**Inductive Specification**

Exercise 1.1

1. $\{3n + 2 \mid n \in N\}$

**Definition (top-down)** *A natural number $n$ is in $S$ if and only if*

*1. $n = 2$, or*

*2. $n - 3 \in S$.*

**Definition (bottom-up)** *Define the set $S$ to be the smallest set contained in $N$ and satisfying the following two properties:*

*1. $2 \in S$, and*

*2. if $n \in S$, then $n + 3 \in S$.*

**Definition (rules of inference)**

$$2 \in S$$

$$\frac{n \in S}{n + 3 \in S}$$

We generate some elements of $S$.

1. $2 \in S$.

2.

$$\frac{2 \in S}{(2 + 3) \in S}$$

3.

$$\frac{5 \in S}{(5 + 3) \in S}$$

2. $\{2n + 3m + 1 \mid n, m \in N\}$

**Definition (top-down)** *A natural number $n$ is in $S$ if and only if*

*1.* $n = 1$*, or*

*2.* $n - 2 \in S$*, or*

*3.* $n - 3 \in S$*.*

**Definition (bottom-up)** *Define the set $S$ to be the smallest set contained in $N$ and satisfying the following three properties:*

*1.* $1 \in S$*, and*

*2. if* $n \in S$*, then* $n + 2 \in S$*, and*

*3. if* $n \in S$*, then* $n + 3 \in S$*.*

**Definition (rules of inference)**

$$1 \in S$$

$$\frac{n \in S}{(n+2) \in S}$$

$$\frac{n \in S}{(n+3) \in S}$$

We generate some elements of $S$.

1. $1 \in S$.

2.

$$\frac{1 \in S}{(1+2) \in S}$$

3.

$$\frac{3 \in S}{(3+2) \in S}$$

4.

$$\frac{5 \in S}{(5+2) \in S}$$

We could have derived $7 \in S$ differently using only property 3.

$$\cfrac{\cfrac{1 \in S}{(1+3) \in S}}{(4+3) \in S}$$

3. $\{(n, 2n+1) | n \in N\}$

**Definition (top-down)** *A pair of natural numbers $(n, m)$ is in the set $S$ if and only if*

*1. the pair is $(0, 1)$, or*

*2. $(n-1, 2(n-1)+1) \in S$.*

**Definition (bottom-up)** *We define $S$ to be the smallest set of pairs whose elements are contained in $N$ and satisfying the following properties:*

*1. $(0, 1) \in S$, and*

*2. if $(n, m) \in S$, then $(n+1, 2(n+1)+1) \in S$.*

**Definition (rules of inference)**

$$(0, 1) \in S$$

$$\frac{(n, m) \in S}{(n+1, 2(n+1)+1) \in S}$$

We generate some elements of $S$.

1. $(0, 1) \in S$.

2.

$$\frac{(0,1) \in S}{(0+1, 2(0+1)+1) \in S}$$

3.

$$\frac{(1,3) \in S}{(1+1, 2(1+1)+1) \in S}$$

4. $\{(n, n^2) | n \in N\}$

**Definition (top-down)** *A pair of natural numbers $(n, m)$ is in the set $S$ if and only if*

*1. the pair is $(0,0)$, or*

*2. $(n-1, n(n-2)+1) \in S$.*

**Definition (bottom-up)** *We define $S$ to be the smallest set of pairs of natural numbers and satisfying the following properties:*

*1. $(0,0) \in S$, and*

*2. if $(n,m) \in S$, then $(n+1, n(n+2)+1) \in S$.*

**Definition (rules of inference)**

$$(0,0) \in S$$

$$\frac{(n,m) \in S}{(n+1, n(n+2)+1) \in S}$$

We generate some elements of $S$.

1. $(0,0) \in S$.

2.

$$\frac{(0,0) \in S}{(1,1) \in S}$$

3.

$$\frac{(1,1) \in S}{(2,4) \in S}$$

Exercise 1.2

1. We know that $n$ increments by 1 starting at 0 and $k$ increments by 7 starting at 1. Therefore the set is $\{(n, 7n+1) | n \in N\}$.

2. We know that $n$ increments by 1 starting at 0 and $k$ doubles starting at 1. Therefore the set is $\{(n, 2^n) | n \in N\}$.