ECON30009/90080 – TUTORIAL 1

This Version: Semester 2, 2025

These tutorial questions are designed to give you a brief math review using tools and solution methods that we will use later on in our subject. It is highly recommended that you attempt these questions before attending your tutorial class. If you find these exercises difficult, or if it has been a long time since you have thought about differentiation or optimization, then there is a wide range of textbooks and other online resources you can use to revise your knowledge. One I would recommend is Alpha Chiang's textbook "Fundamental methods of Mathematical Economics " (any version), especially the chapters on comparative statics, differentiation and optimization (you don't need to read the whole book)

ANSWERS IN RED

Simple math rules

Write down answers to the following:

a) Show what $\exp(\ln x)$ is equal to

$$\exp(\ln x) = x$$

b) Leaving your answer in natural logs, show what $\ln(6) - \ln(2)$ is equal to

$$\ln(6) - \ln(2) = \ln(6/2) = \ln 3$$

c) Write down the derivative of $\ln x$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

d) Write down the derivative of $\ln a^x$ wrt x.

$$\frac{d}{dx} \ln a^x = \frac{d}{dx}[x \ln a] = \ln a$$

e) Write down the derivative of $\exp(ax)$

$$\frac{d}{dx}\exp(ax) = a\exp(ax)$$

f) For the following function $f(x, y) = (x^\alpha + y^\alpha)^{1/\alpha}$, show what $\frac{\partial^2 f(x,y)}{\partial x \partial y}$ is

$$\frac{\partial f(x,y)}{\partial x} = \frac{1}{\alpha}\left(x^\alpha + y^\alpha\right)^{\frac{1-\alpha}{\alpha}} \alpha x^{\alpha-1} = \left(x^\alpha + y^\alpha\right)^{\frac{1-\alpha}{\alpha}} x^{\alpha-1}$$

and differentiating again with respect to y

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} = \frac{1-\alpha}{\alpha}\left(x^\alpha + y^\alpha\right)^{\frac{1-2\alpha}{\alpha}} x^{\alpha-1}\alpha y^{\alpha-1} = (1-\alpha)\left(x^\alpha + y^\alpha\right)^{\frac{1-2\alpha}{\alpha}} (xy)^{\alpha-1}$$

Properities of utility functions

In class, we represent the household's preferences with utility. We assume that the household has a goal, to make himself or herself happy, and that the household does this by maximizing his/her utility.

For $x_1, x_2 > 0$ and $0 < \alpha < 1$, consider the following utility functions

a) Cobb-douglas utility:
$$U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$$

b) Log utility:
$$U(x_1, x_2) = \alpha \ln x_1 + (1-\alpha)\ln x_2$$

c) CRRA utility:
$$U(x_1, x_2) = \alpha\frac{x_1^{1-\sigma} - 1}{1-\sigma} + (1-\alpha)\frac{x_2^{1-\sigma} - 1}{1-\sigma} \quad \text{for } \sigma > 1$$

For each of the above utility functions, show that:

1) Utility is increasing in its arguments

2) Each additional unit of x_1 or x_2 provides a smaller increase in utility.

a) Cobb-douglas

1 Utility is increasing in its arguments:

$$\frac{\partial U(x_1, x_2)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{1-\alpha} > 0$$

$$\frac{\partial U(x_1, x_2)}{\partial x_2} = (1-\alpha) x_1^\alpha x_2^{-\alpha} > 0$$

2 Each additional unit of x_1 or x_2 provides a smaller increase in utility:

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_1^2} = \alpha(\alpha - 1)x_1^{\alpha-2}x_2^{1-\alpha} < 0$$

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_2^2} = -\alpha(1 - \alpha)x_1^{\alpha}x_2^{-\alpha-1} < 0$$

a) Log utility

1 Utility is increasing in its arguments:

$$\frac{\partial U(x_1, x_2)}{\partial x_1} = \frac{\alpha}{x_1} > 0$$

$$\frac{\partial U(x_1, x_2)}{\partial x_2} = \frac{1-\alpha}{x_2} > 0$$

2 Each additional unit of x_1 or x_2 provides a smaller increase in utility:

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_1^2} = -\frac{\alpha}{x_1^2} < 0$$

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_2^2} = -\frac{1-\alpha}{x_2^2} < 0$$

c) CRRA utility

1 Utility is increasing in its arguments:

$$\frac{\partial U(x_1, x_2)}{\partial x_1} = \alpha x_1^{-\sigma} > 0$$

$$\frac{\partial U(x_1, x_2)}{\partial x_2} = (1-\alpha)x_2^{-\sigma} > 0$$

2 Each additional unit of x_1 or x_2 provides a smaller increase in utility:

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_1^2} = -\alpha\sigma x_1^{-(1+\sigma)} < 0$$

$$\frac{\partial^2 U(x_1, x_2)}{\partial x_2^2} = -(1-\alpha)\sigma x_2^{-(1+\sigma)} < 0$$

