Application of Derivatives Part One

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Condition	Event
s<0	Object to the left (below) of the origin
s=0	Object at the origin
s>0	Object to the right (above) of the origin
v<0	Moving to the left (downward)
v=0	At rest
v>0	Moving to the right (upward)
a<0	s-t graph concave down; slow down ; speed up Acceleration directed to the left (downward)
a=0	Constant velocity
a>0	s-t graph concave up; slow down ; speed up Acceleration directed to the right (upward)

Condition	Event
$s \cdot v < 0$	Object moving toward the origin
$s \cdot v > 0$	Object moving away from the origin
$s \cdot a < 0$	Acceleration is directed toward the origin
$s \cdot a > 0$	Acceleration is directed away from the origin
$v \cdot a < 0$	Object is slowing down
$v \cdot a > 0$	Object is speeding up

1. Application of Derivatives Part One

1.1 Sketching

It is important to sketch out the function in the question to reveal all of its qualities (increasing/decreasing intervals, concave up/down, inflection points, etc). There is an algorithm to determine all of the details of the graph.

1. From the original graph:
    - You must first **factor** to check if any **holes** are in the graph.
    - State **VA's** and **Domain**.
    - Find the **x and y intercepts**.
    - Find the **end behaviour**.
    - Look at the behaviour near **zeros** (x-intercepts) and **VA's**. (Remember - do this by looking at multiplicities of zeros)
    - *CAN BE SKIPPED* - Find **positive and negative intervals** between zeros and VA's.

2. From the first derivative:
    - Find **critical points**.
    - *CAN BE SKIPPED* - Find **increasing/decreasing intervals**.

3. From the second derivative:
    - Find possible **inflection points**.
    - Find **concave up/down intervals**.
    - Decide if the possible inflection points found are actual inflection points and classify the critical points using the 2nd derivative test.

Example 1:

Sketch and label all intercepts, asymtotes, crticial points and inflection points. Show all justifying steps.

$$y = (x^{\frac{1}{3}})(x-4)$$

1. Cannot be factored any further. $\therefore$ **no holes, or VA's. Domain** $x \in \Re$

1. Find x and y intercepts:

x-intercept

$$0 = (x^{\frac{1}{3}})(x-4)$$

by property of zeros:

$\therefore (0,0)$ **and** $(4,0)$

y-intercept

$$y = ((0)^{\frac{1}{3}})((0)-4)$$
$$y = (0)(-4)$$
$$y = 0$$
$$\therefore (0,0)$$

1. Find end behaviour: The function is not rational/exponential. $\therefore$ no HA/OA.

2. Find critical points: Find y' (first derivative) and find when it is equal to 0 or DNE.

$$\begin{aligned} y &= (x^{\frac{1}{3}})(x-4) \\ y' &= (x^{\frac{1}{3}})(1) + (x-4)(\tfrac{1}{3})(x^{\frac{-2}{3}}) \\ &= (x^{\frac{-2}{3}})[x + \tfrac{1}{3}(x-4)] \\ &= (x^{\frac{-2}{3}})[x + \tfrac{(x-4)}{3}] \\ &= (x^{\frac{-2}{3}})(\tfrac{3x+x-4}{3}) \\ &= \frac{4(x-1)}{3x^{\frac{2}{3}}} \end{aligned}$$

$$0 = \frac{4(x-1)}{3x^{\frac{2}{3}}}$$
$$0 = 4(x-1)$$

$\therefore$ critical pt. $x = 1$

DNE when denominator = 0

$$0 = 3x^{\frac{2}{3}}$$

$\therefore$ critical pt. $x = 0$

3. Find possible inflection points: Find y'' and find when it is equal to 0 or DNE.

$$\begin{aligned} y' &= x^{\frac{1}{3}} + \tfrac{1}{3}(x)^{\frac{-2}{3}}(x-4) \\ y'' &= \tfrac{1}{3}(x)^{\frac{-2}{3}} + \tfrac{1}{3}[(x)^{\frac{-2}{3}}(1) + (x-4)(\tfrac{-2}{3}(x)^{\frac{-5}{3}})] \\ &= \tfrac{1}{3}(x)^{\frac{-5}{3}}[(x) + (x) + (x-4)(\tfrac{-2}{3})] \\ &= \end{aligned}$$

1.2 Velocity and Acceleration

Condition	Event
s<0	Object to the left (below) of the origin
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v<0	Moving to the left (downward)
v=0	At rest
v>0	Moving to the right (upward)
a<0	s-t graph concave down; slow down ; speed up Acceleration directed to the left (downward)
a=0	Constant velocity
a>0	s-t graph concave up; slow down ; speed up Acceleration directed to the right (upward)

Condition	Event
s· v < 0	Object moving toward the origin
s·v > 0	Object moving away from the origin
s·a < 0	Acceleration is directed toward the origin
s·a > 0	Acceleration is directed away from the origin
v·a < 0	Object is slowing down
v·a > 0	Object is speeding up

Before we start doing problems involving velocity and acceleration, we must make sure we know these key definitions:

Displacement

Displacement is the **change in position** of an object. It is concerned with the initial position of an object to its final position.

$$Displacement = s(t)$$

Velocity

Velocity is speed over time.

$$Velocity = v(t) = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$$

Acceleration

Acceleration is concerned

negative acceleration → decreasing velocity.
position acceleration → increasing velocity.

$$Acceleration = a(t) = \frac{d^2v}{dt^2}$$

Jerk/Turbulence:

$$Jerk/Turbulence = j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

1.3 Other Applications