FNCE90056:
Investment Management

Lecture 5: Arbitrage Pricing Theory and Multifactor Models

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Last time . . .

The Royal Exchange in the City of London was founded as a center of commerce in 1565. In 1698, stockbrokers were expelled from the Royal Exchange for rowdiness. Their solution was to operate in the streets and coffee houses nearby, in particular Jonathan's Coffee House in Change Alley. After a fire swept through Change Alley in 1748, the brokers erected their own building. By 1801, the London Stock Exchange came into existence as a regulated exchange. The picture shown is from the turn of the 20th century.

- The CAPM is a simple and intuitive model of risk and return: β is the only thing you need to know about an asset.
- But it does not seem to work well in the data.
- How can we construct an alternative risk-versus-return (i.e. asset pricing) model that better matches the observed patterns in financial returns?

Today ...

- CAPM is still a very useful benchmark model.
- But, we made a number of assumptions to arrive at the CAPM.
- For example, investors were assumed to have the same ("homogeneous") beliefs about the economy.
- Ultimately, this meant they were happy to hold the same risky portfolio: the market.
- But once we allow investors to have different ("heterogeneous") views on the economy, what is the significance of the market portfolio?
- To address these issues, today we will discuss a different approach to asset pricing called the **Arbitrage Pricing Theory** or, usually, the **APT**.

Arbitrage Pricing Theory

Introduction to the APT

- First proposed by **Ross (1976)**.
- The APT is an approach to determining risk-return tradeoffs based on no-arbitrage.
- It is a multifactor model!
- **The APT is derived from no arbitrage arguments and a statistical model for returns.** This contrasts with the CAPM, an equilibrium (economic) model.
  - To get the APT, we do not have to assume that everyone is optimising, as in the CAPM.
  - This makes the APT a much more robust theory.
- Unlike the CAPM, we need very few assumptions to get the APT.

Assumptions

- Before describing the APT, it is useful to list the necessary assumptions:

    1. All securities follow a factor structure[1] with finite expected values and variances.
    2. Well-functioning capital markets preclude arbitrage opportunities. (Some investors can trade in well-diversified portfolios.)
    3. There are no taxes or transaction costs.

- That is it. We have considerably fewer assumptions than the CAPM.

[1] To be defined next slide . . .

Basic logic of APT

- What makes the APT work?
  - The APT is going to rely on no-arbitrage arguments by deriving restrictions on the prices of securities **relative to each other**.
    - if a can of cola is 3$, a burger is 10$, then a combo of them should be 13$ if there is no arbitrage
    - If asset A is 5$, asset B is 10$, asset C can be replicated by a portfolio of A and B, $R_C = w_A R_A + w_B R_B$, what should be the price of asset C?
  - If there is no arbitrage, identical assets should have same price
    - prices / expected returns of existing assets (and new assets) must satisfy certain relationships
    - **Replication** of an asset with existing assets is the key to apply no-arbitrage argument

1. Factor structure of returns

Start by extending the market index model that we studied earlier.
Assume that all risky security returns are driven by $K \geq 1$ factors:

$$\begin{aligned} r_{i,t} &= \mathbb{E}\left[r_i\right] + \beta_{i,1}F_{1,t} + \beta_{i,2}F_{2,t} + \cdots + \beta_{i,K}F_{K,t} + \epsilon_{i,t} \\ &= \mathbb{E}\left[r_i\right] + \left(\sum_{k=1}^{K} \beta_{i,k}F_{k,t}\right) + \epsilon_{i,t} \end{aligned} \tag{1}$$

- The F's are the factors. We are considering the possibility of multiple factors.
- Assets have different sensitivities to each factor through the β's. The β's are commonly called **factor loadings.**
- The residuals (the ϵ's, the noise) are idiosyncratic and have a mean of 0.

