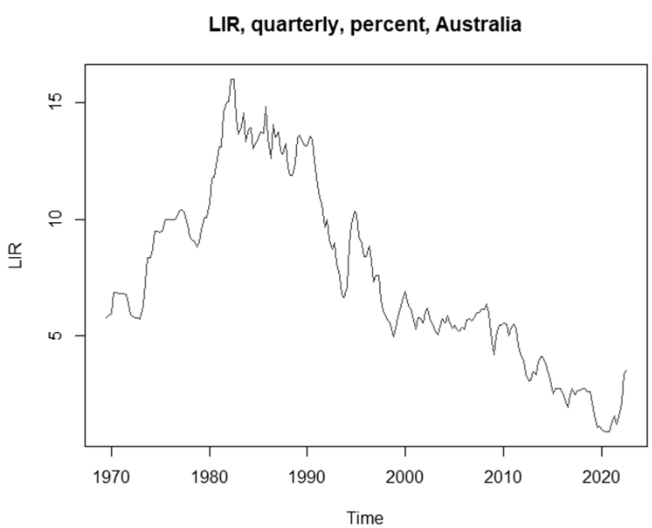
A *SVAR* model, however, cannot be estimated directly due to the contemporaneous relationship among the endogenous variables. Instead, it must be recovered from the corresponding standard *VAR* model, if possible.

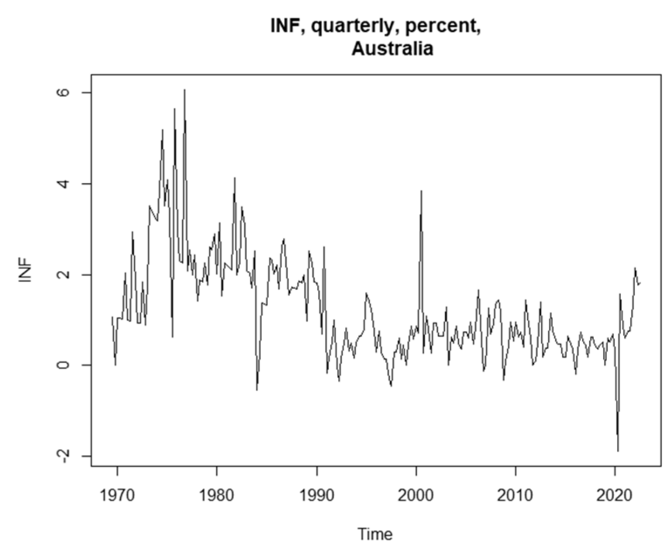
Ex 1:

Consider the quarterly unemployment rate (*UNR*, % of labour force), the long-term interest rate (*LIR*, %), and the consumer price index (*CPI*, %, all groups) in Australia from 1969 Q2 to 2022 Q3 (downloaded from OECD 05/04/2023 and from ABS, 25/03/2023). From *CPI* the rate of inflation is calculated as

$$INF_t = \Delta CPI_t / CPI_{t\text{-}1} \times 100\%.$$



Note: If a *VAR* model has some non-stationary variables, OLS estimation *might* produce spurious, i.e., non-interpretable, regression results and the usual *t*- and *F*-tests *might* be invalid. For this reason, it is usually better to make sure that all variables are stationary (or, cointegrated), especially when one intends to perform inference or study the underlying *SVAR* system.

a) Perform the *ADF* and *KPSS* tests on the level and first difference of each series.

In the tests on *UNR* and *LIR* we use both a drift (intercept) and a trend for the level series, but only a drift for the (first-) differenced series, while in the case of *INF*, we use only a drift for the level and none for the differenced series in the *ADF* tests and a drift for the level and the differenced series alike in the *KPSS* tests.

Without showing the details, the conclusions drawn from these tests at the 10% significance level are summarised in this table:

| | ***UNR*** | | ***LIR*** | | ***INF*** | |
|---|---|---|---|---|---|---|
| | *ADF* | *KPSS* | *ADF* | *KPSS* | *ADF* | *KPSS* |
| *Level* | 1 | 1 | 1 | 1 | 0 | 1 |
| *First diff.* | 0 | 0 | 0 | 0 | 0 | 0 |
| | $I(1)$ | $I(1)$ | $I(1)$ | $I(1)$ | $I(0)$ | $I(1)$ |

The *ADF* and *KPSS* tests alike suggest that *UNR* and *LIR* are *I*(1) variables, but they contradict each other in the case of *INF*. However, based on other unit root tests that are not discussed in this course, there is reason to believe that *INF* is stationary.

