Bayesian hierarchical models

Advanced sampling

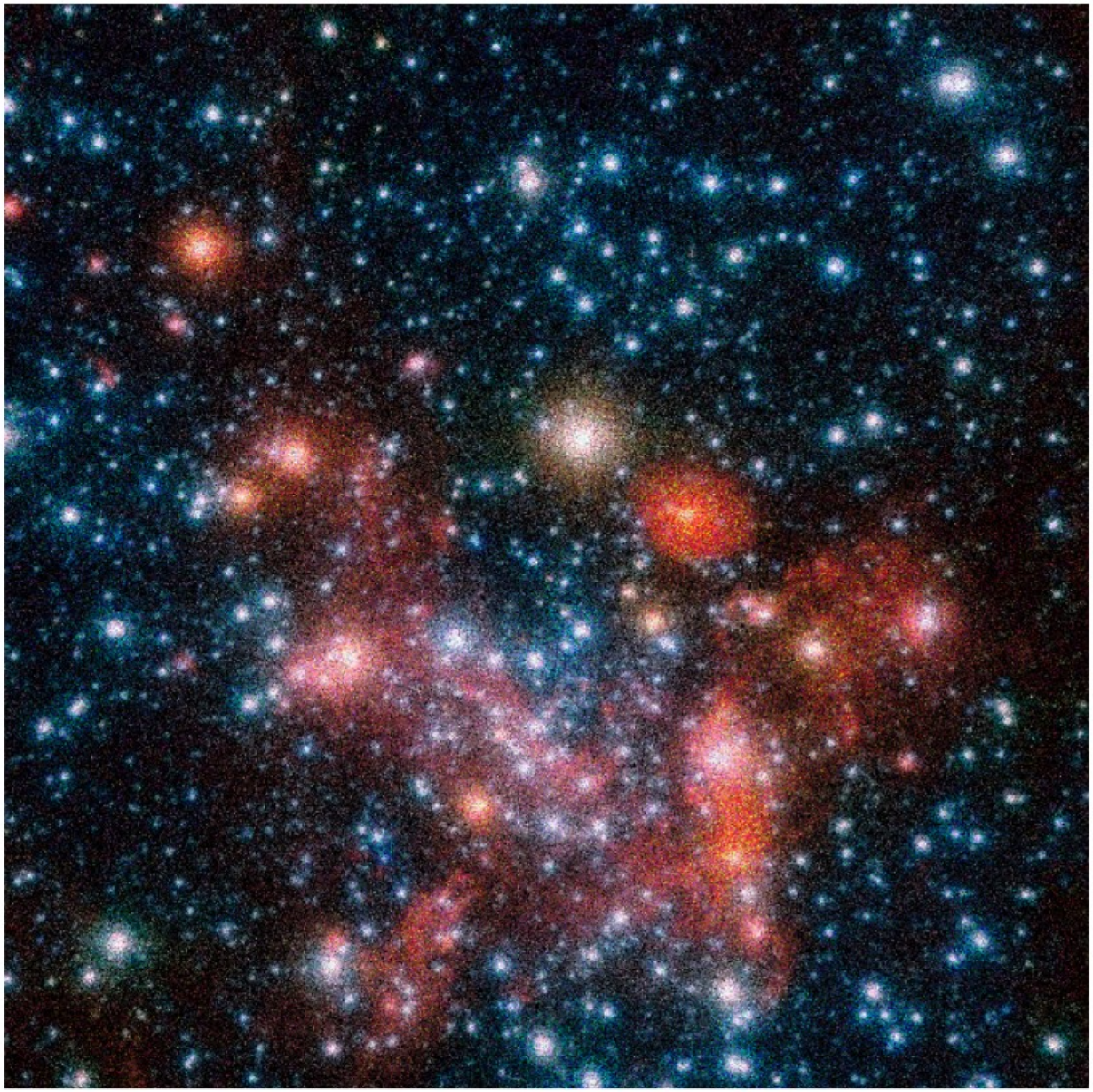
Natalia Porqueres

ICIC school – Sep 2024

True stellar field

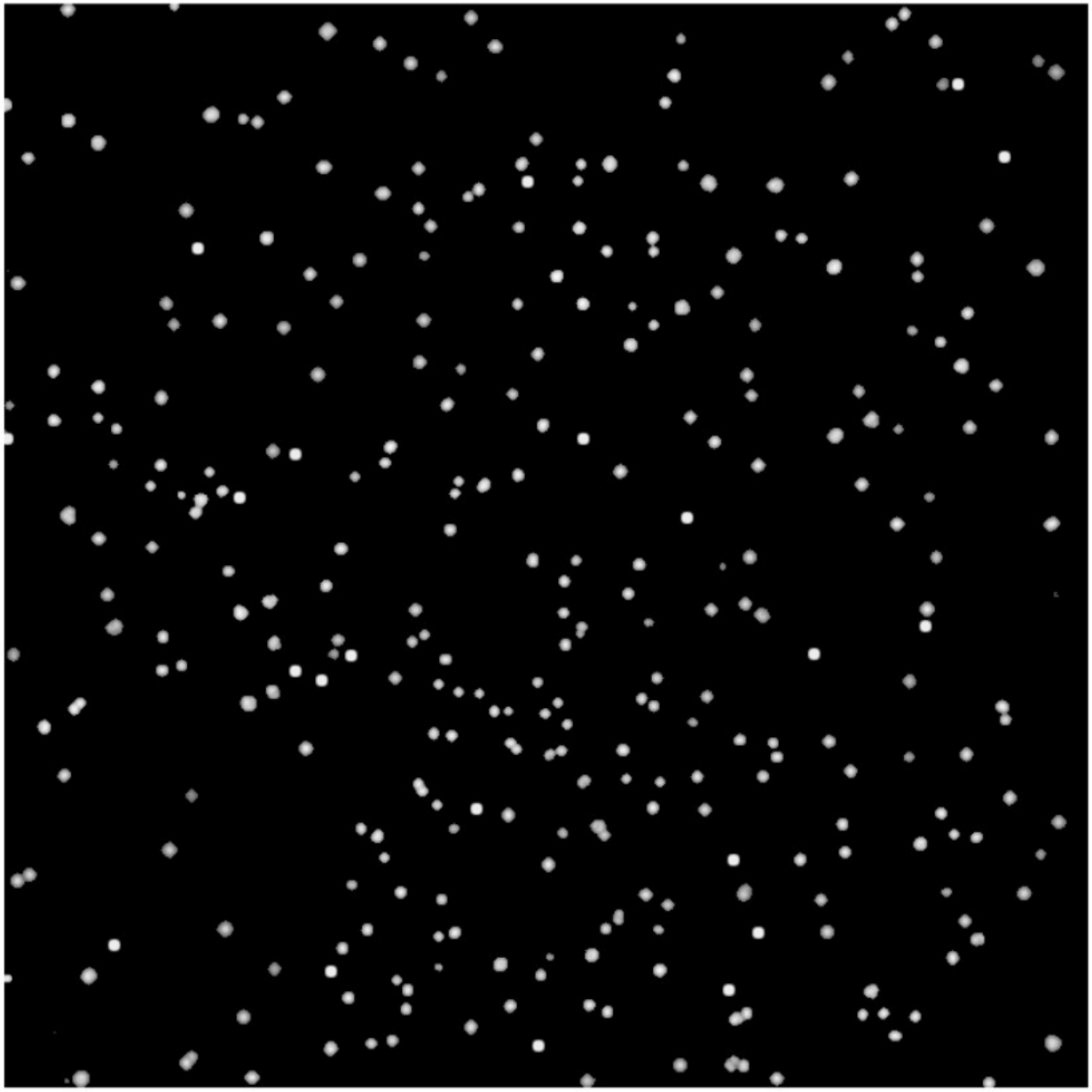
ESO

Stellar field with noise detector



ESO + ES

Assuming chi-square



ES

Weak gravitational lensing

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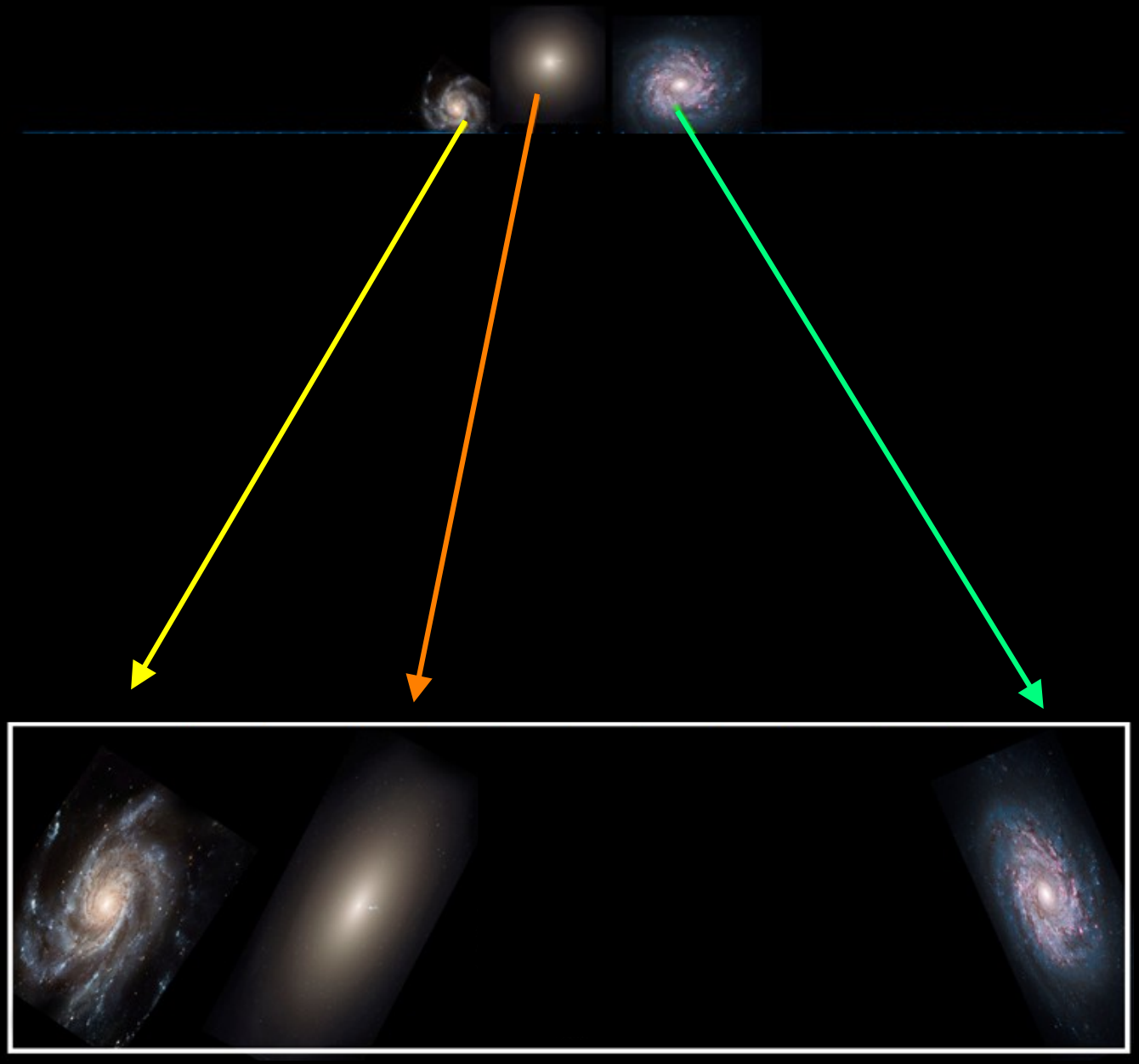
Weak gravitational lensing



