Project 3 Group 2

CS 325 W16

Problem 3

**Part A**

Finding shortest paths using LINDO:

**From a to b:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 1

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to c:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 1

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to d:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 1

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to e:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 1

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to f:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 1

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to g:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 1

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to h:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 1

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to j:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 1

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From a to k:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 1

jl - lm = 0

im + km + lm = 0

END

**From a to l:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 1

im + km + lm = 0

END

**From a to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 1

END

**Part C**

Finding shortest paths from all vertices to vertex m using LINDO:

**From a to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From b to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 1

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From c to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 1

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From d to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 1

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From e to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 1

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From f to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 1

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From g to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 1

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From h to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 1

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From i to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From j to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 1

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

END

**From k to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 1

lm - jl = 0

im + km + lm = 1

END

**From l to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ia + ik + ij + im - ci - ei - fi - hi - ji= 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 1

im + km + lm = 1

END

**Part D**

Finding shortest paths that go through vertex i using two linear programs in LINDO:

**From a to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 1

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to a:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 1

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From b to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 1

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to b:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 1

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From c to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 1

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to c:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 1

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From d to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd =1

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to d:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 1

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From e to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 1

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to e:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci – cf = 0

ad + cd + gd - da - dg - dj – df = 0

be - eh - ec – ei = 1

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj – gk = 0

ah + eh + kh - hi – hk = 0

dj + gj + ij - ji - jk – jl = 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From f to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 1

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to f:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 1

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From g to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 1

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to g:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 1

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From h to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 1

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to h:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 1

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From j to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 1

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to j:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 1

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 0

END

**From k to i:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 1

lm - jl = 0

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to k:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 1

jl - lm = 0

im + km + lm = 0

END

**From l to i: (infeasible)**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 1

im + km + lm = 0

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to l:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 1

im + km + lm = 0

END

**From m to i: (infeasible)**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ab + ac + ad + ah - ba - da - ia = 0

ba + bc + be + bf - ab - cb = 0

cd + cb + cg + ci + cf - ac - bc - ec = 0

da + dg + dj + df - ad - cd - gd = 0

eh + ec + ei - be= 0

fi + fg - bf - cf - df= 0

gd + gj + gk - cg - dg - fg = 0

hi + hk - ah - eh - kh = 0

ji + jk + jl - dj - gj - ij= 0

kh + km - gk - ik - jk= 0

lm - jl = 0

im + km + lm = 1

ci + ei + fi + hi + ji - ia - ik - ij - im= 1

END

**From i to m:**

min 2ab + 3ac + 8ad + 9ah + 4ba + 5bc + 7be + 4bf + 10cd + 5cb + 9cg + 11ci + 4cf + 8da + 2dg + 5dj + 1df + 5eh + 4ec + 10ei + 2fi + 2fg + 2gd + 8gj + 12gk + 5hi + 10hk + 20ia + 6ik + 2ij + 12im + 2ji + 4jk + 5jl + 10kh + 10km + 2lm

ST

ia + ik + ij + im - ci - ei - fi - hi - ji= 1

ba + da + ia - ab - ac - ad - ah= 0

ab + cb - ba - bc - be - bf = 0

ac + bc + ec - cd - cb - cg - ci - cf= 0

ad + cd + gd - da - dg - dj - df= 0

be - eh - ec - ei= 0

bf + cf + df - fi - fg = 0

cg + dg + fg - gd - gj - gk= 0

ah + eh + kh - hi - hk= 0

dj + gj + ij - ji - jk - jl= 0

gk + ik + jk - kh - km = 0

jl - lm = 0

im + km + lm = 1

END