Tutorial 12 Answers

1. The code for the VARs, variance matrix and impulse responses:

```
library(vars)

# VAR(3)
VAR_TGY <- VAR(DY, p=3)
Sigma <- summary(VAR_TGY)$covres
C <- chol(Sigma)
print(round(C,4))
```

```
        DTax   DGovt   DGDP
DTax  2.1271 -0.0938 0.1192
DGovt 0.0000  1.5139 0.1443
DGDP  0.0000  0.0000 0.5724
```

```
# Compute the full set of Impulse Reponses using VAR_TGY:
IRF <- irf(VAR_TGY, ortho=TRUE, n.ahead=8)
```

```
Impulse:     DTax                   DGovt                  DGDP

      DTax  DGovt   DGDP     DTax  DGovt   DGDP    DTax  DGovt  DGDP
 0    2.127 -0.094  0.119    0.000  1.514  0.144   0.000  0.000 0.572
 1   -0.533 -0.086  0.005   -0.138 -0.088 -0.032   0.580 -0.108 0.178
 2    0.508 -0.160  0.005   -0.345  0.243 -0.029   0.638 -0.030 0.176
 3    0.006 -0.162 -0.079   -0.065  0.290 -0.072   0.131 -0.129 0.120
 4    0.009 -0.020  0.012   -0.248  0.061 -0.037   0.361 -0.142 0.058
 5    0.027 -0.097 -0.025   -0.182  0.132 -0.031   0.166 -0.102 0.030
 6    0.029 -0.024  0.001   -0.077  0.097 -0.034   0.124 -0.094 0.031
 7    0.004 -0.022  0.000   -0.133  0.069 -0.015   0.109 -0.090 0.012
 8    0.025 -0.025  0.003   -0.074  0.064 -0.014   0.079 -0.058 0.013
```

2. With the reordering:

```
VAR_TGY <- VAR(DY[,c("DGDP", "DGovt","DTax")], p=3)
Sigma <- summary(VAR_TGY)$covres
```

Technically we don't need to re-estimate the VAR, we could simply reorganise the elements of the matrix Σ to match the re-ordering of the time series. But re-estimation seems easier and less error-prone!

```
C <- chol(Sigma)
print(round(C,4))
```

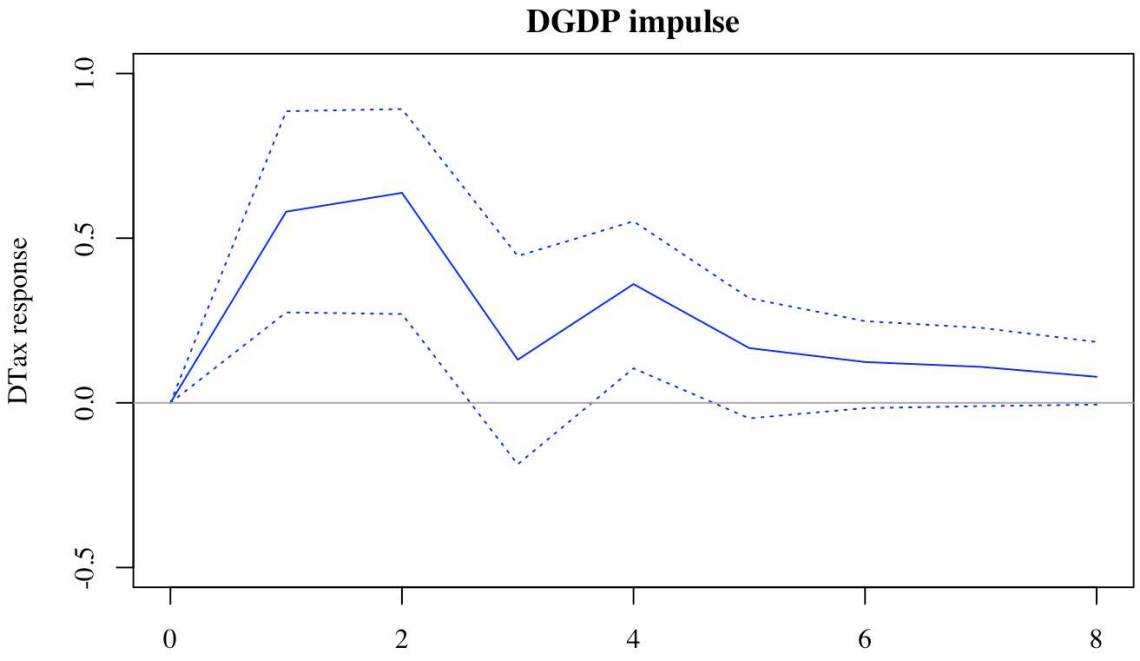
```
        DGDP  DGovt    DTax
DGDP  0.6022 0.3443  0.4212
DGovt 0.0000 1.4772 -0.2332
DTax  0.0000 0.0000  2.0719
```

```
# Compute the full set of Impulse Reponses using VAR_TGY:
IRF2 <- irf(VAR_TGY, ortho=TRUE, n.ahead=8)
```

