FNCE90056: Investment Management

Lecture 4: Empirical Evidence on the CAPM

A/Prof Andrea Lu and Dr Jun Yu

Department of Finance
Faculty of Business and Economics
University of Melbourne

Last time

1. Visualising the efficient frontier
2. Beyond Modern Portfolio Theory – the CAPM
3. Estimating β
4. β for a new firm
5. Rolling β estimation

Today's class

The Amsterdam Stock Exchange, now called Euronext Amsterdam, is considered the oldest stock exchange in the world. It was established in 1602 by the Dutch East India Company to trade its stocks and bonds. Too bad we do not have good data from it — testing the CAPM might be easier.

- The CAPM gives us a simple and elegant way to model the relationship between risk and return.
- The CAPM makes a testable prediction about every asset:

$$\mathbb{E}[r_i] - r_f = \beta_i (\mathbb{E}[r_m] - r_f) \quad (1)$$

- This is "easy" to implement. Estimate via OLS:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \epsilon_{i,t} \quad (2)$$

- Does it "work"?
  - Is $\widehat{\alpha}_i = 0$ for all assets?
  - Do the $\widehat{\beta}_i$ (completely) explain return variation across assets?

Theory vs. practice

- The CAPM is the security market line (SML), relating expected returns (y-axis) to β (x-axis).
- Unfortunately, we don't get to directly observe expected returns: $\mathbb{E}[r_i]$.
- We can't observe β_i either. It has to be estimated (with error).
- How to proceed?

The Market Model

The Market Index models

Let's think about actual returns (which we do observe) instead of expected returns. Write the actual returns, r_i and r_m, as the expected returns plus noise terms, ν_i and ξ — completely random "surprises" that are zero on average:

$$\begin{aligned} r_{i,t} = \mathbb{E}[r_i] + \nu_{i,t} \implies \mathbb{E}[r_i] = r_{i,t} - \nu_{i,t} \\ r_{m,t} = \mathbb{E}[r_m] + \xi_t \implies \mathbb{E}[r_m] = r_{m,t} - \xi_t \end{aligned} \tag{3}$$

(1) tells us what $\mathbb{E}[r_i]$ is supposed to be. Combining (1) and (3), we get:

$$(r_{i,t} - \nu_{i,t}) - r_{f,t} = \beta_i ((r_{m,t} - \xi_t) - r_{f,t}) \tag{4}$$

Define $\epsilon_{i,t} \equiv \nu_{i,t} - \beta_i \xi_t$ and re-arrange (4) a little bit to get the basic **single index model** (or **market model**):

$$r_{i,t} - r_{f,t} = \beta_i (r_{m,t} - r_{f,t}) + \epsilon_{i,t} \tag{5}$$

Why we like market index models

- They're simple, and help us to distinguish between two kinds of risk.
  - **systematic** risk: un-diversifiable risk due to the asset's exposure to the aggregate market/economy
  - **idiosyncratic** risk: diversifiable risk due to the asset's firm-specific exposure

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + \epsilon_{i,t} \tag{2}$$

$$\sigma_i^2 = \underbrace{\beta_i^2 \sigma_m^2}_{\text{systematic}} + \underbrace{\sigma_\epsilon^2}_{\text{idiosyncratic}} \tag{6}$$

- According to the CAPM, we should only be compensated for systematic risks, i.e. covariance with the market.
- They also help us to estimate α_i, any non-zero average return that the index/market cannot "explain". This is the vertical distance (residual) between an asset and the security market line.

Background

- **The CAPM is the most commonly used model for determining the appropriate compensation for risk.** With applications:
  - Portfolio management (managing risk)
  - Evaluating portfolio managers (rank them by alpha)
  - Corporate project valuation ($E[r]$ = discount rate)
  - Cost-of-capital determination in both corporate and regulatory settings
- **Most of the evidence we will see challenges the validity of the CAPM.**
  - The "failure" of the CAPM can be demonstrated by finding assets/portfolios for which (1) does not hold.
  - Identifying these "anomaly" portfolios helps to guide the research into new asset pricing models.
- **What we discuss today is merely the tip of the iceberg.** Properly testing the CAPM is surprisingly difficult, and seemingly contradictory evidence can often be reconciled.

