Stochastic Motion Planning Under Uncertainty Roadmap

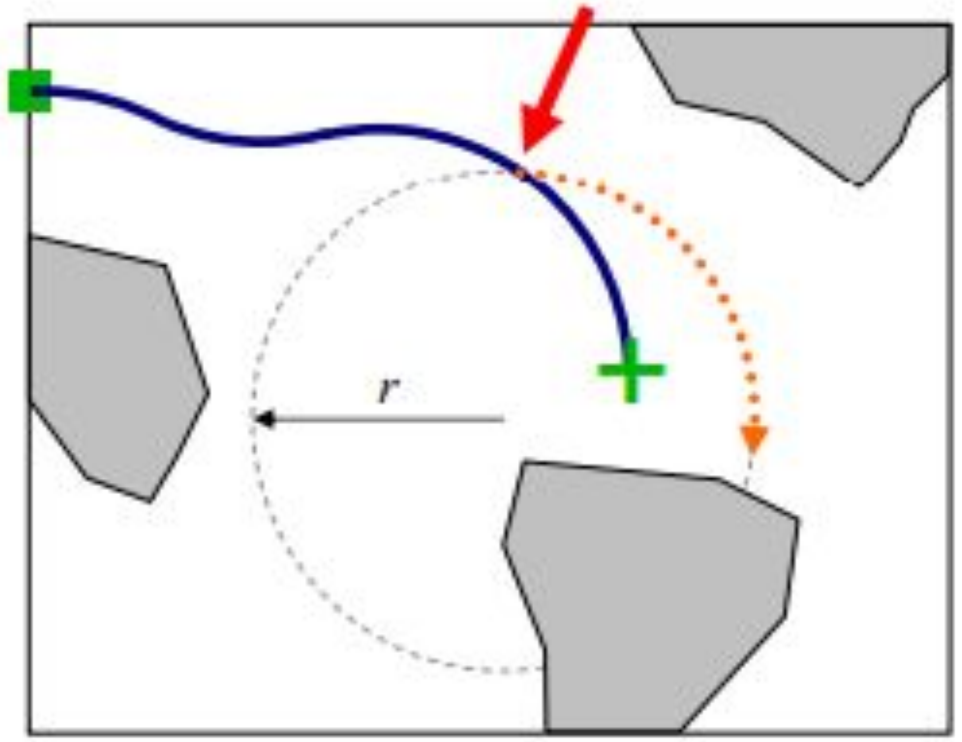
Joshua Han (jh273) & Alan Huang (ah212)

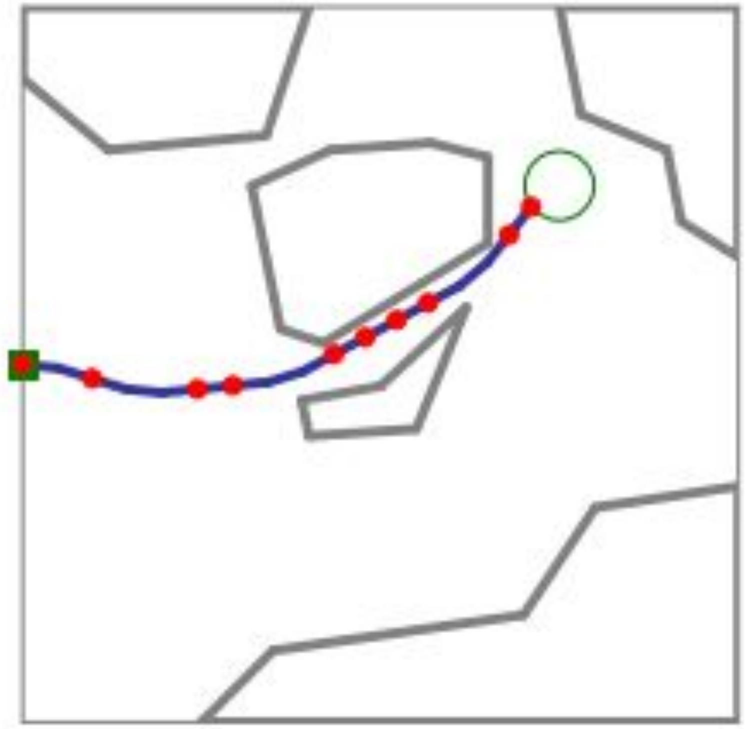
The Challenge of Planning in an Uncertain World

Uncertain Motions can cause ...

- Robots to veer off planned paths

- Collide into obstacles





All images are from "Wolfram Burgard; Oliver Brock; Cyrill Stachniss, "The Stochastic Motion Roadmap: A Sampling Framework for Planning with Markov Motion Uncertainty," in Robotics: Science and Systems III , MIT Press, 2008, pp.233-240."

What is Stochastic Motion Roadmap?

A planner that learns an optimal policy through sampling!

- Constructs a roadmap of sampled points (like PRM).
- Does not connect roadmap vertices directly.
- Instead, edges (v, v') represent the probability P(v' | v, a) to reach v' from v by taking action a.

Understanding Sample Transitions

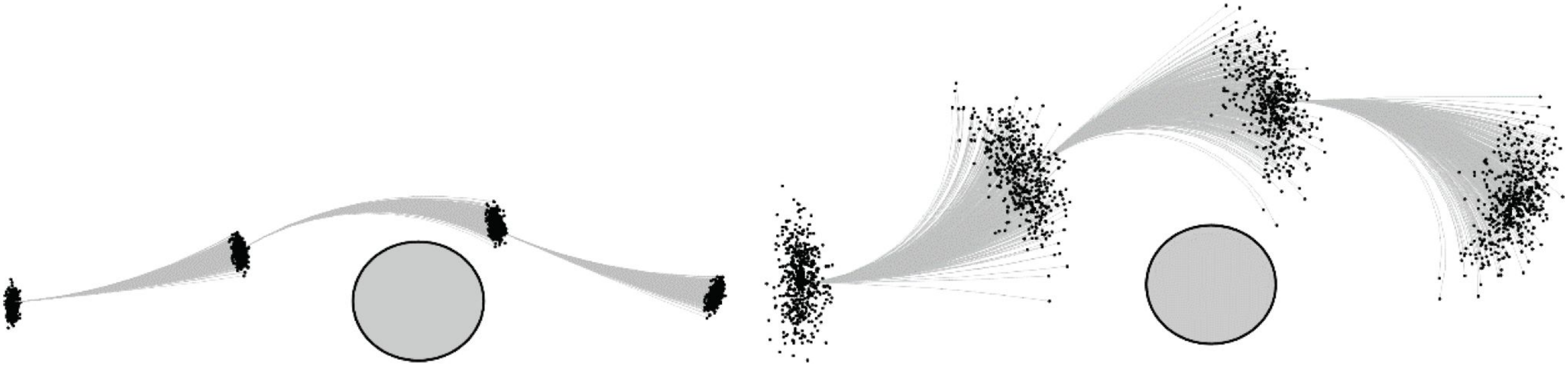
How to build a SMR:

- Given: sample size n, discrete set of controls U, number of sample transitions m.

