ECOM90024 EXAM FORMULA SHEET

Given a random variable X,

Mean: $E[X] = \mu_X$

Variance: $E[(X-\mu_X)^2] = \sigma_X^2$

Skewness: $E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^3\right]$

Kurtosis: $E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^4\right]$

Given a random variable X and a random variable Y

Covariance: $E[(X-E[X](Y-E[Y])] = E[XY] - E[X]E[Y]$

Correlation: $\frac{E[(X-E[X](Y-E[Y])]}{\sqrt{E[([X-E[X])^2]}\sqrt{E[([Y-E[Y])^2]}}$

Given a time series Y_t, for $t = 1,2,\dots,T$,

j-th Autocovariance: $E\left[(Y_t - E[Y_t])(Y_{t-j} - E[Y_{t-j}])\right] = \gamma_j$

j-th Autocorrelation: $\frac{\gamma_j}{\gamma_0} = \rho_j$

<u>Test Statistics</u>

t-test $t = \frac{\hat{\beta}-\beta}{\sigma_{\hat{\beta}}}$

Box-Pierce: $Q_{BP} = T\sum_{\tau=1}^{m}\hat{\rho}^2(\tau) \sim \chi_m^2$

Ljung-Box: $Q_{LB} = T(T+2)\sum_{\tau=1}^{m}\left(\frac{1}{T-\tau}\right)\hat{\rho}^2(\tau) \sim \chi_m^2$

<u>Smoothing</u>

Given a time series $\{y_t\}_{t=1}^{T}$, a moving average of order m is defined as:

$$MA(m)_t = \frac{1}{m}\sum_{j=-k}^{k} y_{t+j}$$

Where $m \geq 3$ is an odd number such that $m = 2k+1$ and thus $k = \frac{1}{2}(m-1)$

A centred moving average of order m is defined as

$$\overline{MA}(m)_t = \frac{1}{2}\left[\frac{1}{m}\sum_{j=-l}^{l-1} y_{t+j}\right] + \frac{1}{2}\left[\frac{1}{m}\sum_{j=-l+1}^{l} y_{t+j}\right]$$

Where $m \geq 2$ is an odd number such that $m = 2l$ and thus $l = \frac{1}{2}m$

Holt's Linear Trend is computed via the following equations:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend Equation: $b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t + hb_t$

Holt's Multiplicative Trend is computed via the following equations:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1}b_{t-1})$

Trend Equation: $b_t = \beta\frac{l_t}{l_{t-1}} + (1-\beta)b_{t-1}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t b_t^h$

Holt's Additive Damped Trend is computed via the following equations:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1} + \phi b_{t-1})$

Trend Equation: $b_t = \beta(l_t - l_{t-1}) + (1-\beta)\phi b_{t-1}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t + (\phi + \phi^2 + \cdots + \phi^h)b_t$

Holt's Multiplicative Damped Trend is computed via the following equations:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1}b_{t-1}^{\phi})$

Trend Equation: $b_t = \beta\frac{l_t}{l_{t-1}} + (1-\beta)b_{t-1}^{\phi}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t b_t^{(\phi+\phi^2+\cdots+\phi^h)}$

<u>Autoregressive Process</u>

An AR(p) process is given by

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim i.i.d.(0, \sigma^2)$$

It is covariance stationary if the roots of:

$$1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p = 0$$

lie ***outside the unit circle*** or equivalently if the roots of:

$$\lambda^p - \phi_1 \lambda^{p-1} - \phi_2 \lambda^{p-2} - \cdots - \phi_{p-1}\lambda - \phi_p = 0$$

lie ***inside the unit circle.***

The autocovariance function for $j = 1,2, \ldots$ is given by

$$\gamma_j = \phi_1 \gamma_{j-1} + \phi_2\, \gamma_{j-2} + \cdots + \phi_p\, \gamma_{j-p}$$

and the variance is given by:

$$\gamma_0 = \phi_1 \gamma_1 + \phi_2\, \gamma_2 + \cdots + \phi_p\, \gamma_p + \sigma^2$$

The autocorrelation function for $j = 1,2, \ldots$ is given by

$$\rho_j = \phi_1 \rho_{j-1} + \phi_2 \rho_{j-2} + \cdots + \phi_p \rho_{j-p}$$

Moving Average Process

An MA(q) process is given by

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

$$\varepsilon_t \sim i.i.d.(0, \sigma^2)$$

It is invertible if the roots of:

$$1 + \theta_1 z + \theta_2 z^2 + \cdots + \theta_q z^q = 0$$

lie ***outside the unit circle*** or equivalently if the roots of:

$$\lambda^q + \theta_1 \lambda^{q-1} + \theta_2 \lambda^{q-2} + \cdots + \theta_{q-1}\lambda + \theta_q = 0$$

lie ***inside the unit circle.***

The autocovariance function for $j = 1,2, \ldots, q$ is given by

$$\gamma_j = \sigma^2\left(\theta_j + \theta_{j+1}\theta_1 + \theta_{j+2}\theta_2 + \cdots + \theta_q \theta_{q-j}\right)$$

and the variance is given by:

$$\gamma_0 = \sigma^2 + \theta_1^2 \sigma^2 + \theta_2^2 \sigma^2 + \cdots + \theta_q^2 \sigma^2$$