Notes on Basic Mathematics

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CHAPTER 1

The integers

1. Rules for addition

We assume proficiency with the addition operator. Observe the rule for addition with 0, that is

$$a + 0 = a. \tag{1}$$

The formula for the addition of negative numbers is as follows,

$$\begin{aligned} a + (-a) &= a + -(a) \\ &= a - a \end{aligned} \tag{2}$$

The statement

$$a + (-a) = 0 \tag{3}$$

shows $-a$ is the additive inverse of a.

The equations above demonstrate the notion of equality and that of introducing proofs.

CHAPTER 2

Properties

1. Multiplication; Associative

The associative property.

$$\begin{aligned} a(bc) &= (ab)c \\ &= abc \\ &= (a)(b)(c) \\ &\vdots \end{aligned}$$

Assume d can be expanded to bc.

$$\begin{aligned} a(d) &= ad \\ &= a(bc) = abc \end{aligned}$$

Expressions within parentheses or have binding with an exponent are to be evaluated before the distributions are applied.

$$\begin{aligned} a(b+c)^2 &= a(b^2 + 2bc + c^2) \\ &\neq (ab + ac)^2 \\ &\neq a(b^2 + c^2) = ab^2 + ac^2 \end{aligned}$$

Expansions mixed with distributions can be syntactically difficult.

$$\begin{aligned} a(b^2 - c^2) &= a(b-c)(b+c) && \text{is equivalent to,} \\ &= a((b-c)(b+c)) && \text{but not to be confused with,} \\ &\neq a(b-c) * a(b+c) \end{aligned}$$

CHAPTER 3

Inequalities

The LHS has a relation to the RHS; it is either greater than, less than, greater than or equal, or less than or equal to.

$$\begin{aligned} 5 &< 10 \\ 10 &> 5 \\ -13 &\leq 3 \\ 3 &\geq 3 \end{aligned}$$

This is an example of an inequality.

$$\begin{aligned} x + 1 &< 5 \\ x &< 5 - 1 \\ x &< 4 \end{aligned}$$

Another.

$$\begin{aligned} & x^2 - 6x \geq 0 && \text{factor expression,} \\ & x(x-6) \geq 0 && \text{observe when zero,} \\ & x = 0, \quad x = 6 \end{aligned}$$

We have found the endpoints on the number line. Now we select a number between the intervals separated by the endpoints. These numbers are picked arbitrarily. Here, the chosen numbers z must follow the rule.

$$\text{endpoints} \begin{cases} 0 \\ 6 \end{cases}$$

$$\text{intervals} \begin{cases} -1 & -\infty < z < 0 \\ 1 & 0 < z < 6 \\ 7 & 6 < z < \infty \end{cases}$$

We then substitute those numbers in each factor to determine the sign of the expression. If the sign of the LHS is correct in relation to the type of inequality then that interval is valid. Since this inequality also determines if the LHS is equal to 0 then we check the endpoints as well.

$$(-\infty, 0] \cup [6, \infty)$$