Given these results, we shall estimate a *VAR* model of the first differences of *UNR* and *LIR*, but the level of *INF*.

- In a *VAR* analysis it is crucial to determine the proper common lag length, because if it is too small or large, the model is incorrectly specified, and the results might be misleading.

  Unfortunately, if the variables in a *VAR* are really interrelated, the common lag length cannot be determined by studying the univariate properties of the data series, and hypothesis testing on individual equations is also insufficient.

  Instead, we can experiment with different lag lengths and choose the one that produces the smallest *BIC* (or some other information criterion), ensuring that the residuals are uncorrelated.

(Ex 1 cont.)

b) Determine the optimal lag length of a *VAR* model of $\Delta UNR$, $\Delta LIR$ and *INF* augmented with a constant and trend with the *VARselect()* function of the *vars* package.

*DUNR = window(diff(UNR), start = c(1969,4), end = c(2022,3))*
*DLIR = window(diff(LIR), start = c(1969,4), end = c(2022,3))*
*INF = window(INF, start = c(1969,4), end = c(2022,3))*
*data = ts(data.frame(DUNR, DLIR, INF),*
*start = c(1969,4), end = c(2022,3)), frequency = 4,*
*library(vars)*
*VARselect(data, type = “both")*

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     4      2      1      4

$criteria
                 1           2           3           4           5
AIC(n) -4.14001927 -4.20352106 -4.22467190 -4.27793990 -4.26758440
HQ(n)  -4.04062340 -4.04448767 -4.00600098 -3.99963146 -3.92963843
SC(n)  -3.89435583 -3.81045955 -3.68421233 -3.59008226 -3.43232870
FPE(n)  0.01592303  0.01494473  0.01463487  0.01388047  0.01403239
                 6           7           8           9          10
AIC(n) -4.23496116 -4.20379836 -4.19237825 -4.16145352 -4.12840310
HQ(n)  -3.83737766 -3.74657735 -3.67551971 -3.58495746 -3.49226951
SC(n)  -3.25230738 -3.07374653 -2.91492835 -2.73660555 -2.55615706
FPE(n)  0.01450868  0.01498324  0.01517554  0.01567856  0.01623915
```

Four information criteria are reported on this printout: *AIC* (Akaike), *HQ* (Hannan-Quinn), *SC* (Schwartz, *SIC* or *BIC*), and *FPE* (*Akaike's Final Prediction Error*).

*SC* takes its smallest value for a single lag, *HQ* selects 2 lags, while *AIC* and *FPE* select 4 lags.

c) Estimate the most parsimonious *VAR*(1) model, as selected by *SC*, and check whether the residuals are serially uncorrelated. If not, increase the length gradually till the residuals become serially uncorrelated.

We can test the *VAR* residuals for serial correlation with the *serial.test()* function. It offers four multivariate generalisations of the portmanteau tests we used earlier (see slide #14, week 3). To keep it simple, we shall always use the Breusch-Godfrey *LM* test for the null hypothesis of no residual serial correlation up to a given lag length.

For example, for $H_0$: no autocorrelation of orders 1-4,

*var1 = VAR(data, p = 1, type = “both")*
*serial.test(vard1, lags.bg = 4, type = "BG")*

```
        Breusch-Godfrey LM test

data:  Residuals of VAR object var1
Chi-squared = 79.433, df = 36, p-value = 4.102e-05
```

$H_0$ is rejected, so a single lag is insufficient.

2 and 3 lags are also insufficient.

*var4 = VAR(data, p = 4, type = “both")*
*serial.test(vard1, lags.bg = 4, type = "BG")*

```
        Breusch-Godfrey LM test

data:  Residuals of VAR object var4
Chi-squared = 42.438, df = 36, p-value = 0.2132
```

$H_0$ is maintained at the 5% level, so the preferred model is *VAR*(4).

The details of the estimated *VAR*(4) are displayed by

*summary(var4)*

The printout has three parts. Let's consider the first two.