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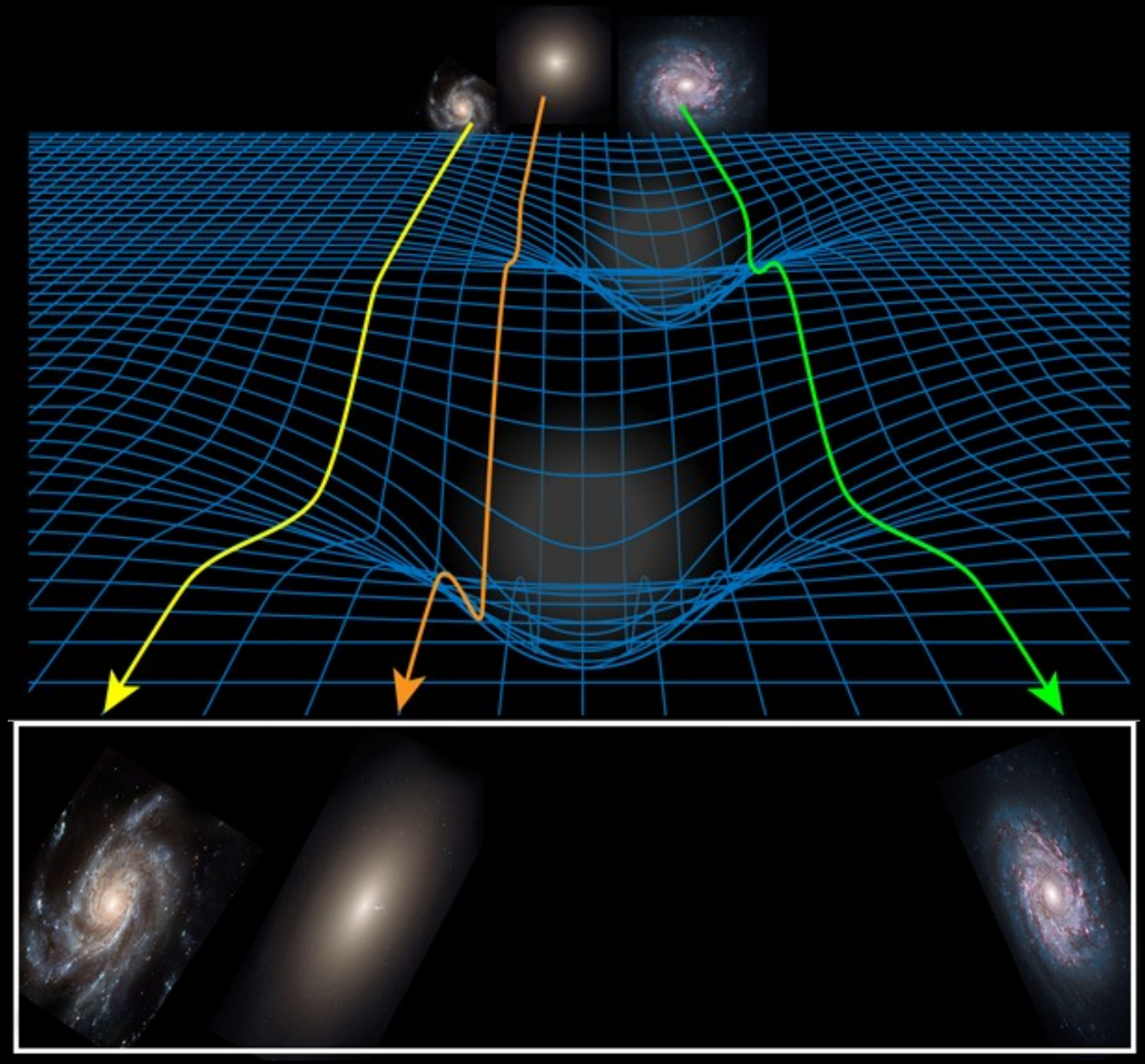
Weak gravitational lensing



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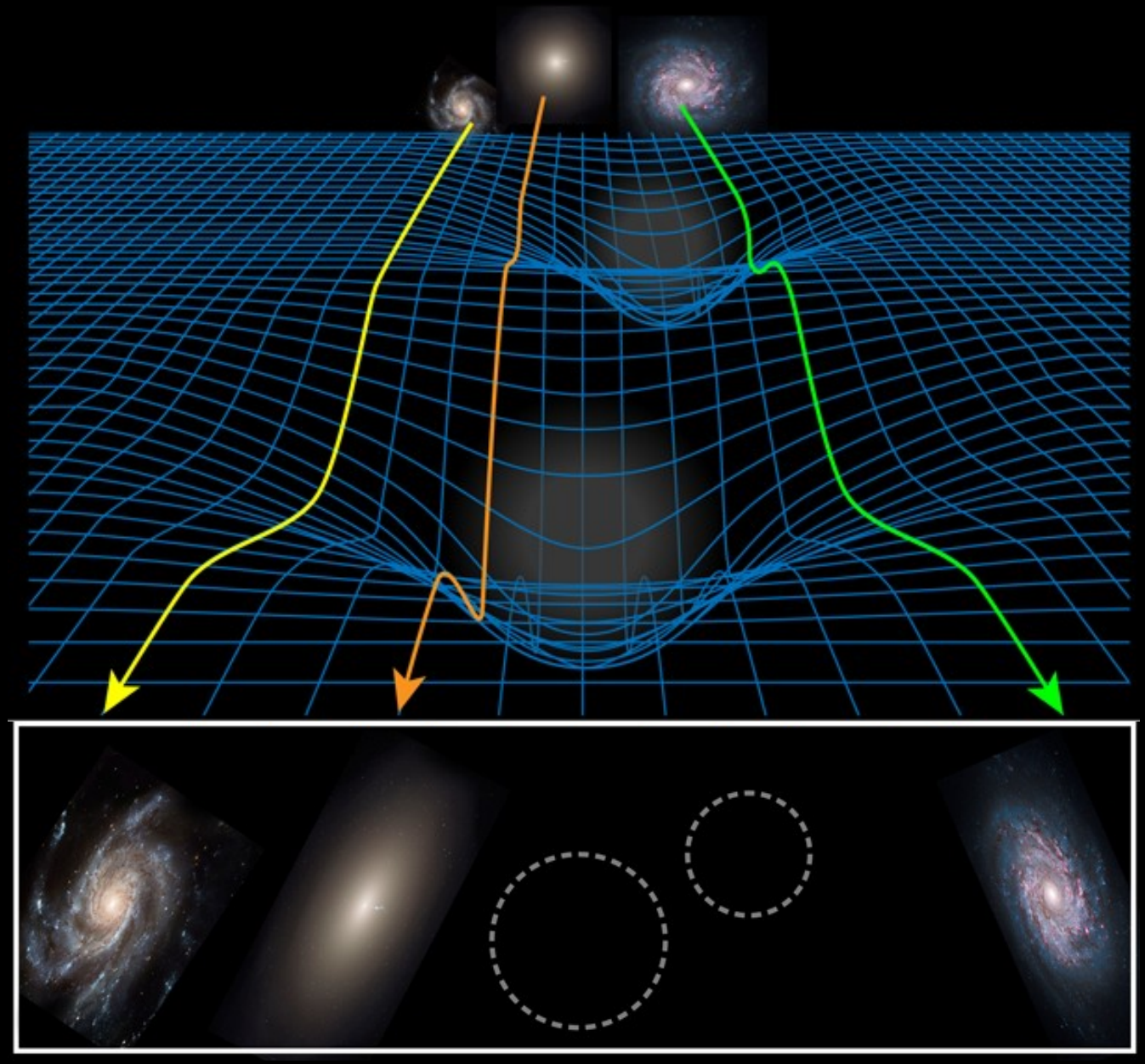

