FNCE90056: Investment Management

Week 7 - Lecture 7: Coupon Bonds

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Introduction

- In last week's lecture (Lecture 6), we studied Zero-coupon Bonds:

  1. Zero-coupon Bonds - Riskless (Discount factor, Interest, PV)

  2. Zero-coupon Bonds - Risky (Credit risk)

- Today in Lecture 7:

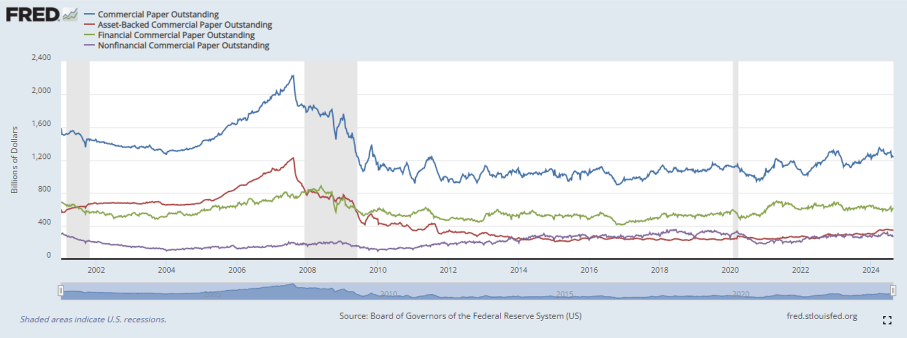
  - For the Thursday stream: Slide 38 - 41 from last week
  - Coupon Bonds - Riskless

[For the Thur class] Commercial paper

Commercial paper (CP), in existence since the 19th century, is an unsecured note issued by a firm for a specific dollar amount with maturity on a specific date.

- The maturity on CP averages 30 days, but may range up to 270 days. Beyond 270 days is rare as the security then needs to be registered with the SEC.
- CP is typically issued at a discount. It is common for firms to roll over their CP.
- 3 categories:
  - Financial - bank holding companies & consumer finance corporations
  - Non-financial - industrial firms & public utilities, largely
  - Asset-backed - issued by “conduits” that include a special-purpose vehicle that manages the assets purchased and the financing via CP.

[For the Thur class] Commercial paper market size



[For the Thur class] Other debt markets

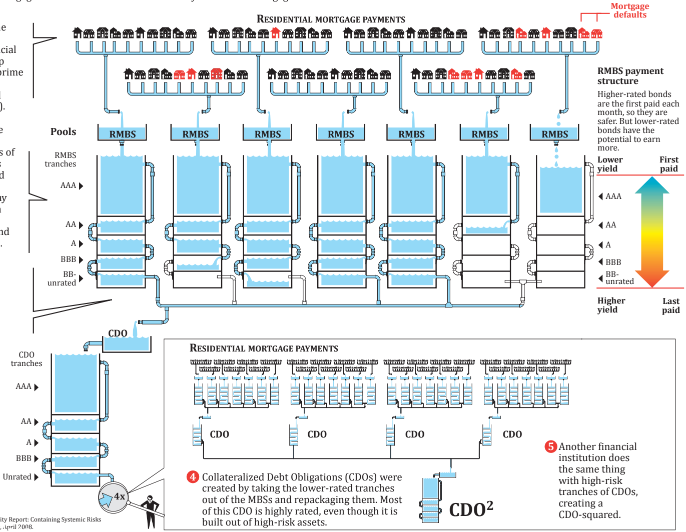
- **Mortgage-backed securities (MBS)**: This is one of the largest debt markets. Mortgage-backed securities are collateralised by pools of residential and non-residential mortgages and sold to investors who then receive claims to the mortgage coupons. These securities present numerous additional risks for investors compared to Treasury securities.
- **Swaps:** A swap is a contract according to which 2 counterparties agree to exchange cash flows in the future. Although considered a derivative market, its sheer size makes it equivalent to a primary market, in the sense that the prices of swaps are really not derived from those of other securities, but rather they depend on the relative size of demand and supply of these contracts by market participants.
- **Government-sponsored enterprise (GSE) Debt**
- **Other Asset-Backed Securities**
- **Sovereign Debt**
- **Other Derivatives:** Options, Futures, Forwards, Credit Derivatives

[For the Thur class] Example: MBS ⟶ CDO

The Theory of How the Financial System Created AAA-rated Assets out of Subprime Mortgages

In the financial system, AAA-rated assets are the most valuable because they are the safest for investors and the easiest to sell. Financial institutions packaged and re-packaged securities built on high-risk subprime mortgages to create AAA-rated assets. The system worked as long as mortgages all over the country and of all different characteristics didn't default all at once. When homeowners all over the country defaulted, there was not enough money to pay off all the mortgage-related securities.