- From starting vertex s in roadmap, simulate the uncertain motion of the robot until you get a valid path to state q. Repeat m times for each possible action $u \in U$.

- For each q, find the nearest vertex t in roadmap. Add edge (s, t, p) to roadmap, where p is the proportion of sample transitions from s that get near t.

Understanding Sample Transitions



(a)Avoiding obstacle with low uncertainty (b)Avoiding obstacle with high uncertainty

Image from "Jin, J., & Chung, W. (2019). Obstacle Avoidance of Two-Wheel Differential Robots Considering the Uncertainty of Robot Motion on the Basis of Encoder Odometry Information. *Sensors*, *19*(2), 289. https://doi.org/10.3390/s19020289"

Querying a SMR as a Markov Decision Process

How do we use a Stochastic Motion Roadmap to find a policy:

- Goal: find a policy that gives the best action for each vertex in roadmap

$$p_s(i) = \max_{u_i} \{E[p_s(j)|i, u_i]\},$$

$$p_s(i) = \max_{u_i} \left\{ \sum_{j \in V} P_{ij}(u_i) p_s(j) \right\}.$$

- Recognize this is a Markov Decision Process, which can formulated as a Bellman equation:

$$J^*(i) = \max_{u_i} \sum_{j \in V} P_{ij}(u_i) \left(g(i, u_i, j) + J^*(j) \right).$$

All images are from "Wolfram Burgard; Oliver Brock; Cyrill Stachniss, "The Stochastic Motion Roadmap: A Sampling Framework for Planning with Markov Motion Uncertainty," in Robotics: Science and Systems III , MIT Press, 2008, pp.233-240."

Problem Space: Needle Steering

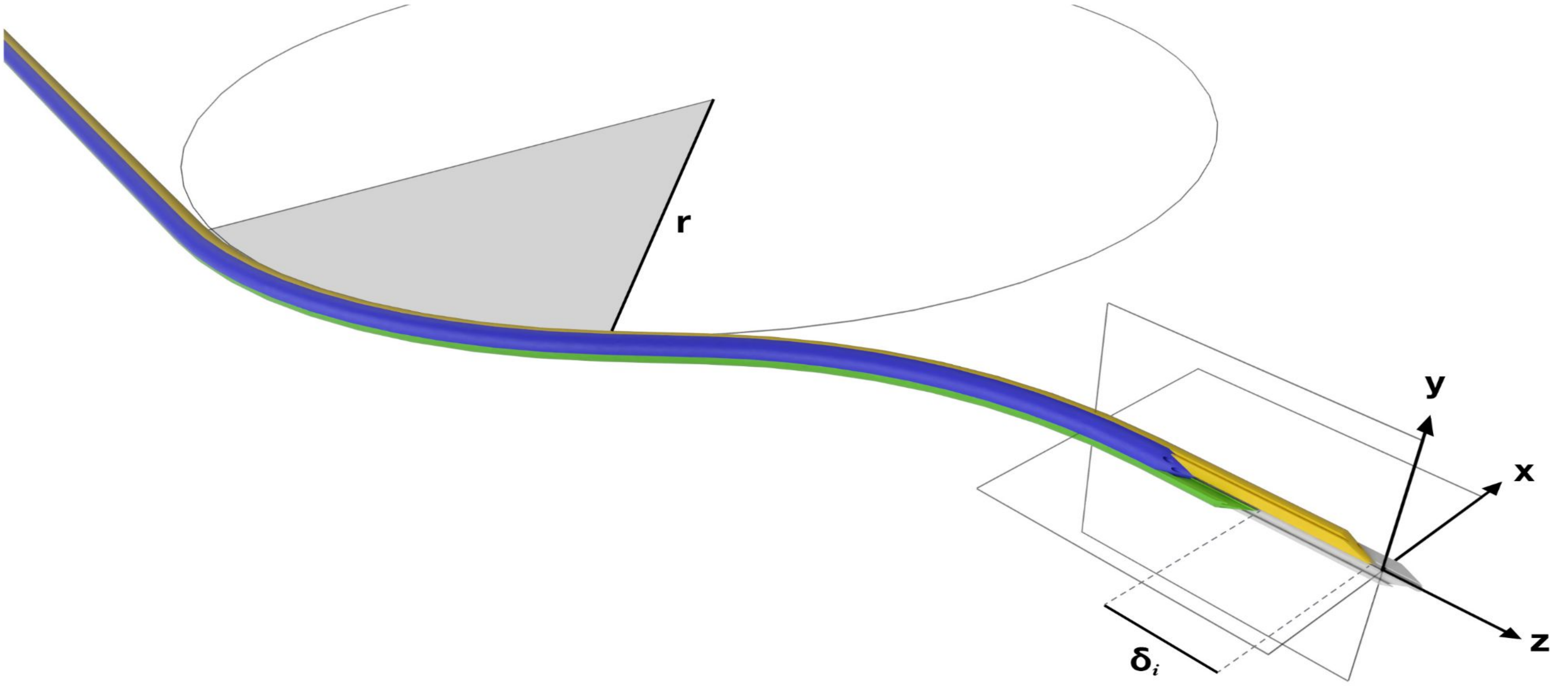
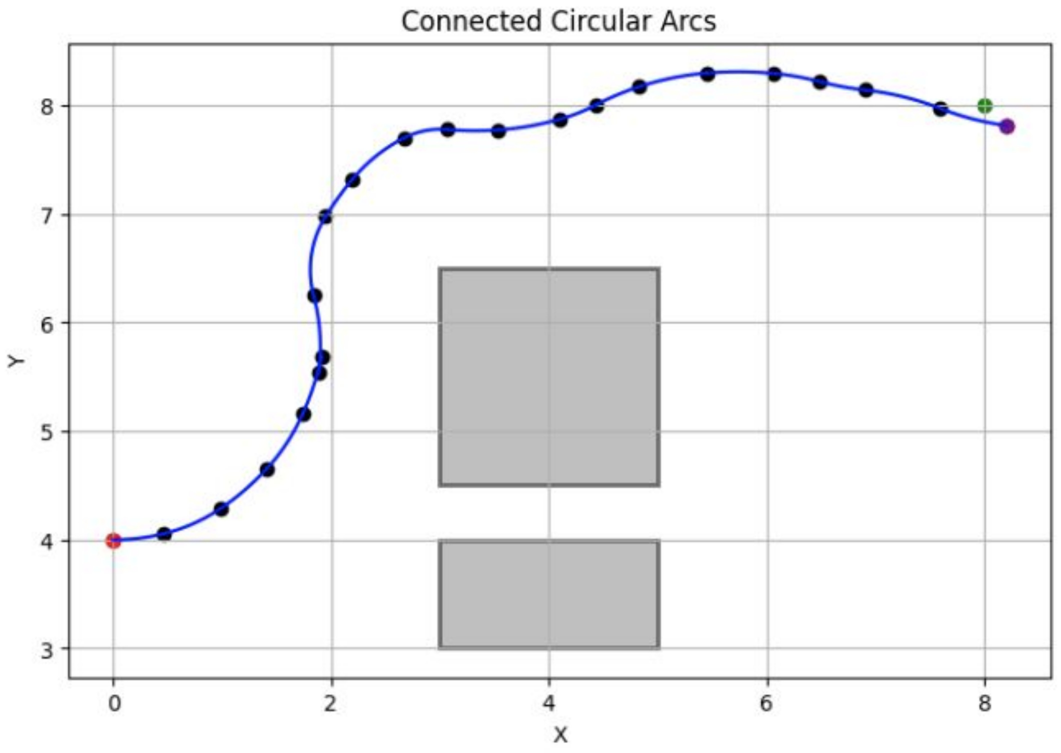