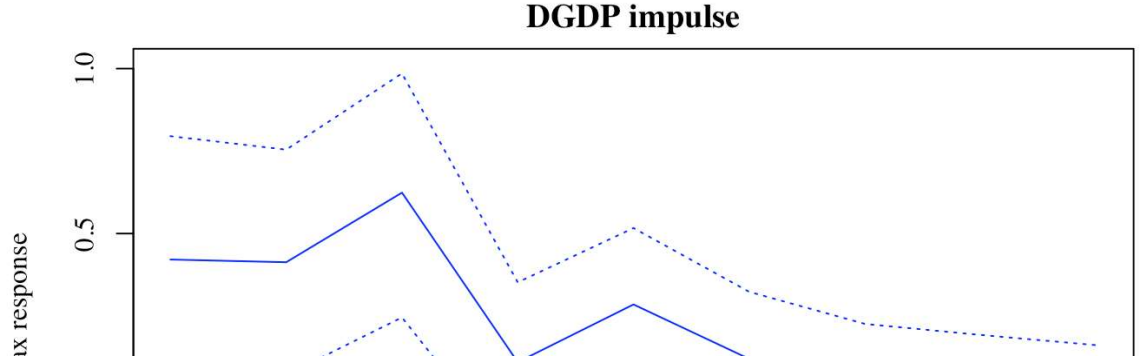
```
Impulse:     DTax                    DGovt                   DGDP

      DTax  DGovt   DGDP     DTax  DGovt   DGDP    DTax  DGovt  DGDP
0    2.072  0.000  0.000   -0.233  1.477  0.000   0.421  0.344 0.602
1   -0.654 -0.066 -0.036   -0.204 -0.052 -0.071   0.413 -0.141 0.163
2    0.335 -0.134 -0.036   -0.531  0.260 -0.067   0.624 -0.002 0.161
3   -0.026 -0.112 -0.107   -0.093  0.327 -0.088   0.110 -0.086 0.081
4   -0.085  0.015 -0.003   -0.321  0.093 -0.050   0.285 -0.124 0.049
5   -0.021 -0.065 -0.033   -0.216  0.161 -0.034   0.120 -0.085 0.016
6   -0.004  0.003 -0.008   -0.105  0.117 -0.039   0.105 -0.071 0.022
7   -0.028  0.002 -0.003   -0.154  0.090 -0.017   0.073 -0.073 0.008
8    0.003 -0.008 -0.001   -0.092  0.077 -0.017   0.062 -0.045 0.009
```

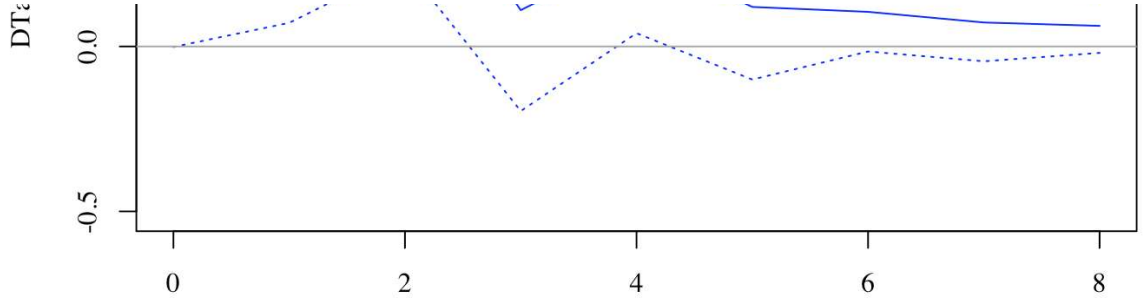
To aid comparisons these impulse responses are printed out with the same organisation as those in question 1. (There is no "right" way to print these out.) Notice the differences in the arrangements of the zeros in the impulses (the $h = 0$ row). For examle, with the re-ordering, the the `DTax` impulse is `(2.072, 0, 0)` to `(DTax, DGovt, DGDP)`, compared to `(2.127, -0.094, 0.119)`. Again there is not "right" or "wrong" about these, they are simply different identifying restrictions.

