# Application of Derivatives Part One by: Joshua Bautista

| Condition | Event |
|---|---|
| s<0 | Object to the left (below) of the origin |
| s=0 | Object at the origin |
| s>0 | Object to the right (above) of the origin |
| v<0 | Moving to the left (downward) |
| v=0 | At rest |
| v>0 | Moving to the right (upward) |
| a<0 | s-t graph concave down   slow down ; speed up<br>Acceleration directed to the left (downward) |
| a=0 | Constant velocity |
| a>0 | s-t graph concave up   slow down ; speed up<br>Acceleration directed to the right (upward) |

| Condition | Event |
|---|---|
| $s \cdot v < 0$ | Object moving toward the origin |
| $s \cdot v > 0$ | Object moving away from the origin |
| $s \cdot a < 0$ | Acceleration is directed toward the origin |
| $s \cdot a > 0$ | Acceleration is directed away from the origin |
| $v \cdot a < 0$ | Object is slowing down |
| $v \cdot a > 0$ | Object is speeding up |

# 1. Application of Derivatives Part One

## 1.1 Sketching

It is important to sketch out the function in the question to reveal all of its qualities (increasing/decreasing intervals, concave up/down, inflection points, etc). There is an algorithm to determine all of the details of the graph.

1. From the original graph:
   - You must first **factor** to check if any **holes** are in the graph.
   - State **VA's** and **Domain**.
   - Find the **x and y intercepts**.
   - Find the **end behaviour**.
   - Look at the behaviour near **zeros** (x-intercepts) and **VA's**. (Remember - do this by looking at multiplicities of zeros)
   - *CAN BE SKIPPED* - Find **positive and negative intervals** between zeros and VA's.

2. From the first derivative:
   - Find **critical points**.
   - *CAN BE SKIPPED* - Find **increasing/decreasing intervals**.

3. From the second derivative:
   - Find possible **inflection points**.
   - Find **concave up/down intervals**.
   - Decide if the possible inflection points found are actual inflection points and classify the critical points using the 2nd derivative test.

### Example 1:

Sketch and label all intercepts, asymtotes, crticial points and inflection points. Show all justifying steps.

$$y = (x^{\frac{1}{3}})(x-4)$$

**1.** Cannot be factored any further. $\therefore$ **no holes, or VA's. Domain** $x \in \Re$

**1.** Find x and y intercepts:

x-intercept

$$0 = (x^{\frac{1}{3}})(x-4)$$

by property of zeros:

$\therefore (0,0)$ **and** $(4,0)$

y-intercept

$$y = ((0)^{\frac{1}{3}})((0)-4)$$
$$y = (0)(-4)$$
$$y = 0$$
$$\therefore (0,0)$$

**1.** Find end behaviour: The function is not rational/exponential. $\therefore$ no HA/OA.

**2.** Find critical points: Find $y'$ (first derivative) and find when it is equal to 0 or DNE.

$$\begin{aligned} y &= (x^{\frac{1}{3}})(x-4) \\ y' &= (x^{\frac{1}{3}})(1) + (x-4)(\tfrac{1}{3})(x^{\frac{-2}{3}}) \\ &= (x^{\frac{-2}{3}})[x + \tfrac{1}{3}(x-4)] \\ &= (x^{\frac{-2}{3}})[x + \tfrac{(x-4)}{3}] \\ &= (x^{\frac{-2}{3}})(\tfrac{3x+x-4}{3}) \\ &= \frac{4(x-1)}{3x^{\frac{2}{3}}} \end{aligned}$$

$$0 = \frac{4(x-1)}{3x^{\frac{2}{3}}}$$
$$0 = 4(x-1)$$

$\therefore$ critical pt. $x = 1$

DNE when denominator = 0

$$0 = 3x^{\frac{2}{3}}$$

$\therefore$ critical pt. $x = 0$

## 1.2 Velocity and Acceleration

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Before we start doing problems involving velocity and acceleration, we must make sure we know these key definitions:

### Displacement
Displacement is the **change in position** of an object. It is concerned with the initial position of an object to its final position.

$$Displacement = s(t)$$

### Velocity
Velocity is speed over time.

$$Velocity = v(t) = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$$

### Acceleration
Acceleration is concerned

negative acceleration $\rightarrow$ decreasing velocity.
position acceleration $\rightarrow$ increasing velocity.

$$Acceleration = a(t) = \frac{d^2v}{dt^2}$$

### Jerk/Turbulence:

$$Jerk/Turbulence = j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

## 1.3 Other Applications