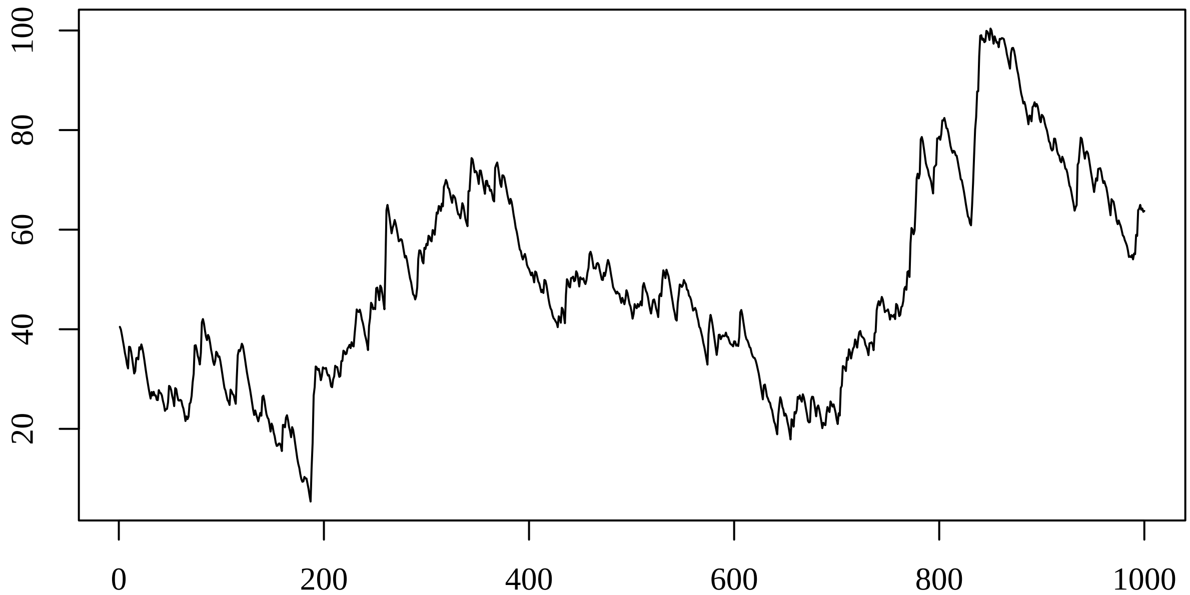
Question 5.

The following time series plot show $n = 1,000$ observations on an asset price P_t. The returns calculated from this price series are calculated ($Y_t = \Delta \log P_t$) and an AR(1)-GARCH(1,1) model is fitted.



The coefficient estimates are as follows:

```
   mu    ar1  omega alpha1  beta1
0.025  0.270  0.785  0.088  0.593
```

To be specific, `mu` is the estimate of the mean of Y_t, `ar1` the coefficient of the AR(1) equation, and `omega`, `alpha1` and `beta1` the coefficients of the GARCH(1,1) equation.

(a) Write out the estimated model in equation form.

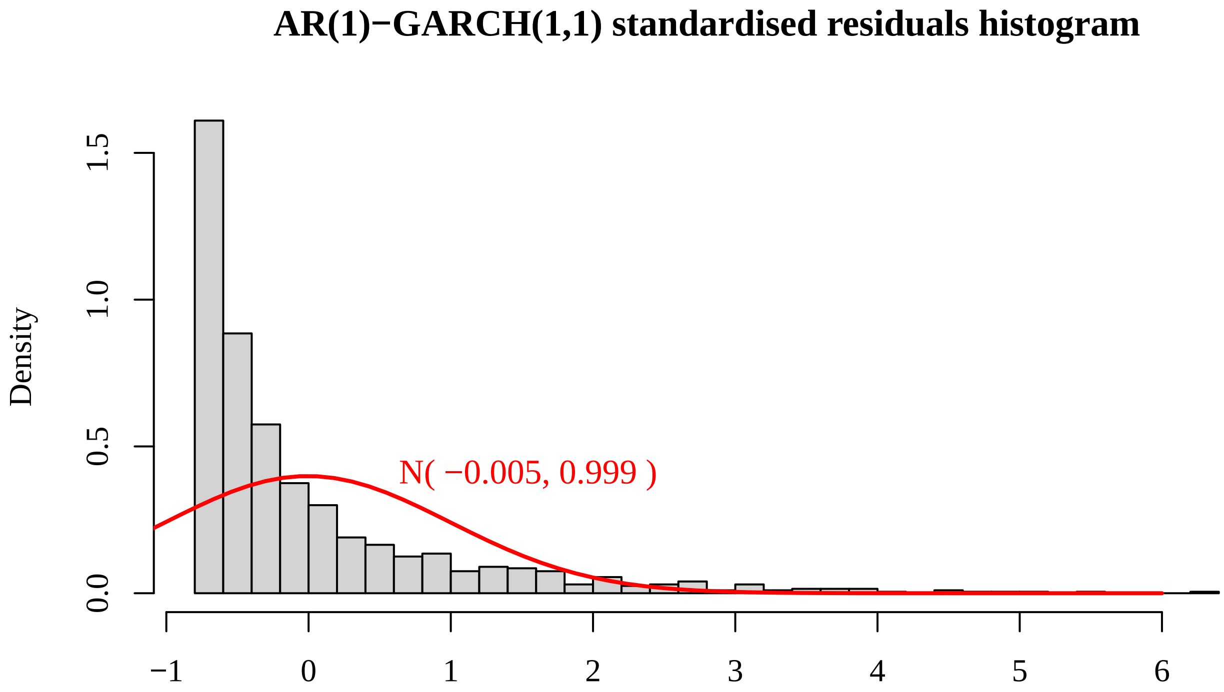
The following statistics were computed from the standardised residuals of the model.