1. Factor structure of returns

- What can be a factor? Think of the factors (the F's) as representing new information about macroeconomic conditions such as
  - interest rates,
  - industrial production,
  - inflation,
  - volatility, etc.
- To make use of (1), we need a few additional assumptions:
  - $\mathbb{E}[F_k] = 0$
  - $\text{Cov}(\epsilon_i, F_k) = 0$
  - $\mathbb{E}[\epsilon_i] = 0$
  - $\text{Cov}(\epsilon_i, \epsilon_j) = 0$
- $\mathbb{E}[F_k] = 0$ simply means that instead of defining F_k directly as e.g. economic growth, we define it as the *surprise* in economic growth, i.e. the difference between what happened and what we were expecting: $F_k \equiv \tilde{F}_k - E[\tilde{F}_k] \Rightarrow E[F_k] = 0$.

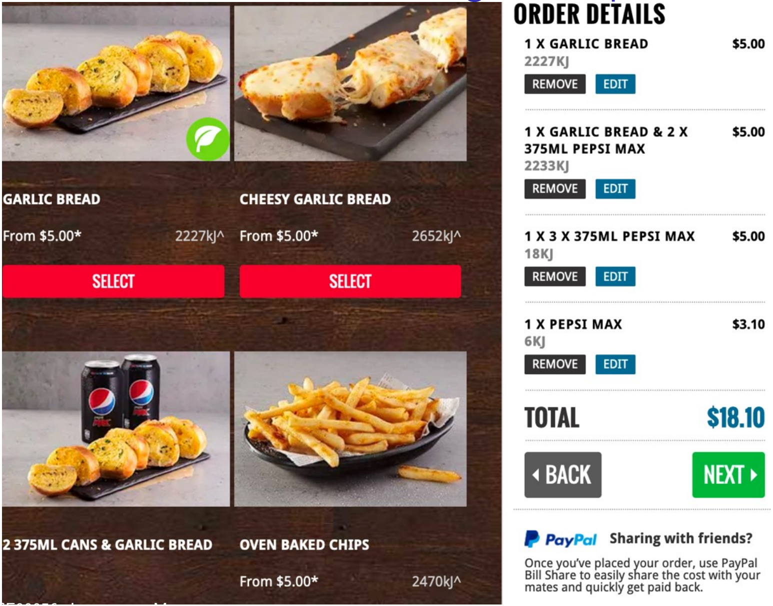
1. Factor structure of returns: Diversified portfolios

- The key to the APT is that some (not every) investor can construct well-diversified portfolios.
- **Definition.** In the context of the APT, a diversified portfolio is just a portfolio that carries no idiosyncratic risk:

$$r_p = \mathbb{E}[r_p] + \sum_{k=1}^{K} \beta_{p,k} F_k \tag{2}$$

- A diversified portfolio is defined with respect to a specific factor model.
  - since idiosyncratic risk ϵ is the residual of a specific factor model
- This contrasts with the notion of an (optimal) diversified portfolio in the context of the CAPM — a portfolio with the lowest total variance for a given expected return.

2. No arbitrage: example



2. No arbitrage: example

- Trading strategy:
  - Buy: 1 Garlic bread + 2 cans Pepsi Max = \$5
  - Sell: 1 Garlic bread = \$5
  - Sell: 2 cans Pepsi Max = \$6.20
  - Profit: (5 + 6.20) − 5 = \$6.20!
- The **Law Of One Price** states that identical assets should have the same price globally.
- An arbitrage (or free lunch) is a violation of the Law of One Price, i.e. when you make a risk-free profit by buying and selling the same thing simultaneously in 2 different markets.
- What happens if you do this trade many times?
- What about transaction costs?

2. No arbitrage: definition

What is an arbitrage opportunity?

An arbitrage opportunity is a “self-financing” trading strategy that either has a:

1. negative (< 0) price, and non-negative (≥ 0) future payoffs; or
2. price of zero, and non-negative (≥ 0) future payoffs with at least one positive payoff with a positive probability.

$\Rightarrow$ “Money-for-Nothing” or “Free Lunch”

Note: Arbitrage opportunities are riskless [like T-Bonds]. But they don’t cost anything today [unlike T-Bonds]...