To show how comparisons of the impulse responses can be done, consider the responses of `DTax` forecasts to a `DGDP` impulse. With the question 1 ordering:



and with the question 2 ordering:

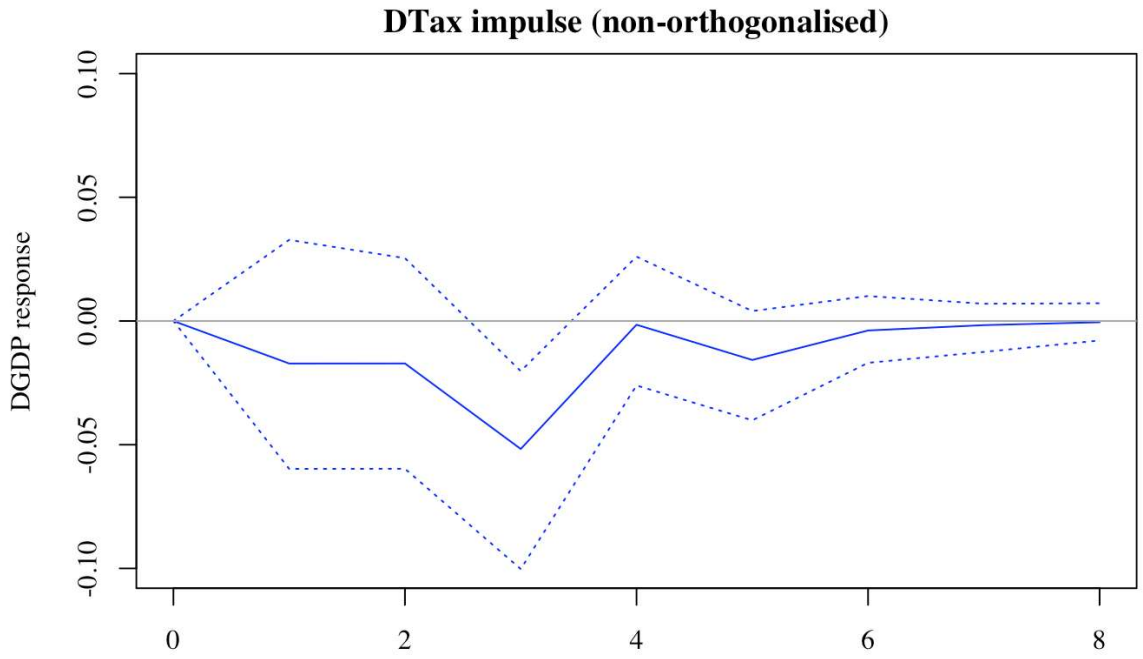




The main difference between these two responses is in the impulse itself, at $h = 0$. In the first ordering the impulse is zero because this is the identifying restriction, while in the second ordering the impulse is positive (0.421). For $h > 1$ the impulse response functions have very similar shapes and significances (the latter determined from whether or not the confidence intervals exclude zero for each h). A positive GDP growth shock is found to have a generally positive effect on tax revenue growth forecasts, especially in the first year (four quarters) the 2nd and 4th quarter forecasts are significantly higher. After one year the effect of the GDP impulse on Tax forecasts decays close enough to zero to be insignificant.

3. The non-orthogonalised impulse response functions:

```
VAR_TGY <- VAR(DY, p=3)
IRF0 <- irf(VAR_TGY, ortho=FALSE, n.ahead=8)
```