1. People all over the country take out mortgages. Financial institutions group hundreds of subprime mortgages into Mortgage Backed Securities (MBSs).
2. The securities are grouped into tranches by levels of risk and earnings potential for bond holders. When everybody can pay their mortgage in full each month, each group of bond holders gets paid.
3. The mortgage payments are collected by a financial institution and payments distributed to bond holders. Higher rated tranches are paid first. When monthly mortgage payments are not made, payments may not reach holders of lower-rated tranches.



Source: IMF, Global Financial Stability Report: Containing Systemic Risks and Restoring Financial Soundness, April 2008.

Today

Today, we'll discuss in greater depth some of the topics we introduced last time.

- Term structure of interest rates
- Coupon-bond Pricing
- Bond yields
- Inflation, nominal vs. real interest rates

Term Structure of Interest Rates

PV and the term structure

- The relationship between interest rates for different maturities is known as the **term structure of interest rates**, or the **yield curve**.

- For a general term structure, the present value formula is:

$$PV = C_1 \times DF_1 + C_2 \times DF_2 + \cdots + C_T \times DF_T = \sum_{t=1}^{T} (C_t \times DF_t) \qquad \text{(lecture 6)}$$

If r_t is compound interest, the discount factor is:

$$DF_t = \frac{1}{(1 + r_t)^t} \qquad \text{(lecture 6)}$$

If r_t is continuously-compounded interest, the discount factor is:

$$DF_t = e^{-r_t \cdot t} \qquad (1)$$

Compounding frequency

Sometimes interest rates are typically quoted in annualized terms, but they may not be compounded on an annual basis: some at higher frequency, e.g. semi-annual.

Example: An interest rate of 5%, compounded annually, gives an annual discount factor of $1/1.05$ for a maturity of 1 year.

Example continued: if the interest rate is compounded n times a year, the annual discount factor for 1 year becomes $1/(1+0.05/n)^n$.

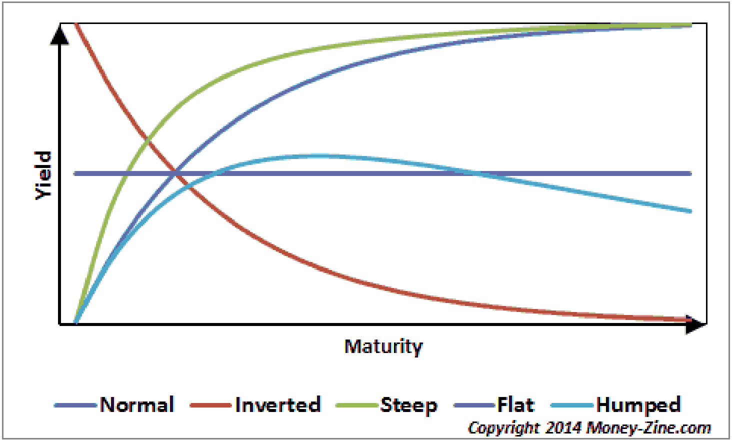
n=1:	Annual	$1/(1+0.05)$	= 1 / 1.05
n=2:	Semi-annual	$1/(1+0.05/2)^2$	= 1 / 1.050625
n=12:	Monthly	$1/(1+0.05/12)^{12}$	= 1 / 1.051162
n=365:	Daily	$1/(1+0.05/365)^{365}$	= 1 / 1.051267
$n=\infty$:	Continuous	$\lim_{n\to\infty} 1/(1+0.05/n)^n$	= 1 / 1.051271

Continuously compounded for t periods:

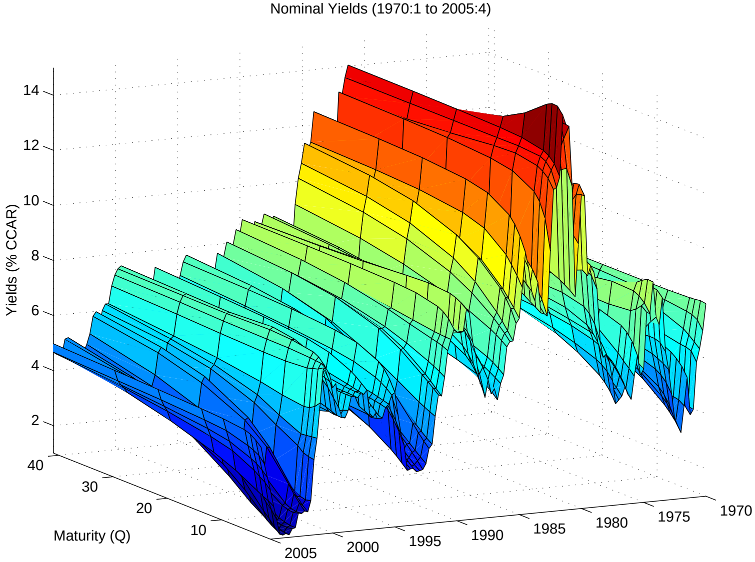
$$\lim_{n\to\infty} 1/(1+r_t/n)^{nt} = 1/e^{r_t \cdot t} = e^{-r_t \cdot t}$$

Examples of term structures

The situation in which the interest rates are the same for all maturities is referred to as a situation in which the term structure is **flat**. But it does not need to be flat...



Historical patterns



Pricing Coupon Bonds

Pricing a coupon bond

Suppose that the term structure of interest rates is as follows: $r_{0.5} = 4\%$, $r_1 = 4.1\%$, $r_{1.5} = 4.3\%$, and $r_2 = 4.5\%$ (annualised, annually compounding rates)

- What is the current price (in percent of par value) of a T-note with 2 years to maturity and a coupon rate of 8% (thus a 4% effective semi-annual coupon rate)?

- Using the general PV formula from lecture 6, we have:

$$P = \frac{4}{(1.040)^{0.5}} + \frac{4}{(1.041)^{1.0}} + \frac{4}{(1.043)^{1.5}} + \frac{104}{(1.045)^2} = 106.76$$

Estimating the term structure

- Since (coupon) bond prices depend on the current term structure, we can use bond prices themselves to recover the underlying term structure, provided we have a sufficient number of bonds with differing maturities and/or coupons.
- This process is called **bootstrapping**.
- It is a series of steps based on elimination of arbitrage.
- Our process goes from:

$$\text{bond prices} \rightarrow \text{discount factors} \rightarrow \text{interest rates}$$

Bootstrapping example

Suppose you observe the following Treasury securities' prices, where the T-notes pay coupons semi-annually:

Security	Coupon Rate	Maturity	Quote	$ Price
T-Bill	-	6 months	4.98	97.51000
T-Bill	-	1 year	5.44	94.56000
T-note	6%	2 years	99.10	$99.3125
T-note	8%	2 years	103.01	$103.03125

Question: What are the 6, 12, 18, and 24 month discount factors and interest rates?

- The term structure means spot rates r_t for all possible dates t. Equivalently, this means DF_t for each t.
- There are 4 payment dates, at $t = 0.5, 1, 1.5$, and 2 years, and we have 4 bonds.

Bootstrapping example

Let's first look at the two zero-coupon Treasury Bills:

Security	Coupon Rate	Maturity	Quote	$ Price
T-Bill	-	6 months	4.98	97.51000
T-Bill	-	1 year	5.44	94.56000

We know that, for zero-coupon bonds: $DF_T = \frac{P}{FV}$

$$
\begin{aligned}
DF_{0.5} &= \frac{97.51}{100} = 0.9751 \\
DF_1 &= \frac{94.56}{100} = 0.9456
\end{aligned}
$$

Bootstrapping example

Now, let's look at the coupon bonds:

Security	Coupon Rate	Maturity	Quote	\$ Price
T-note	6%	2 years	99.10	\$99.3125
T-note	8%	2 years	103.01	\$103.03125

$$
\begin{aligned}
3DF_{0.5} + 3DF_1 + 3DF_{1.5} + 103DF_2 &= 99.3125 \\
4DF_{0.5} + 4DF_1 + 4DF_{1.5} + 104DF_2 &= 103.03125
\end{aligned}
$$

$$
\begin{aligned}
DF_2 &= (0.04 \times 99.3125) - (0.03 \times 103.03125) \\
&= 0.8815625
\end{aligned}
$$

$$
\begin{aligned}
DF_{1.5} &= \frac{99.3125 - (3DF_{0.5} + 3DF_1 + 103DF_2)}{3} \\
&= 0.9164875
\end{aligned}
$$

Bootstrapping example: $DF_t \to r_t$

Date	Discount factor	Annually Compound rate	Continuously compounded rate
0.5	0.9751	0.052	0.050
1.0	0.9456	0.058	0.056
1.5	0.9165	0.060	0.058
2.0	0.8816	0.065	0.063

$$DF_t = 1/(1+r_t)^t \quad or \quad DF_t = e^{-r_t \cdot t}$$

$$r_t = (1/DF_t)^{1/t} - 1 \quad or \quad r_t = -\frac{\ln(DF_t)}{t}$$

$$(1/0.9751)^{1/0.5} - 1 = 5.2\%, \quad or \quad -\frac{\ln(0.9751)}{0.5} = 5.0\%$$

$$(1/0.9456)^{1/1} - 1 = 5.8\%, \quad or \quad -\ln(0.9456) = 5.6\%$$

$$(1/0.9165)^{1/1.5} - 1 = 6.0\%, \quad or \quad -\frac{\ln(0.9165)}{1.5} = 5.8\%$$

$$(1/0.8816)^{1/2} - 1 = 6.5\%, \quad or \quad -\frac{\ln(0.8816)}{2} = 6.3\%$$

Bond Yields

Bond yield

The **yield to maturity** or **internal rate of return (IRR)** is the single rate y that solves the equation that links price with future CFs:

$$P = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{F}{(1+y)^T} \quad (2)$$

- In contrast, the general pricing equation is

$$P = \sum_{t=1}^{T} \frac{C}{(1+r_t)^t} + \frac{F}{(1+r_T)^T} \quad (3)$$

- Unless there are very few payment dates left until maturity, we can only solve for y numerically, not algebraically, e.g. using the Excel solver.

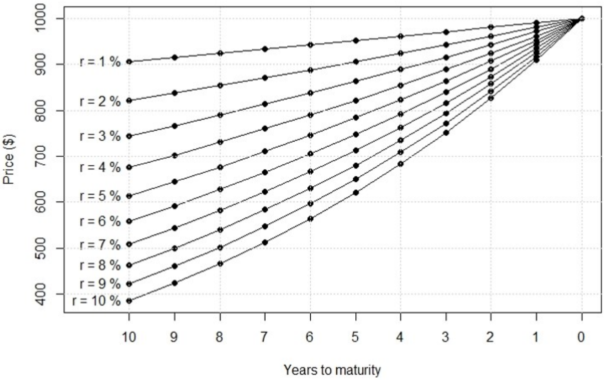
Bond yield

- The **yield of a bond** is a complicated weighted average of the current term structure interest rates.
- For a zero-coupon bond with maturity T, however, we just get $y = r_T$ because all CFs happen at maturity.
- Two bonds with the same maturity but different coupons will in general have different yields. (Consider the last two bonds in our bootstrapping example.)

Relationship between YTM and coupon rate

- When the price is calculated an instant after a coupon payment:
  - Coupon rate $=$ YTM $\quad\Leftrightarrow\quad$ Price $=$ Face value
  - Coupon rate $>$ YTM $\quad\Leftrightarrow\quad$ Price $>$ Par value
  - Coupon rate $<$ YTM $\quad\Leftrightarrow\quad$ Price $<$ Par value
- Price of premium/discount bond will converge to its face value (aka Par value) as it gets closer to the maturity.

Zero-coupon bonds: $PV = \$1,000/(1 + r_T)^T$



- "Pull to par": prices approach the par value as you approach maturity.
- Higher discount rate $\Rightarrow$ lower bond prices.