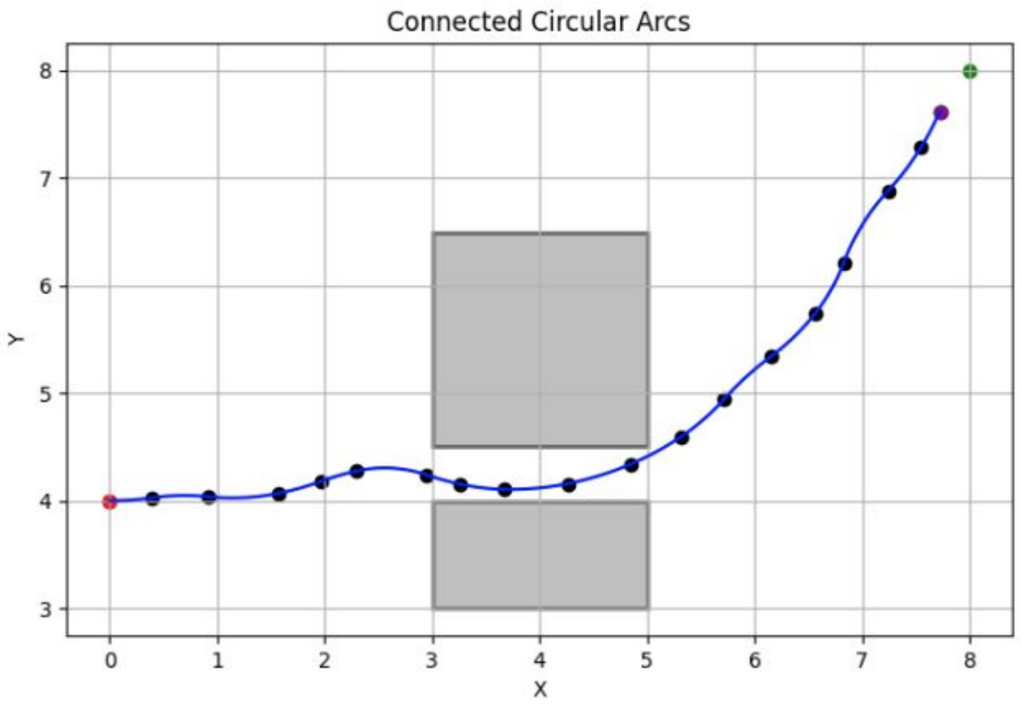
Image from "Matheson, E., & Rodriguez y Baena, F. (2020). Biologically Inspired Surgical Needle Steering: Technology and Application of the Programmable Bevel-Tip Needle. *Biomimetics*, *5*(4), 68. https://doi.org/10.3390/biomimetics5040068"

