ECON30024
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Solution to Tutorial 8

1. (1) Empirical tests of the CAPM can provide a definite answer regarding the validity of the CAPM. True/Fasle? Answer: False.

   (2) Which of the following statements regarding tests of the CAPM is FALSE?

   (a) Times series tests of the CAPM test whether returns (or excess returns) on assets depend linearly and solely on the market return (or excess market return).

   (b) Tests of the CAPM typically use portfolios of assets instead of individual assets.

   (c) Two tests of the CAPM that use different sets of assets may yield different conclusions.

   (d) A times series test of the CAPM involves one regression equation for all assets.

   Answer: (d)

   (3) Which of the following statements regarding similarities and differences between a time series test and a cross-section test of the CAPM is TRUE?

   (a) They both test whether assets' expected returns depend linearly and solely on their betas.

   (b) A cross-section test tests the validity of the SML as predicted by the CAPM, while a time series test tests the validity of a single-factor model for asset returns where the single factor is market return or excess market return.

   (c) They both involve a two-stage procedure.

   (d) They both specify a single regression equation for the chosen assets.

   Answer: (b)

   (4) Suppose you run the following cross-section test of the CAPM

$$\bar{r}_j - \bar{r}_0 = \gamma_0 + \gamma_1 \widehat{\beta}_j + \eta_j, \quad j = 1, 2, \ldots, n$$

   and find that γ_0 is significantly different from zero, and γ_1 is significantly greater than zero, but significantly less than $\bar{r}_M - \bar{r}_0$. What do you conclude from this test?

   (a) This test provides empirical evidence that supports the Sharpe-Lintner CAPM.

   (b) This test provides empirical evidence that supports the Black CAPM.

   (c) This test provides empirical evidence that rejects the CAPM.

   (d) We cannot conclude anything from the test.

   Answer: (b)

(5) In the Fama-French three-factor model, the three factors include

i) the excess market return

ii) the beta coefficients of portfolios

iii) the market value of equity

iv) the difference between returns on small ME portfolio and big ME portfolio

v) the difference between returns on high BE/ME portfolio and low BE/ME portfolio

vi) the book-to-market ratio

(a) all of the above

(b) i), iv) and v)

(c) ii), iii) and vi)

(d) i), iii) and vi)

Answer: (b)

2. That's not possible. Suppose that's possible. Denote an arbitrage portfolio of A and B as (y_A, y_B), then the zero initial outlay condition implies that

$$y_A + y_B = 0.$$

Without loss of generality, let $y_A > 0$, then $y_B = -y_A < 0$.

Then the payoff on the portfolio is given by

$$r_A y_A + r_B y_B = r_A y_A + r_B(-y_A) = (r_A - r_B)y_A.$$

Recall that

$$r_A = b_{A0} + b_{A1}F_1, \quad r_B = b_{B0} + b_{B1}F_1,$$

then

$$r_A - r_B = b_{A0} - b_{B0} + (b_{A1} - b_{B1})F_1.$$

Because the factor loadings of A and B are different, $b_{A1} - b_{B1} \neq 0$. Because F_1 is a random variable, $(b_{A1} - b_{B1})F_1$ cannot be nonnegative in all states of the world. As a result, the payoff on the portfolio cannot be nonnegative in all states of the world, i.e., the portfolio cannot be risk free.

3. The standard method is to calculate a regression with the excess return on company A's shares as the dependent variable and the excess return on the market portfolio as the independent or explanatory variable. We can also plot a scatter diagram of company A's excess return against the excess return on the market portfolio.

The regression estimates are that the beta-coefficient (i.e. the slope coefficient in the regression) equals, approximately, 1.383, with a standard error equal to 0.081. See the following output from an OLS regression.[1]

[1]This table was generated from an old software, so the exact numbers in the table might be slightly different from the output you obtain from R studio or other software.

Note that the t statistic and the p value for the beta-coefficient are 17.10 and 0 respectively. Thus, the beta-coefficient is significantly different from zero, suggesting that excess market return is a significant factor in determining the return on Company A's stock. The intercept term is estimated to be 0.0586 with a standard error of 0.4216. Its t statistic and p value are -0.14 and 0.893 respectively, so the intercept is not significantly different from zero.