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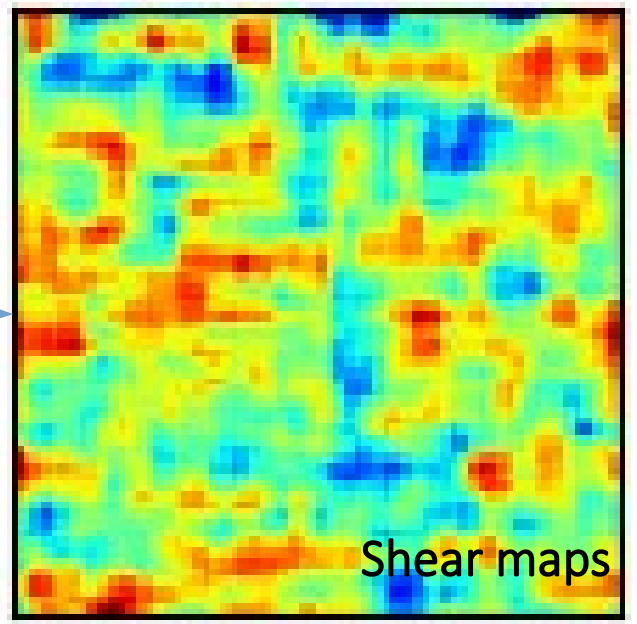
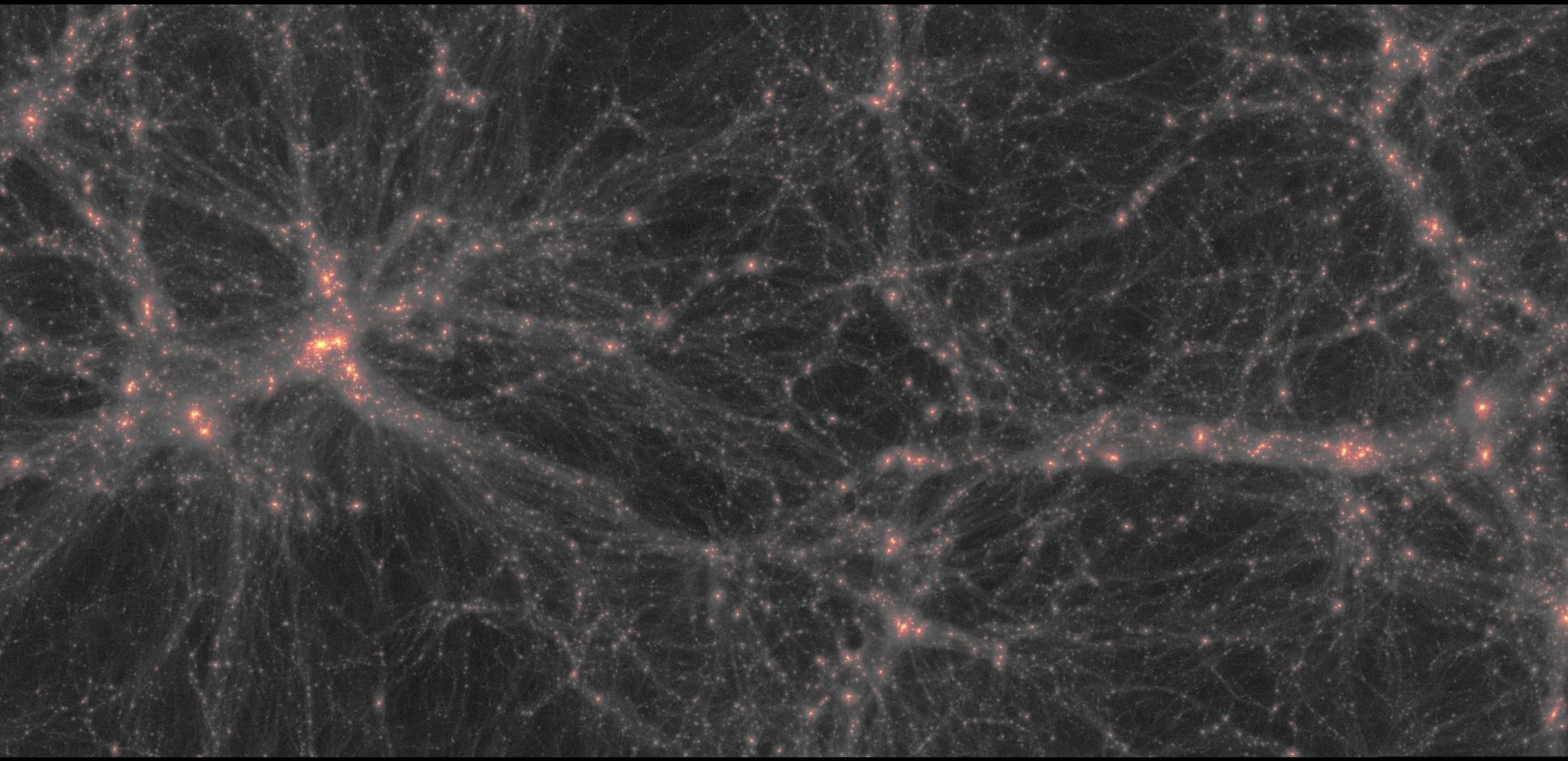
Building shear maps from the clean data



Average galaxies shape in pixels

The cosmic structures



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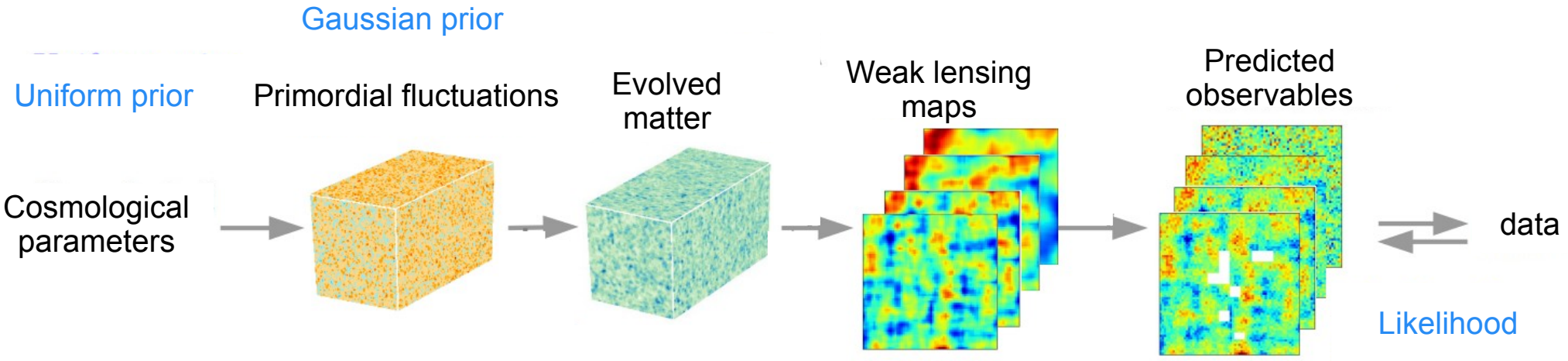
A BHM with simulations



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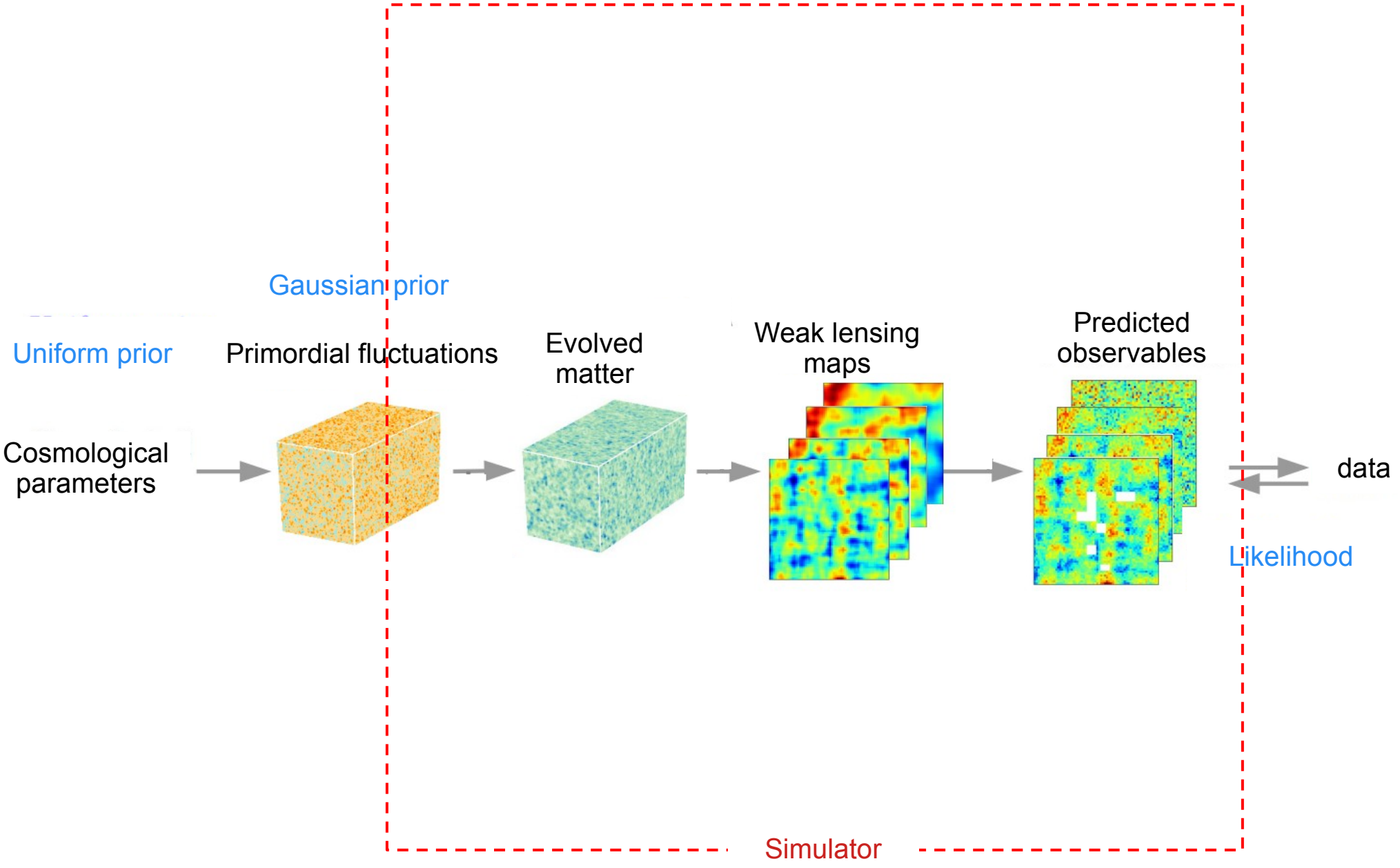

