ECON30024
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Solution to Tutorial 9

1. (1) Which of the following statements is FALSE?

    (a) The CAPM and APT can be interpretted as special versions of the NPV asset pricing theory in a static setting.

    (b) Asset prices determined by the NPV relationships are compatible with the arbitrage principle.

    (c) According to the NPV theory, the equilibrium price of a finitely-lived asset under certainty is uniquely given by the net present value of its dividends in all future periods and its maturity value.

    (d) According to the NPV theory, the equilibrium price of an indefinitely-lived asset under certainty is uniquely given by the net present value of its dividends in all future periods.

    Answer: (d)

   (2) The arbitrage principle under uncertainty requires that every asset yields an expected return equal to the risk-free rate. True/False?

    Answer: False

   (3) The NPV equation under uncertainty (give the form of the equation) presumes that investors are risk neutral and have homogeneous beliefs about the probability distributions of dividends. True/False

    Answer: True

   (4) Company A paid a dividend of $3.00 yesterday, and these dividends are expected to grow at the rate of 5% in the long run. The beta of Company A's stock is 0.95, the expected return on the market is 15%, and the risk free rate is 10% at present. Using the Gordon growth model to find the fair price of one share of Company A's stock. (need to be more precise)

    (a) $ 30.77

    (b) $ 32.31

    (c) $ 31.5

    (d) $ 63

    (e) $ None of the rest

    Answer: (b)

    According to the Gordon growth model, the equilibrium or fair price of Company A's share price is given by

$$p_t = \frac{E_t d_{t+1}}{\mu - g}.$$

Here, g is the growth rate of dividends, $g = 5\%$. μ is the required rate of return on the stock, which is given by the CAPM prediction:

$$\mu = 10\% + 0.95(15\% - 10\%) = 10\% + 4.75\% = 14.75\%.$$

$E_t d_{t+1}$ is the expected dividend in next period, which is given by $3(1+5\%) = \$3.15$. So

$$p_t = \frac{3.15}{14.75\% - 5\%} = \$32.31.$$

(5) Which of the following statements is TRUE?

(a) Financial bubbles are not compatible with the arbitrage principle.

(b) Financial bubbles cannot exist if asset markets are in equilibrium.

(c) Shiller's variance bounds test uses prices predicted by the NPV theory if investors have perfect foresight about future dividends as benchmark prices to assess the volatility of US stock prices.

(d) As Ponzi schemes can yield unusually high returns to early investors, investors should participate in them as early as possible.

Answer: (c)

2. Denote the risk free rate from $t+i$ to $t+i+1$ as r^f_{t+i+1}, $i = 0, \ldots, N-1$. Then by the arbitrage principle under certainty, the equilibrium rate of return on any asset from $t+i$ to $t+i+1$ must equal the risk free rate r^f_{t+i+1}:

$$r^f_{t+i+1} = \frac{(d_{t+i+1} + p_{t+i+1}) - p_{t+i}}{p_{t+i}} = \frac{d_{t+i+1} + p_{t+i+1}}{p_{t+i}} - 1$$

$$\Rightarrow \quad p_{t+i} = \frac{d_{t+i+1} + p_{t+i+1}}{1 + r^f_{t+i+1}}.$$

This equation holds for any $i = 0, 1, \ldots, N-1$:

$$\begin{aligned}
p_t &= \frac{d_{t+1} + p_{t+1}}{1 + r^f_{t+1}} \quad \text{when } i = 0 \\
p_{t+1} &= \frac{d_{t+2} + p_{t+2}}{1 + r^f_{t+2}} \quad \text{when } i = 1 \\
p_{t+2} &= \frac{d_{t+3} + p_{t+3}}{1 + r^f_{t+3}} \quad \text{when } i = 1 \\
&\cdots \\
p_{t+N-1} &= \frac{d_{t+N} + p_{t+N}}{1 + r^f_{t+N}} \quad \text{when } i = N-1
\end{aligned}$$

Now starting from the equation for p_t, successively use the equations for p_{t+1}, ..., p_{t+N-1} to substitute out p_{t+1}, ..., p_{t+N-1}:

$$
\begin{aligned}
p_t &= \frac{d_{t+1}+p_{t+1}}{1+r^f_{t+1}} = \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{p_{t+1}}{1+r^f_{t+1}} \\
&= \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{1}{1+r^f_{t+1}}\left[\frac{d_{t+2}+p_{t+2}}{1+r^f_{t+2}}\right] \\
&= \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{d_{t+2}}{(1+r^f_{t+1})(1+r^f_{t+2})} + \frac{p_{t+2}}{(1+r^f_{t+1})(1+r^f_{t+2})} \\
&= \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{d_{t+2}}{(1+r^f_{t+1})(1+r^f_{t+2})} + \frac{1}{(1+r^f_{t+1})(1+r^f_{t+2})}\left[\frac{d_{t+3}+p_{t+3}}{1+r^f_{t+3}}\right] \\
&= \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{d_{t+2}}{(1+r^f_{t+1})(1+r^f_{t+2})} + \frac{d_{t+3}}{(1+r^f_{t+1})(1+r^f_{t+2})(1+r^f_{t+3})} \\
&\qquad + \frac{p_{t+3}}{(1+r^f_{t+1})(1+r^f_{t+2})(1+r^f_{t+3})} \\
&= \cdots \\
&= \frac{d_{t+1}}{1+r^f_{t+1}} + \frac{d_{t+2}}{(1+r^f_{t+1})(1+r^f_{t+2})} + \frac{d_{t+3}}{(1+r^f_{t+1})(1+r^f_{t+2})(1+r^f_{t+3})} + \cdots \\
&\qquad + \frac{p_{t+N}}{(1+r^f_{t+1})(1+r^f_{t+2})\cdots(1+r^f_{t+N})}
\end{aligned}
$$

