ECON30009/90080 – Tutorial 7

This Version: Semester 2, 2025

This tutorial is designed to get you used to solving search models of unemployment. To aquaint ourselve with the model, we will consider static (one-period) versions of the search model in this tutorial, although you should also familiarize yourself with solving the two-period model.

Question 1

Consider the search model of unemployment. The economy only lasts for one period. Households get utility from consuming their income at the end of the period. We will assume log utility from consumption. Conditional on being employed, the household inelastically supplies one unit of labour to the firm and receives an exogenous wage equal to $\bar{w}$. If non-employed, the household produces home goods worth h. There is no disutility to working. There is a measure 1 of households in the population.

A job is a single firm-worker pair. A firm needs a worker to produce. Specifically, a matched firm-worker pair produces output $y > \bar{w} > h$. An unmatched firm must pay a vacancy posting cost of κ to post a vacancy. The unmatched firm receives a lump-sum subsidy S, for each vacancy it creates. Search is random and new firms fill their vacancies with probability $q(\theta)$ and households find jobs with probability $p(\theta)$ where $\theta = v/u$. Let the matching function in this economy be given by

$$M = \xi \frac{uv}{(u^\alpha + v^\alpha)^{1/\alpha}}$$

where $0 < \alpha < 1$. All individuals are initially non-employed in the economy at the start of the period.

Suppose there exists a government that wants to incentivize job creation by providing a subsidy S to firms for each vacancy created. This implies the total amount of subsidies are equal to Sv. The government runs a balanced budget and finances the subsidy by levying a lump-sum tax, τ, on all households.

a Write down the government budget constraint
Answer:

$$Sv = \tau$$

b Write down the value of the non-employed, the value of the employed and value of the matched firm at the end of the period.
Answer:
Since there is only 1 peroid in this model, we will drop the time sub-scripts. The value of the non-employed at the end of the period is given by:

$$V^U = \ln(h - \tau)$$

The value of the employed at the end of the period is:

$$V^E = \ln(\bar{w} - \tau)$$

and the value of the matched firm at the end of the period is:

$$V^F = y - \overline{w}$$

We can already see from the end-of-period values that since $\bar{w} > h$, the household would always want to search since the expected value of searching is a convex combination of the value of employment and non-employment ($[1 - p(\theta)]V^U + p(\theta)V^E$), which is strictly greater than the value of just non-employment.

c Write down the value of a vacancy at the start of the period.
Answer:
At the start of the period, the value of a vacancy is given by:

$$\widetilde{V}^V = -\kappa + S + q(\theta)V^F$$

Note that as long as the firm creates a vacancy, it gets the subsidy S from the government.

d Assume free entry of firms. Show that θ and thus vacancies v are positively related to the subsidy S
Answer:
Under free entry, the value of a vacancy is driven to zero. And so we have:

$$\kappa - S = q(\theta)V^F$$

We know from the matching function that $q(\theta)$ is given by:

$$q(\theta) = \frac{M}{v} = \frac{\xi uv}{v(u^\alpha + v^\alpha)^{1/\alpha}} = \frac{\xi}{(1 + \theta^\alpha)^{1/\alpha}}$$

And we know $V^F = y - \overline{w}$. Substituting these pieces of information into the free entry equation, we have:

$$\kappa - S = \frac{\xi(y - \overline{w})}{(1 + \theta^\alpha)^{1/\alpha}}$$

Re-arranging to make θ the subject of the equation:

$$\theta = \left(\left[\frac{\xi(y - \overline{w})}{\kappa - S} \right]^\alpha - 1 \right)^{1/\alpha}$$

Clearly, since $-S$ is in the denominator on the right hand side of the equation, θ is positively related to S.

Further we know that all non-employed search the labour market at the beginning of the period, implying $v = \theta$ as there is a measure 1 of households in the population. Thus, vacancies are also positively related to S. Intuitively, the subsidy reduces the cost of posting a vacancy. Thus more firms create vacancies and enter the labour market. As they create more vacancies, the ratio of vacancies to unemployed job-seekers, θ increases, implying more competition among firms for the same set of job-seekers. Consequently, $q(\theta)$ is lower. Firms keep entering the labour market until the value of a vacancy is driven to zero.

e Show that the unemployment rate is declining with the subsidy S.
Answer:
The unemployment rate at the end of period 1 is given by all the unemployed who failed to find a job, i.e.,

$$u_1 = 1 - p(\theta)$$

From the matching function we know that:

$$p(\theta) = \frac{M}{u} = \frac{\xi uv}{u(u^\alpha + v^\alpha)^{1/\alpha}} = \frac{\xi\theta}{(1+\theta^\alpha)^{1/\alpha}}$$

Further we know that

$$\begin{aligned}
\frac{dp(\theta)}{d\theta} &= \xi\left[\frac{1}{(1+\theta^\alpha)^{1/\alpha}} - \frac{1}{\alpha}\frac{\theta}{(1+\theta^\alpha)^{1/\alpha}}\frac{\alpha\theta^{\alpha-1}}{(1+\theta^\alpha)}\right] \\
&= \xi\frac{\theta}{(1+\theta^\alpha)^{1/\alpha}}\left[\frac{1}{\theta} - \frac{\theta^{\alpha-1}}{1+\theta^\alpha}\right] \\
&= p(\theta)\frac{1}{1+\theta^\alpha} \\
&> 0
\end{aligned}$$

Since θ is positively related to S, then it must be that $p(\theta)$ is also positively related to S, i.e., $\frac{dp(\theta)}{dS} = \frac{dp(\theta)}{d\theta}\frac{d\theta}{dS} > 0$. Since the unemployment rate is declining in $p(\theta)$, a higher subsidy which encourages more vacancy creation and raises the job-finding rate of workers must lead to lower unemployment. So $\frac{du_1}{dS} < 0$.

