ECOM90024
Forecasting in Economics and Business
Tutorial 4 Solutions

1.) The updating equations for Holt's multiplicative trend model are given by:

Level Equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1}b_{t-1})$

Trend Equation: $b_t = \beta \dfrac{l_t}{l_{t-1}} + (1-\beta)b_{t-1}$

Forecasting Equation: $\hat{y}_{t+h|t} = l_t b_t^h$

a.) Rewrite the above level and trend equations in their error correction forms and describe the role that the parameters, α and β play in updating the level and trend when new information arrives.

To derive the error correction form of the level equation, we write:

$$l_t = l_{t-1}b_{t-1} + \alpha\big(y_t - (l_{t-1}b_{t-1})\big) = \hat{y}_{t|t-1} + \alpha e_t$$

Where $e_t = y_t - (l_{t-1}b_{t-1})$. Therefore, the level is adjusted by the forecast error with weight α.

The error correction form of the trend equation is given by:

$$b_t = b_{t-1} + \beta\left(\frac{l_t - l_{t-1}b_{t-1}}{l_{t-1}}\right) = b_{t-1} + \alpha\beta\frac{e_t}{l_{t-1}}$$

Thus, the trend is adjusted by the forecast error relative to the level with weight $\alpha\beta$.

b.) Let $\alpha = 0.2$ and $\beta = 0.4$ and let the initial values of the trend and level be given by $l_0 = 1$ and $b_0 = 0.1$. Then, suppose that you have the following time series data set:

$$\{1,4,9,20,23\}$$

Using Excel, compute the smoothed time series according to the equations provided above.

See Excel spreadsheet.

c.) Using Excel, calculate the h = 4 step ahead point forecasts according to equations provided above equations.

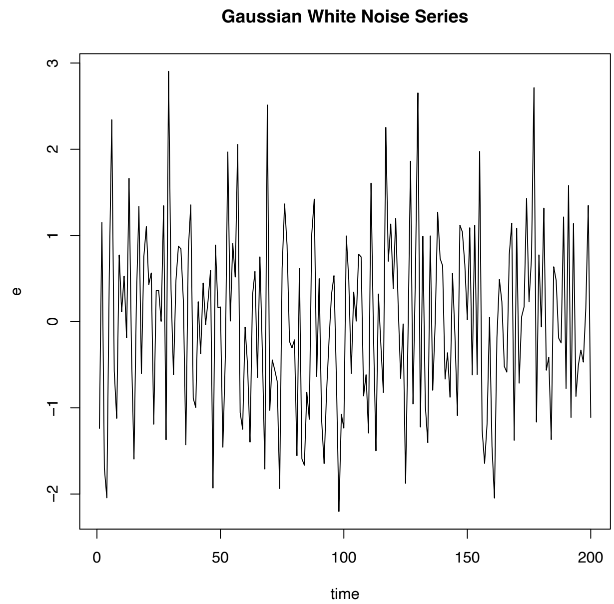
See Excel spreadsheet.

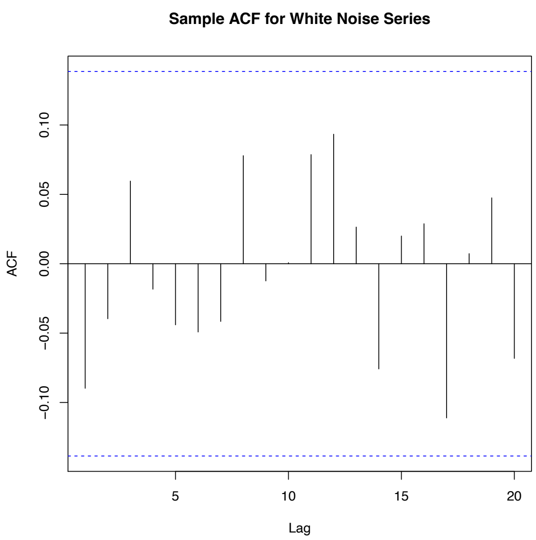
2.) Using the ***rnorm*** command in R, generate and plot Gaussian white noise series that comprises of 200 observations. These will be a set of observations from the following data generating process,