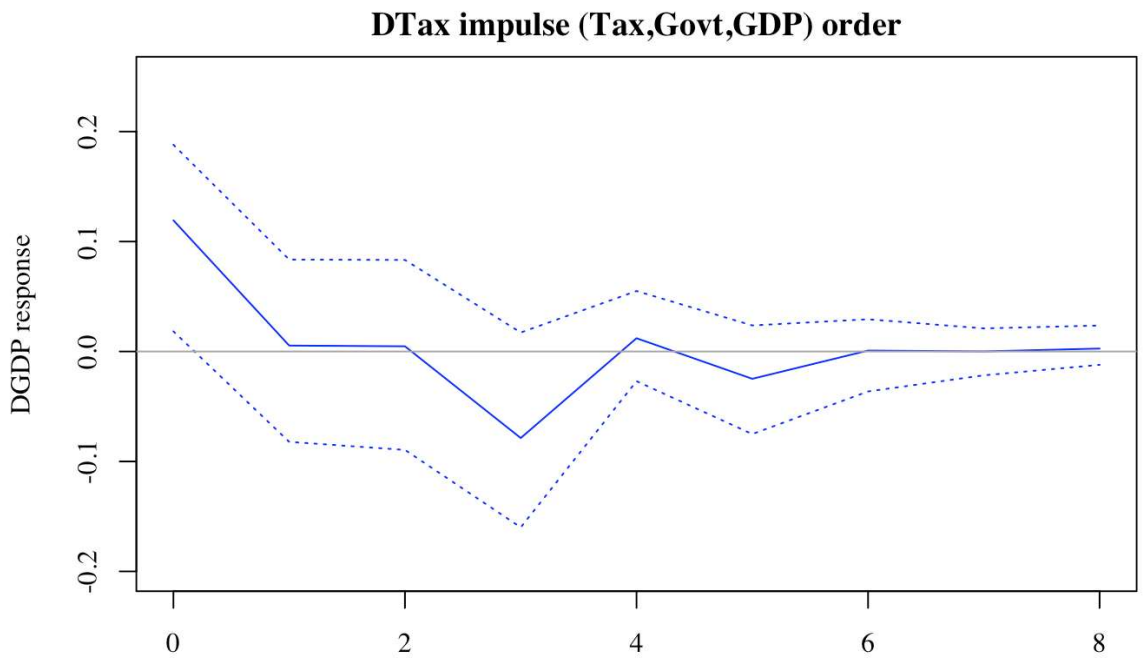


There is evidence of a significant downwards revision to growth forecasts 3 quarters after a +1 unit tax growth impulse. We may interpret this as some informal evidence for Granger causality from `DTax` to `DGDP`.

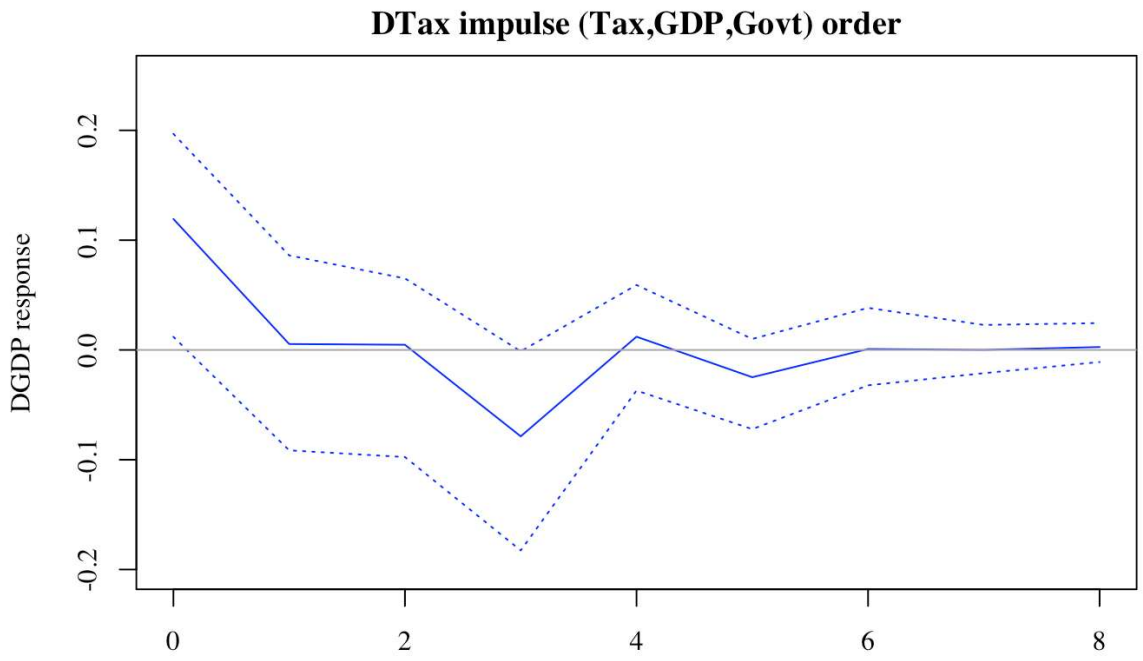
To what extent is this finding robust to impulse orthogonalisation and ordering? Here is a "brute force" approach — try every ordering and see what differences emergy. (We may call this a "sensitivity analysis" to the ordering.)

