Economics of Finanical Markets
ECON30024, Shuyun May Li

Assignment 2 (100 marks)

(due: May 19, 2025, Monday, 4pm)

Read the instructions very carefully. Requirements that are clearly stated here will not be answered through emails or online discussion. As for Assignment 1, please also include a page in your uploaded file to briefly record your group meetings and the role of each group member in completing the assignment.

1. (70 marks) This question asks you to choose any three individual stocks in the Australian Securities Exchange that have been traded in past 10 years and form your own optimum portfolio.

   (a) (5 marks) Find end-of-month prices data for the three stocks you have chosen over the period December 2014 - December 2024, and compute their monthly returns for January 2015 - December 2024. Report what stocks you have chosen and the data source for their prices if you use a different data source than given in footnote 1.[1]

   (b) (10 marks) Report the sample average returns and sample standard deviations of returns on each stock, and the sample correlations among the returns on the three stocks. These statistics provide estimates for μ_j, σ_j, and ρ_{ij}, for $i, j = 1, 2, 3$.[2]

   (c) (15 marks) Pick any two stocks out of the three stocks you have chosen. Follow my solution to Exercise_Topic4 Q4 to do the following:

      (i) Plot the portfolio frontier with the two risky stocks in Excel or any other software. Report the figure.

      (ii) Suppose that you have a mean-variance objective as follows

$$G(\mu_P, \sigma_P^2) = \mu_P - \alpha\sigma_P^2.$$

      Choose a value for α that best represents your degree of risk aversion. Find your optimum portfolio, denoted as E_2. Report the proportions of each stock in E_2 and $(\sigma_{E_2}, \mu_{E_2})$.

[1] The library database, DatAnalysis Premium, contains prices data for individual stocks. You can access the database via the website: http://cat.lib.unimelb.edu.au/record=e1000952 S31. When you log into the database, click "Company Reports", input the company's name, then click the "price history" tab, you'll be able to search the end-of-month **adjusted prices** for this company over the period: **31/12/2014 - 31/12/2024**. Then you can calculate monthly returns on each stock as you did for Assignment 1, Q2.

[2] It's possible that the stocks you choose may have negative sample average returns. I suggest you to choose stocks with **positive sample average returns** to work with so that the portfolio frontier you are to construct would look nicer. You should also try to **avoid the case** that one stock in your 3 stocks is obviously not attractive in the sense that it has lower average return but higher variance of return than either of the other two.

(iii) Plot your indifference curves on the figure from (i) and depict the optimum portfolio E_2. Report the new figure.

(d) (30 marks) Now work with the three stocks you have chosen. Construct the portfolio frontier with the three stocks by numerically solve a minimum variance problem as described in Topic 4, Section 3 (recall that the solution to this problem defines the portfolio frontier with n risky assets).[3] You can proceed as follows.

(i) Write down the minimum variance problem that defines the portfolio frontier with 3 risky assets and report it.

(ii) In Excel or any other software, let a_1, a_2 vary between 0 and 1 and let $a_3 = 1 - a_1 - a_2$ (need to make sure a_3 is within 0 and 1), and calculate the μ_P and σ_P^2 corresponding to each combination of (a_1, a_2, a_3), using the estimated μ_j, σ_j, and ρ_{ij} values you have found in part (b). No need to report values of (a_1, a_2, a_3) and the corresponding (σ_P, μ_P) values, just report how you vary a_1 and a_2 and how many combinations of (a_1, a_2, a_3) you have considered.[4]

(iii) Plot all the combinations of (σ_P, μ_P) obtained in the (σ_P, μ_P) space. Note that each point (σ_P, μ_P) represents a budget feasible portfolio consisting of the three stocks. When many points are plotted, a left boundary of these points would emerge, which would be your constructed portfolio frontier with the 3 risky stocks. The wider range and the more combinations of (a_1, a_2, a_3) you consider, the more accurate your constructed portfolio frontier would be. Report this figure.

(iv) On the same figure from (iii), plot the portfolio frontier with the two stocks that you have constructed in part (c) (using different colours to distinguish). Do you observe the portfolio frontier with three stocks is located to the left of the portfolio frontier with two stocks? Report this new figure, and comment on the relative positions of the two portfolio frontiers.

If you feel that the procedure above is too cumbersome, you can also use a solver function that is provided in Excel or any other software that can solve the minimum variance problem for you. That is, for each given μ_P, you let the software solve the minimum variance problem to give you a corresponding value of σ_P^2.