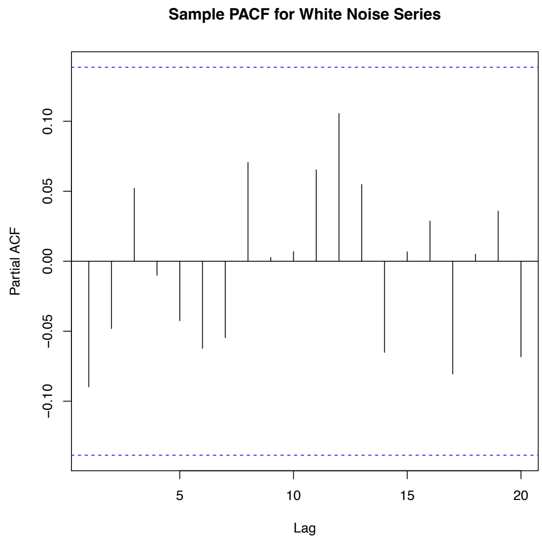
$$Y_t = \varepsilon_t$$

$$\varepsilon_t \sim_{iid} N(0,1)$$

Then, using the ***acf*** and ***pacf*** commands in R, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated series. Do they accord with the properties of the underlying data generating process?







The sample ACF and PACF should be consistent with the white noise stochastic process in that no dependence can be seen at any lag.

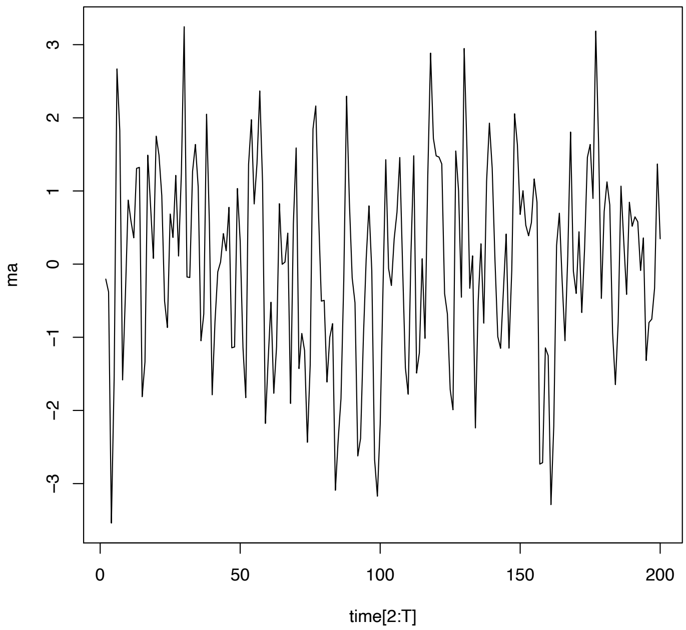
3.) Using the data that you've generated in question 2, generate and plot a series that represents a set of observations from the following MA(1) process,