Law of One Price: Coupon bond = zero + annuity

Coupon bond:

	$\$C$	$\$C$	...	$\$C$	$\$(C+F)$
$t=0$	1	2	...	$T-1$	T

Zero:

	$\$F$
$t=0$	T

Annuity:

	$\$C$	$\$C$	...	$\$C$	$\$C$
$t=0$	1	2	...	$T-1$	T

$$\textbf{PV [coupon bond] = PV[zero + annuity]} = \frac{F}{(1+y)^T} + \frac{C}{y}\left(1 - \frac{1}{(1+y)^T}\right)$$

Proof of annuity formula

$$\textbf{PV of annuity: } A = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \ldots + \frac{C}{(1+y)^T}$$
$$= \frac{C}{1+y}\left(1 + \frac{1}{1+y} + \ldots + \frac{1}{(1+y)^{T-1}}\right)$$

$$\begin{aligned} A &\equiv \frac{1}{y}\left(A(1+y) - A\right) \\ &= \frac{1}{y}C\left(1 + \cancel{\frac{1}{1+y}} + \ldots + \cancel{\frac{1}{(1+y)^{T-1}}}\right) \\ &\quad -\frac{1}{y}C\left(\cancel{\frac{1}{1+y}} + \ldots + \cancel{\frac{1}{(1+y)^{T-1}}} + \frac{1}{(1+y)^T}\right) \\ &= \frac{C}{y}\left(1 - \frac{1}{(1+y)^T}\right) \end{aligned}$$

Semi-annual reporting

- By convention, bond yields are often reported using a semi-annual compounding convention. The yield, y, in (2) is a simple interest rate.
- To convert to the effective semi-annual yield, solve

$$\left(1 + \frac{y_{\text{semi-annual}}}{2}\right)^2 = 1 + y \tag{4}$$

for the semi-annual yield.

Inflation, Nominal vs Real Interest Rates

Nominal vs real rates

- Suppose you invest in the bond market for 1 year. Your payoff is $1 + r_1$ dollars for each dollar invested. Does this mean you are better off than at the start of the year?
- The answer depends on what happened to **inflation**.
- Suppose that the 1-year rate of inflation is i_1. Then in order to buy the same amount of goods you could have purchased with a dollar today you will need $1 + i_1$ dollars a year from now. This means that your actual return, measured in today's dollars is

$$R_1 = \frac{1 + r_1}{1 + i_1} - 1 = \frac{r_1 - i_1}{1 + i_1} \tag{5}$$

R_1 is the **real rate of return**, as opposed to r_1, which is a **nominal rate**.

Computing the real rate

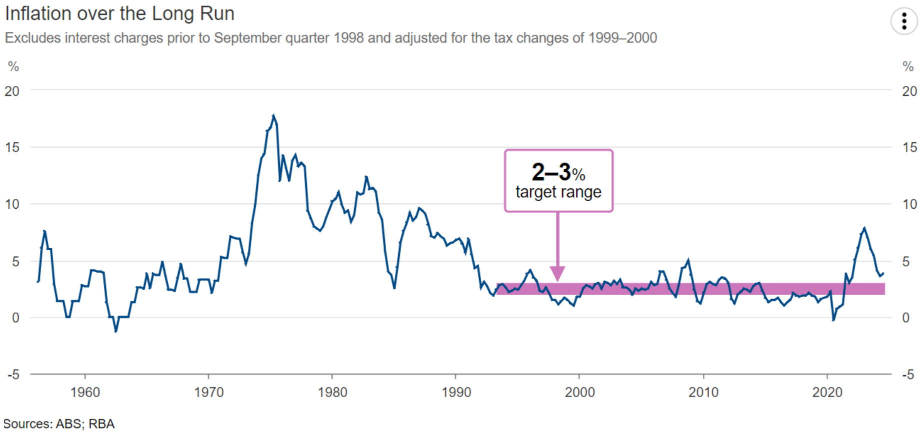
- People often calculate the real rate of return R_1 to be the difference between the nominal rate of return r_1 and the inflation rate i_1:

$$R_1 \approx r_1 - i_1 \tag{6}$$

  This is only **approximately** true, and only for small i_1.[1]

- It is always appropriate to use (5).