Testing the CAPM

Implications of the CAPM

To see if the CAPM "works", we focus on testing the implications of the CAPM. Namely, if it is true, then all securities should lie on the SML:

$$\mathbb{E}\left[r_i\right] - r_f = \beta_i \left(\mathbb{E}\left[r_m\right] - r_f\right) \tag{1}$$

- The relation between $E[r_i]$ and β_i is linear.
- Only β_i explains differences in expected returns among securities.
- An asset with $\beta = 0$ should have an expected return equal to the risk-free rate.
- An asset with $\beta = 1$ should have an expected return equal to the market.

β is unobservable

Can you see a β in the market without estimation? No...

$$\mathbb{E}[r_i] - r_f = \beta_i (\mathbb{E}[r_m] - r_f) \tag{1}$$

The CAPM relates two unobservable quantities, $\mathbb{E}[r_i]$ and $\mathbb{E}[r_m]$, via an unobservable ratio, $\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2}$.

The CAPM can't be tested without a way to measure betas.

That's why we estimate beta (and alpha) using:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \epsilon_{i,t} \tag{2}$$

This results in $\widehat{\alpha}_i$ and $\widehat{\beta}_i$, estimates of the true parameters, α_i and β_i. (But that's as close as we're going to get.)

Other issues

- What should we use for the market portfolio? Usually a value-weighted equity index (S&P 500, ASX 200) but ignores issues like:
  - Only 1/3 of non-government tangible assets are owned by the corporate sector.
  - Among the corporate assets, only 1/3 are financed by equity.
  - What about intangible assets such as human capital?
  - International markets?
- Roll (1977) argues that without a precisely measured market proxy, we cannot properly test the CAPM. (And therefore, any "failures" may be due to an imprecise proxy.) This became known as the "Roll critique".
- Shanken (1987) shows that, unless you believe the market proxy to be exceptionally poor ($\rho < 0.7$), we can still reject the CAPM as an exact model of asset returns.
- Measurement errors in β.

Initial Results

Fama and MacBeth (1973)

Fama and MacBeth (1973) regressions provide a way to empirically test the validity of the CAPM:

1. Estimate (2) to get $\widehat{\alpha}_i$ and $\widehat{\beta}_i$ for a given asset. This is known as a **time-series test**, because (2) is a regression for a single asset's returns *over time*.
2. Using $\widehat{\beta}_i$ estimated from the first step, see if they explain the differences in returns *across* stocks. This is a **cross-sectional test** of CAPM. For <u>each month</u>, t, we estimate a_t and b_t in the regression

$$r_{i,t} - r_f = a_t + b_t \widehat{\beta}_i + \delta_{i,t} \tag{7}$$

   - the left-hand-side variable is the excess returns of all assets
   - the right-hand-side variable is $\widehat{\beta}_i$, the estimated betas of all assets

- Finally, compute the averages: $\widehat{a} = \frac{1}{T}\sum_{t=1}^{T} \widehat{a}_t$ and $\widehat{b} = \frac{1}{T}\sum_{t=1}^{T} \widehat{b}_t$
- If the CAPM is true, $\widehat{a}$ should be zero and $\widehat{b}$ should be the risk premium on the market. Compare (1) and (7) to see why this is true.