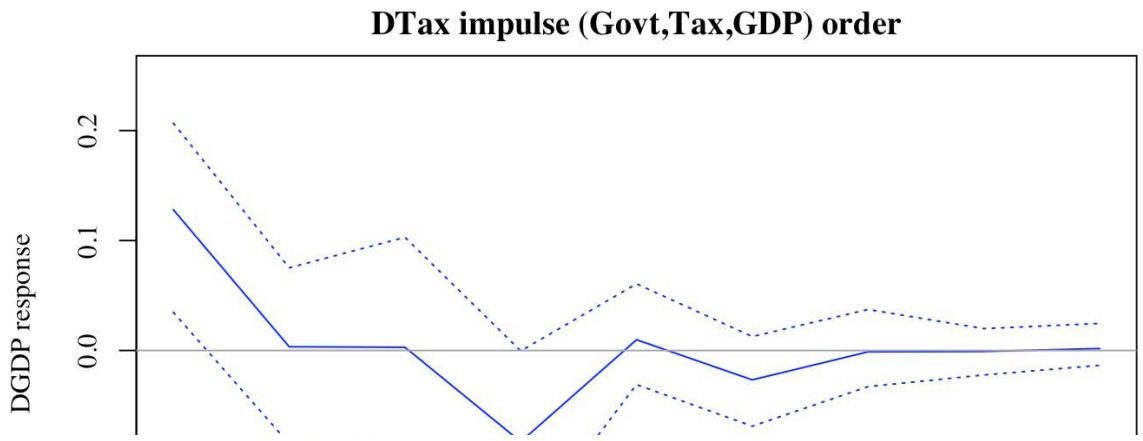
```
VAR_TGY <- VAR(DY[,c("DTax", "DGovt","DGDP")], p=3)
```

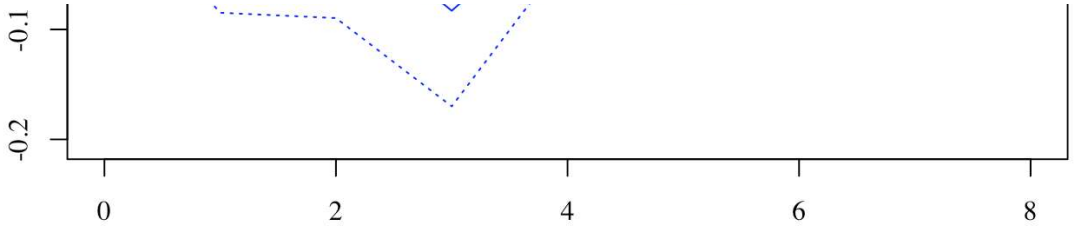
```
IRF1 <- irf(VAR_TGY, ortho=TRUE, n.ahead=8)
```