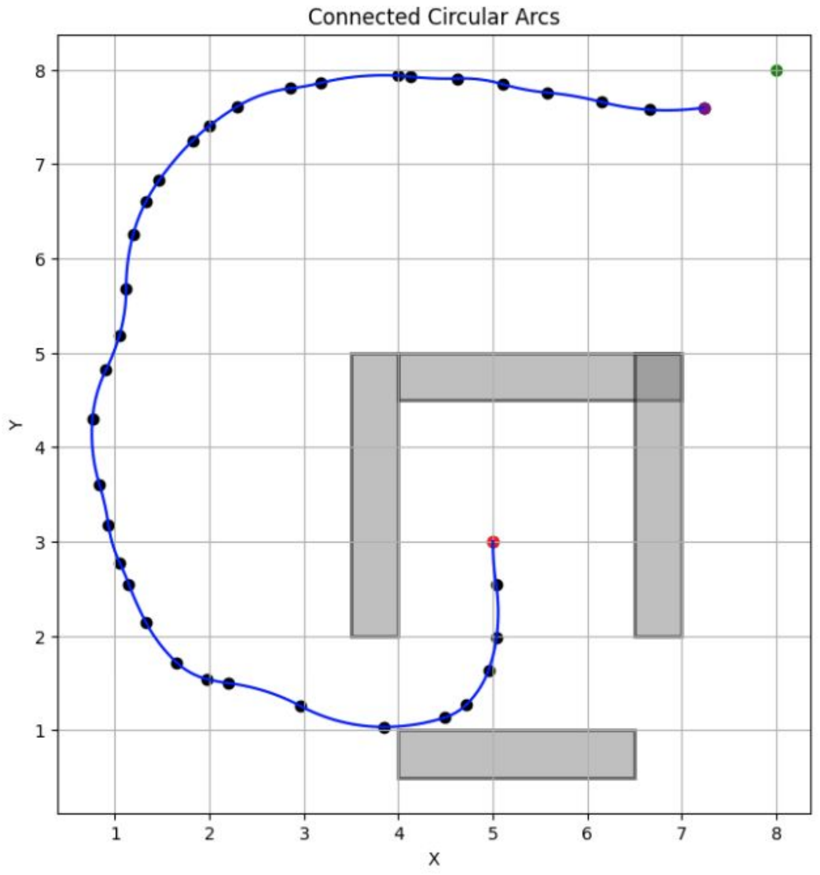
Experiments

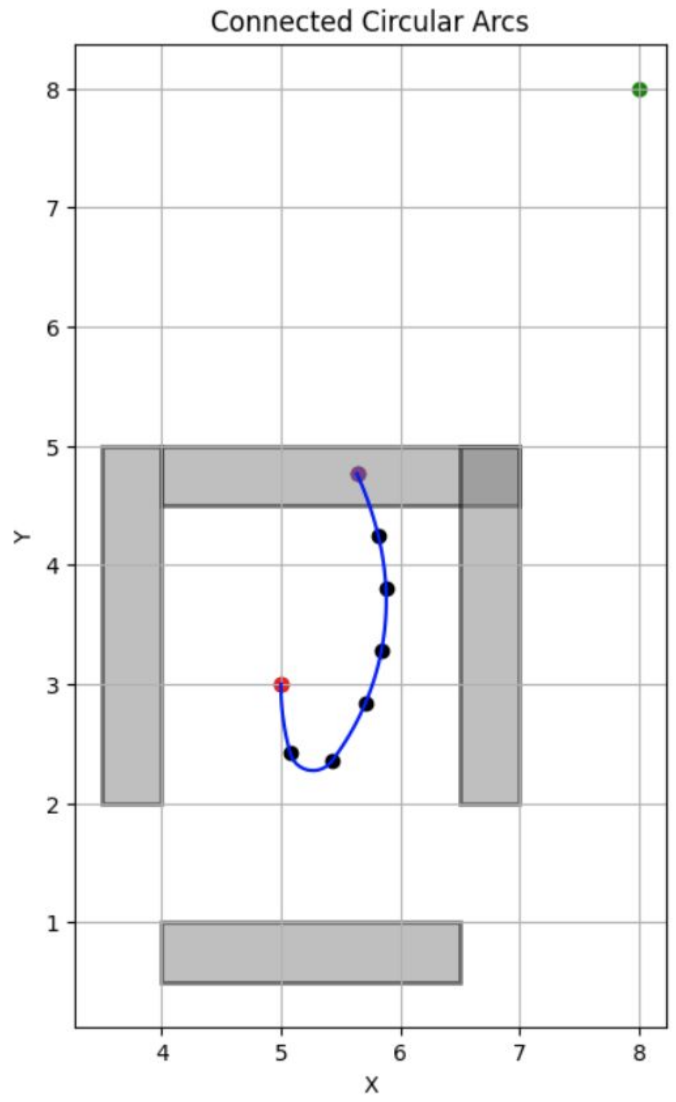
- **Experiment #1: Build SMR on different environments and analyze the expected probability of success.**
- **Experiment #2: Simulate paths with different standard deviations for the Gaussian random motion of the needle.**
- **Experiment #3: Compare actual vs expected probabilities of success for different sample sizes.**