$$Y_t = \varepsilon_t + 0.9\varepsilon_{t-1} \qquad t = 2, 3, \dots, 200$$

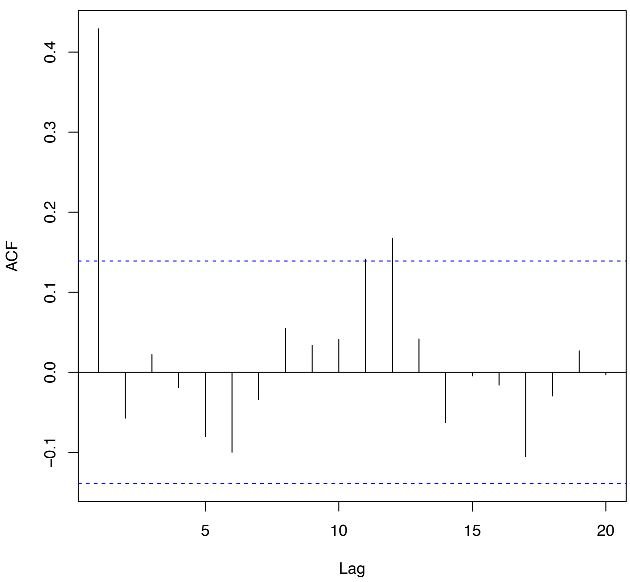
$$\varepsilon_t \sim_{iid} N(0,1)$$

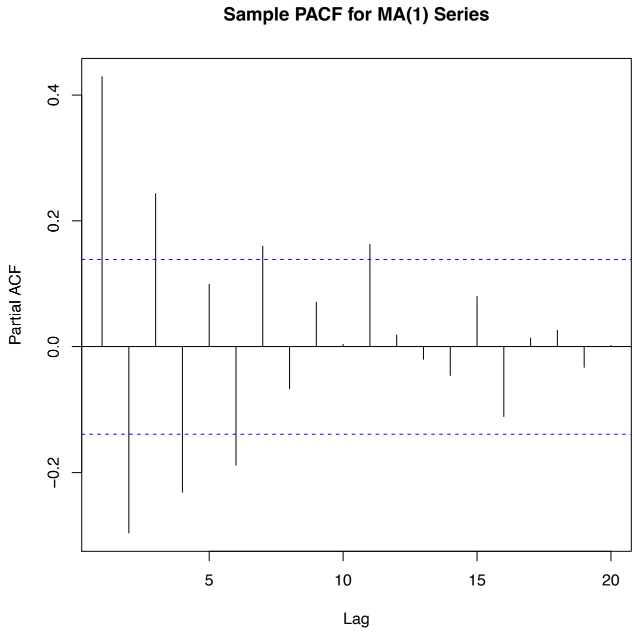
Then, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated MA(1) series. Discuss your findings.

First Order Moving Average Series



Sample ACF for MA(1) Series





In this case, we observe a single significant autocorrelation at displacement 1 and a decaying pattern in the PACF where the sign alternates from one displacement to the next. This is typical of an MA(1) process where the MA(1) parameter is positive.

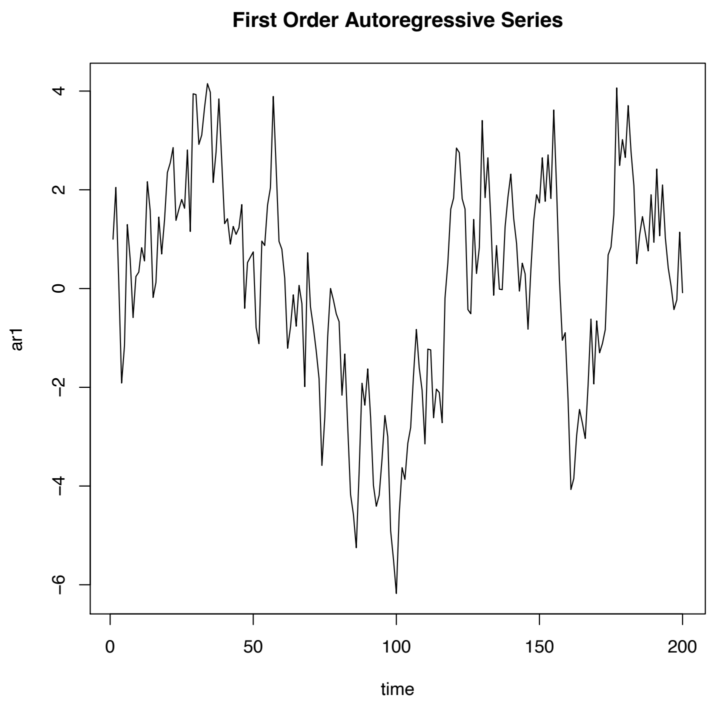
4.) Using the data that you've generated in question 2, generate and plot a series that represents a set of observations from the following AR(1) process,