Size Effect

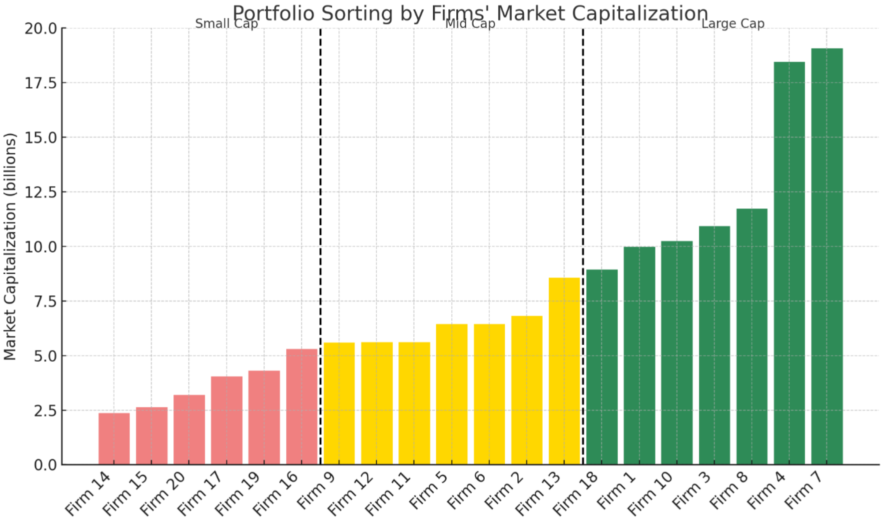
"Discovery" 1: size

Another way to interpret the CAPM (see (1) again) is that only market beta should explain expected returns.

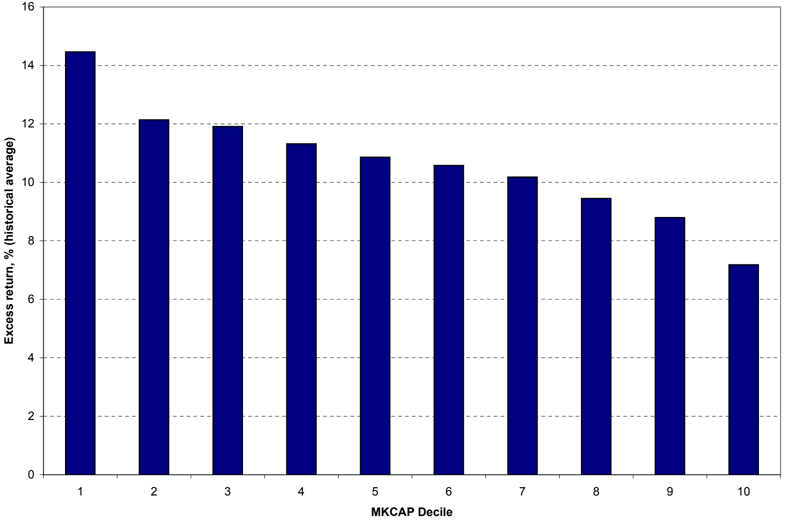
Keim (1981) and Banz (1981) found that firm size, defined as total market capitalisation, also explained differences in expected returns.

- In particular, small-cap stocks tended to outperform large cap stocks in the data.
- To get a handle on this effect, we can build portfolios of stocks by sorting stocks based on past market capitalisations.
    1. We use *past* firm size, because we can't form portfolios for month t using information that only becomes available during month t. But firm size in month $t-1$ is nearly as good — firm size is quite persistent.
    2. We use *portfolios* as "test assets", because expected returns are measured more accurately for portfolio than for individual stocks.

Univariate Portfolio Sorting on Size

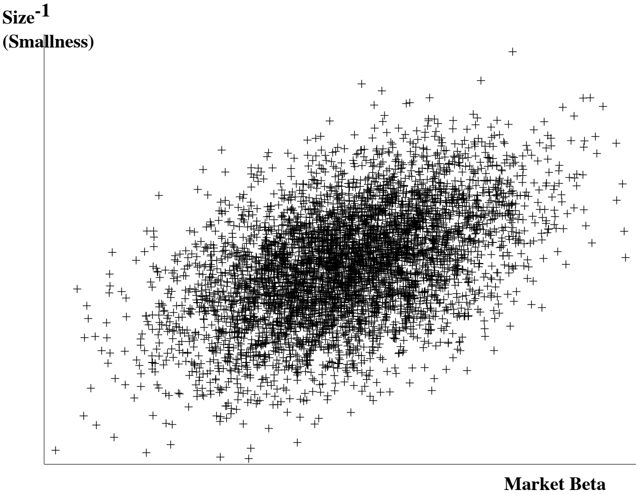


Size effect: returns

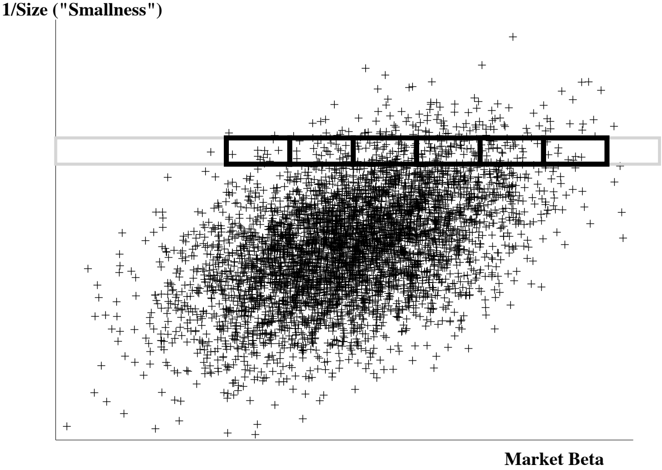


Is this consistent with the CAPM? Does beta explain these differences?