Two key features of arbitrage opportunities: zero/negative costs, riskless gain

2. No arbitrage: Underlying mechanism

- As long as our portfolios are diversified, consistent with (2), we can ignore idiosyncratic risk.
- In the world of no-arbitrage, if 2 of these portfolios have the same factor loadings but different expected returns (arbitrage), then what happens?
  - Investors will immediately buy the high-expected-return portfolio, and sell the low-expected-return portfolio.
  - This will increase the price (decrease expected return) of the high-expected-return portfolio, and decrease the price (increase expected return) of the low-expected-return portfolio, until the 2 portfolios have the same expected returns in equilibrium.
  - This no-arbitrage mechanism ensures that 2 portfolios with same factor loadings will have same expected returns.
- The portfolios returns are **entirely** attributable to factor risk: this is APT.

APT SML

Multifactor security market line

In the APT:

1. Returns on assets i come from factors plus noise, as in (1).
2. But diversified portfolios p have no noise, so returns depend only on factors, as in (2).
3. Expected returns come only from factors, an "SML" that gives $\mathbb{E}[r_p]$ in terms of factor loadings and factor risk premia:

$$\mathbb{E}[r_p] = r_f + \beta_{p,1}\lambda_1 + \beta_{p,2}\lambda_2 + \cdots + \beta_{p,K}\lambda_K \tag{3}$$

   - The λ's are **factor risk premia** as they tell you how much extra return you receive by holding a portfolio exposed to the factors.
   - In the CAPM there is just one λ, the slope of the SML, $E[r_m] - r_f$.
   - Portfolios with no risk (all betas are 0) must return riskfree rate r_f.

4. SML holds only **approximately** for individual assets and imperfectly diversified portfolios, with idiosyncratic risk.

APT's SML: Single factor APT example

- Suppose there is just one economic factor F ($K = 1$); it proxies for economic innovation (EI):

$$r_i = \mathbb{E}[r_i] + \beta_i F_{EI} + \epsilon_i,$$

where again $\mathbb{E}[F_{EI}] = 0$, which simply means that we have normalised the factor so that only surprises (the unanticipated changes) can move prices.

- Now build 2 well-diversified portfolios. By examining just these two, we will be able to price ALL assets
  - determine the expected returns: given any β, with r_f and λ_{EI}, we can find $\mathbb{E}[r_i] = r_f + \beta_i \lambda_{EI}$.
- One portfolio H loads on high levels of economic innovation, while the other portfolio L loads on low levels. All we really need is $\beta_H \neq \beta_L$:

$$r_H = \mathbb{E}[r_H] + \beta_H F_{EI},$$
$$r_L = \mathbb{E}[r_L] + \beta_L F_{EI}.$$

- Using no-arbitrage, we will now price all risky securities!

APT's SML: Single factor APT example

- Create a combination of H and L that has a β of zero . We must determine the weights w_H on H and $1 - w_H$ on L:

$$w_H \beta_H + (1 - w_H)\beta_L = 0 \implies w_H = -\frac{\beta_L}{\beta_H - \beta_L}$$

  - By doing so, we are replicating the risk-free asset

- This portfolio has $\beta_p = 0$, no exposure to the factor, and no noise:

$$\begin{aligned} r_p &= \underbrace{w_H \, \mathbb{E}[r_H] + (1 - w_H) \, \mathbb{E}[r_L]}_{\mathbb{E}[r_p]} + \underbrace{0}_{\beta_p F_{EI}} \\ &= \left(-\frac{\beta_L}{\beta_H - \beta_L}\right) \mathbb{E}[r_H] + \left(1 - \left(-\frac{\beta_L}{\beta_H - \beta_L}\right)\right) \mathbb{E}[r_L] \\ &= \frac{\beta_H \, \mathbb{E}[r_L] - \beta_L \, \mathbb{E}[r_H]}{\beta_H - \beta_L} \end{aligned}$$

APT's SML: Single factor APT example

- By no-arbitrage, this portfolio p must have the same return as the risk-free asset (no-arbitrage argument). It has zero β.

$$r_f = \frac{\beta_H \mathbb{E}[r_L] - \beta_L \mathbb{E}[r_H]}{\beta_H - \beta_L}$$

- Re-arranging, we get

$$\frac{\mathbb{E}[r_L] - r_f}{\beta_L} = \frac{\mathbb{E}[r_H] - r_f}{\beta_H}$$

- We picked these 2 well-diversified portfolios arbitrarily, so this ratio is the same for any well-diversified portfolio.
- Define $\lambda_{EI} \equiv \frac{\mathbb{E}[r_i] - r_f}{\beta_i}$. Rearranging:

$$\mathbb{E}[r_i] = r_f + \beta_i \lambda_{EI}$$

λ_{EI} is the slope of the SML.