```
VAR Estimation Results:
=========================
Endogenous variables: DUNR, DLIR, INF
Deterministic variables: both
Sample size: 208
Log Likelihood: -393.534
Roots of the characteristic polynomial:
0.7646 0.6939 0.6939 0.6746 0.6746 0.6587 0.6587 0.6499 0.6499 0.6452 0.4869 0.4869
```

The first part shows, among others, the lengths of the estimated characteristic roots. They  are all smaller than one (i.e., well inside the unit circle), indicating that this *VAR* is stable.

The second part of the *VAR* printout shows the three OLS regressions. They are displayed on the next two slides.

As you can see on those slides, each regression is significant, but most of the coefficients are insignificant individually. This is not unusual; *VAR* models tend to be overparameterized.

Do not worry about interpreting the coefficients, not even the significant ones, because in standard *VAR*s the individual equations have hardly any economic content.

```
Estimation results for equation DUNR:
=====================================
DUNR = DUNR.l1 + DLIR.l1 + INF.l1 + DUNR.l2 + DLIR.l2 + INF.l2
+ DUNR.l4 + DLIR.l4 + INF.l4 + const + trend

          Estimate Std. Error t value Pr(>|t|)
DUNR.l1  0.3614241  0.0723371   4.996  1.3e-06 ***
DLIR.l1 -0.1024902  0.0441571  -2.321   0.0213 *
INF.l1   0.0060234  0.0290317   0.207   0.8359
DUNR.l2  0.0751548  0.0780543   0.963   0.3368
DLIR.l2 -0.0150584  0.0462580  -0.326   0.7451
INF.l2   0.0513597  0.0281547   1.824   0.0697 .
DUNR.l3 -0.0252969  0.0780371  -0.324   0.7462
DLIR.l3 -0.0200965  0.0461055  -0.436   0.6634
INF.l3   0.0409572  0.0282696   1.449   0.1490
DUNR.l4 -0.0478108  0.0722235  -0.662   0.5088
DLIR.l4  0.0475578  0.0459863   1.034   0.3023
INF.l4   0.0022962  0.0275215   0.083   0.9336
const   -0.1722896  0.0988612  -1.743   0.0830 .
trend    0.0004723  0.0005259   0.898   0.3702
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘


Residual standard error: 0.3021 on 194 degrees of freedom
Multiple R-Squared: 0.3024,     Adjusted R-squared: 0.2557
F-statistic: 6.469 on 13 and 194 DF,  p-value: 3.785e-10
```

```
Estimation results for equation DLIR:
=====================================
DLIR = DUNR.l1 + DLIR.l1 + INF.l1 + DUNR.l2 + DLIR.l2 + INF
+ DUNR.l4 + DLIR.l4 + INF.l4 + const + trend

          Estimate Std. Error t value Pr(>|t|)
DUNR.l1 -3.522e-01  1.166e-01  -3.020  0.00287 **
DLIR.l1  1.991e-01  7.120e-02   2.796  0.00569 **
INF.l1   4.612e-02  4.681e-02   0.985  0.32578
DUNR.l2  9.589e-02  1.259e-01   0.762  0.44707
DLIR.l2 -1.663e-01  7.459e-02  -2.230  0.02692 *
INF.l2   2.706e-02  4.540e-02   0.596  0.55187
DUNR.l3 -8.144e-04  1.258e-01  -0.006  0.99484
DLIR.l3  1.121e-01  7.434e-02   1.508  0.13310
INF.l3   5.864e-02  4.558e-02   1.286  0.19984
DUNR.l4 -2.409e-01  1.165e-01  -2.068  0.03995 *
DLIR.l4 -1.804e-01  7.415e-02  -2.433  0.01586 *
INF.l4  -2.426e-02  4.438e-02  -0.547  0.58515
const   -1.488e-01  1.594e-01  -0.933  0.35182
trend    2.872e-05  8.479e-04   0.034  0.97302
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘


Residual standard error: 0.4871 on 194 degrees of freedom
Multiple R-Squared: 0.1662,     Adjusted R-squared: 0.1104
F-statistic: 2.975 on 13 and 194 DF,  p-value: 0.0005432
```

