ECOM40006/ECOM90013 Econometrics 3
Department of Economics
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Week 6 Tutorial Exercise

Semester 1, 2025

1. Take the opportunity to ask any questions that you may have about the lecture material or previous tutorial questions.

2. An estimator $\hat{\theta}$ is said to be consistent for a parameter θ iff $\hat{\theta} \xrightarrow{p} \theta$. Let $Y_1, Y_2, \ldots, Y_n$ denote a simple random sample from a population with probability density function

$$f(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Show that the sample mean $\overline{Y}$ is a consistent estimator of $\theta/(\theta+1)$.
   Hint: First derive the mean of the population and then remember that laws of large numbers are your friends.

3. Let $Y_1, Y_2, \ldots, Y_n$ denote a simple random sample of size n from a Normal population with mean μ and variance σ^2. Assuming that $n = 2k$ for some integer k, one possible estimator of σ^2 is

$$\hat{\sigma}^2 = \frac{1}{2k} \sum_{j=1}^{k} (Y_{2j} - Y_{2j-1})^2.$$

   (a) Show that $\hat{\sigma}^2$ is an unbiased estimator for σ^2.

   (b) Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.

4. Let $Y_1, Y_2, \ldots, Y_n$ be a sequence of independent random variables with $\mathrm{E}[Y_i] = \mu$ and $\mathrm{Var}[Y_i] = \sigma_i^2$. Notice that not all the σ_i^2's need be equal.

   (a) What is $\mathrm{E}\left[\overline{Y}_n\right]$?

   (b) What is $\mathrm{Var}\left[\overline{Y}_n\right]$?

   (c) Under what condition (on the σ_i^2's) can the following theorem be applied to show that $\overline{Y}_n$ is a consistent estimator for μ?
   Theorem: An unbiased estimator $\hat{\theta}_n$ for θ is a consistent estimator of θ if

$$\lim_{n\to\infty} \mathrm{Var}\left[\hat{\theta}_n\right] = 0.$$

5. If $Y_1, Y_2, \ldots, Y_n$ denote a simple random sample of size n from a population with a gamma distribution with parameters α and β, show that $\overline{Y}$ converges in probability to some constant and find the constant, when

$$f(y \mid \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} e^{-y/\beta}, \quad 0 < y < \infty.$$

Hint: Recall that

$$\int_0^\infty e^{-y/\beta} y^{\alpha-1} \, \mathrm{d}y = \beta^\alpha \Gamma(\alpha),$$

and explore the behaviour of $\mathrm{E}[Y]$ and $\mathrm{Var}[Y]$.