Assignment 2 Questions

The first published research to consider the effects of trend breaks on unit root tests was Perron, P. (1989), "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis", *Econometrica* 57, 1361–1401. In one of his analyses he considered the effect of allowing for a trend break at 1973 for the OPEC oil price shock when testing for a unit root in postwar US real GNP, 1947 to 1986. In this assignment we will update his analysis by using data on U.S. real GDP per capita (a common measure of "productivity") until 2019 and by allowing for a second trend break for the GFC in 2008.
(There is no need to attempt to read Perron's paper to complete this assignment, but it would be reasonable to look through his sections 1 and 2 to get a sense for his contribution.)

Data file: USRealGDPPerCapita.csv

Read in the data file and create a time series object `Y` for the log of the quarterly seasonally adjusted real per capita GDP series from 1947q1 to 2019q4.

1. Perron's paper uses a different parameterisation of a break in trend from that we have earlier used when modelling retail sales. We will begin by relating these two parameterisations. For the first break for the oil price shock in 1973, Perron's set up (which is quite common for this situation) can be written

$$Y_t = \beta_0 + \beta_1 \,\text{Time}_t + \beta_3 \, DU_{1,t} + \beta_4 \, DT_{1,t} + Z_t \qquad \textbf{(1 Break)}$$

where

$$DU_{1,t} = \begin{cases} 0, & \text{Time}_t \le 1973 \\ 1, & \text{Time}_t > 1973 \end{cases} \quad DT_{1,t} = \begin{cases} 0, & \text{Time}_t \le 1973 \\ \text{Time}_t, & \text{Time}_t > 1973 \end{cases}$$

a. Set up the Time_t variable and estimate the trend regression

$$Y_t = \beta_0 + \beta_1 \,\text{Time}_t + Z_t \qquad \textbf{(Linear)}$$

Report the coefficient estimates, and create a time series plot of `Y` with the fitted trend line included. Also create a plot of the residuals from the regression, i.e. the deviations from the trend.

b. Set up the $DU_{1,t}$ and $DT_{1,t}$ variables and estimate the trend regression (1 Break). Report the coefficient estimates, and create a time series plot of `Y` with the fitted trend line included. Also create a plot of the residuals from the regression, i.e. the deviations from the trend.

c. Now consider the trend model with two breaks:

$$\begin{aligned} Y_t = \beta_0 + \beta_1 \,\text{Time}_t + \beta_3 \, DU_{1,t} + \beta_4 \, DT_{,t} \\ + \beta_5 \, DU_{2,t} + \beta_6 \, DT_{2,t} + Z_t \end{aligned} \qquad \textbf{(2 Breaks)}$$

where the GFC break dummy variables are

$$DU_{2,t} = \begin{cases} 0, & \text{Time}_t \le 2008 \\ 1, & \text{Time}_t > 2008 \end{cases} \quad DT_{2,t} = \begin{cases} 0, & \text{Time}_t \le 2008 \\ \text{Time}_t, & \text{Time}_t > 2008 \end{cases}$$

Set up the $DU_{2,t}$ and $DT_{2,t}$ variables and estimate the trend regression **(2 Breaks)**. Report the coefficient estimates, and create a time series plot of `Y` with

$(2\ \text{Breaks})$. Report the coefficient estimates, and create a time series plot of Y with the fitted trend line included. Also create a plot of the residuals from the regression, i.e. the deviations from the trend.

d. The trend regression we used for retail sales was restricted to enforce continuity at the breakpoint. Regressions $(1\ \text{Break})$ and $(2\ \text{Breaks})$ do not enforce continuity, instead they are generalised to permit a "jump" up or down at the breakpoint(s). (If required, see Figures 2 and 3 of Perron for a visual comparison.)

i. What parameter restriction is required to enforce continuity at a breakpoint? In particular give the conditions required for continuity at 1973 and 2008.

ii. Show how imposing this parameter restriction produces a trend specification that has the same general form as that used for the GFC trend break in the retail sales modelling earlier in semester.

iii. Use your answer to (i) to provide estimates of the extent of any "jumps" at 1973 and 2008 implied by your estimates of $(2\ \text{Breaks})$. Do the results support Perron's imposition of continuity of the trend function at 1973? What about 2008?

2. Perron's theoretical contribution to unit root testing was to demonstrate the dependence of the critical values of the ADF test on the specification of the break in the trend function (the type and timing of break). His results do not cover the case of $(2\ \text{Breaks})$, so to implement a unit root test with this deterministic trend specification it is necessary to use simulation methods to obtain critical/p-values.

We will carry out unit root tests with trend specifications (Linear), $(1\ \text{Break})$ and $(2\ \text{Breaks})$. To this end define

$$
\begin{aligned}
X_t^{(0)} &= (\ 1,\ \text{Time}_t\ )' \\
X_t^{(1)} &= (\ 1,\ \text{Time}_t,\ DU_{1,t},\ DT_{1,t}\ )' \\
X_t^{(2)} &= (\ 1,\ \text{Time}_t,\ DU_{1,t},\ DT_{1,t}\ ,\ DU_{2,t},\ DT_{2,t}\ )'
\end{aligned}
$$

so that the three trend specifications can be represented

$$
Y_t = X_t^{(j)\prime}\beta^{(j)} + Z_t, \qquad \textbf{(Trend)}
$$

for $j = 0, 1, 2$ respectively corresponding to (Linear), $(1\ \text{Break})$ and $(2\ \text{Breaks})$. The specification for Z_t will as usual be an AR model

$$
Z_t = \phi_1^{(j)} Z_{t-1} + \ldots + \phi_p^{(j)} Z_{t-p^{(j)}} + U_t. \qquad \textbf{(AR)}
$$

In practice it is possible that the lag order p may differ for different trend specifications. This is especially likely if there is misspecification in any trend function, for example if (Linear) omits important breaks that leaves additional autocorrelation in Z_t that needs to be approximated by additional lags. Even if the lag order does not vary with j, the values of the coefficients $\phi_1, \ldots, \phi_p$ likely will vary.

a. For each $j = 0, 1, 2$, carry out a lag order specification search in the usual way to choose AR lag orders $p^{(j)}$.

b. For each $j = 0, 1, 2$, compute the ADF t test statistic for a unit root in Z_t.

c. For each $j = 0, 1, 2$, carry out a simulation of the null distribution of the ADF t statistic. Compute the 5% critical value of the test for each j.

d. Use your simulation from part (c) to compute p-values for each of the three ADF statistics.

e. Conclude the unit root test for each of the three cases.

f. Discuss implications of your conclusions in (e).