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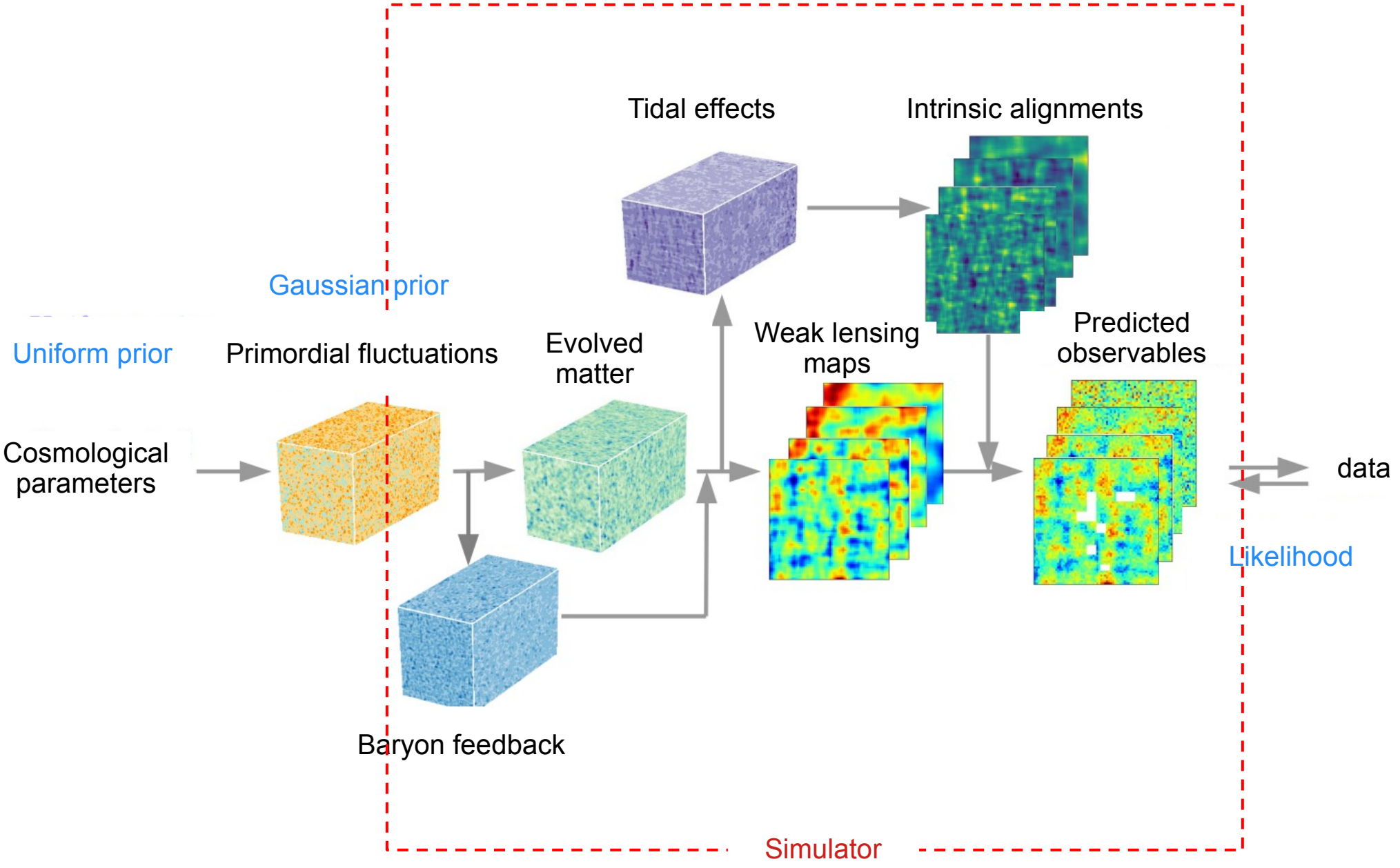
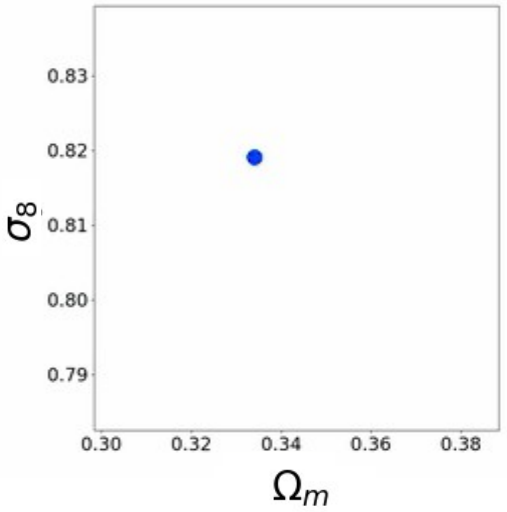


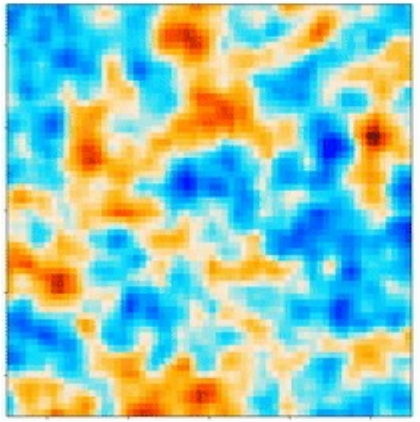
The samples

Cosmological parameters



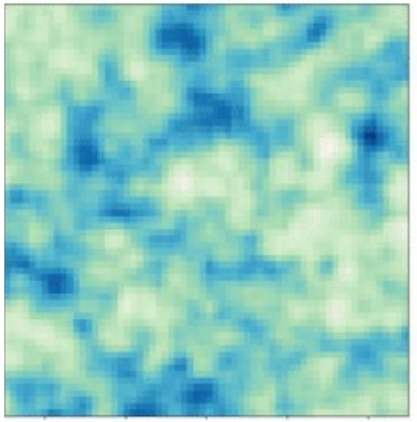
sampled

Primordial fluctuations

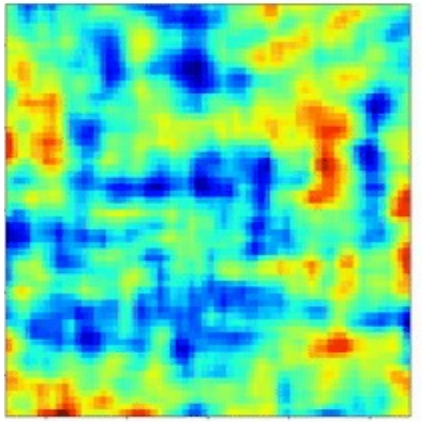


sampled

Evolved matter



Predicted lensing



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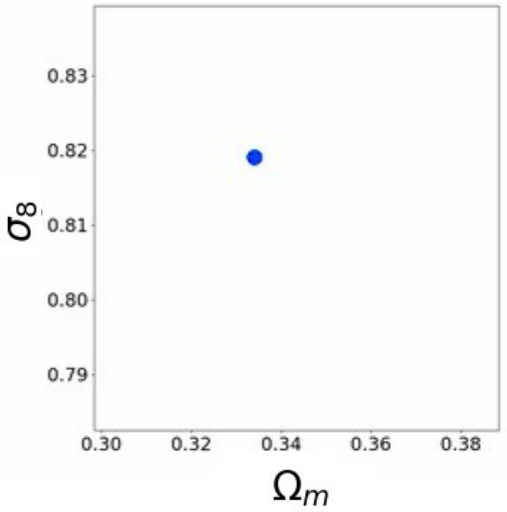

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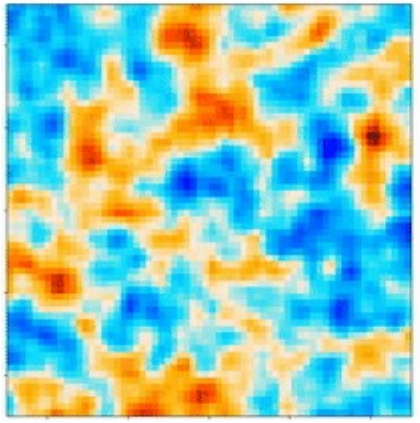

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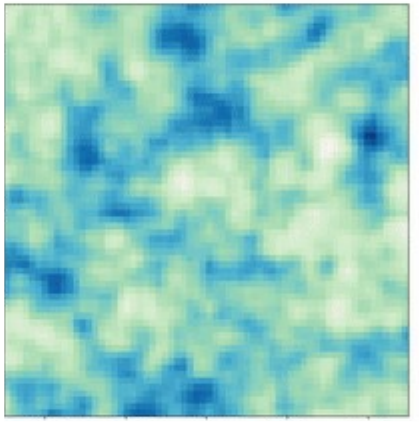