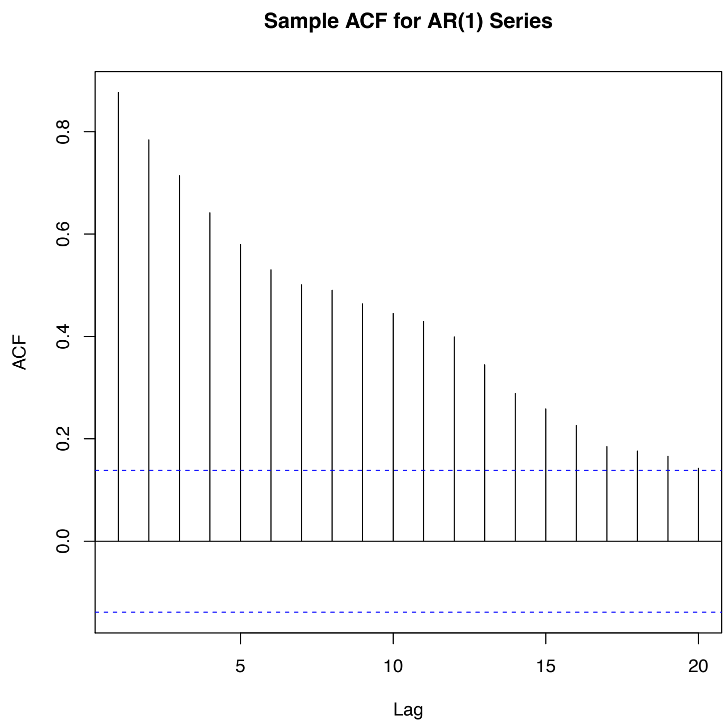
$$Y_t = 0.9Y_{t-1} + \varepsilon_t \qquad t = 2, 3, \dots, 200$$