Experiments 1

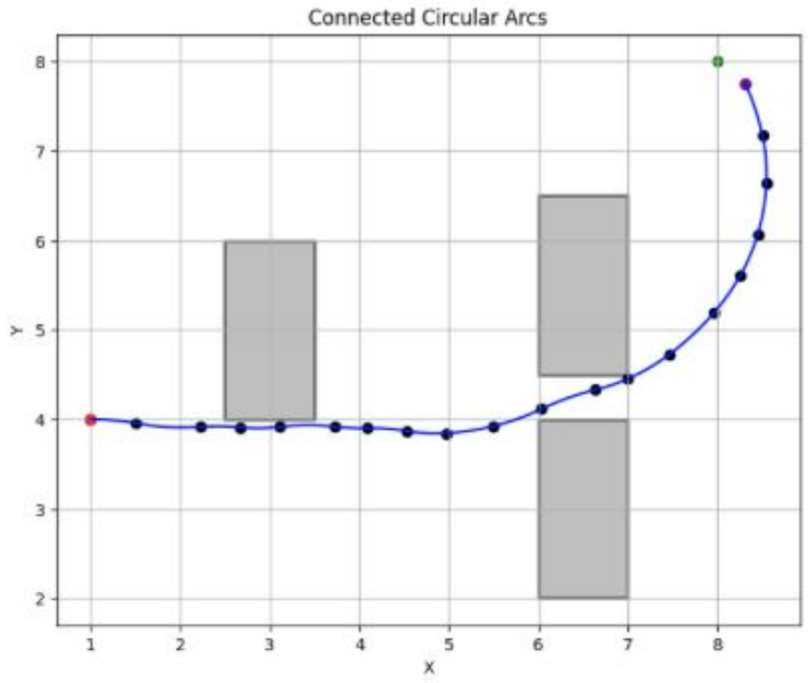


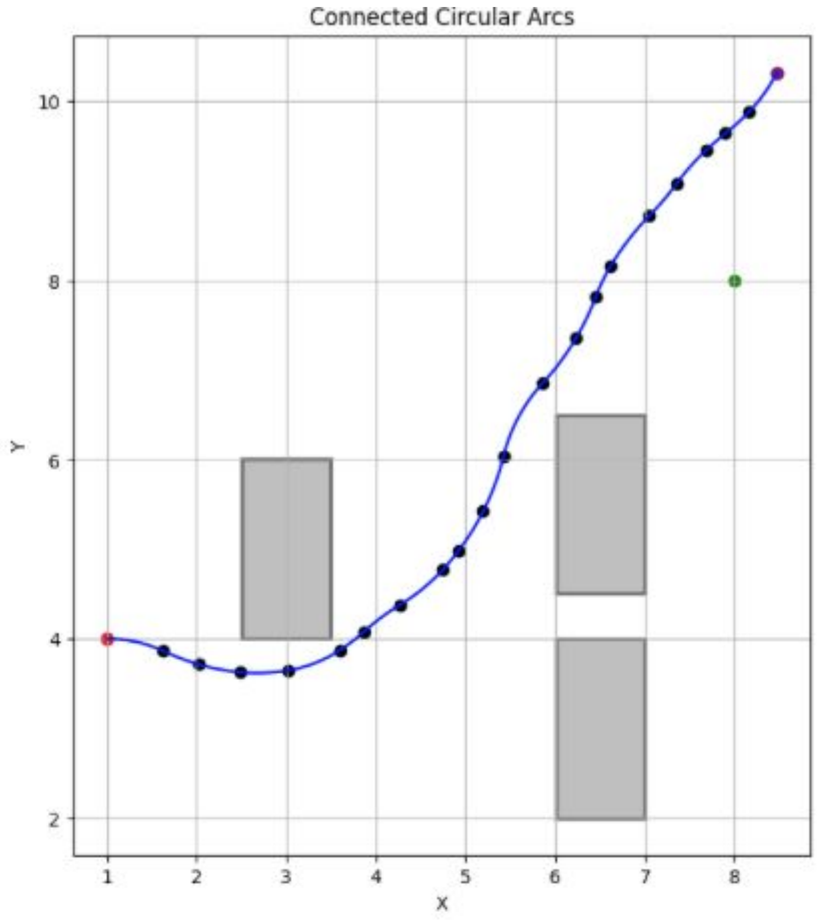


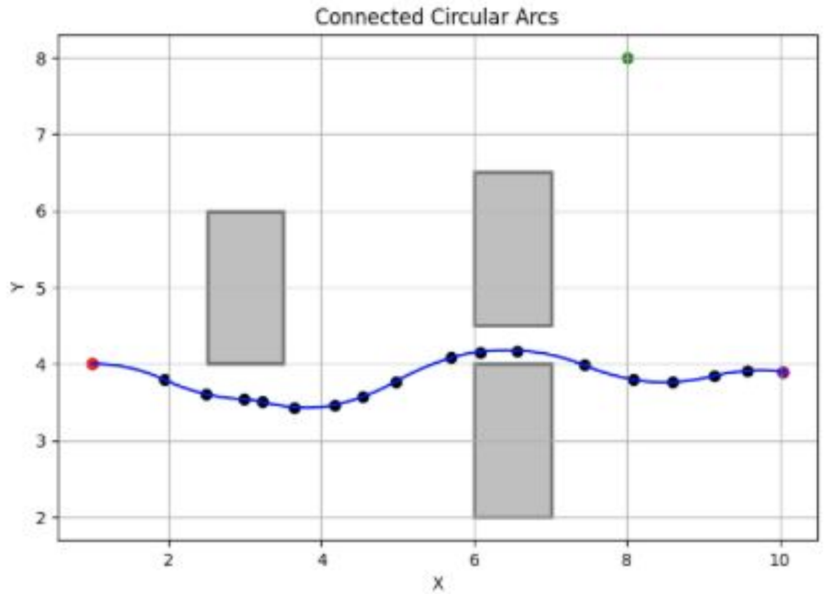


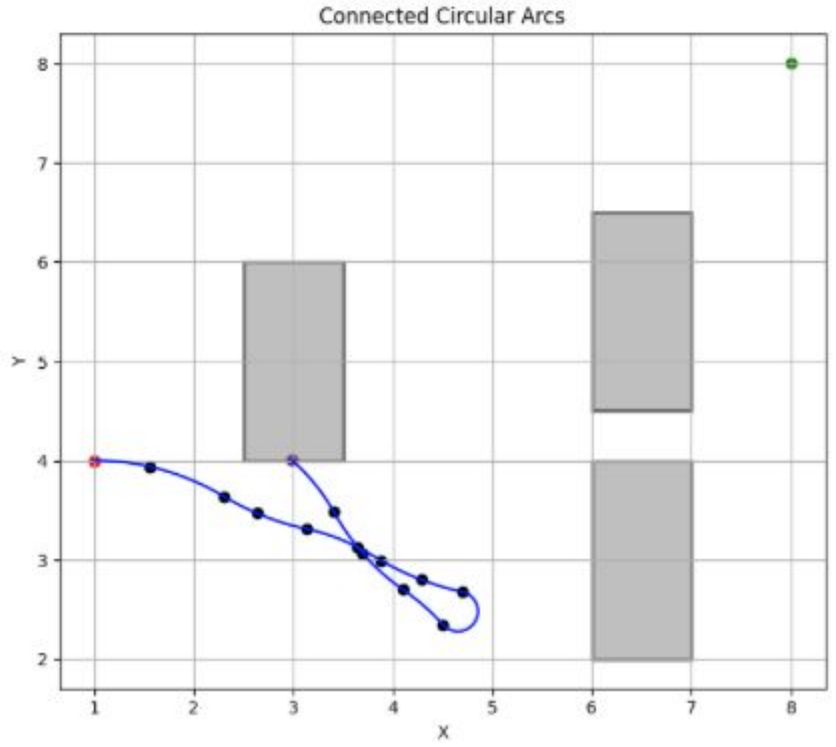


Experiments 1



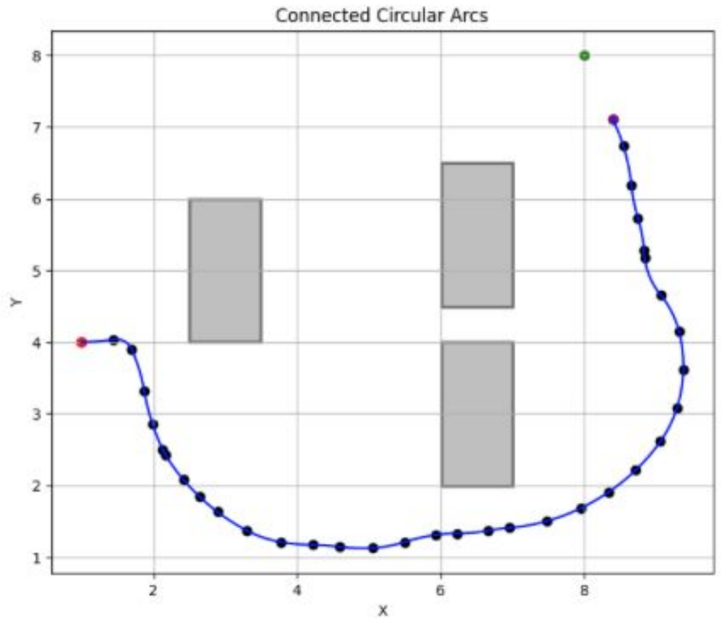


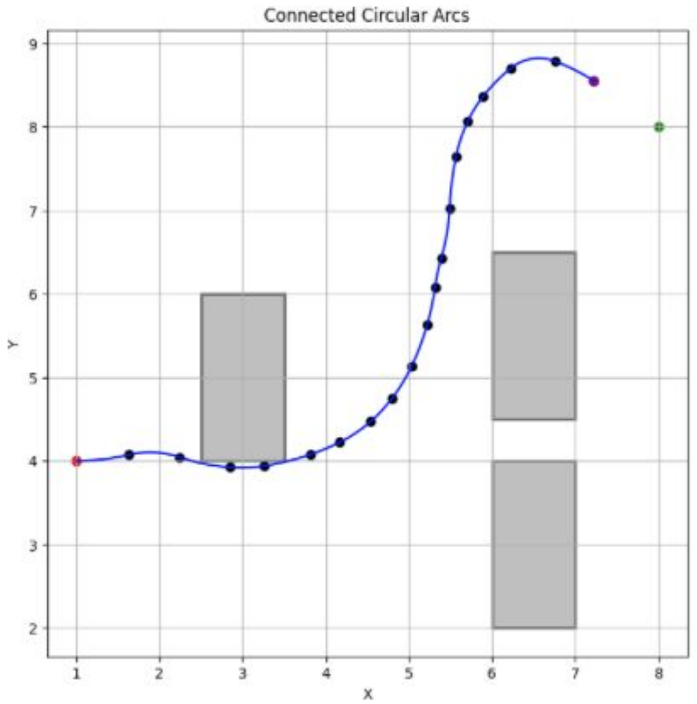


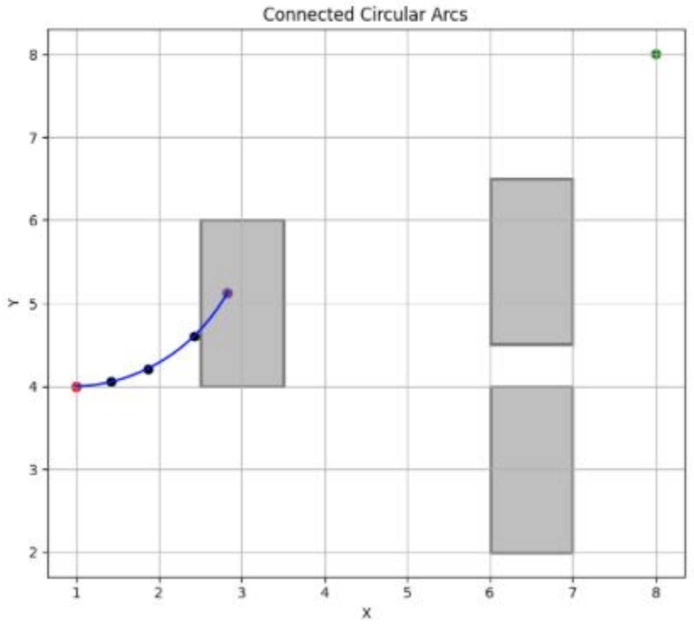


Experiments 2

High Uncertainty:

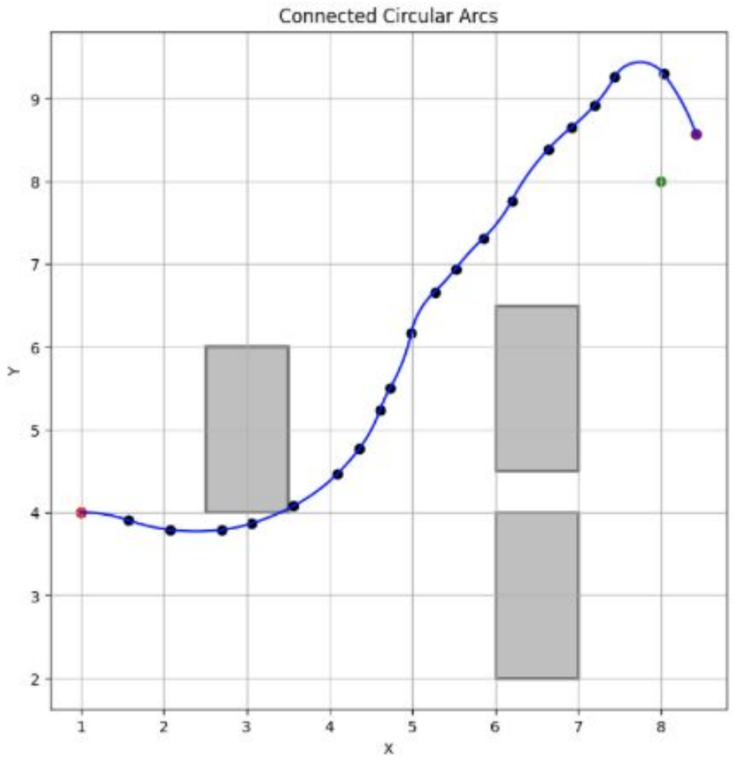


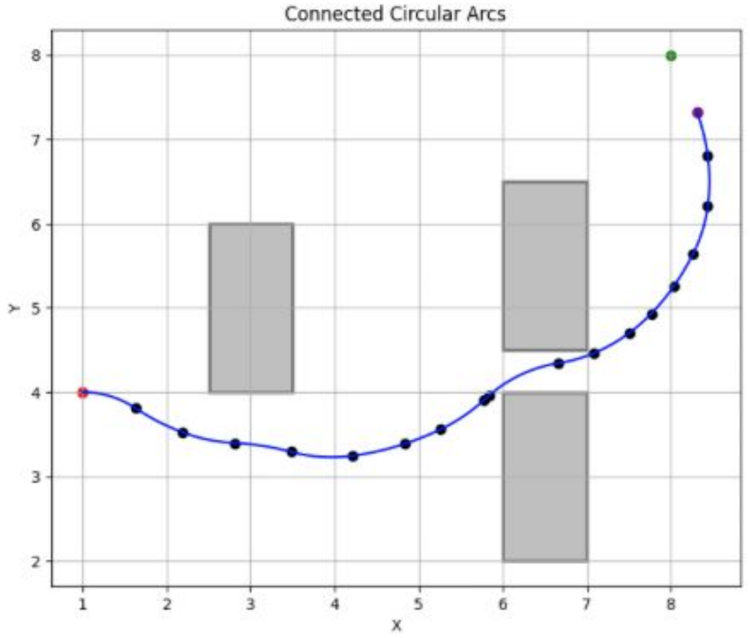


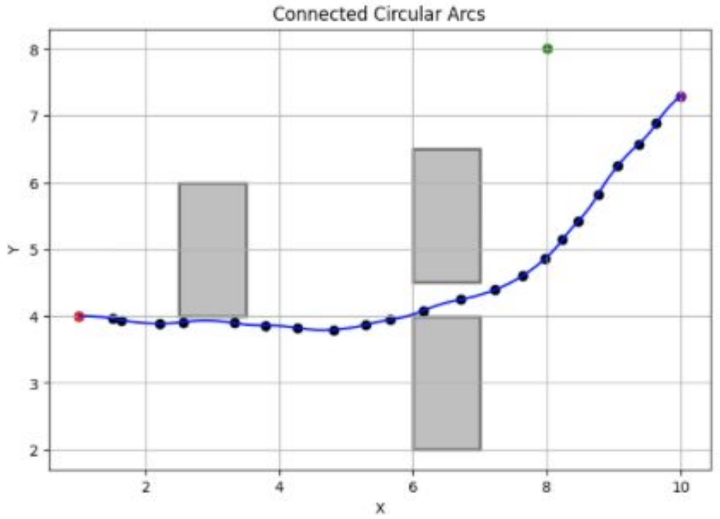


- More Variety Solution
- Run faster
- Lower Probability

Low Uncertainty:



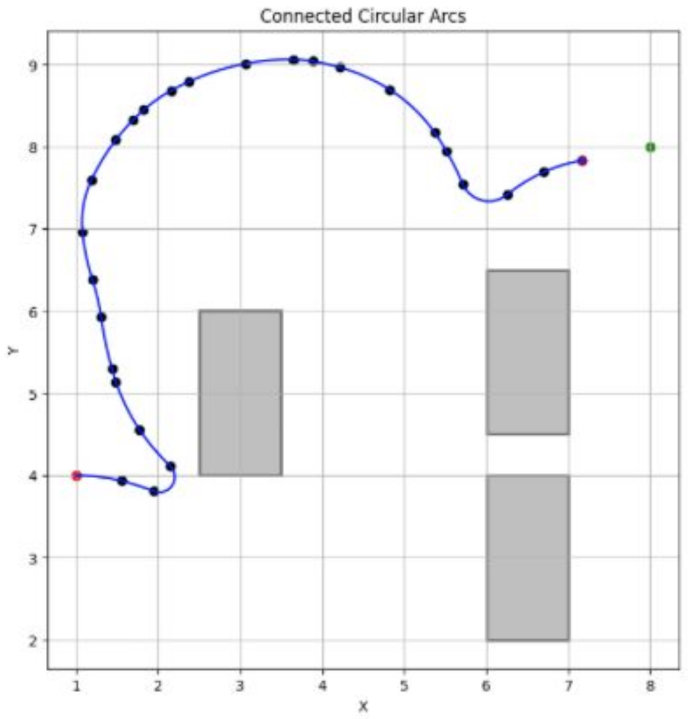


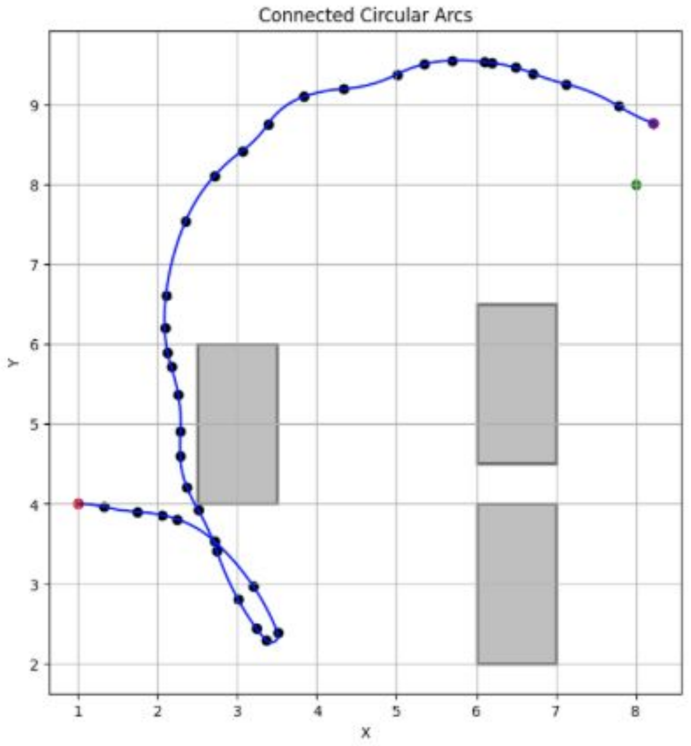


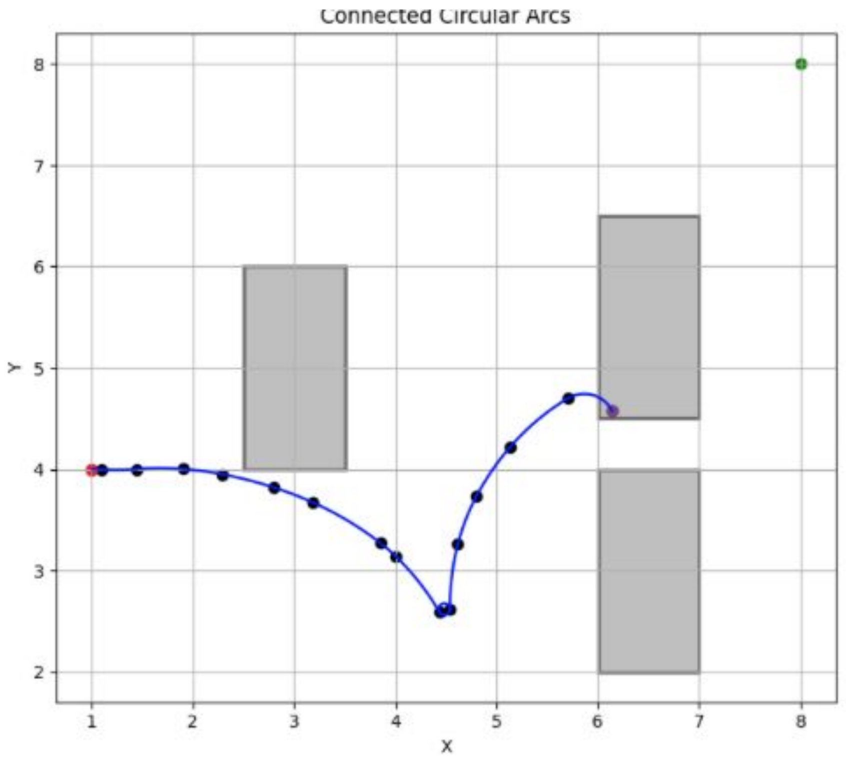
- Run Slower
- Much Smooth Path
- Lower Probability

Experiments 2

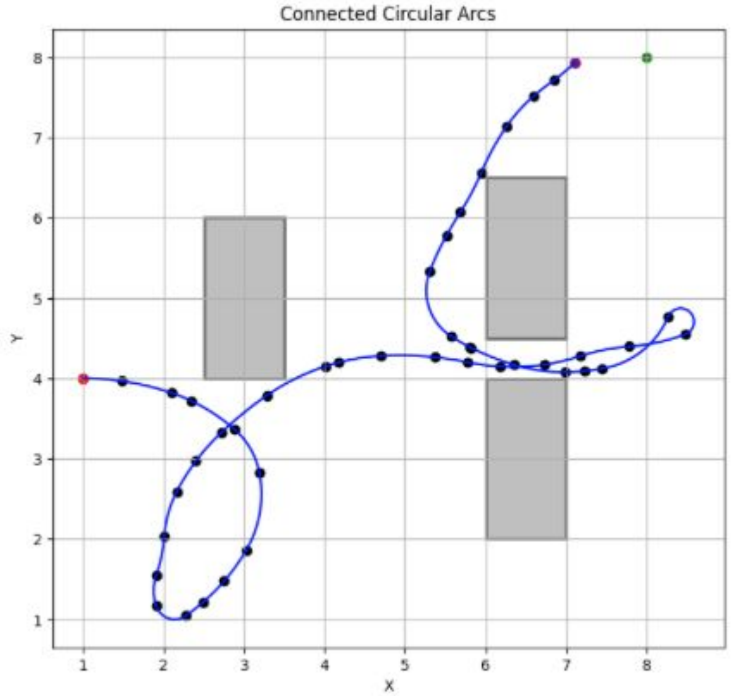
Left Control Higher Uncertainty :