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.08651  0.7687
Lag[2]                    0.97189  0.7567
Lag[5]                    1.76399  0.7779
H0 : No serial correlation

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3] 0.0004424 0.500 2.000  0.9832
ARCH Lag[5] 0.4603905 1.440 1.667  0.8953
ARCH Lag[7] 0.5228336 2.315 1.543  0.9763
```

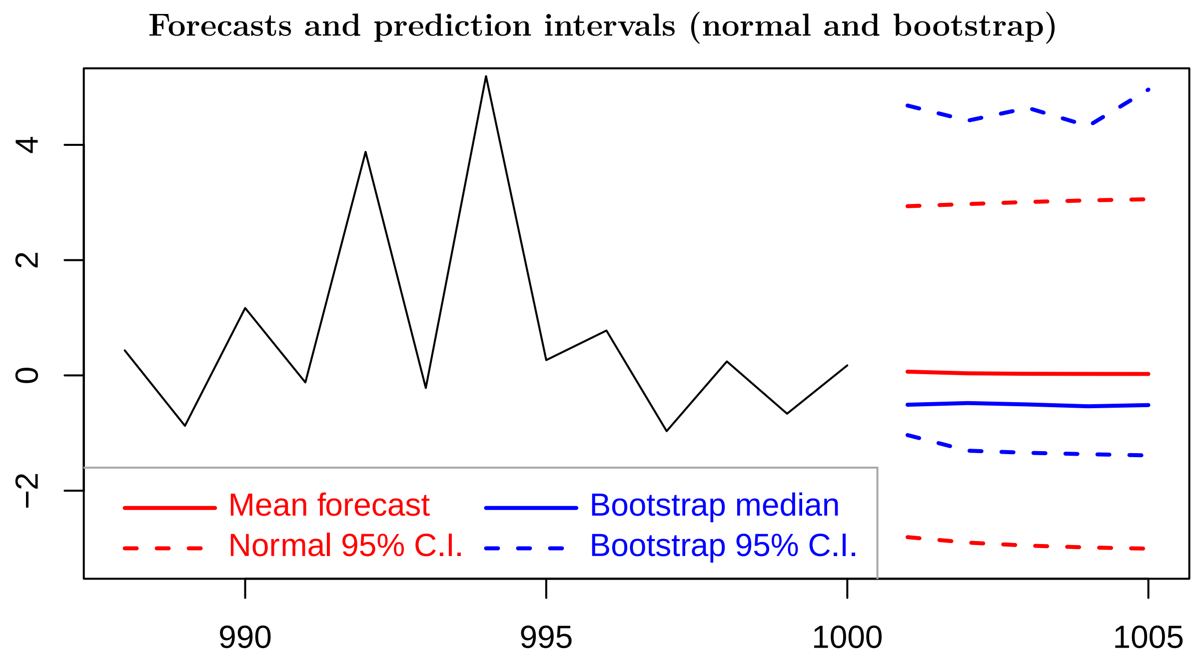
(b) What do these tests suggest about the specification of the model? Explain.

Following are some graphical and descriptive statistics relevant to the distribution of the standardised residuals.



Mean	Std.Dev.	Skewness	Kurtosis
−0.005	1.000	2.318	9.386

(c) What do you conclude about the normality or otherwise of the standardised residuals?



The plot above and output table below show up to 5-step-ahead forecasts and prediction intervals from the model. The usual conditional mean point forecasts are denoted `Mean`, with prediction intervals computed assuming conditional normality denoted `L_Norm` and `U_Norm`. The bootstrap was also applied to produce 5,000 replications over the forecast period, with the 2.5%, 50% and 97.5% quantiles of these replications denoted `L_Boot`, `Median` and `U_Boot` respectively.

h	L_Norm	L_Boot	Median	Mean	U_Norm	U_Boot	Sigma
1	-2.806	-1.035	-0.509	0.065	2.936	4.682	1.465
2	-2.901	-1.306	-0.480	0.036	2.973	4.420	1.499
3	-2.953	-1.343	-0.504	0.028	3.009	4.641	1.521
4	-2.985	-1.367	-0.536	0.026	3.037	4.327	1.536
5	-3.006	-1.388	-0.515	0.025	3.056	4.959	1.546

(d) What are the interpretations of the two confidence intervals? Why do they differ, and which would you prefer for this application?

(e) Return to the model in equation form in part (a). Show how this can be rearranged to produce estimated equations for $\widehat{E}(Y_t|\mathscr{Y}_{t-1})$ and $\widehat{\text{var}}(Y_t|\mathscr{Y}_{t-1})$ written in terms of lags of the observed time series Y_t on the right hand side (i.e. not lags of Z_t or U_t) which could be used for computational purposes.