Source	SS	df	MS	Number of obs =	10
Model	249.434624	1	249.434624	F(1, 8) =	292.40
Residual	6.82438048	8	.85304756	Prob > F =	0.0000
				R-squared =	0.9734
				Adj R-squared =	0.9700
Total	256.259004	9	28.4732227	Root MSE =	.92361

ABCplc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
MktRet	1.38261	.0808552	17.10	0.000	1.196158	1.569063
_cons	-.0586141	.4215806	-0.14	0.893	-1.030781	.9135526

These results therefore provide evidence supporting the Black CAPM, while rejecting the Sharpe-Lintner CAPM (which also implies zero intercept in the regression).

4. (a) The SML can be estimated by regressing the average rates of return on each asset on their estimated beta coefficients. We can also make a rough sketch of $\bar{r}_j$ against the estimated beta-coefficients for the three assets. Formal OLS estimates yield the table below.

   From the table, the intercept (an estimate of the risk-free rate) is 0.1166 and the slope (an estimate of the excess return on the market portfolio) is 0.04845.

Source	SS	df	MS	Number of obs =	3
Model	.001518213	1	.001518213	F(1, 1) =	4.36
Residual	.000348454	1	.000348454	Prob > F =	0.2844
				R-squared =	0.8133
				Adj R-squared =	0.6267
Total	.001866667	2	.000933333	Root MSE =	.01867

ExcessR	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Beta	.0484536	.0232131	2.09	0.284	-.2464963	.3434035
_cons	.1165979	.0262968	4.43	0.141	-.217534	.4507299

(b) The Black CAPM predicts that the slope of this regression is positive while the

Sharpe-Lintner CAPM also predicts that the intercept of the regression equals the risk free rate. The regression results in the table above seem to show reasonable support for a positive slope but the intercept seems to be significantly different from the average risk free rate,[2] hence providing support for the Black CAPM.

From the regression it is possible to calculate predicted expected returns for the three assets:

$$\begin{aligned}
\text{Asset 1}: &\ 0.1166 + (1.50)(0.048454) = 0.189281 < \bar{r}_1 \\
\text{Asset 2}: &\ 0.1166 + (0.40)(0.048454) = 0.135982 < \bar{r}_2 \\
\text{Asset 3}: &\ 0.1166 + (1.20)(0.048454) = 0.174745 > \bar{r}_3
\end{aligned}$$

These results suggest that according to the estimated SML, asset 1 and 2 are underpriced while asset 3 is overpriced. This could just reflect inaccuracy in the estimation of the SML. If we believe in the estimated SML, an investment strategy might be to short-sell asset 3 and buy asset 1 and 2.

(c) You would include E/P along with the estimated beta-coefficients in the regression with the average rate of return as the dependent variable. That is, regress the average rates of return on an intercept (constant), the estimated beta-coefficients, and E/P values.

If the regression find that the coefficient on E/P is not significantly different from zero, this result suggests that E/P is not important in influencing $\bar{r}_j$ once the beta- coefficient is taken into account.

This result tends to support the CAPM which predicts that E/P should not appear in the regression. But, of course, there could be other variables which might be significant and thus cast doubt on the CAPM.

5. Despite mixed findings from the empirical testing of the CAPM, the CAPM remains the most established asset pricing theory. So it's often served as the benchmark model for assessing asset market efficiency.

   The CAPM does not explicitly assume market efficiency. However, if asset markets are efficient, some key assumptions for the CAPM are more likely to hold (and hence the CAPM prediction is more likely to hold). For example, if all information is quickly reflected in asset prices which are observable to all investors, then investors are more likely to form the same beliefs. Asset prices are also more likely to clear the market, as they respond quickly to changes in demand and supply. In this sense, we might say the CAPM implicitly assume market efficiency.

[2] Because of the small sample size, we do not do a formal test here, but you are asked to do a formal test of the CAPM in Assignment 2