

Not having a good time with the `mathpartir` package here.

Exercise 4.1

Calls to `g` declare a reference `counter`, set its contents, then return its contents. The information between calls is lost because `counter` is different on each call. That is, the reference data structure represents a different location. The first program was made such that the environment of the closure had access to `counter`, and thus was able to reference that variable.

Exercise 4.2

$$\frac{(\text{value-of } \text{exp}_1 \ \rho \ \sigma_0) = (val_1, \sigma_1)}{(\text{value-of } (\text{zero?-exp } \text{exp}_1) \ \rho \ \sigma_0) = \begin{array}{ll} ((\text{bool-val } \#t), \sigma_1) & \text{if } [val_1] = 0 \\ ((\text{bool-val } \#f), \sigma_1) & \text{if } [val_1] \neq 0 \end{array}}$$

Exercise 4.3

$$\frac{\begin{array}{l} (\text{value-of } \text{exp}_1 \ \rho \ \sigma_0) = (val_1, \sigma_1) \\ (\text{value-of } \text{exp}_2 \ \rho \ \sigma_1) = (val_2, \sigma_2) \end{array}}{(\text{value-of } (\text{call-exp } \text{exp}_1 \ \text{exp}_2) \ \rho \ \sigma_0) = (\text{apply-procedure } (\text{expval} \rightarrow \text{proc } val_1) \ val_2 \ \sigma_2)}$$

$$\frac{val_1 = (\text{procedure } \text{var } \text{body } \rho)}{(\text{apply-procedure } val_1 \ val_2 \ \rho_0) = (\text{value-of } \text{body } [var = val_2] \rho \ \sigma_0)}$$

Exercise 4.4

$$\begin{array}{c}
(\text{value-of } exp_1 \ \rho \ \sigma_0) = (val_1, \sigma_1) \\
\vdots \\
(\text{value-of } exp_n \ \rho \ \sigma_{n-1}) = (val_n, \sigma_n) \\
\hline
(\text{value-of } (\text{begin } exp_1 \dots exp_n) \ \rho \ \sigma_0) = (val_n, \sigma_n)
\end{array}$$

Exercise 4.5

$$\begin{array}{c}
(\text{value-of } exp_1 \ \rho \ \sigma_0) = (val_1, \sigma_1) \\
\vdots \\
(\text{value-of } exp_n \ \rho \ \sigma_{n-1}) = (val_n, \sigma_n) \\
\hline
(\text{value-of } (\text{list-exp } exp_1 \dots exp_n) \ \rho \ \sigma_0) \\
= ((\text{pair-val } val_1 \ (\dots (\text{pair-val } val_n \ (\text{emptylist-val})) \dots)), \sigma_n)
\end{array}$$

Exercise 4.6

$$\begin{array}{c}
(\text{value-of } exp_1 \ \rho \ \sigma_0) = (l, \sigma_1) \\
(\text{value-of } exp_2 \ \rho \ \sigma_1) = (val, \sigma_2) \\
\hline
(\text{value-of } (\text{setref-exp } exp_1 \ exp_2) \ \rho \ \sigma_0) = (val, [l = val]\sigma_2)
\end{array}$$

Exercise 4.7

$$\begin{array}{c}
(\text{value-of } exp_1 \ \rho \ \sigma_0) = (l, \sigma_1) \\
(\text{value-of } exp_2 \ \rho \ \sigma_1) = (val, \sigma_2) \\
\hline
(\text{value-of } (\text{setref-exp } exp_1 \ exp_2) \ \rho \ \sigma_0) = (\sigma_0(l), [l = val]\sigma_2)
\end{array}$$

Exercise 4.8

`newref`, `deref`, and `setref!` take linear time.

Exercise 4.9

`newref` uses `new-store-longer-by-one` which takes linear time because I could not find a built-in procedure for copying vectors. `deref` takes as much time as `vector-ref`. `setref!` takes as much time as `vector-length`.

```
(define empty-store
  (lambda ()
    (make-vector 0)))

;; initialize-store! : () -> Sto
;; usage: (initialize-store!) sets the-store to the empty-store
(define initialize-store!
  (lambda ()
    (set! the-store (empty-store))))

;; reference? : SchemeVal -> Bool
(define reference?
  (lambda (v)
    (integer? v)))

;; new-store-longer-by-one : Sto -> Sto
(define new-store-longer-by-one
  (lambda (store)
    (let ((new-store (make-vector (+ 1 (vector-length store)))))
      (letrec ((inner
                  (lambda (current-index stop)
                     (if (= current-index stop)
                         new-store
                         (begin (vector-set!
                                new-store
                                current-index
```

```

                                (vector-ref store current-index))
                                (inner (+ current-index 1) stop))))))
    (inner 0 (vector-length store))))))

;; newref : ExpVal -> Ref
(define newref
  (lambda (val)
    (let* ((next-ref (vector-length the-store))
           (new-store (new-store-longer-by-one the-store)))
      (vector-set! new-store next-ref val)
      (set! the-store new-store)
      (when (instrument-newref)
        (eopl:printf
         "newref: allocating location ~s with initial contents ~s~
         next-ref val))
      next-ref)))

;; deref : Ref -> ExpVal
(define deref
  (lambda (ref)
    (vector-ref the-store ref)))

;; setref! : Ref * ExpVal -> Unspecified
(define setref!
  (lambda (ref val)
    (if (> (vector-length the-store) ref)
        (vector-set! the-store ref val)
        (report-invalid-reference ref the-store))))

