Lecture 8 R scripts

Direct approximation

This example of direct approximation of the posterior is the case of an inverse variance component. The joint pdf $p(\mathbf{y}, \tau)$ is

$$\frac{\tau^{n/2}}{(2\pi)^{n/2}} e^{-\frac{\tau}{2}[(n-1)s^2+n(\bar{y}-\mu)^2]} \times \frac{\beta^\alpha \tau^{\alpha-1} e^{-\beta\tau}}{\Gamma(\alpha)} \propto \tau^{n/2+\alpha-1} e^{-\frac{1}{2}\tau[(n-1)s^2+n(\bar{y}-\mu)^2+2\beta]}$$

and values of hyper-parameters and sufficient statistics are $\mu = 5$, $\bar{y} = 4.88$, $n = 10$, $s^2 = 1.23$ and $\alpha = \beta = 1$.

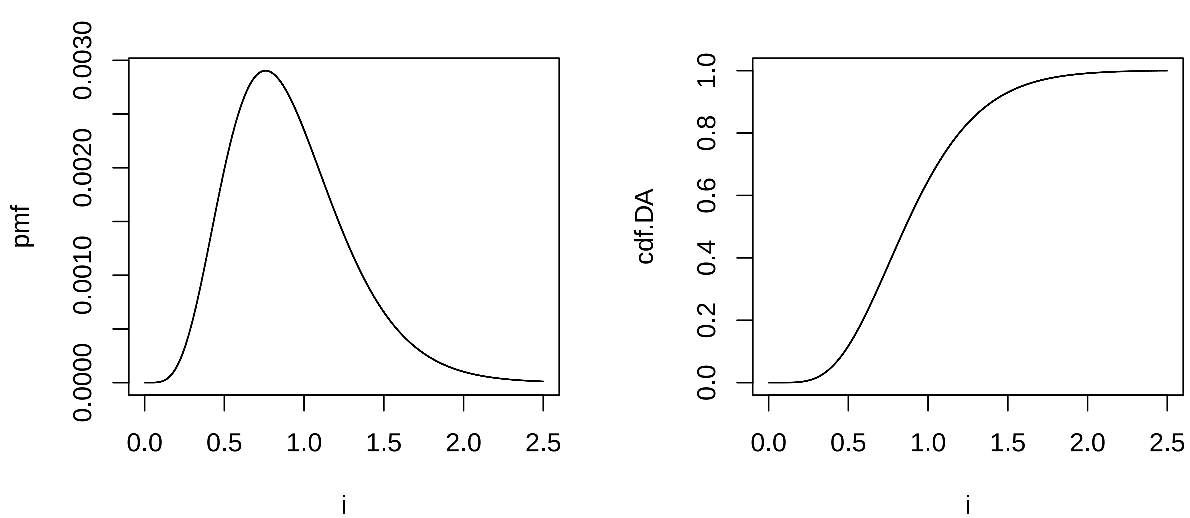
```r
set.seed(123456)
  #Hyper-parameters and sufficient statistics.
mu= 5; bary= 4.88; n=10; s2 = 1.23; a=b=1
N<-1000; i<-0:(N-1)/(N-1)*2.5
  #Building a grid of points to evaluate the un-normalised density

Qt<-function(x){x^(0.5*n+a-1)*exp(-0.5*x*((n-1)*s2+n*(bary-mu)^2+2*b))}
  #Un-normalised density

DA<-sapply(i,FUN=Qt,simplify=TRUE)
 #Evaluate un-normalised density at selected points

pmf<-DA/sum(DA) #Normalise

cdf.DA<-cumsum(pmf) #Construct empirical cdf.
par(mfrow=c(1,2));plot(i,pmf,type="s");plot(i,cdf.DA,type="s");par(mfrow=c(1,1))
```



```
sam.u<-runif(10000)

  #Generate random variable using inverse cdf method.
ind.post<-sapply(sam.u, FUN=function(x) which(abs(cdf.DA - x)== min(abs(cdf.DA -x))))
sam.post<-i[ind.post]

 #Histogram of random sample drawn using direct approximation
hist(sam.post,breaks=100,freq=FALSE,xlab=expression(tau),
     main='Histogram of Direct approximation posterior sample')

#Overlapping the analytic posterior over histogram
curve(dgamma(x,0.5*n+a,0.5*(n-1)*s2+0.5*n*(bary-mu)^2+b),0,2.5,
  xlab=expression(tau),ylab=expression(paste(p(tau,' |y'))), add=TRUE)
```