Budget constraints

Using the following pieces of information. Write down what the lifetime budget constraint of the individual looks like. Some notation that we will use: c_1 is consumption in period 1, c_2 is consumption in period 2. y_1 is income in period 1, y_2 is income in period 2. Let $R = (1 + r)$ be the gross rate of return on savings s.

a) Suppose $y_2 = 0$, $y_1 > 0$. Write down the budget constraint in period 1, write down the budget constraint in period 2. Also derive the lifetime budget constraint. Draw what the lifetime budget constraint looks like in (c_1, c_2) space. On your graph, label the endowment point E, which is the point if individuals had zero savings and consumed out of their income in that period only. On the same graph, mark out the highest possible consumption the household could have in period 1. On the same graph, mark out the highest possible consumption the household could have in period 2. Finally, write down what the slope of the lifetime budget constraint is.

   – budget constraint in period 1

$$c_1 + s = y_1$$

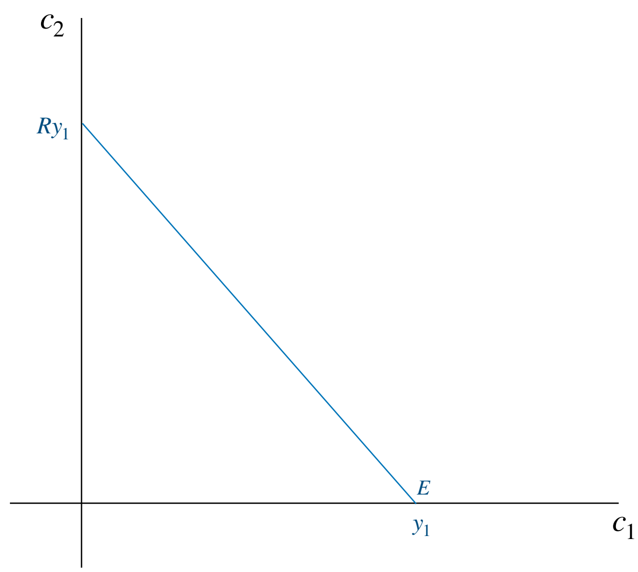
   – budget constraint in period 2

$$c_2 = Rs$$

   – lifetime budget constraint

$$c_1 + \frac{c_2}{R} = y_1$$

   – Slope of budget constraint: $\frac{dc_2}{dc_1} = -R$



b) Suppose $y_1 = 0, y_2 > 0$. In addition, households face a lump-sum tax T_2 in period 2. Write down the budget constraint in period 1, write down the budget constraint in period 2. Also derive the lifetime budget constraint. Draw what the lifetime budget constraint looks like in (c_1, c_2) space, and mark out the same points as in part a).

– budget constraint in period 1

$$c_1 + s = 0$$

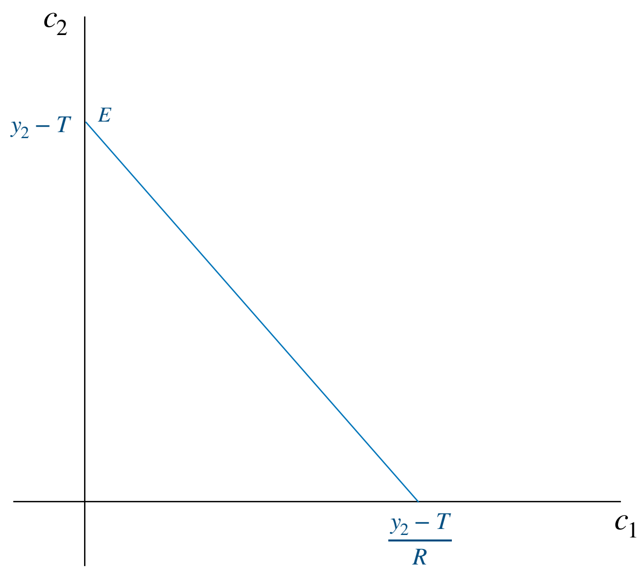
– budget constraint in period 2

$$c_2 = y_2 - T + Rs$$

– lifetime budget constraint

$$c_1 + \frac{c_2}{R} = \frac{y_2 - T}{R}$$

– Slope of budget constraint: $\frac{dc_2}{dc_1} = -R$


c_2
$y_2 - T$
E
$\frac{y_2 - T}{R}$
c_1