Size vs. β



Double sorts



Fama and French (1992) form test portfolios using a double sort, first on size and then on $\widehat{\beta}$.

Fama and French (1992)

Fama and French (1992) find that the relation between average returns and β __within__ a size decile is generally negative.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

This table is the ultimate source for many news stories and research proclaiming that beta is dead.

Issues

- Other measures of firm size such as book value of assets or number of employees have no predictive power.
- The size effect has become weaker since 1980.

Value Effect

"Discovery" 2: value

Graham and Dodd (1934), in *Security Analysis*, noticed that value stocks tend to outperform growth stocks on average.

- A **value stock** is a stock with a low market price relative to the book value of assets.
- A **growth stock** is a stock with a high market price relative to the book value of assets.
- Value stocks are characterised by some as being "undervalued" by the market, while growth stocks are characterised as being "glamour" stocks that are "overvalued."[1]
- Later work argues that these two types of stocks simply have different risk characteristics.[2]

[1] We will talk more about market efficiency later in the lecture, but note that language like this presumes some degree of *inefficiency*.

[2] See, in particular, Fama and French (1996).

Value vs. β

- Suppose we sort all stocks into 10 portfolios based on the ratio of the book value of equity to the market value of equity, with rebalancing occurring yearly.
- Here are the characteristics of these portfolios:

	10 portfolios sorted on book-to-market equity, 1926-2007										
Sort	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R_i) - r_f$	6.77	8.01	7.85	7.98	8.93	9.37	9.66	11.41	12.12	13.17	6.40
	(2.22)	(2.13)	(2.07)	(2.34)	(2.18)	(2.39)	(2.59)	(2.70)	(2.95)	(3.62)	(2.57)
σ	19.98	19.16	18.59	21.06	19.60	21.46	23.29	24.22	26.48	32.49	23.10
α	-1.01	0.42	0.53	-0.23	1.38	1.13	0.92	2.46	2.43	1.97	2.98
(t)	(0.74)	(0.60)	(0.59)	(0.73)	(0.78)	(0.87)	(1.04)	(1.14)	(1.34)	(1.84)	(2.30)
β_{MKT}	1.01	0.98	0.95	1.06	0.98	1.06	1.13	1.16	1.25	1.45	0.44
(t)	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.04)	(0.06)	(0.07)	(0.05)	(0.10)	(0.11)
R^2(%)	89.80	92.74	91.86	89.94	88.02	87.23	83.44	80.80	79.38	70.35	12.89

- If CAMP hold, we have

$$E(R_{Hi}) - E(R_{Lo}) = \underbrace{\alpha_{Hi-Lo}}_{\text{should be } 0} + (\beta_{Hi} - \beta_{Hi})\left[E(R_m) - R_f\right]$$