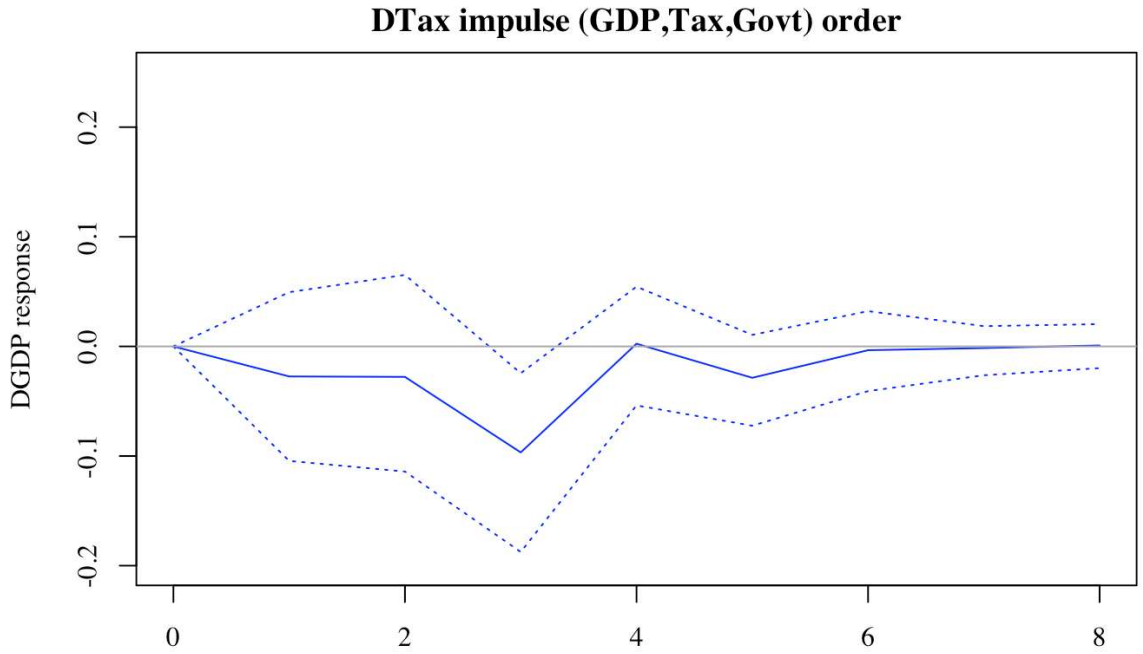


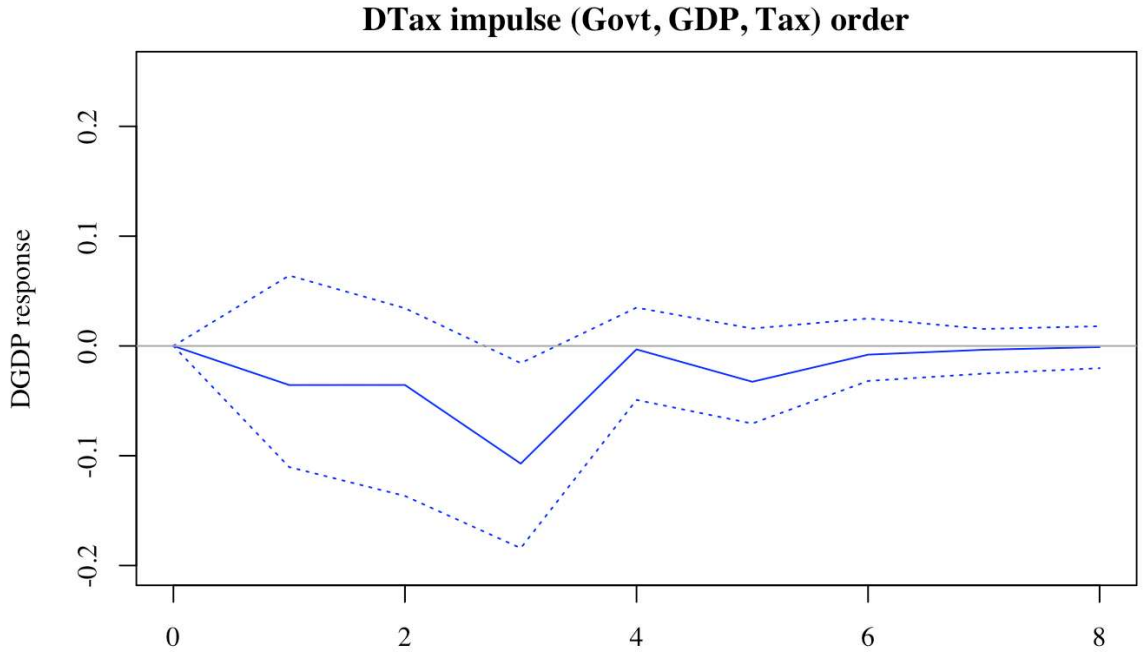

```
VAR_TGY <- VAR(DY[,c("DTax","DGDP", "DGovt")], p=3)
```

