

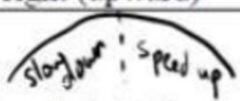
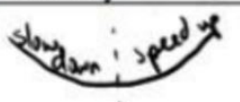
# Application of Derivatives

## Part One

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Condition	Event
$s < 0$	Object to the left (below) of the origin
$s = 0$	Object at the origin
$s > 0$	Object to the right (above) of the origin
$v < 0$	Moving to the left (downward)
$v = 0$	At rest
$v > 0$	Moving to the right (upward)
$a < 0$	<p><i>s-t graph concave down</i> </p> <p>Acceleration directed to the left (downward)</p>
$a = 0$	Constant velocity
$a > 0$	<p><i>s-t graph concave up</i> </p> <p>Acceleration directed to the right (upward)</p>

Condition	Event
$s \cdot v < 0$	Object moving toward the origin
$s \cdot v > 0$	Object moving away from the origin
$s \cdot a < 0$	Acceleration is directed toward the origin
$s \cdot a > 0$	Acceleration is directed away from the origin
$v \cdot a < 0$	Object is slowing down
$v \cdot a > 0$	Object is speeding up

# 1. Application of Derivatives Part One

## 1.1 Sketching

It is important to sketch out the function in the question to reveal all of its qualities (increasing/decreasing intervals, concave up/down, inflection points, etc). There is an algorithm to determine all of the details of the graph.

- From the original graph:
  - You must first **factor** to check if any **holes** are in the graph.
  - State **VA's** and **Domain**.
  - Find the **x and y intercepts**.
  - Find the **end behaviour**.
  - Look at the behaviour near **zeros** (x-intercepts) and **VA's**. (Remember - do this by looking at multiplicities of zeros)
  - \*CAN BE SKIPPED\*** - Find **positive and negative intervals** between zeros and VA's.
- From the first derivative:
  - Find **critical points**.
  - \*CAN BE SKIPPED\*** - Find **increasing/decreasing intervals**.
- From the second derivative:
  - Find possible **inflection points**.
  - Find **concave up/down intervals**.
  - Decide if the possible inflection points found are actual inflection points and classify the critical points using the 2nd derivative test.

**Example 1:**

Sketch and label all intercepts, asymptotes, critical points and inflection points. Show all justifying steps.

$$y = (x^{\frac{1}{3}})(x - 4)$$

1. Cannot be factored any further.  $\therefore$  **no holes, or VA's. Domain**  $x \in \mathbb{R}$

1. Find x and y intercepts:

$$\begin{aligned} &\text{x-intercept} \\ 0 &= (x^{\frac{1}{3}})(x - 4) \\ &\text{by property of zeros:} \\ &\therefore (0, 0) \text{ and } (4, 0) \end{aligned}$$

$$\begin{aligned} &\text{y-intercept} \\ y &= ((0)^{\frac{1}{3}})((0) - 4) \\ y &= (0)(-4) \\ y &= 0 \\ &\therefore (0, 0) \end{aligned}$$

1. Find end behaviour: The function is not rational/exponential.  $\therefore$  no HA/OA.

2. Find critical points: Find  $y'$  (first derivative) and find when it is equal to 0 or DNE.

$$\begin{aligned} y &= (x^{\frac{1}{3}})(x - 4) \\ y' &= (x^{\frac{1}{3}})(1) + (x - 4)(\frac{1}{3})(x^{-\frac{2}{3}}) \\ &= (x^{-\frac{2}{3}})[x + \frac{1}{3}(x - 4)] \\ &= (x^{-\frac{2}{3}})[x + \frac{(x-4)}{3}] \\ &= (x^{-\frac{2}{3}})(\frac{3x+x-4}{3}) \\ &= \frac{4(x-1)}{3x^{\frac{2}{3}}} \end{aligned}$$

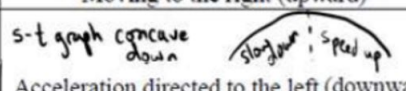
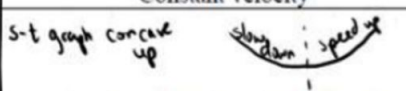
$$\begin{aligned} 0 &= \frac{4(x-1)}{3x^{\frac{2}{3}}} \\ 0 &= 4(x-1) \\ &\therefore \text{critical pt. } x = 1 \end{aligned}$$

$$\begin{aligned} &\text{DNE when denominator} = 0 \\ 0 &= 3x^{\frac{2}{3}} \\ &\therefore \text{critical pt. } x = 0 \end{aligned}$$

3. Find possible inflection points: Find  $y''$  and find when it is equal to 0 or DNE.

$$\begin{aligned} y' &= x^{\frac{1}{3}} + \frac{1}{3}(x)^{-\frac{2}{3}}(x - 4) \\ y'' &= \frac{1}{3}(x)^{-\frac{2}{3}} + \frac{1}{3}[(x)^{-\frac{2}{3}}(1) + (x - 4)(\frac{-2}{3}(x)^{-\frac{5}{3}})] \\ &= \frac{1}{3}(x)^{-\frac{5}{3}}[(x) + (x) + (x - 4)(\frac{-2}{3})] \\ &= \end{aligned}$$

## 1.2 Velocity and Acceleration

Condition	Event	Condition	Event
$s < 0$	Object to the left (below) of the origin	$s \cdot v < 0$	Object moving toward the origin
$s = 0$	Object at the origin	$s \cdot v > 0$	Object moving away from the origin
$s > 0$	Object to the right (above) of the origin	$s \cdot a < 0$	Acceleration is directed toward the origin
$v < 0$	Moving to the left (downward)	$s \cdot a > 0$	Acceleration is directed away from the origin
$v = 0$	At rest	$v \cdot a < 0$	Object is slowing down
$v > 0$	Moving to the right (upward)	$v \cdot a > 0$	Object is speeding up
$a < 0$	 Acceleration directed to the left (downward)		
$a = 0$	Constant velocity		
$a > 0$	 Acceleration directed to the right (upward)		

Before we start doing problems involving velocity and acceleration, we must make sure we know these key definitions:

### Displacement

Displacement is the **change in position** of an object. It is concerned with the initial position of an object to its final position.

$$\text{Displacement} = s(t)$$

### Velocity

Velocity is speed over time.

$$\text{Velocity} = v(t) = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$$

### Acceleration

Acceleration is concerned

negative acceleration  $\rightarrow$  decreasing velocity.

position acceleration  $\rightarrow$  increasing velocity.

$$\text{Acceleration} = a(t) = \frac{d^2v}{dt^2}$$

### Jerk/Turbulence:

$$\text{Jerk/Turbulence} = j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

## 1.3 Other Applications