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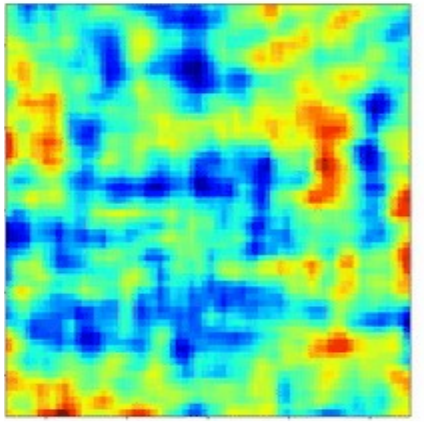
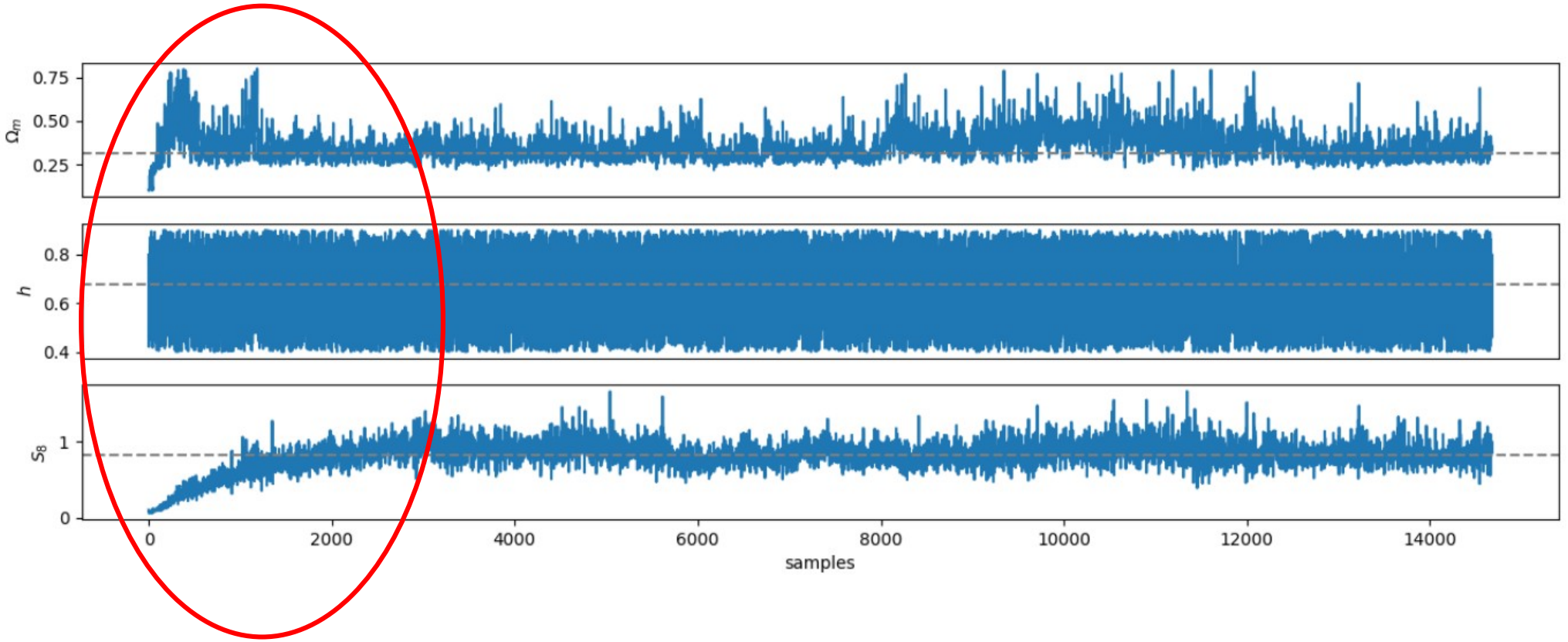


10^6 parameters!

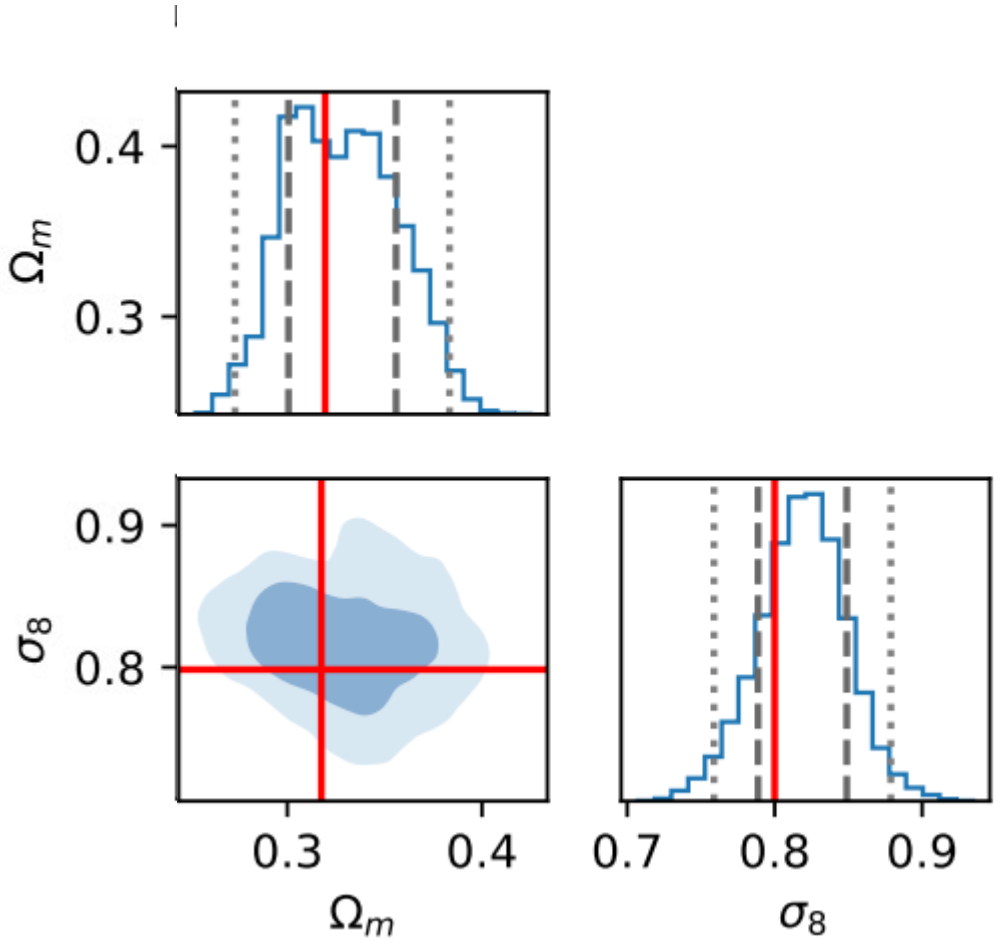
Bayesian Hierarchical Models

- Split the problem into steps, where the full data description is made of sub-models
- BHM links the sub-models and propagates the uncertainty from one step to the next
- We know the conditional probability of each step
- **BHM builds a complete model of the data and propagate the uncertainties in a principled way**
- In practice: sub-models introduce “latent” variables
- BHM can have many parameters.

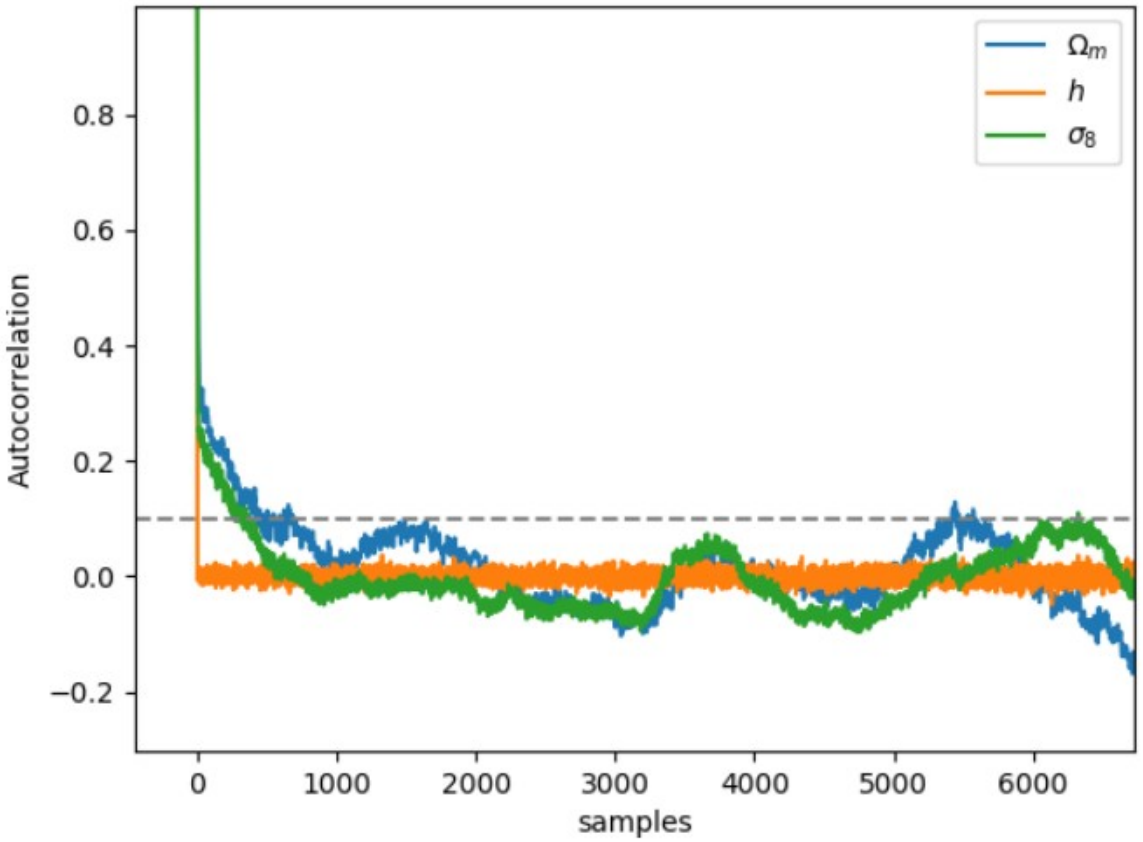
Testing results: trace plots



Testing results: Corner plots



Testing results: Correlation length



$$C_n(\theta) = \frac{1}{N-n}\sum_{i=0}^{N-n}\frac{(\theta^i - \langle\theta\rangle)(\theta^{i+n} - \langle\theta\rangle)}{\mathrm{Var}(\theta)}$$

Effective sample size:

$$N_{\mathrm{eff}} = \frac{N}{\sum_{t=-\infty}^{\infty}\rho_t} = \frac{N}{1+2\sum_{t=1}^{\infty}\rho_t}.$$

Testing results: Convergence tests

Gelman-Rubin test:

Run M chains with different starting point and N samples.