- However, Hi-Lo long-short portfolio has significantly positive α

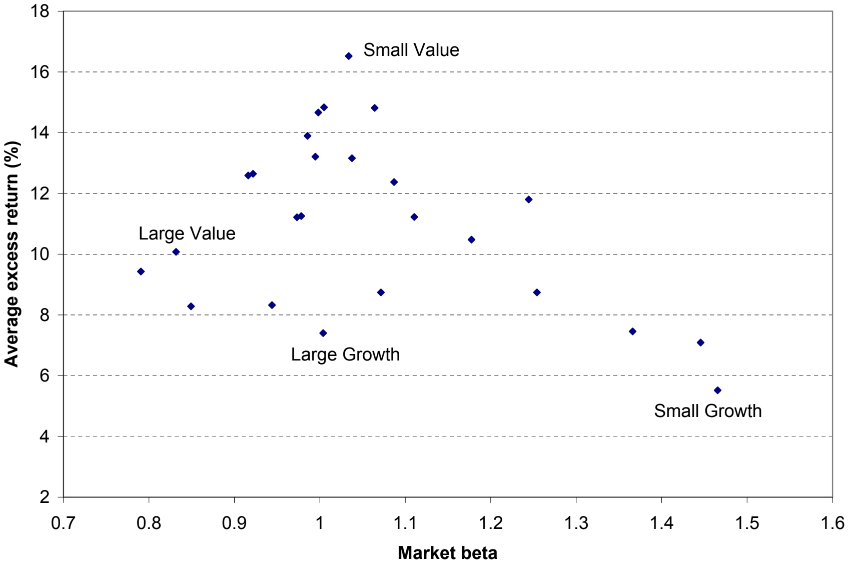
Value vs. β: post-1960 subsample

Since 1960, the results are even stronger.

	10 portfolios sorted on book-to-market equity, 1962-2007										
Sort	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R_i) - r_f$	3.95	5.59	6.07	6.29	6.28	7.28	8.33	8.67	9.64	11.11	7.16
	(2.67)	(2.42)	(2.39)	(2.37)	(2.22)	(2.21)	(2.21)	(2.19)	(2.37)	(2.73)	(2.28)
σ	18.01	16.35	16.14	16.00	15.01	14.90	14.92	14.78	16.01	18.46	15.39
α	-2.07	-0.04	0.54	0.95	1.38	2.38	3.63	4.05	4.66	5.71	7.78
(t)	(1.05)	(0.73)	(0.76)	(0.95)	(1.01)	(0.92)	(1.10)	(1.12)	(1.23)	(1.71)	(2.44)
β_{MKT}	1.10	1.03	1.01	0.97	0.89	0.90	0.86	0.84	0.91	0.99	-0.11
(t)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)
R^2(%)	86.02	91.34	90.27	85.63	81.92	83.31	76.58	75.20	74.27	65.82	1.07

- Hi-Lo long-short portfolio has significantly positive α:
  - meaning CAPM beta difference between Hi and Lo portfolio can not fully explain their expect return difference

Size and value (1962–2007)



A critique of size and value "anomalies"

Berk (1995) makes the following observations:

- The value of a growing perpetuity (initial dividend D which grows at rate g, discount rate R) is

$$P = \frac{D}{1+R} + \frac{(1+g)D}{(1+R)^2} + \frac{(1+g)^2 D}{(1+R)^3} + \cdots$$

$$P = \frac{D}{R-g} \implies R = g + \frac{D}{P}$$

- If the discount rate, R, is the expected return, then price and expected returns have an inverse relationship! This implies that the "size effect" is not an anomaly, but rather a predictable phenomenon.
- Also note that $\frac{D}{P} \approx \frac{B}{M}$ $(B_{t+1} = B_t + E_t - D_t)$, so there's room for a value effect, too.
  - Assuming that $\frac{D}{E} = p$ and $ROE \approx \frac{E}{B}$ are stable, $\frac{D}{P} \approx p \times ROE \times \frac{B}{M}$
  - Higher D/P means a higher R

What isn't the CAPM capturing?

Examples include:

- **Distress Risk.** Value stocks tend to be stocks that have underperformed in the past. They could be in financial distress making them riskier.
  - Financial distress refers to a situation where a company is struggling to meet its financial obligations – such as paying interest, repaying debt, or covering operating expenses, and there is a heightened risk of default or bankruptcy.
- **Liquidity Risk.** Small stocks are more illiquid and may thus command a higher premium.
  - Liquidity of an asset refers to how quickly and easily it can be converted into cash at (or close to) its fair market value without significantly affecting its price.

(suggesting that there are other systematic risks out there other than just the market risk...)

Unconditional vs. conditional CAPM

More importantly, we've been expressing the CAPM as an **unconditional** model:

$$\mathbb{E}\left[r_i\right] - r_f = \beta_i \left(\mathbb{E}\left[r_m\right] - r_f\right) \tag{1}$$

But this is a simplification. It is properly expressed as a **conditional** model:

$$\mathbb{E}_t\left[r_i\right] - r_{f,t} = \beta_{i,t} \left(\mathbb{E}_t\left[r_m\right] - r_{f,t}\right) \tag{8}$$

The expected returns and β are potentially time-varying. (The difference between (8) and (1) is all the t subscripts.)

