ECON30024
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Solution to Tutorial 6

1. (1) Suppose the risk free rate is 5 percent and the expected return on the market portfolio is 8 percent. Consider a portfolio of asset 1 and asset 2 in equal proportions, where asset 1's beta is 0.8 and asset 2's beta is 1.2. Then what is the risk premium on this portfolio as predicted by the CAPM?

   (a) 2.4 percent
   (b) 3.6 percent
   (c) 3 percent
   (d) 8 percent
   (e) None of the rest

   Answer: (c)
   As beta is linearly additive, the portfolio has a beta of 1: $(0.5)(0.8)+(0.5)(1.2) = 1$, and hence its risk premium is given by

$$\mu_j - r_0 = \beta(\mu_M - r_0) = \mu_M - r_0 = 3\%$$

   (2) Is the following statement true or false?
   In the Black CAPM, all investors' efficient frontiers in equilibrium are represented by the CML.

   (a) True
   (b) False

   Answer: (b)

   (3) Suppose an asset locates below the SML. Is this asset over-priced or under-priced, according to the CAPM?

   (a) over-priced
   (b) under-priced

   Answer: (a)
   Its expected return is lower than predicted by the CAPM, so its price is higher than predicted by the CAPM, i.e., the asset is over-priced.

   (4) The arbitrage principle asserts that every portfolio with a zero initial outlay has a zero payoff in all states? Is this statement true or false?

   (a) True
   (b) False

Answer: False

The arbitrage principle asserts that every arbitrage portfolio, i.e. a portfolio that has zero initial outlay and is **risk free**, has a zero payoff in all states.

(5) Factor models of asset returns specify the rate of return on any asset as a linear function of some systematic factors. Which of the following variables can serve as such factors?

(a) GDP growth rate
(b) The return on S&P 500 index fund
(c) The term spread
(d) Price-to-earnings ratio
(e) All of the rest

Answer: (e)

2. This is the "Black CAPM" for which the SML is defined by

$$\mu_j = \omega + \beta_j(\mu_M - \omega),$$

where ω is the expected rate of return on a zero-beta portfolio.

Suppose the three assets locate on the SML in the Black CAPM, then we can find the value of ω and μ_M in the same way as we did for Q3(a) in Tutorial 5. Then we would get $\omega = 0.03$, and $\mu_M - \omega = 0.05$, i.e., $\mu_M = 0.05$ (same values as before). The only difference is that ω appears instead of a risk-free rate r_0.

If we don't know whether these assets locate on the SML or not, to construct the SML in the Black CAPM, we need to construct a zero-beta portfolio, which has a zero correlation with the market portfolio, then use the average observed rate of return on this portfolio or conjectured expected return on this portfolio as ω.

3. In the absence of arbitrage opportunities, for any portfolio that uses zero initial capital and for which the payoffs are non-negative in every state (i.e. is risk-free), then the payoffs must be zero in every state. This requirement places restrictions on the pattern of observable asset prices, i.e. links asset prices.

Consider an arbitrage portfolio (x_A, x_B, x_C), by definition, it satisfies the following conditions:

Zero initial outlay:

$$5x_A + 4x_B + p_C x_C = 0. \tag{1}$$

Risk free:

$$9x_A + 12x_c \geq 0$$
$$4x_A + 8x_B + 6x_C \geq 0$$

The arbitrage principle implies that the arbitrage portfolio must have a zero payoff in every state:

$$9x_A + 12x_c = 0 \tag{2}$$
$$4x_A + 8x_B + 6x_C = 0 \tag{3}$$

Eq. (2) and (3) imply that

$$x_A = 16x_B$$
$$x_C = -12x_B.$$

Substituting these expressions in the zero outlay condition:

$$(5)(16)x_B + 4x_B + p_C(-12x_B) = 0$$
$$\Rightarrow (80 + 4 - 12p_C)x_B = 0$$
$$\Rightarrow 80 + 4 - 12p_C = 0$$
$$\Rightarrow p_C = 84/12 = 7.$$

The arbitrage principle implies that the price of asset C must be $p_C = 7$ in equilibrium.