Calculate the mean and std in each chain: $\hat{\theta}_m = \frac{1}{N_m} \sum_i^{N_m} \theta_i^m$

Calculate the meand and std of all chains: $\hat{\theta} = \frac{1}{M} \sum_m^M \hat{\theta}_m$

Compute how the means vary around the joint

$$B = \frac{N}{M-1} \sum_{m=1}^{M} (\hat{\theta}_m - \hat{\theta})^2$$

Compute the average variance of the chains

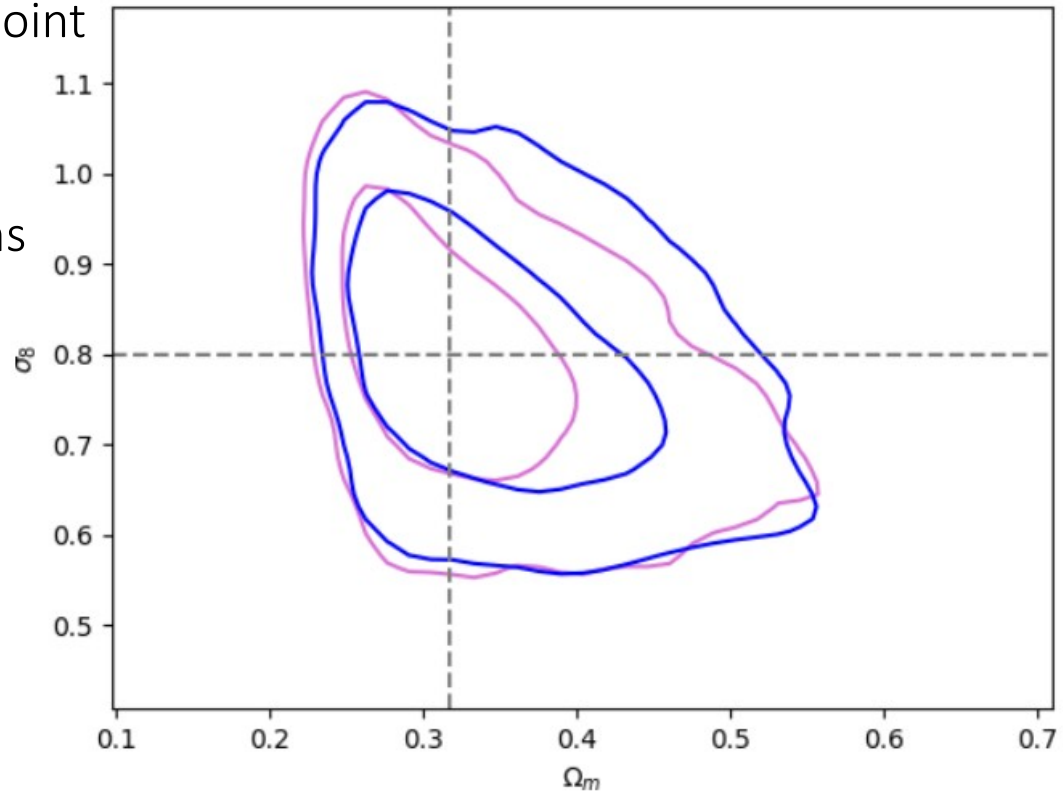
$$W = \frac{1}{M} \sum_{m=1}^{M} \sigma_m^2$$

Compute unbiased estimator of the true variance

$$\hat{V} = \frac{N-1}{N} W + \frac{M+1}{MN} B$$

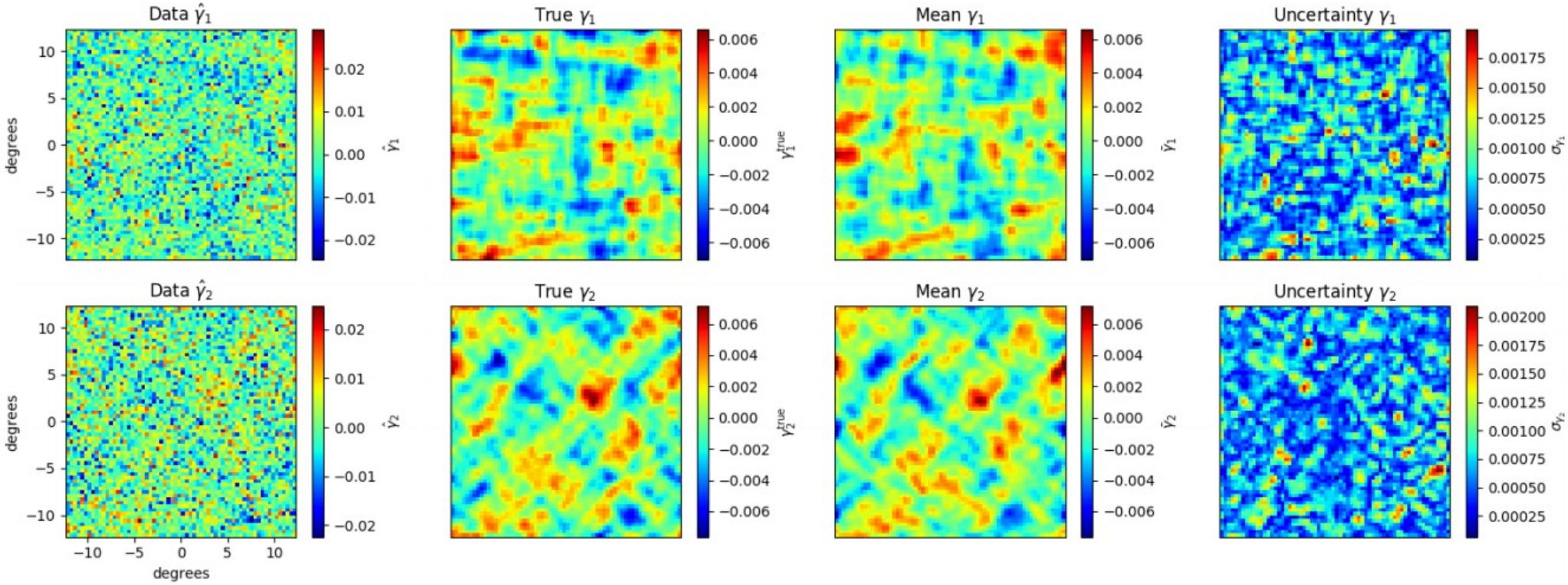
The chain has converged if

$$R = \sqrt{\hat{V}/W} \approx 1$$



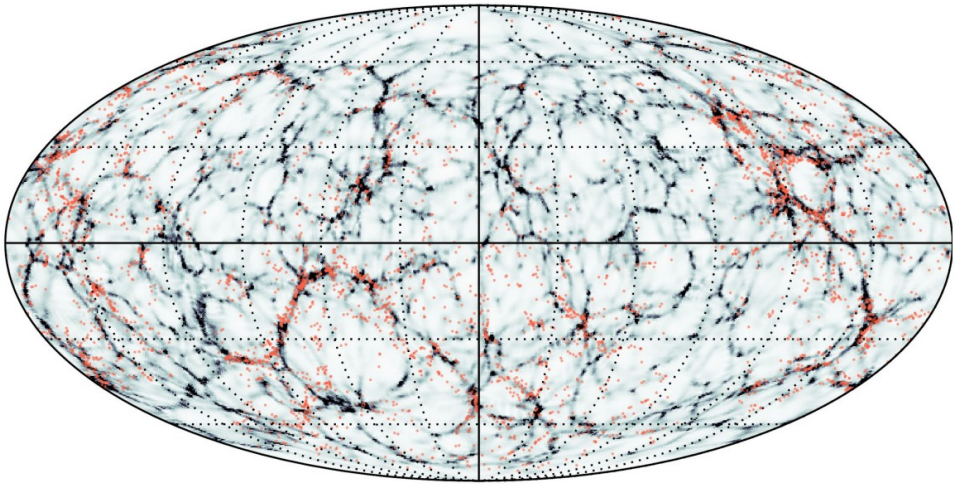
Posterior predictive tests

Posterior predictive tests: Can the inferred quantities explain the data?



Validating the results of a real data analysis

Posterior predictive tests and cross-validation with independent measurements



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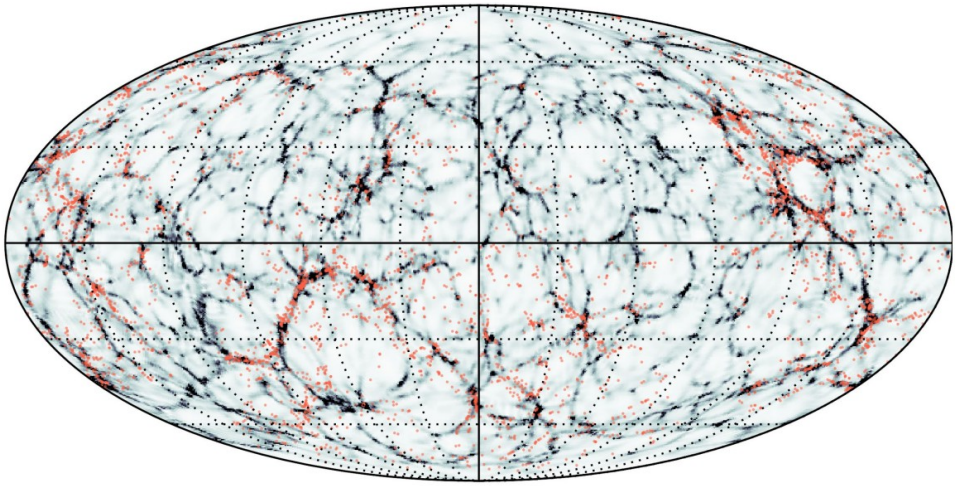
Posterior predictive tests and cross-validation with independent measurements

- do we get the clusters we know are there?



