ECON30009/90080 – Tutorial 8

This Version: Semester 2, 2025

This tutorial is designed to get you used to solving models with money in the utility function.

Question 1: Household problem only

Suppose the household lives for two periods and has the following household preferences

$$U(c_1, c_2, m_1, m_2) = (1-\beta)\ln c_1 + (1-\gamma)\ln m_1 + \beta \ln c_2 + \gamma \ln m_2$$

where c_t and m_t refer to consumption and real money balances in period t, respectively, for $t \in \{1, 2\}$. The parameters $0 < \beta < 1$ and $0 < \gamma < 1$ affect the weights households put on the utility derived from consumption and holding real money balances in each period. The household is born with nominal money balances, M_0 and nominal bonds, B_1. Each unit of B_1 has a gross nominal return of $1+i_1$. The household receives exogenous income y_t each period for $t \in \{1, 2\}$. In period 1, households can choose how much to consume, how much money to hold, and how much to save in a bond that has a gross nominal return of $1 + i_2$. In period 2, the household can choose how much to consume and how much money to hold.

a Write down the nominal budget constraint of the household in period 1.

b Write down the nominal budget constraint of the household in period 2.

c Write down the lifetime budget constraint of the household in nominal terms.

d Derive an IS curve from the household problem.

Question 2: Money in the utility function model

Consider the following money in the utility function model. Household's preferences are given by:

$$U(c_1, c_2, m_1, m_2) = c_1^{1-\beta} c_2^{\beta} + m_1^{1-\gamma} m_2^{\gamma}$$

where $m_t = M_t/P_t$, $0 < \beta < 1$ and $0 < \gamma < 1$. The household is born with physical capital a_1 *and* nominal money holdings, M_0. Price P_0 is given and exogenous. The household rents out her physical capital at the nominal rental rate $P_t R_t$, and earns the nominal wage rate $P_t w_t$ for each unit of labour supplied. Households have no disutility from labour and inelastically supply 1 unit of labour. Households also receive dividend income from firms each period as well as a nominal transfer $P_t \tau_t$ each period. There is a measure $N = 1$ of households in the economy.

Firms produce output according to a Cobb-Douglas production $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$. Firms rent capital and hire labour to produce. The monetary authority follows an exogenous money supply rule $M_{t+1}^s = \theta M_t^s$ where θ is a parameter that governs whether money supply is growing ($\theta > 1$), contracting ($\theta < 1$) or constant ($\theta = 1$). There is no other government spending in the economy. Since the government earns revenue from printing money, it runs a balanced budget by transferring the money supply created to the households, that is $P_t \tau_t = M_t^s - M_{t-1}^s$.

a Derive the household's lifetime budget constraint in real terms.

b Set up the household's problem and derive the optimality conditions of the household.

c Set up the firm's problem and derive the firm's optimality conditions.

d Show that real output, real investment and real consumption do not depend on money supply.

e Suppose $\theta = 1$. Derive an expression for i_2 in terms of parameters of the model only.