```
Estimation results for equation INF:
=====================================
INF = DUNR.l1 + DLIR.l1 + INF.l1 + DUNR.l2 + DLIR.l2 + INF.
+ DUNR.l4 + DLIR.l4 + INF.l4 + const + trend

         Estimate Std. Error t value Pr(>|t|)
DUNR.l1  0.245066   0.174259   1.406 0.161224
DLIR.l1  0.433296   0.106374   4.073 6.75e-05 ***
INF.l1   0.090467   0.069937   1.294 0.197357
DUNR.l2 -0.145827   0.188032  -0.776 0.438961
DLIR.l2  0.138963   0.111435   1.247 0.213889
INF.l2   0.118765   0.067824   1.751 0.081514 .
DUNR.l3 -0.431367   0.187990  -2.295 0.022825 *
DLIR.l3  0.172930   0.111067   1.557 0.121105
INF.l3   0.144958   0.068101   2.129 0.034551 *
DUNR.l4  0.131549   0.173985   0.756 0.450512
DLIR.l4  0.071096   0.110780   0.642 0.521777
INF.l4   0.244798   0.066299   3.692 0.000289 ***
const    0.963553   0.238155   4.046 7.52e-05 ***
trend   -0.004021   0.001267  -3.174 0.001749 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘


Residual standard error: 0.7278 on 194 degrees of freedom
Multiple R-Squared: 0.6166,     Adjusted R-squared: 0.5909
F-statistic:    24 on 13 and 194 DF,  p-value: < 2.2e-16
```

d) Use the estimated *VAR*(4) model to forecast *DUNR*, *DLIR* and *INF* 1- 5 quarters ahead.

Like *AR* forecasts, *VAR* forecasts can be developed with the (multivariate) recursive substitution method. We do the calculations only with *R.*

We can use the already familiar *forecast()* function from the *forecast* library.

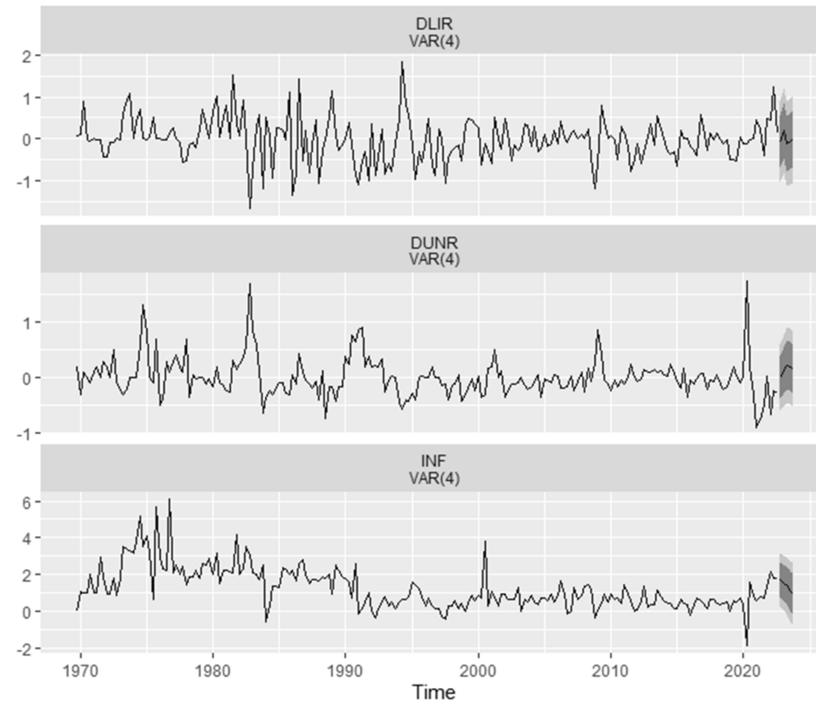
*library(forecast)*
*var4_ea = forecast(var4, h = 5, plot = TRUE)*
*print(var4_ea)*

