

ECON30009/90080 – TUTORIAL 5

This Version: Semester 2, 2025

These questions are designed to give you practice solving the OLG model when government spending is exogenous.

Question 1: Government spending financed by a payroll tax

Consider the OLG model we discussed in class. There exists a government that needs to spend an exogenous amount $G_t = G$ each period. This government spending goes towards financing a public good that households get some utility from. Specifically let the preferences of a household be given by

$$U(c_t^y, c_{t+1}^o, G) = \ln c_t^y + \beta \ln c_{t+1}^o + \gamma \ln G$$

where $0 < \beta < 1$ and $\gamma > 0$. The government runs a balanced budget each period and fully finances its government spending within period with a payroll tax on firms, i.e., for each unit of labour employed, the firms pay $\tau_t^w w_t$ as tax revenue.

As per lecture slides, households work when young and retire when old. They inelastically supply 1 unit of labour when they are young, and receive wage and dividend income. Households when young can choose to save in an asset a_{t+1} and the gross return to a unit of savings is equal to $1 + r_{t+1}$. Households when old consume their savings. Firms use capital and labour in production and the per unit cost of these inputs is given by R_t and w_t , respectively. Output is produced via a Cobb-Douglas production function, $Y_t = zK_t^\alpha L_t^{1-\alpha}$ where $0 < \alpha < 1$.

- a) Set up the household problem and derive the optimality conditions of the household.
- b) Set up the firm's problem and derive the optimality conditions of the firm. Explain how the firm's labour demand varies the payroll tax, **holding all else constant**.
- c) Write down the government budget constraint.
- d) Derive a transition equation for k_{t+1} in terms of pre-determined k_t , exogenous variables z and $g = G/N$, as well as parameters of the model.

- e) Does k_{t+1} vary positively or negatively with more government spending? Provide some intuition as to why k_{t+1} varies with g in that manner.
- f) Is the welfare of the household necessarily lower with higher government spending? Explain how your answer depends on the size of γ .

Question 2: Government spending financed by lump-sum tax on young

Suppose the government spends $G_t = G$ each period. Government spending in this case is wasteful. The government runs a balanced budget and finances its spending G_t in each period by issuing a lump-sum tax only on young households. The rest of the problem is standard. The household has log utility given by $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$. The household works when young and inelastically supplies one unit of labour. She/he collects wage income and dividend income when young and pays the lump-sum tax. The young household can invest an asset, a_{t+1} that has a return of $1 + r_{t+1}$ when old. Households when old consume their savings. Firms use capital and labour in production and the per unit cost of these inputs is given by R_t and w_t , respectively. Output is produced via a Cobb-Douglas production function, $Y_t = zK_t^\alpha L_t^{1-\alpha}$ where $0 < \alpha < 1$.

- a) Write down the government budget constraint
- b) Set up the firm's problem and derive the firm's optimality conditions
- c) Set up the household's problem and derive the household's optimality conditions
- d) Solve for k_{t+1} , c_t^y and c_t^o in terms of k_t, z, g and parameters of the model, α, β .
- e) Suppose at date t , G increases permanently to G' where $G' > G$. What happens to investment per person k_{t+1} , and consumption per person (c_t^y and c_t^o) at date t . What happens to the growth path of k_t when government spending rises. How does that affect the path of consumption spending when young and old? Provide some intuition for your answer.