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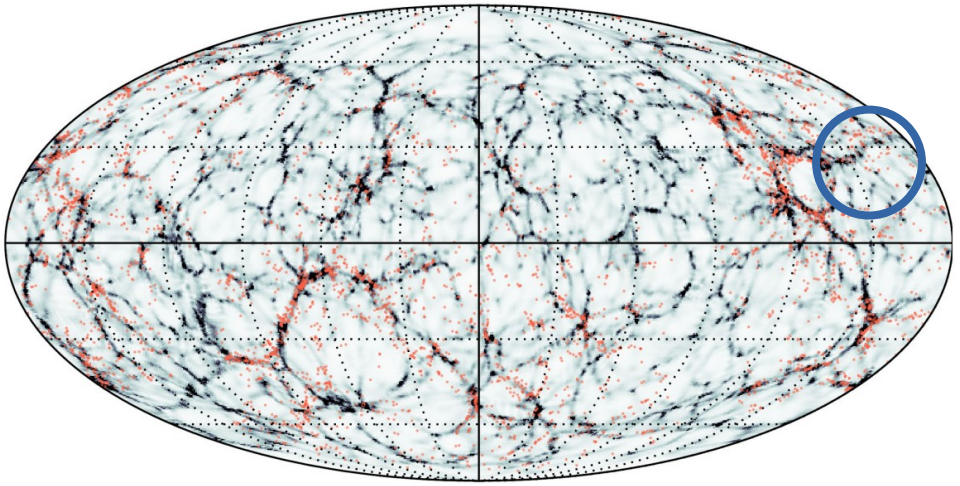

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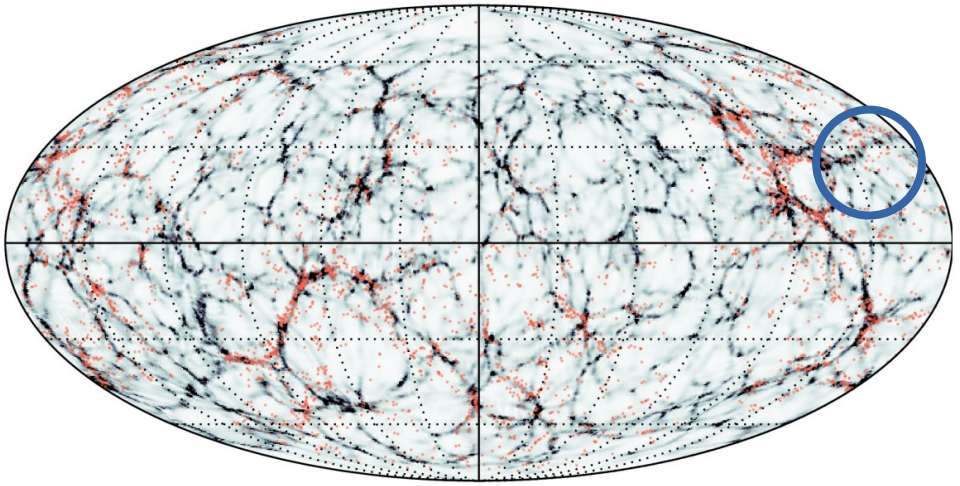

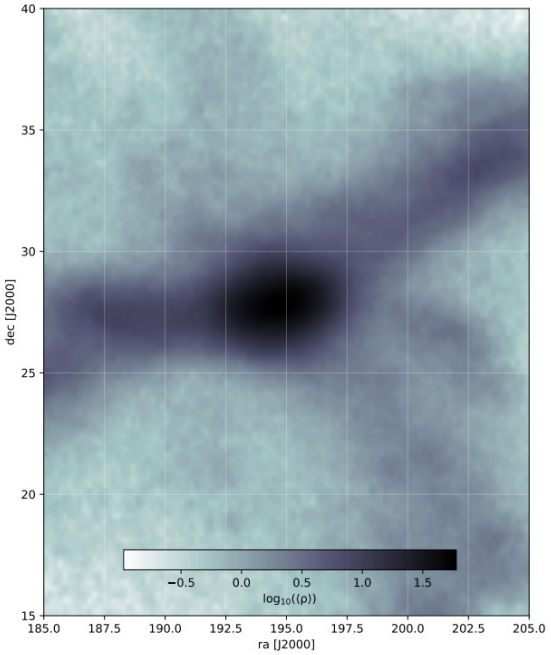

Jasche & Lavaux 2019

Validating the results of a real data analysis

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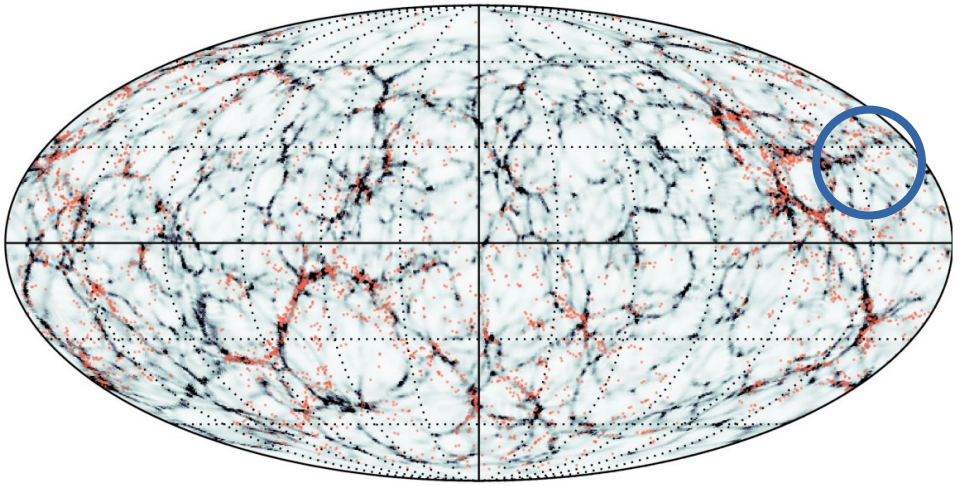
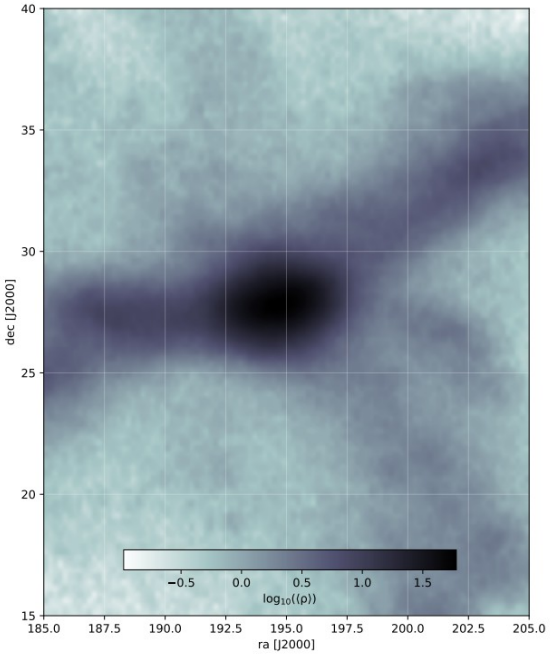
- do we get the expected mass profiles?





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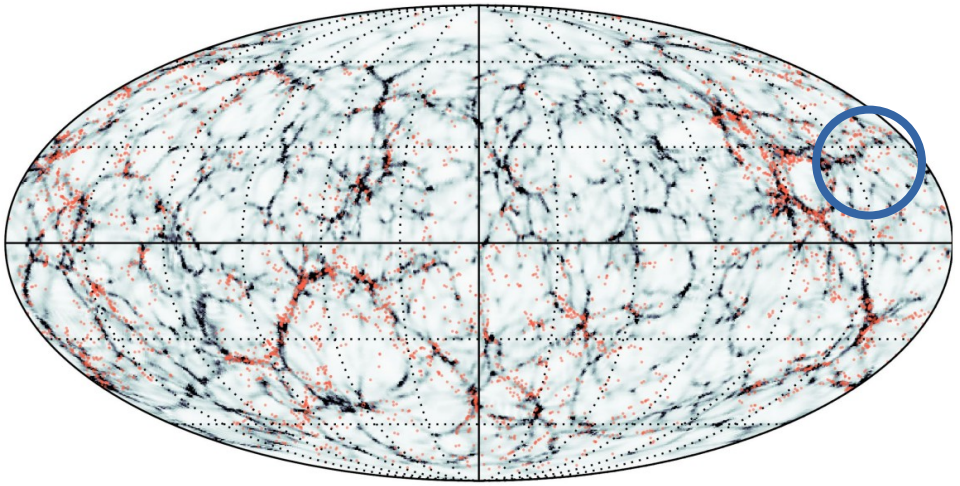