This makes the true CAPM much harder to test. We can (and do) reject the unconditional CAPM, in (1), but this does not mean that the conditional CAPM, in (8), is not valid.

This is the Hansen and Richard (1987) critique.

Momentum Effect

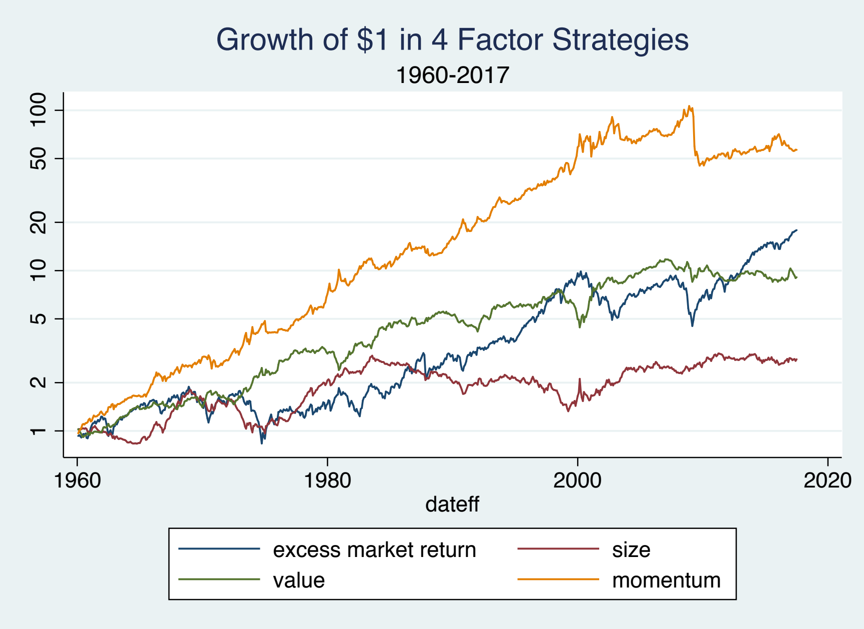
"Discovery" 3: momentum

Jegadeesh and Titman (1993) showed that firms with high (low) past returns tend to have high (low) returns in the subsequent months. They form portfolios based on past annual return with monthly rebalancing:

	10 portfolios sorted on previous 12 month returns (1926-2007)										
Sort	Lo	2	3	4	5	6	7	8	9	Wi	Wi-Lo
$E(R_i) - r_f$	0.31	5.18	5.12	6.72	6.80	7.58	8.68	10.30	11.47	15.37	15.07
	(3.70)	(3.13)	(2.70)	(2.50)	(2.31)	(2.26)	(2.17)	(2.09)	(2.20)	(2.52)	(2.94)
σ	33.24	28.15	24.24	22.41	20.77	20.33	19.51	18.78	19.78	22.64	26.44
α	-11.52	-5.08	-3.91	-1.77	-1.18	-0.39	1.09	3.05	3.97	7.48	19.01
(t)	(1.65)	(1.31)	(1.12)	(0.94)	(0.79)	(0.66)	(0.70)	(0.70)	(0.81)	(1.33)	(2.44)
β_{MKT}	1.53	1.33	1.17	1.10	1.03	1.03	0.98	0.94	0.97	1.02	-0.51
(t)	(0.08)	(0.07)	(0.06)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.06)	(0.13)
R^2(%)	74.95	78.61	82.24	85.01	87.30	90.93	89.64	88.34	85.09	71.90	13.05

This is inconsistent with the CAPM, Hi-Lo long-short portfolio has significantly positive α:

- meaning CAPM beta difference between Hi and Lo portfolio can not fully explain their expect return difference


