11

Cointegration

Allows you to construct regressions using non-stationary variables if they have stationary linear combination - a cointegrated r/ship. These relationships are often referred to as equilibrium r/ships.

↳ Non-stationary ($I(1)$) variables might have a stationary ($I(0)$) linear combination. Specifically, variables x & y are said to be $CI(d,b)$ if:

i. Each is integrated of order d

ii. They have at least one non-trivial linear combination that is integrated of order $(d-b)$, where $d \geq b > 0$.

Cointegration rules:

1. If x_{1t} & x_{2t} are $CI(1,1)$, then so are x_{1t} & $x_{2,t-i}$ for any i
2. Up to a scalar, cointegrate variables share the same stoch. trends
3. Two $I(1)$ variables can have at most 1 lin. ind. coint. vector.
   ↳ In general, the cointegration rank (r), or number of linearly independent coint. vecs cannot exceed integration order
4. Sample sizes need to be relatively large otherwise OLS estimations of B cointegration vector will be biased

For a bivariate system, solve so the dependent variable is ε_t, the RHS is your error correction term.

↳ It's B is referred to as the speed of adjustment coefficient.

↳ At least one must be different from zero

↳ Given $B_1 > 0$, stability requires $-2 < \alpha_1$ and $\alpha_2 < 2$

↳ This is because if the r/ship is cointegrated, regressing one variable over another will produce white noise residuals

The basic bivariate VECM for $CI(1,1)$ is:

$$\boxed{\begin{aligned} \Delta y &= \alpha_1 (y_{t-1} - \beta_1 z_{t-1}) + \varepsilon_{1t} \\ \Delta z &= \alpha_1 (y_{t-1} - \beta_1 z_{t-1}) + \varepsilon_{2t} \end{aligned}}$$

It is denoted a VECM(0) because it does not have lagged differences

It has several key features:

i. This VECM(0) is equivalent to the following VAR(1) model:

$$\boxed{\begin{aligned} y_t &= (1+\alpha_1) y_{t-1} - \alpha_1 \beta_1 z_{t-1} + \varepsilon_{1t} \\ z_t &= \alpha_2 y_{t-1} + (1 - \alpha \beta_1) z_{t-1} + \varepsilon_{2t} \end{aligned}}$$

A level VAR(1), but a restricted one (3 ind. params. not 4)

→ It can be shown every VEC(M) model has an equivalent VAR(p+1) representation

ii. As y & z are $I(1)$ & no lag is necessary, ε_{1t}, ε_{2t}, Δy_{t-1} & Δz_{t-1} are all stationary

iii. This parameterisation allows for two different types of dynamics

↳ Adjustment to LR equilibrium via the lagged EC term

↳ Additional SR dynamics captured by lagged first diffs (AR distributed lags)

iv. One of the speed of adjustment coeffs can be zero

↳ e.g. if $\alpha_1 < 0$ & $\alpha = 0$ equilibrium As occurs thru y, not z

↳ vice versa if $\alpha_1 = 0$ & $\alpha_2 < 0$.