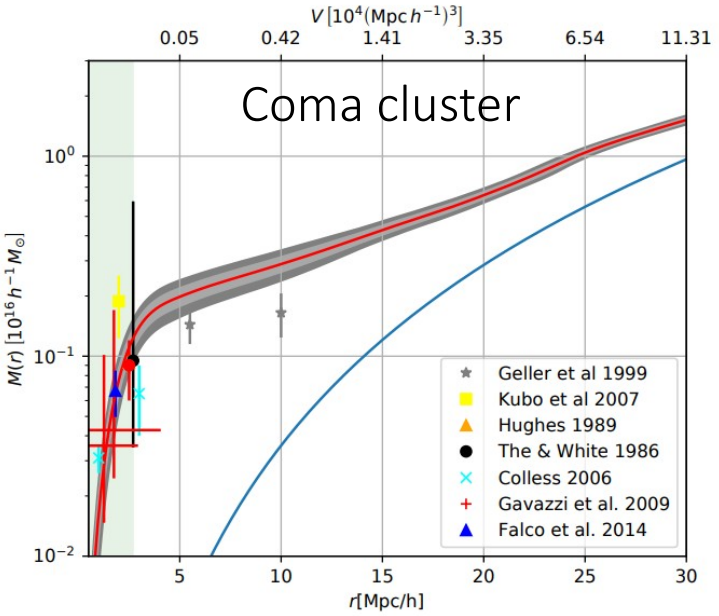

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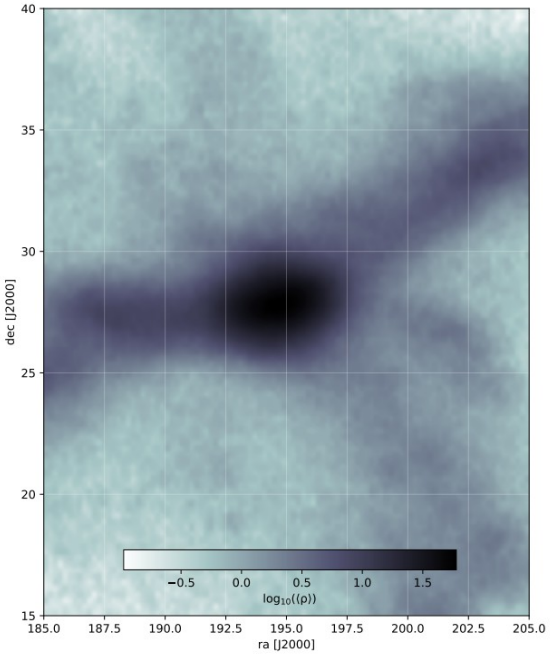

Jasche & Lavaux 2019

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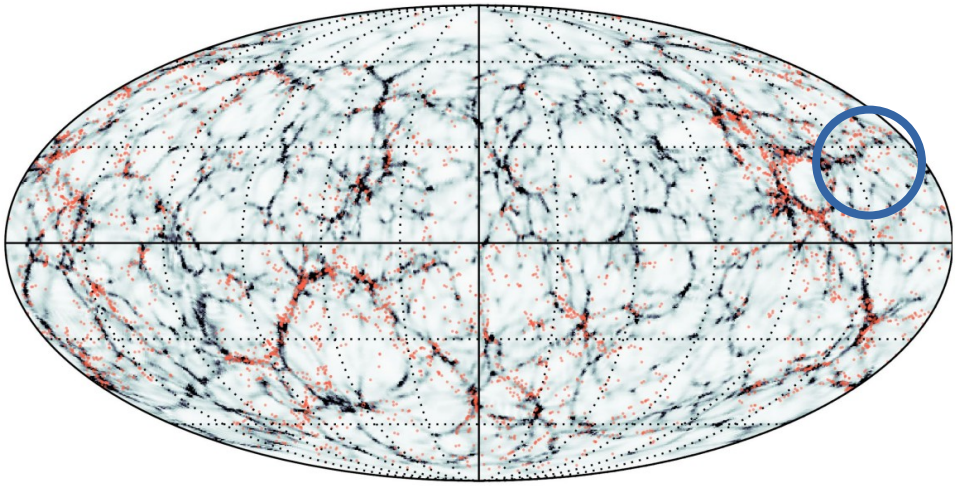
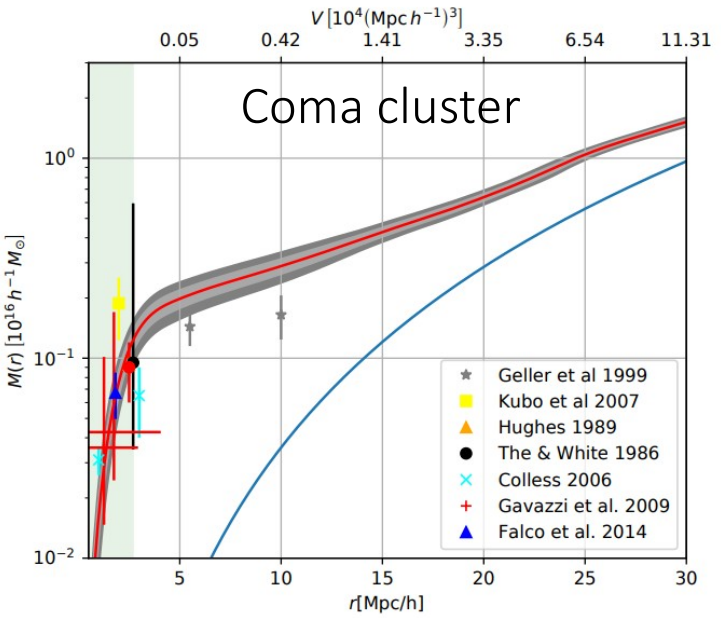
- are the inferred IC compatible with CMB?





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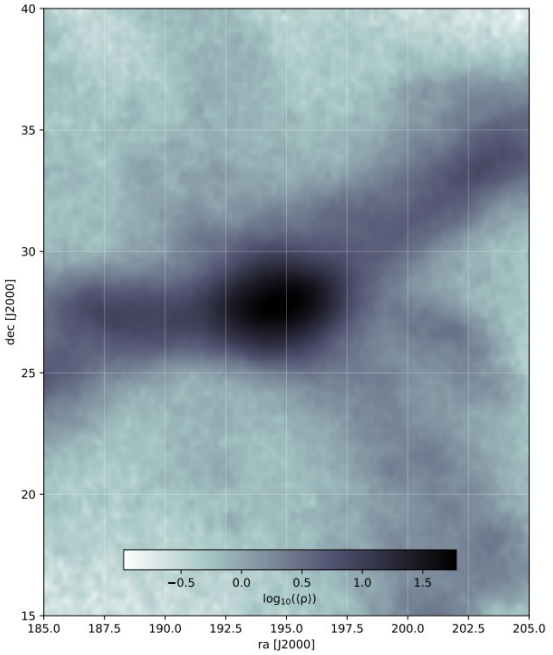
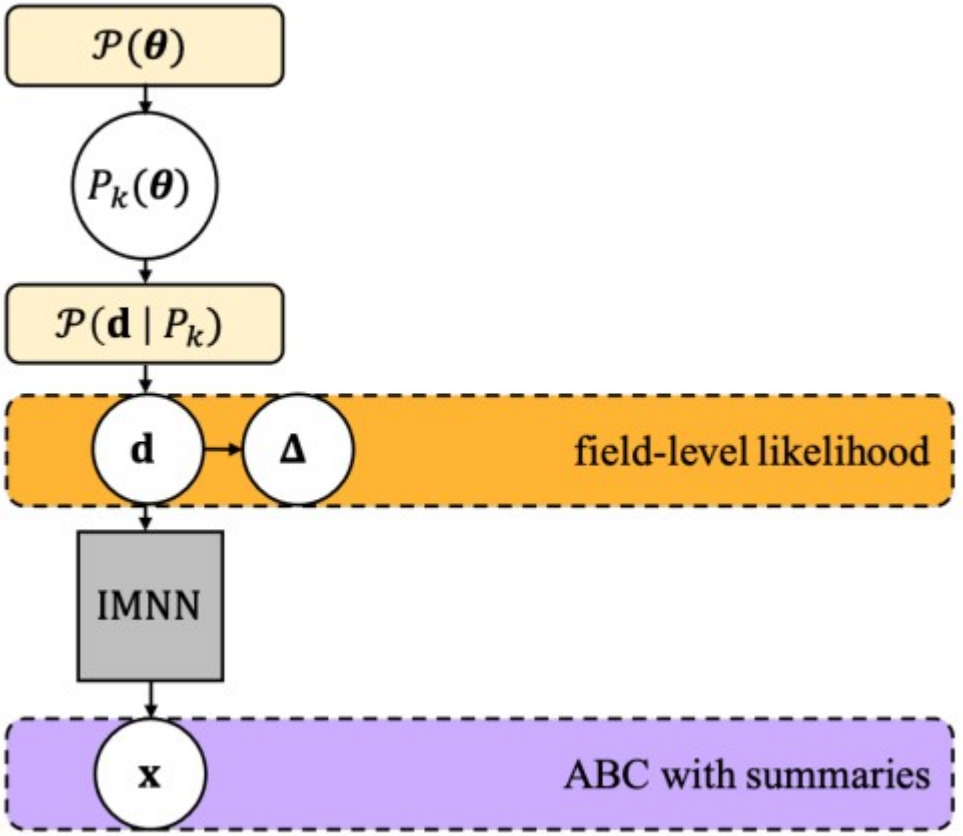


Jasche & Lavaux 2019

Natalia Porqueres

Can we find an alternative with the same info?



Makinen+ 2021

Two approaches to inference with simulators

Bayesian Hierarchical Model (aka explicit inference)

Treat the simulator as a probabilitic model to sample full posterior distribution

(e.g. Jasche+2013, Huiyuan+2014, Seljak+2017, Modi+2018, Schmidt+2019, Porqueres+2022, Boruah+2022)

Simulation-based inference (aka implicit inference, likelihood-free inference)

Treat the simulator as a black box to draw random samples

(e.g. Fluri+2018, Charnock+2018, Makinen+2021, Gupta+2019, Jeffrey+2020, Leclercq2022, Dai+2022)