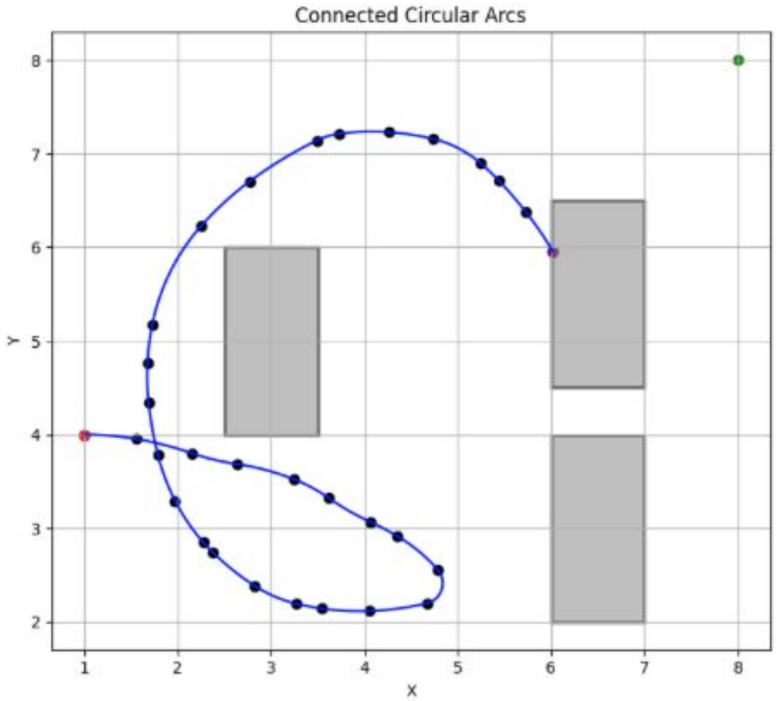


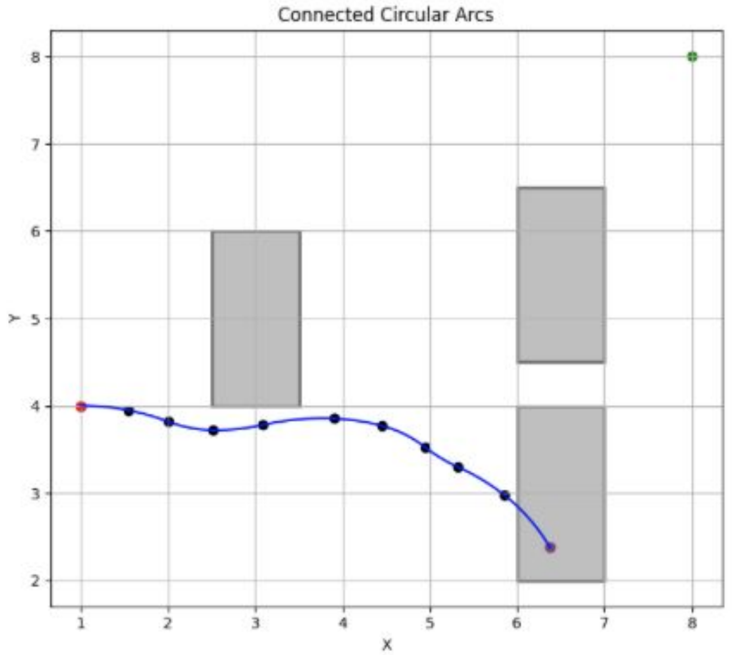




Right Control Higher Uncertainty :

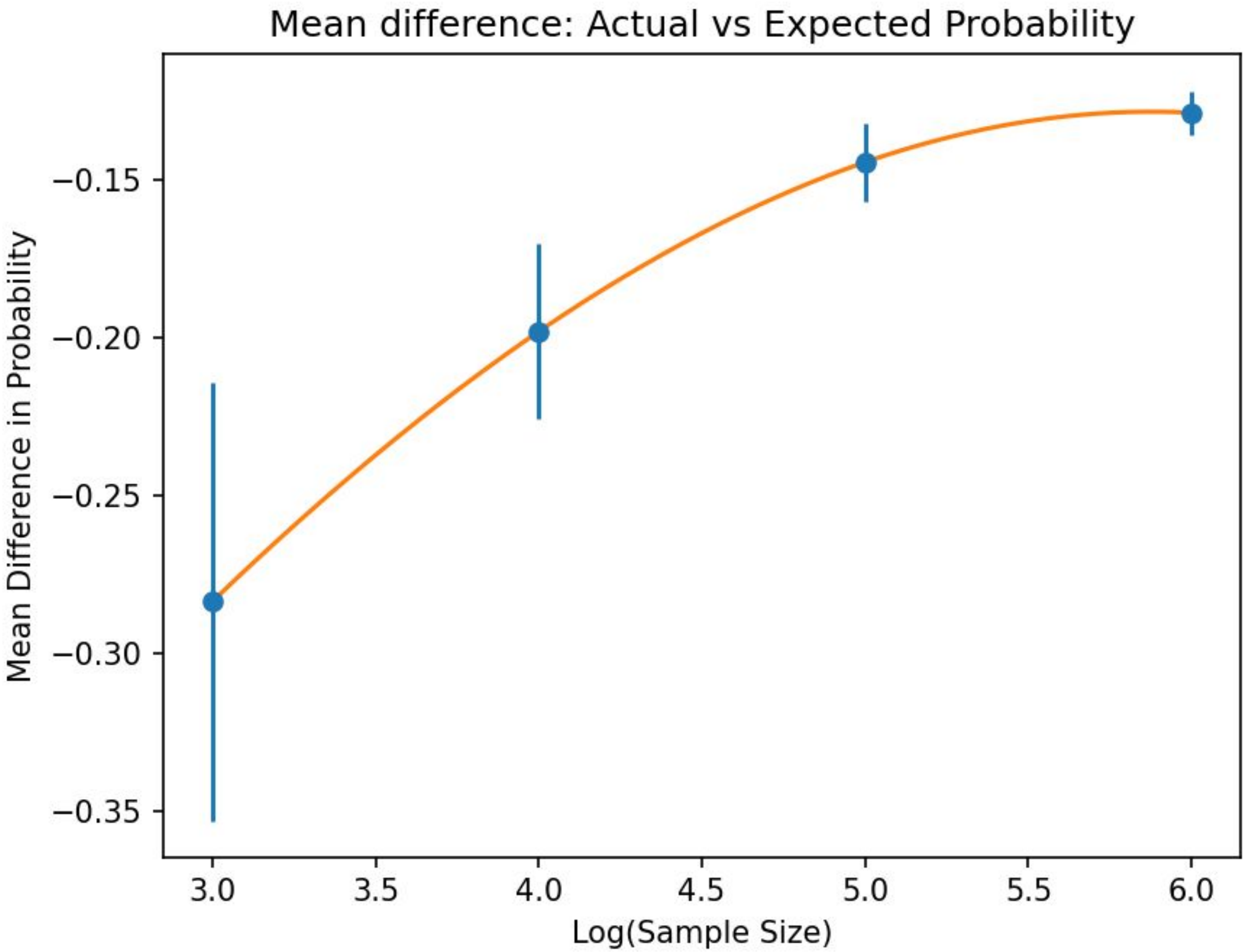






- Distortion Paths
- Lower Probability
- Rely on Environment

Results: The impact of sample size on SMR performance



Conclusion

- **SMR finds an optimal policy from a roadmap of sample states and sample transitions**
- **When planning under uncertainty, the path changes with each attempt. While solutions may emerge, reaching the goal is not guaranteed in every test.**
- **High uncertainty introduces path variety and encourages exploration but often results in distortion. On the other hand, low uncertainty operates more slowly, yielding smoother and more controlled transitions.**
- **In certain environments, imbalanced control uncertainty can be advantageous, occasionally uncovering novel solutions that would otherwise remain unexplored.**
- **Increasing sample size leads to better estimation of expected probability of success on average, but with diminishing returns.**

Questions