Implications of the single factor example

- For every well-diversified portfolio, its risk premium per unit of factor risk equals the same constant λ_{EI}.
- Rearranging, we arrive at the SML:

$$\begin{array}{lccccccc} \text{APT:} & \mathbb{E}[r_i] & = & r_f & + & \beta_i & \times & \lambda_{EI} \\ \text{CAPM:} & \mathbb{E}[r_i] & = & r_f & + & \beta_i & \times & (\mathbb{E}[r_M] - r_f) \end{array}$$

  which under APT holds exactly for well-diversified portfolios (and *approximately* for individual securities).
- Why is this a big deal? Because with the SML, you can now price every asset in the economy, all relative to these two, H and L.
- Using H and L, we replicated an asset with $\beta = 0$. We can use the same procedure to replicate an asset with $\beta = 1$.
- For an asset with $\beta = 1$, its risk premium will be λ_{EI}.

SML Examples

Determining the SML: 1 factor

- In a single factor model, we only need 2 well-diversified portfolios to "price" all other assets. We used H and L. Suppose they are:

Diversified Portfolio	$E(r_i)$	$\beta_{i,EI}$
H	18%	1.5
L	8%	0.5

- Step 1: combine H and L to get zero beta:

$$(w_H \times 1.5) + ((1 - w_H) \times 0.5) = 0 \implies w_H = -0.5, w_L = 1.5$$

- Step 2: set that portfolio's return equal to r_f:

$$r_f = (-0.5 \times E(r_H)) + (1.5 \times E(r_L)) = (-0.5 \times 18\%) + (1.5 \times 8\%) = 3\%$$

- Step 3: now find λ, the risk premium for a beta of 1:

$$\begin{array}{lcccccccc} \text{SML:} & E(r) & & r_f & & \beta & & \lambda & \\ \text{Use H:} & 18\% & = & 3\% & + & (1.5 & \times & \lambda) & \\ \text{Use L:} & 8\% & = & 3\% & + & (0.5 & \times & \lambda) & \end{array}$$

λ is 10%.

Determining the SML: 2 factors

- Suppose now there are 2 factors: economic innovation (EI) and monetary policy (MP). An analyst gives you the following information on some well-diversified funds:

Fund	$E(r_i)$	$\beta_{i,EI}$	$\beta_{i,MP}$
BRI	14%	1.5	0.0
SAC	14%	2.0	-1.0
APL	5%	0.0	0.0
SOR		1.0	0.6

- What is the SML?

$$
\begin{array}{lccccccccccccc}
\text{SML:} & E(r) & & & & \beta_{EI} & & & & \beta_{MP} & & \\
\text{Use BRI:} & 14\% & = & r_f & + & (1.5 & \times & \lambda_{EI}) & + & (0.0 & \times & \lambda_{MP}) \\
\text{Use SAC:} & 14\% & = & r_f & + & (2.0 & \times & \lambda_{EI}) & + & (-1.0 & \times & \lambda_{MP}) \\
\text{Use APL:} & 5\% & = & r_f & + & (0.0 & \times & \lambda_{EI}) & + & (0.0 & \times & \lambda_{MP})
\end{array}
$$

- What is the expected return according to the APT on the fund SOR with $\beta_{SOR,EI} = 1$ and $\beta_{SOR,MP} = 0.6$?