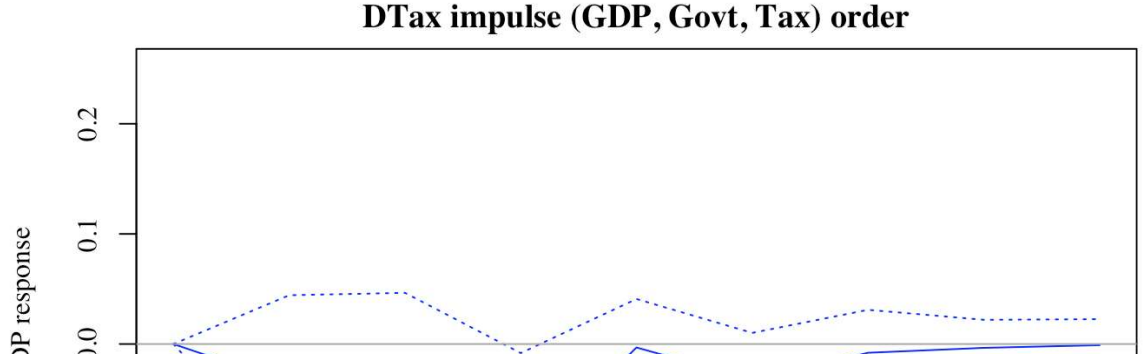


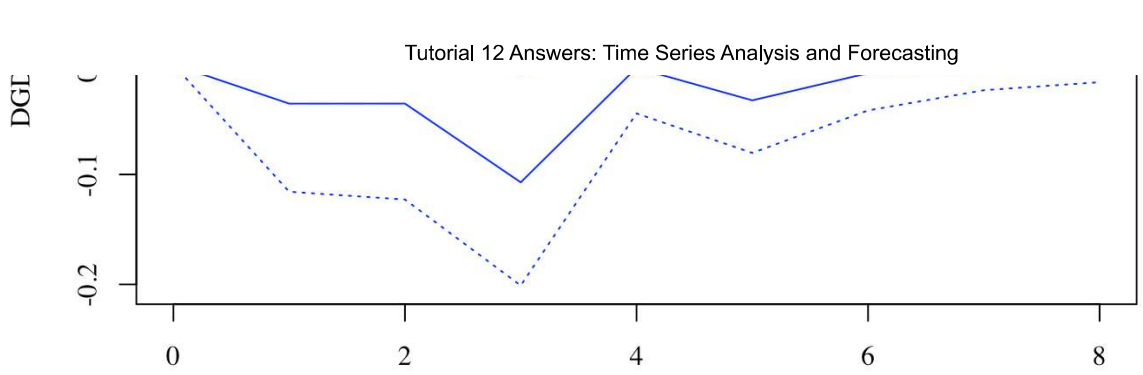



-0.1
-0.2
0
2
4
6
8


DTax impulse (GDP,Tax,Govt) order
DGDP response
0.2
0.1
0.0
-0.1
-0.2
0
2
4
6
8


DTax impulse (Govt, GDP, Tax) order
DGDP response
0.2
0.1
0.0
-0.1
-0.2
0
2
4
6
8


DTax impulse (GDP, Govt, Tax) order
0.2
0.1



If a Tax impulse is restricted by the ordering to have zero contemporaneous impulse to GDP then there is some evidence (two out three orderings) of a negative response of GDP at the 3rd quarter. If the ordering permits the Tax impulse to have a contemporaneous GDP impulse (which turns out to be positive), finding of a significant response at the third quarter occurs in only one instance, although directionally the negative response remains clearest at this horizon. So there is not a definitive answer to this question, but we may say there is some mixed evidence for tax increases having a limited negative effect on future GDP growth (noting that Government spending is included and therefore controlled in this analysis).

Is there an agreed "correct" ordering? Generally not, and this type of sensitivity analysis is one approach. In this context macroeconomists may also attempt to order variables according to the plausibility or otherwise of contemporaneous effects. Examples:

- An increase in government spending definitionally increases GDP (Y=C+I+G+X-M), so putting `DGovt` prior to `DGDP` in the ordering could make sense on these grounds. Conversely economic growth increases may take some time to result in lower government welfare and support expenditure (eg unemployment benefits), which may be another argument for this ordering.

- An increase in economic growth can contemporaneously increase tax revenue to the extent that higher sales and wages may immediately (i.e within 3 months) result in higher tax revenue through PAYG and GST mechanisms, even though other indirect taxes (eg company tax) could take longer and be reflected in later impulse responses. This would imply `GDP` could come before `Tax` in the ordering.

Putting these together would imply a (Govt,GDP,Tax) ordering. But of course this logic will not be agreed upon by everybody, and pragmatically it is valuable to check the sensitivity of the results to the ordering. Also results can be importantly dependent on the choice of variables in the model — expanding beyond this small collection of time series may lead to different results (plus many more potential orderings!). This is just scratching the surface of the work on identifying impulses and conducting inference on subsequent responses…