Growth of $1 in 4 Factor Strategies
1960-2017
100
50
20
10
5
2
1
1960
1980
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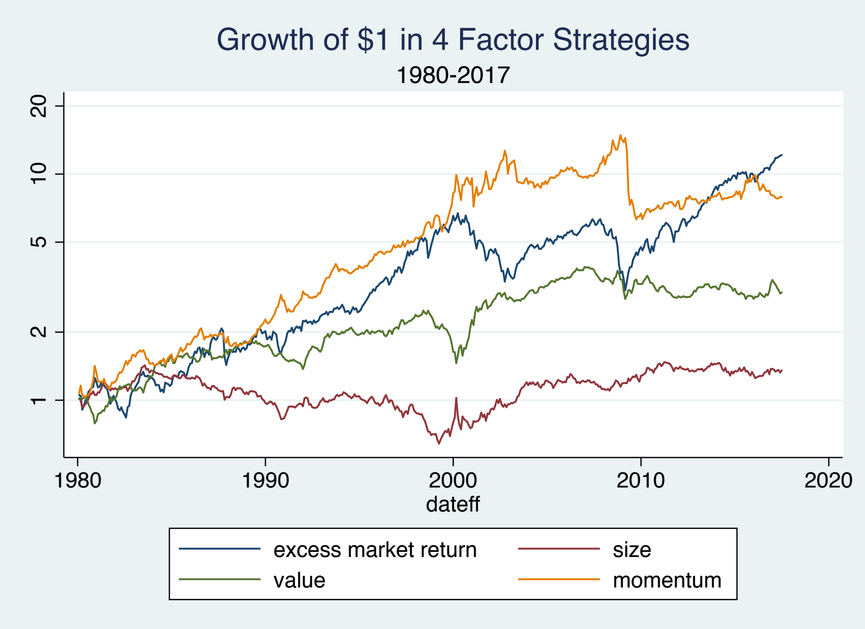
excess market return
size
value
momentum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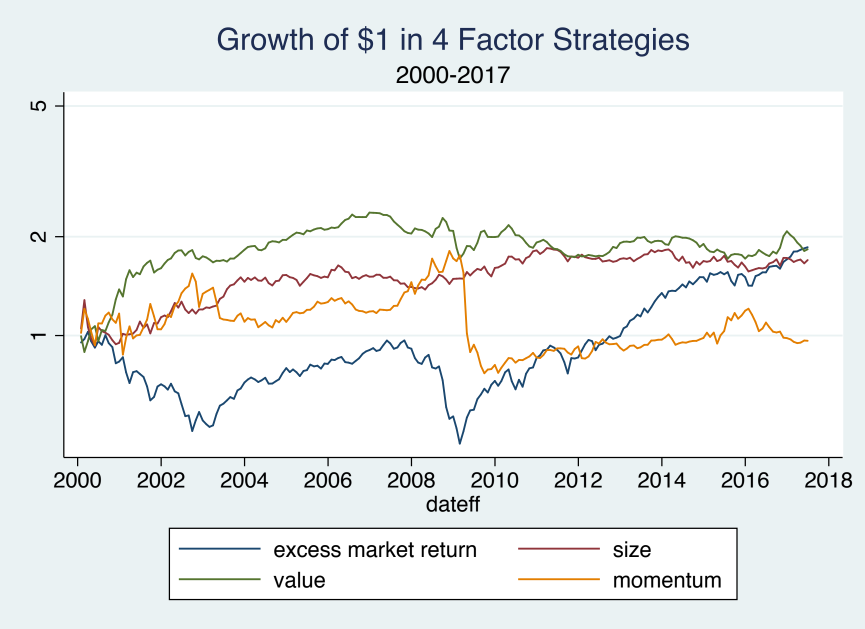
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2018
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Implications

Efficient Market Hypothesis (EMH)

Paul Samuelson (1965) and **Eugene Fama (1965)** laid the theoretical groundwork for the modern interpretation of market efficiency.

- Market efficiency means that asset prices incorporate available information about asset values
- Why should they (asset prices) reflect information?
  - because of competition and free entry.
  - If we could easily predict that stock prices will rise or decline tomorrow with some information, we'd all try to buy and sell today. Then today's prices will change until they reflect that information.
  - As a result, price changes between today and tomorrow can only be driven by unexpected new information (unknown today)
    - today's available information can not predict future prices / returns
    - then expected returns should only reflect risk premia, not predictable mispricings (relative to a benchmark of risk premium)

Efficient Market Hypothesis

Fama (1970) provides three forms of the Efficient Market Hypothesis (EMH) depending on the scope of available information:

1. **weak-form efficiency** asserts that current prices reflect all relevant information embedded in past prices. Technical analysis of trends, support levels, etc. should not be profitable.
   - price changes must be unpredictable from historical prices
2. **semistrong-form efficiency** asserts that current prices reflect all relevant publicly available information. Fundamental analysis of securities should not be profitable.
3. **strong-form efficiency** asserts that current prices reflect all relevant information, public or private. Insider trading should not be profitable.