Calculations

Fund	$E(r_i)$	$\beta_{i,EI}$	$\beta_{i,MP}$
BRI	14%	1.5	0.0
SAC	14%	2.0	-1.0
APL	5%	0.0	0.0
SOR	$r_f + 1.0\lambda_{EI} + 0.6\lambda_{MP}$	1.0	0.6
Bond	r_f	0.0	0.0
F_{EI}	$r_f + 1.0\lambda_{EI}$	1.0	0.0
F_{MP}	$r_f + 1.0\lambda_{MP}$	0.0	1.0

- Step 1: Riskless rate: $r_f = 5\%$ using APL.
- Step 2: Solve for λ's: $\lambda_{EI} = \dfrac{14\% - 5\%}{1.5} = 6\%$ using BRI
  $\lambda_{MP} = 5\% + (2 \times 6\%) - 14\% = 3\%$ using SAC.
- Step 3: Use r_f and λ's to compute $E[r]$ for SOR:

$$E[r_{SOR}] = 5\% + (1 \times 6\%) + (0.6 \times 3\%) = 12.8\%$$

APT in Practice

Introduction

- The APT can be used in place of the CAPM for
  - pricing assets
  - performance evaluation
  - risk management

- But, what are the systematic factors (F's) that drive returns?[2]
  1. macroeconomic approach
  2. fundamental approach
  3. statistical approach

[2] Recall that the CAPM told us not only how many factors there were (only 1), but also told us exactly what the factor was (the market).

Macroeconomic Approach

Observable macro factors

- Economic intuition drove researchers/practitioners to develop multi-factor models with pre-specified macroeconomic factors F such as:
  - the market portfolio,
  - the growth rate of industrial production and other business cycle variables,
  - the default premium to capture credit spreads (the yield spread of Baa-Aaa corporates),
  - the term premium to capture interest rate dynamics (10-year minus 1-year Treasury yield spread),
  - expected and unexpected changes in CPI to capture inflation dynamics.
- An early implementation of such a factor model was Chen, Roll, and Ross (1986).
- Later, macro-based APT models were used in industry for asset allocation and risk management applications.

Fundamental Approach

Fundamental factor models

- These models use company and industry attributes as well as market data as factors. Some examples:
  - price-to-earnings ratio, market-to-book ratio, projected earnings growth,
  - membership in specific industries (e.g. durable goods),
  - trading activity/liquidity,
  - leverage,
  - size,
  - price momentum.

- These are risk factors beyond the market portfolio from the CAPM that represent systematic co-movements in historical stock returns.
  - A factor is not unique to one stock. It's something common across many assets
  - If many stocks have nonzero loadings β_{ij}, then when factor F_j moves, those stocks will tend to move together.
  - This induces systematic co-movement across securities' returns.

Implementing the fundamental approach

- Form portfolios of stocks sorted on characteristics (e.g. value vs. growth). With portfolio r_k as a factor,
  - estimate the price of risk λ_k,
  - construct the factor "surprise" by $F_k = r_k - E(r_k)$.
- How to select firm characteristics?
  - Differences in characteristics should be associated with differences in expected returns.
    - *Value firms have higher returns than growth firms.*
  - Firms with the same characteristics should also move together.
    - *Value firms co-move more with other value firms than with growth firms.*
- These portfolios should typically satisfy the following:
  1. They capture systematic movements in stock returns. In other words, a lot of stocks load on these portfolios.
  2. These portfolios will typically be mispriced by the CAPM.

Fama-French 3 factor model

- When we looked at the performance of the CAPM in real data, we found that when it failed, it failed in a systematic way.
- We saw that value firms and small firms had higher returns than those implied by the CAPM.
- One possible explanation is that value firms and small firms are exposed to sources of systematic risk that are not captured by the existing proxies of the market portfolio, e.g. the S&P500.
- Motivated by this, Fama and French (1992, 1993, 1996) constructed a 3-factor model of the APT.

Fama-French 3 factor model, continued

- This model has three factors:

$$E(r_i) - r_f = \beta_{i,m}E(r_m - r_f) + \beta_{i,SMB}E(r_{SMB}) + \beta_{i,HML}E(r_{HML})$$

1. Aggregate Market Index (the market)
   - ★ The return on the stock market in excess of the return on a one-month T-Bill: $F_m = r_m - r_f$.
2. Aggregate Firm Size Index (SMB)
   - ★ The return on small capitalisation firms in excess of the return on large capitalisation firms: $F_{SMB} = r_{small} - r_{big}$.
3. Financial Distress Index (HML)
   - ★ The return on stocks with high book-to-market (book equity relative to market equity) in excess of the return on stocks with low book-to-market: $F_{HML} = r_{high} - r_{low}$.