Question 2

Consider the search model of unemployment and again assume that the economy only lasts one period. Households get utility from consuming their income at the end of the period. We will assume log utility from consumption. There is a measure 1 of households in the population, but now there are two types of households, A and B. Type A households, if employed, produce $y_A > y_B$, where y_B refers to the output produced by type B if employed. Both types of households observe the same level of home-production h if non-employed.

The proportion of type A households in the economy is given by η_A while the proportion of type B households in the economy is given by $\eta_B = 1 - \eta_A$. Employed households earn a wage of $\overline{w}y_j^\gamma$ where $0 < \gamma < 1$, $0 < \overline{w} < 1$ and $j \in \{A, B\}$. Further, $\overline{w}y_j^\gamma > h$ so that trivially all households want to participate in the labour market. The rest of the set-up of the model is the same as in **Question 1** of this tutorial, except there is no government, and therefore no tax or subsidy.

a Write down the end-of-period value of the non-employed. Write down the end-of-period value of the employed for each type $j \in \{A, B\}$. Write down the value of the firm matched to a type j worker at the end of the period.
The value of the non-employed at the end of period 1 is given by:

$$V^U = \ln h$$

The value of the employed of type A at the end of period 1 is given by:

$$V_A^E = \ln(\overline{w}y_A^\gamma)$$

The value of the employed of type B at the end of period 1 is given by:

$$V_B^E = \ln(\overline{w}y_B^\gamma)$$

The value of the matched firm attached to a worker of type A at the end of the period is:

$$V_A^F = y_A - \overline{w}y_A^\gamma$$

The value of the matched firm attached to a worker of type B at the end of the period is:

$$V_B^F = y_B - \overline{w}y_B^\gamma$$

b Write down the value of a vacancy. Assume free entry and solve for θ and thus v.
The firm when posting a vacancy has to take into account that conditional on meeting a job-seeker, there is some probability that the job-seeker is type A and some probability that the job-seeker is type B. This implies that the value of posting a vacancy at the start of the period is given by:

$$\widetilde{V}^V = -\kappa + q(\theta)\left[\eta_A V_A^F + (1-\eta_A)V_B^F\right]$$

Under free entry, the value of a vacancy gets driven to zero.

We already know the form of $q(\theta)$:

$$q(\theta) = \frac{M}{v} = \frac{\xi uv}{v(u^\alpha + v^\alpha)^{1/\alpha}} = \frac{\xi}{(1+\theta^\alpha)^{1/\alpha}}$$

And we know the values of the firm matched to each type of worker. We can substitute that information into the free entry equation:

$$\kappa = \frac{\xi}{(1+\theta^{\alpha})^{1/\alpha}}\left[\eta_A(y_A - \overline{w}y_A^{\gamma}) + (1-\eta_A)(y_B - \overline{w}y_B^{\gamma})\right]$$

Make θ the subject of the equation:

$$\theta = \left(\left[\frac{\xi\left\{\eta_A(y_A - \overline{w}y_A^{\gamma}) + (1-\eta_A)(y_B - \overline{w}y_B^{\gamma})\right\}}{\kappa}\right]^{\alpha} - 1\right)^{1/\alpha}$$

Since all wages are higher than h for all types, all households search the labour market, and this implies $v = \theta$.

c How does the composition of job-seekers affect the number of vacancies in the economy? Do firms create more vacancies when η_A is higher? Provide some brief intuition with your answer.
Answer
Note that one can show that $\frac{d(y-\overline{w}y^{\gamma})}{dy} = 1 - \overline{w}\gamma y^{\gamma-1} > 0$ for $0 < \gamma < 1$, $0 < \overline{w} <$. So profits are higher if you match with a type A worker, i.e., $V_A^F > V_B^F$.

This implies that if η_A is higher, the expected benefit of creating a vacancy is larger, since more job-seekers are of type A. Thus more firms enter the labour market and vacancy creation increases. We can see this by writing the expression for θ as:

$$\theta = \left(\left[\frac{\xi\left\{\eta_A\overbrace{(y_A - \overline{w}y_A^{\gamma} - [y_B - \overline{w}y_B^{\gamma}])}^{>0} + y_B - \overline{w}y_B^{\gamma}\right\}}{\kappa}\right]^{\alpha} - 1\right)^{1/\alpha}$$

Since θ is positively related to η_A, firms create more vacancies when there are more A types in the economy.