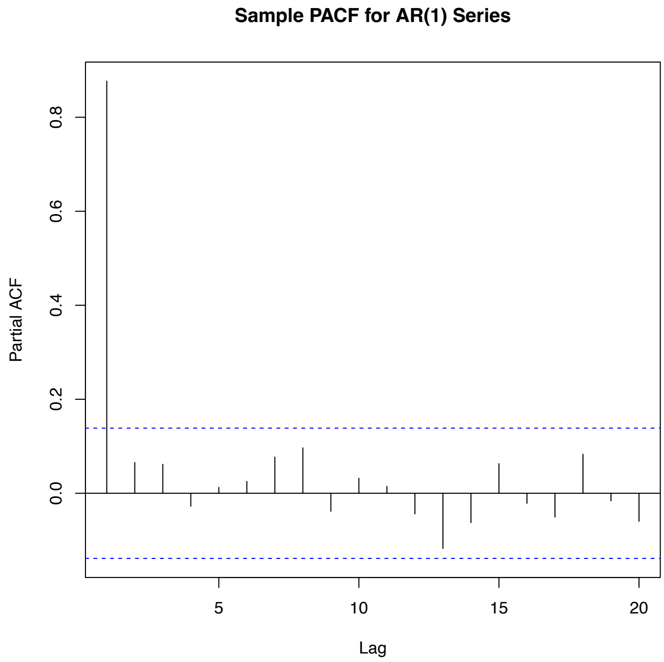
$$Y_1 = 1$$

$$\varepsilon_t \sim_{iid} N(0,1)$$

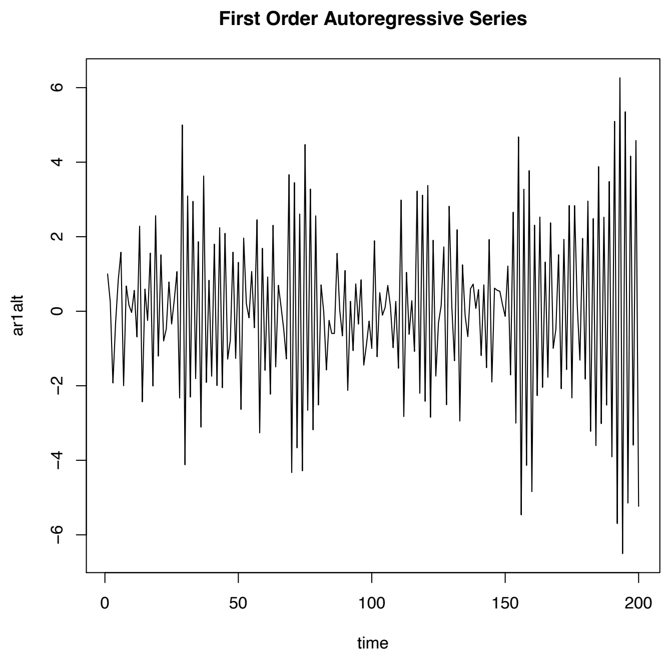
Then, generate and plot the sample autocorrelation and partial autocorrelation functions associated with your generated AR(1) series. Repeat the exercise using an autocorrelation coefficient of -0.9. Discuss your findings.

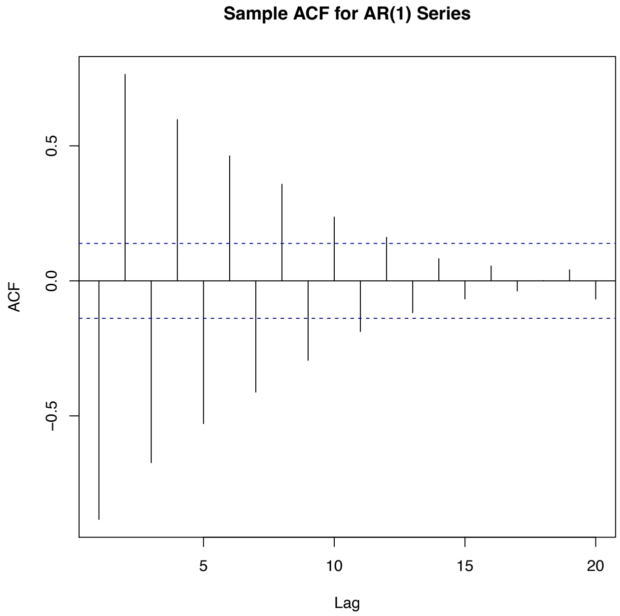

First Order Autoregressive Series
ar1
4
2
0
−2
−4
−6
0
50
100
150
200
time

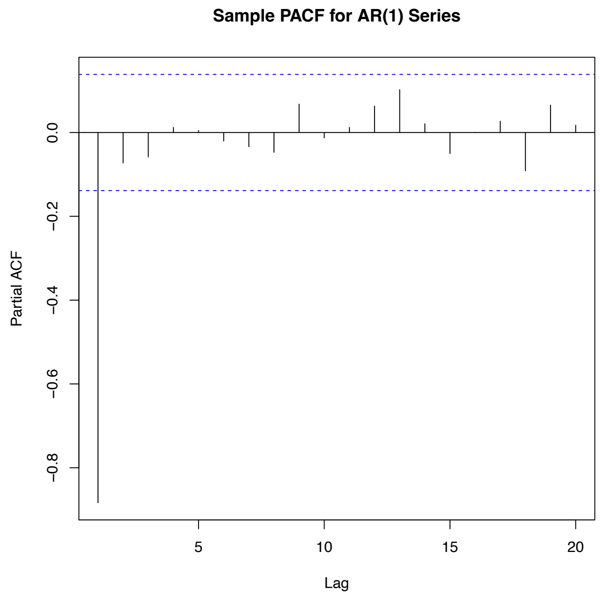

Sample ACF for AR(1) Series
ACF
0.8
0.6
0.4
0.2
0.0
5
10
15
20
Lag



In this case, we observe gradual one-sided decay in the ACF and an abrupt cutoff in the PACF after the first displacement. This is typical of a stationary AR(1) process where the AR(1) parameter is positive.







When the autoregressive coefficient is negative, the sample autocorrelation function displays a decaying oscillation while the partial autocorrelation function again displays an abrupt cutoff at the first lag.