- The Fama-French factors carry significant risk premiums (1926-2011).

$$\begin{aligned}
\lambda_m &= E(r_m - r_f) && = 7.4\% \\
\lambda_{SMB} &= E(r_{small} - r_{big}) && = 2.9\% \\
\lambda_{HML} &= E(r_{high} - r_{low}) && = 4.6\%
\end{aligned}$$

Fama-French-Carhart 4-factor model

- Carhart (1997) advocated augmenting the Fama-French three factor model with a momentum factor.
- Defining momentum (MOM) as the return spread of stocks between the highest and lowest quintiles of return performance over -7 months to -1 months gives the Carhart (1997) model:

$$E[r_i] - r_f = \beta_{i,m} E[r_m - r_f] + \beta_{i,SMB} E[r_{SMB}] + \beta_{i,HML} E[r_{HML}] + \beta_{i,MOM} E[r_{MOM}].$$

- The momentum factor carries a significant risk premium (1926-2011).

$$\begin{aligned}
\lambda_m &= E(r_m - r_f) && = 7.4\% \\
\lambda_{SMB} &= E(r_{small} - r_{big}) && = 2.9\% \\
\lambda_{HML} &= E(r_{high} - r_{low}) && = 4.6\% \\
\lambda_{MOM} &= E(r_{winner} - r_{loser}) && = 8.4\%
\end{aligned}$$

- The Fama-French-Carhart 4-factor model is used a great deal in evaluating the performance of fund managers.

Statistical Approach

Data mining

- Principal components analysis (PCA) is a statistical technique that extracts common factors from a cross-section of stock returns:

$$r_i = E(r_i) + \beta_{i,1}F_1 + \beta_{i,2}F_2 + \cdots + \beta_{i,K}F_K + \epsilon_i.$$

  - PCA aims to describe the data, r_i, as accurately as possible, using a small number of systematic factors F_k.
  - One way to think of PCA is as extracting the "most relevant" systematic information from the data. The remainder, the ϵ_i, it classifies as "noise" or unsystematic information.
  - The basic process is to take the variance-covariance matrix of security returns and extract factors that maximally explain the variances.
- Problem: Hard to economically interpret these purely statistical factors.

Conclusion

Summary

We considered 3 ways to obtain factors:

- Macroeconomic factors
- Factors based on characteristics or firm fundamentals
- Factors extracted statistically

What's left? Our original lecture 1 problem has one more dimension:

	Holdings by instrument, %			Equity holdings by area, %								Bond holdings by currency, %				
	Equities	Bonds	Cash	United States	Other Americas	Britain	Germany	France	Other Europe	Japan	Other Asia	Dollar	Yen	Sterling	Euro-zone	Others
Robeco Group	50	50	0	53	3	10 ▲	4	6 ▲	14 ▲	7 ▼	3 ▼	33 ▲	15 ▼	5 ▲	45 ▲	2 ▼
Julius Baer PB	54	39	7 [1]	60	0	4 ▲	5	3	11 ▼	12	5	77 ▼	0	0	15 ▲	8 ▲
Commerz Int. CM	53 ▲	47 ▼	0	49 ▲	0	7 ▲	6	4	20	12 ▼	2 ▼	30	28 ▲	3 ▼	37 ▲	2 ▼
Credit Suisse PB	43	45	12 [1]	51 ▲	0	9 ▲	7	5	9 ▼	14 ▲	5	88	0	0	12	0
Lehman Brothers	60 ▲	35 ▼	5	45 ▼	2 ▲	10 ▲	11 ▲	10 ▲	11 ▼	9 ▼	2	40 ▲	12	6 ▲	37 ▼	5 ▲
Standard Life	60	40	0	52 ▼	1	11 ▲	5	6	13 ▲	10 ▼	2 ▲	28 ▼	25 ▼	4 ▼	42 ▲	1 ▼
Daiwa	50 ▼	45 ▲	5	39 ▼	2 ▲	9 ▲	7 ▼	9	16 ▲	14 ▼	4	35 ▲	18 ▼	6 ▲	37 ▼	4 ▼

Bonds.