Let the discount factor be denoted by

$$
\delta_{t+i} = \frac{1}{(1+r^f_{t+1})(1+r^f_{t+2})\cdots(1+r^f_{t+i})} \quad \text{for } i = 1, 2, \ldots, N
$$

then we reach equation (4) on the lecture slide:

$$
p_t = \left(\sum_{i=1}^{N} \delta_{t+i} d_{t+i}\right) + \delta_{t+N} p_{t+N}.
$$

It's clear that the equilibrium price p_t of a finitely-lived asset is uniquely determined by this equation, because the dividends $d_{t+1}, d_{t+2}, \ldots, d_{t+N}$ and the maturity value of the asset p_{t+N} are all finite numbers (i.e., p_t is the sum of $N+1$ finite numbers).

3. The absence of arbitrage condition, equation (5), is given by

$$
p_t = \frac{d_{t+1}+p_{t+1}}{1+r^f}.
$$

Now we show that asset price sequence as defined by equation (20)

$$
p_t = \sum_{i=1}^{\infty} \delta_{t+i} d_{t+i} + b_t = \frac{d_{t+1}}{1+r^f} + \frac{d_{t+2}}{(1+r^f)^2} + \frac{d_{t+3}}{(1+r^f)^3} + \cdots + b_t,
$$

where the b_t sequence satisfies $b_{t+1} = (1 + r^f)b_t$, satisfies equation (5).

As (20) holds for any t, it holds for period $t+1$ as well:

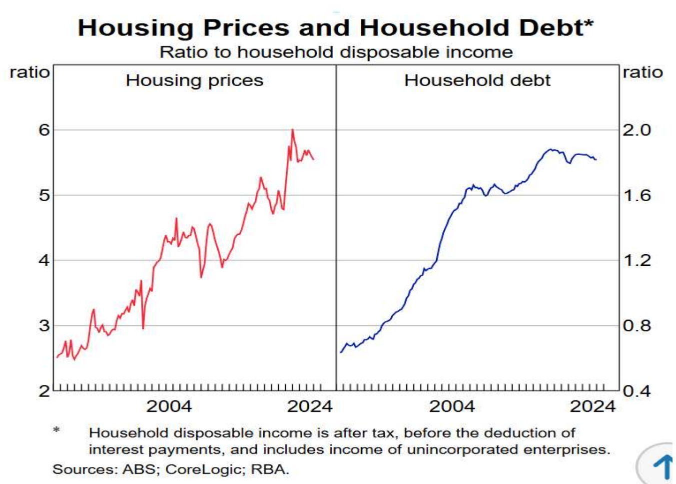
$$p_{t+1} = \sum_{i=1}^{\infty} \frac{d_{t+1+i}}{(1+r^f)^i} + b_{t+1} = \frac{d_{t+2}}{1+r^f} + \frac{d_{t+3}}{(1+r^f)^2} + \frac{d_{t+4}}{(1+r^f)^3} + \cdots + b_{t+1}$$

So

$$\begin{aligned} \frac{d_{t+1} + p_{t+1}}{1+r^f} &= \frac{d_{t+1} + \frac{d_{t+2}}{1+r^f} + \frac{d_{t+3}}{(1+r^f)^2} + \frac{d_{t+4}}{(1+r^f)^3} + \cdots + b_{t+1}}{1+r^f} \\ &= \frac{d_{t+1}}{1+r^f} + \frac{d_{t+2}}{(1+r^f)^2} + \frac{d_{t+3}}{(1+r^f)^3} + \cdots + \frac{b_{t+1}}{1+r^f} \\ &= \frac{d_{t+1}}{1+r^f} + \frac{d_{t+2}}{(1+r^f)^2} + \frac{d_{t+3}}{(1+r^f)^3} + \cdots + b_t \\ &= p_t. \end{aligned}$$

This is exactly equation (5). This derivation also makes it clear why the bubble term needs to satisfy $b_{t+1} = (1 + r^f)b_t$. So equation (20) provides many other solutions for the equilibrium price of the indefinitely lived asset.

4. (a) The following figure depicts the housing price price and household debt in Australia (Source: RBA chart pack)



The housing price has indeed grown fast in past few decades. There are economic forces behind it: strong domestic demand for properties driven by population growth and strong economic growth since early 1990s, strong demand for Australian properties from overseas, favourable tax policies like negative gearing and capital gains discount that encourage housing investment, slow growth in housing supply due to restrictions on land development as well as low productivity in the construction sector, etc. I tend to believe the high housing price is not a bubble. That being said, the rapid growth of housing price during several episodes could be a housing bubble (considering, for example, the housing boom spurred by low interest rate between 2020 and 2022).

(b) Under the pay-as-you-go social security, workers pay social security tax and receive benefits when retired. In a given period, proceeds collected from current workers are used to pay benefits to retirees. Rolling over debt means borrowing new debt to pay or service old debt. These practices do resemble one aspect of a Ponzi scheme: returns to early investors come from new investors. However, these practices do not involve a fraud, so probably should not be called a public Ponzi scheme.