Josh Gribbon

I pledge my honor I have abided by the Stevens Honor system.

Section 1.5

4. Let P (x, y) be the statement “Student x has taken class y,” where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x \exists y P(x, y)$ Some student has taken each CS course

b) $\exists x \forall y P(x, y)$ Some student has taken every CS course

c) $\forall x \exists y P(x, y)$ All students take some CS course

d) $\exists y \forall x P(x, y)$ Some CS course is taken by every student

e) $\forall y \exists x P(x, y)$ Every course is taken by some student

f) $\forall x \forall y P(x, y)$ Every course is taken by every student

8. Let Q(x, y) be the statement “student x has been a contestant on quiz show y.” Express each of these sentences in terms of Q(x, y), quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

a) There is a student at your school who has been a contestant on a television quiz show.

$\exists x,y\ Q(x, y)$

b) No student at your school has ever been a contestant on a television quiz show.

$\neg \exists x,y\ Q(x, y)$

c) There is a student at your school who has been a contestant on Jeopardy and on Wheel of Fortune.

$\exists x\ (Q(x, \text{Jeopardy}) \land Q(x, \text{Wheel of Fortune}))$

d) Every television quiz show has had a student from your school as a contestant.

$\exists x \forall y\ Q(x, y)$

12. Let I (x) be the statement “x has an Internet connection” and C(x, y) be the statement “x and y have chatted over the Internet,” where the domain for the variables x

and y consists of all students in your class. Use quantifiers to express each of these statements.

a) Jerry does not have an Internet connection.

$\neg I(\text{Jerry})$

b) Rachel has not chatted over the Internet with Chelsea.

$\neg C(\text{Rachel}, \text{Chelsea})$

c) Jan and Sharon have never chatted over the Internet.

$\neg C(\text{Jan}, \text{Sharon})$

d) No one in the class has chatted with Bob.

$\neg \exists x\, C(x, \text{Bob})$

e) Sanjay has chatted with everyone except Joseph.

$\forall x[(x \equiv \neg \text{Joseph}) \rightarrow C(x, \text{Sanjay})]$

f) Someone in your class does not have an Internet connection.

$\exists x\, \neg I(x)$

Section 1.6

4. What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

Simplification

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

Addition

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

Modus ponens

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

Addition

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Hypothetical syllogism

8. What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”

Modus tollens

24. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x Q(x)$	Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

Steps 4 and 6, and thus 7, are not valid. The premise works for all x when both parts are there. If you split it up, then they might not be true. For example x could be numbers, P(x) could x is positive, Q(x) could be x is not positive. So for all x, the premise is true, but P(x) and Q(x) are not true for all x individually.

Section 1.7

10. Use a direct proof to show that the product of two rational numbers is rational.

$a,b,c,d \in Z$	a,b,c,d are integers
$b,d \neq 0$	b,d are non-zero integers
$x,y \in Q$	x,y are rational numbers
$x=a/b$	def of rational numbers
$y=c/d$	def of rational numbers
$x*y = (a*c)/(b*d)$	
$x*y \in Q$	the product of integers in an integer, the quotient of

integers is rations, so the product of rational numbers is rational