Application of Derivatives Part One

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Condition	Event
s<0	Object to the left (below) of the origin
s=0	Object at the origin
s>0	Object to the right (above) of the origin
v<0	Moving to the left (downward)
v=0	At rest
v>0	Moving to the right (upward)
a<0	s-t graph concave down; slow down ; speed up; Acceleration directed to the left (downward)
a=0	Constant velocity
a>0	s-t graph concave up; slow down ; speed up; Acceleration directed to the right (upward)

Condition	Event
$s \cdot v < 0$	Object moving toward the origin
$s \cdot v > 0$	Object moving away from the origin
$s \cdot a < 0$	Acceleration is directed toward the origin
$s \cdot a > 0$	Acceleration is directed away from the origin
$v \cdot a < 0$	Object is slowing down
$v \cdot a > 0$	Object is speeding up

1. Application of Derivatives Part One

1.1 Sketching

It is important to sketch out the function in the question to reveal all of its qualities (increasing/decreasing intervals, concave up/down, inflection points, etc). There is an algorithm to determine all of the details of the graph.

1. From the original graph:
   - You must first **factor** to check if any **holes** are in the graph.
   - State **VA's** and **Domain**.
   - Find the **x and y intercepts**.
   - Find the **end behaviour**.
   - Look at the behaviour near **zeros** (x-intercepts) and **VA's**. (Remember - do this by looking at multiplicities of zeros)
   - *CAN BE SKIPPED* - Find **positive and negative intervals** between zeros and VA's.

2. From the first derivative:
   - Find **critical points**.
   - *CAN BE SKIPPED* - Find **increasing/decreasing intervals**.

3. From the second derivative:
   - Find possible **inflection points**.
   - Find **concave up/down intervals**.
   - Decide if the possible inflection points found are actual inflection points and classify the critical points using the 2nd derivative test.

Example 1:

Sketch and label all intercepts, asymtotes, crticial points and inflection points. Show all justifying steps.

$$y = (x^{\frac{1}{3}})(x-4)$$

1. Cannot be factored any further. $\therefore$ **no holes, or VA's. Domain** $x \in \Re$

1. Find x and y intercepts.

<u>x-intercept</u>

$$0 = (x^{\frac{1}{3}})(x-4)$$

by property of zeros:

$\therefore (0,0)$ **and** $(4,0)$

<u>y-intercept</u>

$$y = ((0)^{\frac{1}{3}})((0)-4)$$
$$y = (0)(-4)$$
$$y = 0$$
$$\therefore (0,0)$$

1.2 Velocity and Acceleration

Condition	Event
s<0	Object to the left (below) of the origin
s=0	Object at the origin
s>0	Object to the right (above) of the origin
v<0	Moving to the left (downward)
v=0	At rest
v>0	Moving to the right (upward)
a<0	s-t graph concave down; slow down ; speed up; Acceleration directed to the left (downward)
a=0	Constant velocity
a>0	s-t graph concave up; slow down ; speed up; Acceleration directed to the right (upward)

Condition	Event
$s \cdot v < 0$	Object moving toward the origin
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Before we start doing problems involving velocity and acceleration, we must make sure we know these key definitions:

Displacement

Displacement is the **change in position** of an object. It is concerned with the initial position of an object to its final position.

$$Displacement = s(t)$$

Velocity

Velocity is speed over time.

$$Velocity = v(t) = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$$

Acceleration

Acceleration is concerned

negative acceleration $\rightarrow$ decreasing velocity.
position acceleration $\rightarrow$ increasing velocity.

$$Acceleration = a(t) = \frac{d^2v}{dt^2}$$

Jerk/Turbulence:

$$Jerk/Turbulence = j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

1.3 Other Applications