EMH really is the intellectual foundation of modern empirical finance

Where does this leave us?

Expected returns are not just driven by CAPM beta: also by size, value, and momentum.

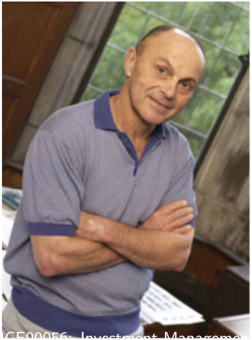
Is the CAPM completely useless?!

- No, but the CAPM does not capture all aspects of expected returns.
- These anomalies provide us a set of risk-return trade-offs that other asset pricing models need to deliver.
- There are some risk factors that are not properly accounted for by the CAPM. That is why we have multifactor risk-return models, our next topic.

Efficient markets vs. CAPM

- Failure of CAPM $\neq$ Inefficiency: CAPM can "fail" and markets can still be efficient.
  - Market efficiency means prices reflect all available information so that no trading strategy can systematically earn abnormal risk-adjusted returns (mispricing relative to a benchmark).
  - The CAPM is one specific equilibrium model linking expected returns to market beta, and is just one way of risk-adjustment (benchmark)
- And the "failure" is, in most cases, a test of the *unconditional* CAPM
  - Some folks get this wrong.
- In the next lecture, we will discuss **multifactor models** and the **arbitrage pricing theory** (APT), which similarly offer structured frameworks for understanding the relationship between risk and returns without abandoning market efficiency.

Eugene Fama



Nobel prize in 2013, for his (1970) work on efficient-market hypothesis

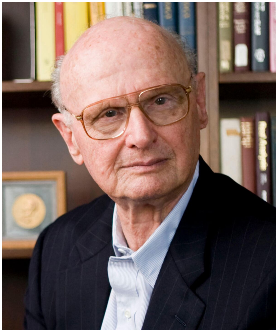
- Born in 1939 (age 84)
- PhD: U of Chicago (1964)
- Work: University of Chicago, Booth School of Business
- Industry: Dimensional Fund Advisors
- He proposed definitions for market efficiency and demonstrated that the notion of market efficiency could not be rejected without an accompanying rejection of the model of market equilibrium.

Lars Hansen

Nobel prize in 2013, for his (1982) work on econometric techniques to test modern financial models

- Born in 1952 (age 70)
- PhD: U of Minnesota(1978) (supervised by Christopher Sims)
- Work: U of Chicago
- Industry: Financial Engines (NASDAQ:FNGM)
- He developed the econometric technique generalized method of moments (GMM) and applied it to empirical analysis of asset prices.

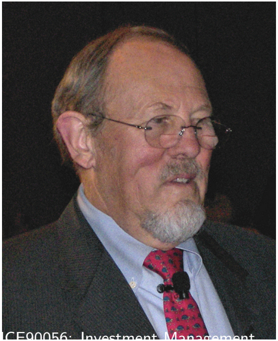
Harry Markowitz



Nobel prize in 1990, for his (1952) work on mean-variance portfolio optimization

- August 24, 1927 - June 22, 2023
- PhD: U of Chicago (1954)
- Work: UC San Diego
- Industry: Arbitrage Management Company (with Paul Samuelson and Robert Merton)
- He set out to measure the relationships among a diverse assortment of stocks to construct the most efficient portfolio, and to chart what he called a "frontier", where no additional return can be obtained without also increasing risk.

William Sharpe



Nobel prize in 1990, for his (1964) co-invention of the CAPM

- Born in 1934 (age 89)
- PhD: UCLA (1961) (supervised by Harry Markowitz)
- Work: Stanford
- Industry: Financial Engines (NASDAQ:FNGM)
- He developed the CAPM and Sharpe ratio.

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