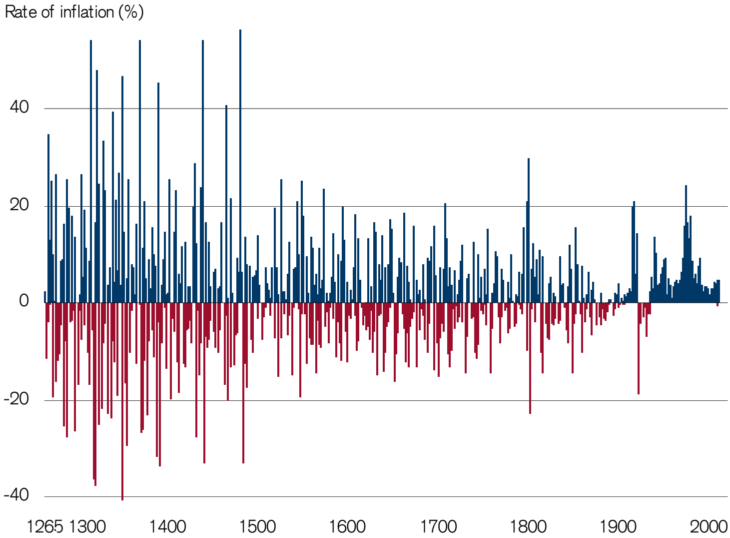
[1] The expression in (6) is exactly correct under continuous compounding.



- Inflation = average % increase in prices from one year to the next.
- RBA's inflation target: 2-3% per year on average over time.

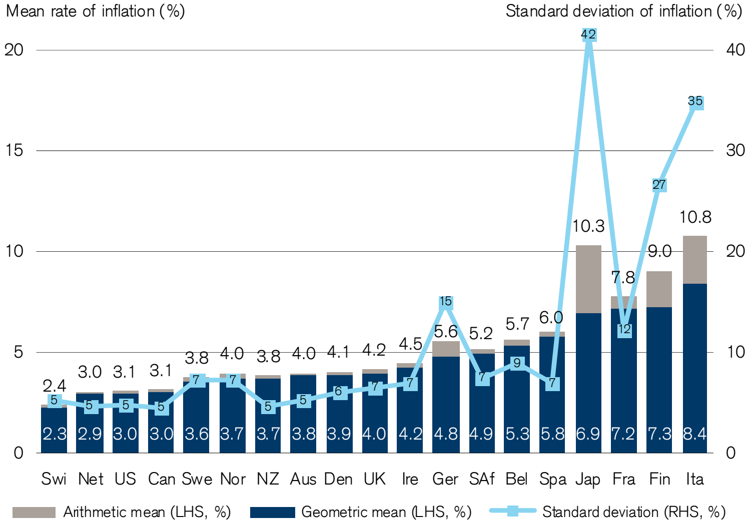
Annual inflation rates in the United Kingdom, 1265–2011

Source: Officer and Williamson (2011)



Annual inflation rates in the Yearbook countries, 1900–2011

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, Triumph of the Optimists; authors' updates



Importance of real rates

- Suppose you're in Germany at the beginning of 1923. Someone offers you a 1-year German Treasury bill denominated in Marks with a face value of M10,000,000 ($\approx$ \$550) at the "bargain" price of M5,000,000.
- Since this implies a rate of return of

$$r_1 = \frac{10,000,000}{5,000,000} - 1 = 100\%, \tag{7}$$

you accept. Does this turn out to be a good deal?

- The inflation rate in Germany in 1923 was about $4,530,000,000\%$, so the real return on your investment was

$$R_1 = \frac{1+1}{1+45,300,000} - 1 \approx -99.9999956\%. \tag{8}$$

- The investment was almost worthless in real terms at the end of 1923.

Summary

Discount factors. The discount factor is the value today of 1 dollar in the future. Discount factors decrease with the time horizon and also vary over time.

Interest rates. The promised rate of return of an investment, an interest rate needs a compounding frequency to be well-defined. They are typically quoted on an annualised basis.

Term structure of interest rates. The term structure of interest rates is the relation between interest rates and maturity. Investment horizons affect the interest rate to be received on an investment or paid on a loan.

Bond yields. A bond's yield is its internal rate of return. It tells you the price of a bond. It does not convey information about the term structure unless it is the yield on a zero-coupon bond.

Inflation. Inflation must always be treated consistently. Use real discount rates for real cash flows.