```
DUNR
        Point Forecast       Lo 80     Hi 80      Lo 95     Hi 95
2022 Q4    0.004304234 -0.3828676 0.3914760 -0.5878239 0.5964324
2023 Q1    0.130567425 -0.2884463 0.5495811 -0.5102587 0.7713936
2023 Q2    0.208251886 -0.2252107 0.6417145 -0.4546720 0.8711757
2023 Q3    0.205296679 -0.2335999 0.6441933 -0.4659378 0.8765311
2023 Q4    0.164553872 -0.2770792 0.6061869 -0.5108656 0.8399733

DLIR
        Point Forecast       Lo 80     Hi 80      Lo 95     Hi 95
2022 Q4    -0.06755118 -0.6918459 0.5567435 -1.0223275 0.8872252
2023 Q1     0.18891905 -0.4690971 0.8469352 -0.8174298 1.1952679
2023 Q2    -0.10730136 -0.7681924 0.5535897 -1.1180470 0.9034443
2023 Q3    -0.05654765 -0.7231300 0.6100347 -1.0759974 0.9629021
2023 Q4    -0.01843418 -0.6958541 0.6589857 -1.0544585 1.0175902

INF
        Point Forecast        Lo 80    Hi 80       Lo 95    Hi 95
2022 Q4       1.709217  0.776526458 2.641908  0.28278990 3.135644
2023 Q1       1.545674  0.566549077 2.524798  0.04823178 3.043116
2023 Q2       1.438848  0.430220077 2.447477 -0.10371555 2.981413
2023 Q3       1.074403  0.007449973 2.141356 -0.55736074 2.706166
2023 Q4       0.954270 -0.185989396 2.094529 -0.78960624 2.698146
```

*autoplot(var4_ea)*




Note, however, that these forecasts are for $\Delta$*UNR*, $\Delta$*LIR* and *INF*. To obtain the ex ante forecasts for *UNR* and *LIR*, recall the definition of the difference operator.

$$\Delta y_{T+1} = y_{T+1} - y_T \longrightarrow y_{T+1} = y_T + \Delta y_{T+1}$$

$$y_{T+2} = y_{T+1} + \Delta y_{T+2} = y_T + \Delta y_{T+1} + \Delta y_{T+2}$$

$$y_{T+h} = y_T + \sum_{i=1}^{h} \Delta y_{T+1} \quad , \quad h = 1, 2, \dots$$

$$\longrightarrow \quad E_T(y_{T+h}) = E_T(y_{T+h-1}) + E_T(\Delta y_{T+h}) = y_T + \sum_{i=1}^{h} E_T(\Delta y_{T+i})$$

Hence, the ex ante forecast prepared in period *T* for *h* periods ahead is the sum of the last available observation and the cumulative sum of the predicted first differences from period *T* to period *T* + *h*.

We can generate these forecasts with the *cumsum()* function of *R* which returns the cumulative sum of a time series or vector.

```
DUNR_ea = var4_ea$forecast$DUNR$mean
UNR_ea = ts(UNR[length(UNR)] + cumsum(DUNR_ea),
            start = c(2022, 4), end = c(2023, 4), frequency = 4)
DLIR_ea = var4_ea$forecast$DLIR$mean
LIR_ea = ts(LIR[length(LIR)] + cumsum(DLIR_ea),
            start = c(2022, 4), end = c(2023, 4), frequency = 4)
INF_ea = var5_ea$forecast$INF$mean
```

The ex ante forecasts for *UNR*, *LIR* and *INF*:

*print(cbind(UNR_ea, LIR_ea, INF_ea))*

