Introduction

- So I will be talking about the Hidden Beauty of the Double Pendulum, that may sound strange now but I will take you through everything and by the end I hope you will appreciate what I mean.
- Let's dive straight in with Chaos Theory.

Chaos Theory

- Some of you may have heard of Chaos Theory already, maybe you have heard of the slightly sadistic butterfly that starts a hurricane on the other side of the world with one flap of its wings.
- Whilst this is not entirely true this indicates towards the fundamental principle of chaos theory, that a small change in the starting conditions of a system leads to a large change in the final result.
- Or take the turbulence coming from the tip of an airplane wing, another example of a chaotic system.
- This may sound abstract and confusing but it occurs throughout nature and physics so allow me to show you an example of this in action, namely the double pendulum.

Introduction to the double pendulum

- To start with let's think of a conventional single pendulum. Now this is quite a predictable system, you attach a mass to a string, pull it aside and it will swing back and forth. This predictable motion is quite useful for clocks and timekeeping but it also quite boring and for us looking at chaos theory not very useful.
- So why not attach two pendulums end to end with a join in between and see what happens (indicate towards real life example)
- Now we can see the interesting curves that emerge from this system.
- Notice how also if we slightly change the initial conditions such as the angle of both the pendulums this leads to a large divergence in the outcome which can be seen as a very different curve being drawn out.

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Finding a measurable parameter

- These curves are interesting and demonstrate the crucial part of this system but they don't provide the scientist with a measurable quantity. So in order to find something to measure to really understand this system we need to dig a little deeper.
- Now watching these pendulums swing you can observe when the bottom pendulum flips over the vertical at certain points during the motion.
- Now we can measure the time it takes from releasing the pendulum to the first flip that occurs and call this the "flip time."
- This is the measurable quantity that we need to analyse this system and the changes in outcomes that result in different initial positions.

Approach to measurement

- With this in mind we can begin to measure different flip times for different starting positions of both pendulums based on their angles.
- However we hit a problem, there are many starting positions and only one of us with a stopwatch and it would be immensely boring to sit here and measure the flip time over and over and probably also very inaccurate as it is hard to position the pendulums exactly.
- So we need to rethink our approach, rather than run the experiment in real life we can run a repeated computer simulation to determine the flip time from *every* possible position.
- To do this we need the *characteristic equations* of the system so here comes the maths, you'll love this (pause) bear with me.

Deriving the equations of the system

Lagrangian definition

- We start with the coordinates of both pendulums and defining them using a bit of trigonometry with the initial angles and lengths of both pendulums, as you can see here.
- Then we use a mathematical tool called the Lagrangian. Now conceptually this tool is quite simple, all it does is take the difference in the potential energy of a system and the kinetic energy of a system.
- So we define the Lagrangian taking the gravitational potential energy of both pendulums which depends on the masses and the lengths and also the kinetic energy which depends on their rotational speed and their linear speed.
- We can then simplify this overall expression just in terms of angles using the coordinates we found earlier.

Using the Lagrangian

- To find the characteristic equations that we wanted, we can now differentiate this Lagrangian expression to find the rates of change of 4 things.
- The first two equations that we get tell us how the angles of both pendulums change in the next time step based on their speed and current position.
- The second two equations tell us how the speeds change based on the current speed and position.

Creating a simulation and measuring

- Now we have the 4 characteristic equations needed we can create a computer simulation to run our experiment and measure the flip time.
- Using a process called numerical integration in a programming language called Python using their Scipy package we can simulate the movement of the double pendulum using the characteristic equations.
- Here is an example of the motion on the simulation.

Measuring the flip time

- Finally we can run our simulation for every possible starting position and measure the flip time, to display this information we produce a colour map which shows the starting angle of the pendulums on each axis and plots the flip time as a colour, the darker the colour the longer the flip time. Here is the result.
- Now you can see what I meant by the hidden beauty and the true nature of chaos is revealed. There are still distinct sections and patterns that are being followed here but the chaotic nature produces sudden changes in the flip time based on a small change in the starting position at these regions.