5 Forecasting theory & evaluation

Period	Errors	
1	e_1	ex-post in sample forecast
2	e_2	
⋮	⋮	
t^*	e_{t^*}	
t^*+1	e_{t^*+1}	ex-post out of sample forecast
⋮	⋮	
T	e_T	
$T+1$	—	ex-ante forecast
⋮	⋮	
$T+h$	—	

Mean Errors:

$$ME = \frac{1}{T-t^*}\sum_{t=t^*+1}^{T} e_t$$

+ very simple, preserves U.M & shows if avg. f'cast is above (>0) or below (<0) true values
− assigns same weight to each error, small or large, & pos/neg errors might offset each other

note: ME, MAE & RMSE are all sensitive to U.M - no good for comparing TS w/ diff. U.Ms or intervals.
↳ Find three are relative measures

↳ $P = \frac{e_t}{y_t} = \frac{y_t - \hat{y}_t}{y_t} \times 100\%$ $\hat{y}$ = forecast, y = observation

Mean absolute errors:

$$MAE = \frac{1}{T-t^*}\sum_{t=t^*+1}^{T} |e_t|$$

+ easy to interpret, preserves U.M & overcome offsetting ME issue.
− same as above bar offsetting

Root mean squared errors:

$$RMSE = \sqrt{\frac{1}{T-t^*}\sum_{t=t^*+1}^{T} e^2}$$

+ preserves U.M, no offsetting pos/neg errors, large errors are penalised

Mean per cent errors:

$$MPE = \frac{1}{T-t^*}\sum_{t=t^*+1}^{T} P_t$$

Mean absolute per cent errors:

$$MAPE = \frac{1}{T-t^*}\sum_{t=t^*+1}^{T} |P_t|$$

+ same as above but relative measures
− If any $y_t = 0$ they are undefined
∴ only use if measurement scale = ratio

Mean absolute scaled errors:

$$MASE = \frac{1}{T-t^*}\sum_{t=t^*+1}^{T}\frac{|e_t|}{MAE_{NF}}, \quad MAE_{NF} = \frac{1}{t^*}\sum_{t=1}^{t^*}|y_t - y_{t-1}|$$

↳ scales f/cast errors to the naive f/cast over training period.

↳ Rules:
- ↳ MASE is never negative
- ↳ MASE = 0 if & only if $e_t = 0$, i.e. each out of sample is perf.
- ↳ MASE < 1 if model performs better than NF
- ↳ MASE = 0 if model performs like NF
- ↳ MASE > 1 if model performs worse than NF

Point f/cast → single number → can't verify confidence/reliability
Interval f/cast → range of numbers → size of range indicates confidence
Density f/cast → estimate of prob. dist → i.e. annual GDP ~ N(1.80, 2.03)

External regressors / exogenous predictors

→ Providing an external regressor to ARMA model does a linear regression w/ ARMA errors

↳ Doing this might improve sample regression eqn
↳ To verify if worthwhile, do statistical test on suitability of linear regression, compare resids etc.