[3] In current economic research, the economic models researchers build are becoming more and more sophisticated in order to capture as many important features of the data as possible. As a result, most models cannot be solved analytically and have to be solved numerically on computers. I use Matlab all the time to do numerical computing in my own research. Numerical computing is also very useful in other business areas like finance, as well as science and engineering. Hope this question can give you a little bit taste of this powerful tool. You will be taught various numerical methods if you choose to do further studies in economics.

[4] When you consider different combinations of (a_1, a_2, a_3), you need to make sure the combinations you choose cover a wide range of possible combinations. That is, each of a_1, a_2, a_3 can take values that are closer to 0, closer to 1, and between 0 and 1. Then your portfolio frontier would be more accurate. You need to find ways to choose the combinations to achieve this (Excel provides random number generator functions that can help you). Also, you should consider a relatively large number of combinations (at least a few hundreds) to achieve reasonable accuracy.

Then you plot the various combinations of (σ_P, μ_P), which will also give you the portfolio frontier with three stocks.

(e) (10 marks) Use the same mean-variance objective as in Part (c)-(ii).

(i) Calculate the values of $G(\mu_P, \sigma_P^2)$ corresponding to the combinations of (σ_P,μ_P) that you have calculated in part(d)-(ii). Find the combination of (σ_P,μ_P) that gives you the highest value of $G(\mu_P, \sigma_P^2)$, which would be your approximated optimum portfolio of the three stocks, denote it as E_3. Report the proportions of each stock in E_3 and $(\sigma_{E_3}, \mu_{E_3})$, and compare the expected values and variances of the returns on E_2 and E_3 and the values of your mean-variance objective at E_2 and E_3. Do the differences make sense to you?

(ii) Plot your indifference curves on the figure from (d)-(iii) and depict the optimum portfolio E_3. Report this new figure.

2. (30 marks) This question asks you to do a simple test of the CAPM using data on stocks in the Australian Securities Exchange. Using the S&P/ASX 200 index as the market proxy, and the interest rate on 3-month BABs/NCDs as the risk free rate.[5]

(a) (5 marks) Data collection: Find end-of-month adjusted price data for 12 individual stocks over the period December 31 2014 - December 31 2024 and find the S&P/ASX 200 index over the same period (from the same database as described in footnote 1).[6] Convert these prices into monthly returns for January 2015 - December 2024. Download the risk-free rate over the same period and convert the annualised interest rates into monthly interest rates. Report what stocks you have chosen (but do not include your data series in your report) and how you've converted the annualised interest rates into monthly interest rates.

(b) (5 marks) For each of the 12 stocks, run a time series regression

$$r_{jt} - r_{0t} = \alpha_j + \beta_j(r_{Mt} - r_{0t}) + \varepsilon_{jt}, \quad j = 1, 2, \ldots, 12,$$

No need to report the regression results from each regression. Just use a table to list your estimates of β_j.

(c) (20 marks) Use the estimated betas from (b) and sample average excess returns on the stocks to run the following cross-section regression:

$$\bar{r}_j - \bar{r}_0 = \gamma_0 + \gamma_1\widehat{\beta}_j + \eta_j, \quad j = 1, 2, \ldots, 12,$$

[5]The interest rate on 3-month BABs/NCDs is given in the RBA table: Interest Rates and Yields - Money Market - Monthly - F1.1, https://www.rba.gov.au/statistics/tables/. The entries in this table are annualised intereste rates (which refer to returns over a year), so you need to convert them into monthly interest rates (which refer to returns over a month) as required in part (a). As the ASX is highly integrated into the global financial market, you may consider using S&P 500 index for the US stock market as the market proxy and the US one month or three treasury bill rate as the risk free rate to check the robustness of your results, but you are not required to do so.

[6]Recall that you used the S&P/ASX 200 index in Assignment 1. Follow the same instruction to find price data for this index in the database

(i) Report the regression results, in particular, the estimates of γ_0 and γ_1. How does the estimate of γ_1 compare with the sample average excess market return?

(ii) Plot the average excess returns on each stock against their estimated betas, and plot the estimated regression line on the same figure. Report the figure.

(iii) Test $H_0 : \gamma_0 = 0$ ($H_A : \gamma_0 \neq 0$), and $H_0 : \gamma_1 \leq 0$ ($H_A : \gamma_1 > 0$).[7] Report your testing results.

(iv) Briefly discuss whether your results provide evidence supporting or against the standard CAPM or the Black CAPM.[8]

[7] You are not required to conduct a joint test here. Just test each hypothesis separately. If you are not familiar with one-sided test, you may have a look at this website for a brief explanation: https://www.statology.org/left-tailed-test-vs-right-tailed-test/

[8] Note that the test you perform is a very simple test, with only 12 individual stocks instead of portfolios covering all stocks. So do not take the results you get very seriously. Just interpret whatever you get based on the discussions in Topic 7.