```
           UNR_ea   LIR_ea   INF_ea
2022 Q4 3.519157 3.442449 1.709217
2023 Q1 3.649725 3.631368 1.545674
2023 Q2 3.857977 3.524067 1.438848
2023 Q3 4.063273 3.467519 1.074403
2023 Q4 4.227827 3.449085 0.954270
```

Note:

a) Recall that in a *VAR* model each equation relates one of the endogenous variables to the same set of lagged endogenous variables.

If the lag length is allowed to vary across equations, the model is called *near VAR* and it should be estimated with seemingly unrelated regression (*SUR*) method instead of OLS.

b) If necessary, *VAR* models can be augmented with exogenous variables and with extra deterministic terms, such as seasonal dummy variables.

# GRANGER (PREDICTIVE) CAUSALITY

- Although econometric models based on economic theory are supposed to describe cause-effect relationships, they cannot prove causality directly because mathematical dependence and real-life causality are not equivalent.

  In econometrics the concept of causality is closely related to the idea of succession in time (David Hume's problem of induction, 1739), meaning that *a cause always precedes its effect*.

$\longrightarrow$ Granger causality means *precedence*, i.e., a situation when one time series variable consistently and predictably changes before another variable does.

Granger causal relationships can be expected to improve forecasts in the sense that if variable $Z$ is causal to variable $Y$, current and/or lagged values of $Z$ should contain some extra information that is not contained in the current and lagged values of $Y$ but helps forecast $Y$.

$\longrightarrow$ $Z$ is said to Granger cause $Y$, if $y_{t+1}$ is conditional on $z_t$, $z_{t-1}$, $z_{t-2}$, …, so the latter help predict the former.

- To formalise the discussion, consider two (weakly) stationary variables, $Y$ and $Z$, and suppose that the information set available in period $t$ includes the current and lagged values of the two variables, i.e., $\Omega_t = \{y_t, y_{t-1}, \ldots; z_t, z_{t-1}, \ldots\}$.

  Let $E(y_{t+1} | \Omega_t)$ be a one-period-ahead optimal linear forecast prepared for $y_{t+1}$ in period $t$, $e_{t+1} = E(y_{t+1} | \Omega_t) - y_{t+1}$ the corresponding forecast error, and $\sigma^2(e_{t+1} | \Omega_t)$ the conditional forecast error variance.

  Given this setup, we can define two types of causality: Granger causality and instantaneous causality. They are based on the idea that the quality of forecasts can be captured with the forecast error variance (the smaller the better).

  a) Granger causality: $Z$ is Granger causal to $Y$ (denoted as $Z \rightarrow Y$) if and only if $y_{t+1}$ can be predicted better when the information set includes current and past values of $Z$.

  $$\longrightarrow \quad \boxed{\sigma^2\left(e_{t+1} \mid \Omega_t\right) < \sigma^2\left(e_{t+1} \mid \Omega_t - \{z_{t-i}\}\right)}$$

  This is one-way causality for one period ahead from $Z$ to $Y$.

$Z$ and $Y$ are said to have a two-way (or feedback) Granger causal relationship (denoted as $Z \leftrightarrow Y$) if there is one-way causality for one period ahead from $Z$ to $Y$ and there is also one-way causality for one period ahead from $Y$ to $Z$.

b) Instantaneous causality: $Z$ is instantaneously causal to $Y$ (denoted as $Z - Y$) if and only if $y_{t+1}$ can be predicted better when in addition to current and past $Z$ values the future value $z_{t+1}$ is used as well.

$$\longrightarrow \quad \boxed{\sigma^2\left(e_{t+1} \mid \Omega_t + z_{t+1}\right) < \sigma^2\left(e_{t+1} \mid \Omega_t\right)}$$

This is a mutual relationship, i.e., if $Z$ is instantaneously causal to $Y$, $Y$ is also instantaneously causal to $Z$.

<u>Note</u>:

a) The definition of Granger causality implies causality for one-period ahead.

In the bivariate case this is a ‘comprehensive’ definition because if there is no causality for one period ahead, there is no causality for more periods ahead either.

In a multivariate case, however, it might be insufficient.

For example, it might happen that $X$ does not cause $Y$ one-period ahead, but it causes $Z$ which in turn causes $Y$, implying a two-period-ahead causal relationship from $X$ to $Y$, via $Z$.

b) Granger causality between stationary variables can be tested for in individual equations. It is, however, usually preferred to test for simple causality in every possible directions using *VAR* models.

c) Whether a causal relationship appears to be Granger causality or instantaneous causality might depend on the frequency of the data.

In addition, the implementation of Granger causality might be affected by the frequency of the data.

← In the case of some high frequency (daily, weekly or monthly) data, it can be reasonably to assume that changes in one variable exert effect on the other variable(s) in later periods only, but in the case of annual data this assumption does not necessarily hold.

d) In a multivariate system, a variable is considered an endogenous variable if the other variables jointly Granger cause it, and it is exogenous otherwise.

- Granger causality can be tested with the general *F*-test or the Wald chi-square test on all lags of a variable (or several variables) jointly. Under the null hypothesis all these lags have zero coefficients, while under the alternative hypothesis some lag(s) has (have) non-zero coefficient(s).

  The *F*-test requires normally distributed error terms, but in large samples it is equivalent to the Wald test because

$$\boxed{\lim_{m_2 \to \infty} m_1 F \sim \chi^2_{m_1}}$$

(Ex 1 cont.)

e) Test for Granger causality between $\Delta UNR$, $\Delta LIR$ and $INF$ in the $VAR(4)$ model at the 5% significance level.

   Like most tasks, Granger causality tests can be performed in several ways in *R*. Probably the most convenient option is the *granger_causality()* function of the *bruceR* package.

