ECOM90024
Forecasting in Economics and Business
Tutorial 7

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1. Let $x_1, x_2, \ldots, x_n$ be a set of realizations of from an i.i.d. sequence $X_1, X_2, \ldots, X_n$ in which each X_i is characterized by a Poisson distribution function:

$$P(X = x;\ \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

   a.) Using words, provide an explanation of the parameter λ and the role it plays in determining the shape of the distribution function.

   b.) Write down the likelihood and log-likelihood function associated with the set of realizations $x_1, x_2, \ldots, x_n$.

   c.) Given the realizations $x_1, x_2, \ldots, x_n$, what is the maximum likelihood estimate of the parameter λ?

   d.) Using the rpois() function in ***R***, generate a set of 500 independent realizations from a Poisson random variable where $\lambda = 2$. Using these realizations, compute the maximum likelihood estimate of the parameter λ. Does your estimate conform to expectations?

   e.) Repeat part d an additional 499 times. You will have 500 samples of 500 observations from which you will obtain 500 estimates of the parameter λ. Plot a histogram of the estimates and discuss its shape. (***Hint: Try writing a loop in R:*** https://www.r-bloggers.com/how-to-write-the-first-for-loop-in-r/)

2. Suppose that you are analyzing a time series model that behaves according to an ARMA(1,1) process,

$$Y_t = \phi Y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$$

$$\varepsilon_t \sim_{i.i.d.} (0, \sigma^2)$$

a.) Given the information set $\Omega_t = \{Y_t, Y_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots\}$, write down the expressions for the one-step, two-step and h-step ahead forecasts.

b.) Write down the expressions for the one-step, two-step and h-step ahead forecast errors and their associated variances. What happens to the forecast error variance when $h \to \infty$

c.) Using the data contained in *tute7.csv*, estimate an ARMA(1,1) model in ***R*** using the *mle()* function and compare your estimates with those produced by the *Arima()*

function. Then, use the estimates and the formulas that you derived in part b) to compute h-step ahead 95% interval forecasts for $h = 1,2, \ldots, 10$. Compare your intervals with those produced by the *forecast()* function and describe what happens to the interval forecasts as h increases.