```
library(bruceR)
granger_causality(var4)
```

Granger Causality Test (Multivariate)

$F$ test and Wald $\chi^2$ test based on VAR(4) model:

| | F | df1 | df2 | p | | Chisq | df | p | |
|---|---|---|---|---|---|---|---|---|---|
| DUNR <= DLIR | 1.67 | 4 | 194 | .160 | | 6.66 | 4 | .155 | |
| DUNR <= INF | 2.20 | 4 | 194 | .070 | . | 8.81 | 4 | .066 | . |
| DUNR <= ALL | 2.03 | 8 | 194 | .045 | * | 16.24 | 8 | .039 | * |
| DLIR <= DUNR | 3.52 | 4 | 194 | .008 | ** | 14.09 | 4 | .007 | ** |
| DLIR <= INF | 1.13 | 4 | 194 | .345 | | 4.50 | 4 | .342 | |
| DLIR <= ALL | 2.23 | 8 | 194 | .027 | * | 17.83 | 8 | .023 | * |
| INF <= DUNR | 2.10 | 4 | 194 | .083 | . | 8.39 | 4 | .078 | . |
| INF <= DLIR | 5.43 | 4 | 194 | <.001 | *** | 21.72 | 4 | <.001 | *** |
| INF <= ALL | 4.81 | 8 | 194 | <.001 | *** | 38.45 | 8 | <.001 | *** |

Three sets of $F$ and Wald-test results are reported, one for each endogenous variable.

Each set consists of three Granger causality tests for

i. $H_0$: $X \nrightarrow Y$,
ii. $H_0$: $Z \nrightarrow Y$,
iii. $H_0$: $(X, Z) \nrightarrow Y$.

The third test in each block (*ALL*, on the printout) is a multivariate Granger causality test. It can be used to find out whether the variable on the left-hand side is jointly Granger-caused by all other variables, and hence it is an endogenous variable within the given system.

In this case the $F$- and chi-square tests have very similar $p$-values and lead to the same conclusions at the 5% significance level:

$$\Delta LIR \nrightarrow \Delta UNR,\ INF \nrightarrow \Delta UNR,\ (\Delta LIR,\ INF) \rightarrow \Delta UNR$$

$$\Delta UNR \rightarrow \Delta LIR,\ INF \nrightarrow \Delta LIR,\ (\Delta UNR,\ INF) \rightarrow \Delta LIR$$

$$\Delta UNR \nrightarrow INF,\ \Delta LIR \rightarrow INF,\ (\Delta UNR,\ \Delta LIR) \rightarrow INF$$

→ In this system all three variables are endogenous as each variable is Granger caused by the other two.

In addition, at the 5% significance level, the quarterly change of the unemployment rate Granger causes the quarterly change of the long-term interest rate, and the quarterly change of the long-term interest rate Granger causes the rate of inflation.

At the 10% significance level, there is also two-way Granger causal relationship between the rate of inflation and the quarterly change of the unemployment rate.

# WHAT SHOULD YOU KNOW?

- Structural vector autoregression and reduced form vector autoregression
- Estimation of *VAR* systems
- Forecasting with *VAR* systems
- Granger causality and instantaneous causality
- Endogenous and exogenous variables
- Testing for Granger causality

# BOARD OF FAME

**David Hume (1711-1776):**

Scottish  Enlightenment philosopher, historian, economist, librarian, and essayist

Librarian at the University of Edinburgh

*A Treatise of Human Nature*