```

Exercise 4.10

```

;; the-grammar
(expression
  ("begin" expression (arbno ";" expression) "end")

```

```

begin-exp)

;; value-of
(begin-exp (exp1 rest-exps)
  (letrec ((begin-inner
    (lambda (exps final-val)
      (if (null? exps)
          final-val
          (begin-inner (cdr exps)
                        (value-of (car exps)
                                  env))))))
    (begin-inner rest-exps (value-of exp1 env))))

```

Exercise 4.11

```

;; the-grammar
(expression
  ("list" "(" (separated-list expression ",") ")")
  list-exp)

;; value-of
(list-exp (exps)
  (letrec ((list-inner
    (lambda (exps)
      (if (null? exps)
          (emptylist-val)
          (pair-val (value-of (car exps) env)
                     (list-inner (cdr exps)))))))
    (list-inner exps)))

(define-datatype expval expval?
  (num-val
    (value number?))
  (bool-val

```

```

    (boolean boolean?))
(proc-val
  (proc proc?))
(ref-val
  (ref reference?))
(pair-val
  (val1 expval?)
  (val2 expval?))
(emptylist-val))

```

Exercise 4.12

The fragment of the interpreter given in the book does not have enough context. The meaning of `apply-store` is unclear and `deref` takes only one argument which makes the program look like the store is still a global variable.

```

;; value-of-program : Program -> ExpVal
(define value-of-program
  (lambda (pgm)
    (cases program pgm
      (a-program (exp1)
        (cases answer (value-of exp1
                                (init-env)
                                (init-store))
          (an-answer (val store)
                     val)))))))

;; value-of : Exp * Env -> Answer
(define value-of
  (lambda (exp env store)
    (cases expression exp
      (const-exp (num) (an-answer (num-val num) store))
      (var-exp (var)
        (an-answer (apply-env env var)

```

```

store))
(diff-exp (exp1 exp2)
  (cases answer (value-of exp1 env store)
    (an-answer (v1 store1)
      (cases answer (value-of exp2
                          env
                          store1)
        (an-answer (v2 store2)
          (an-answer
            (num-val
              (- (expval->num v1)
                (expval->num v2)))
            store2))))))
(zero?-exp (exp1)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (an-answer
        (if (zero? (expval->num val1))
          (bool-val #t)
          (bool-val #f))
        store1))))
(if-exp (exp1 exp2 exp3)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (if (expval->bool val1)
        (value-of exp2 env store1)
        (value-of exp3 env store1))))))
(let-exp (var exp1 body)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (value-of body
        (extend-env var val1 env)

```

```

                                store1))))
(proc-exp (var body)
  (an-answer (proc-val (procedure var body env))
    store))
(call-exp (rator rand)
  (cases answer (value-of rator env store)
    (an-answer (proc-val store1)
      (cases answer (value-of rand
                                env
                                store1)
        (an-answer (arg store2)
          (apply-procedure
            (expval->proc proc-val)
            arg
            store2))))))
(letrec-exp (p-names b-vars p-bodies letrec-body)
  (value-of letrec-body
    (extend-env-rec* p-names
      b-vars
      p-bodies
      env)
    store))
(begin-exp (exp1 exps)
  (letrec
    ((begin-inner
      (lambda (e1 es store)
        (cases answer (value-of e1 env store)
          (an-answer (v1 store1)
            (if (null? es)
              (an-answer v1 store1)
              (begin-inner
                (car es)

```



```

(cdr es)
store1))))))
(begin-inner exp1 exps store))
(newref-exp (exp1)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (let ((newref-pair
              (newref val1 store1)))
        (let ((ref (car newref-pair))
              (store2 (cdr newref-pair)))
          (an-answer (ref-val ref)
                     store2))))))
(deref-exp (exp1)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (let ((ref1 (expval->ref val1)))
        (an-answer (deref store1 ref1)
                   store1))))))
(setref-exp (exp1 exp2)
  (cases answer (value-of exp1 env store)
    (an-answer (val1 store1)
      (cases answer (value-of exp2
                              env
                              store1)
        (an-answer (val2 store2)
          (an-answer
            (num-val 23)
            (setref
              (expval->ref val1)
              val2
              store2))))))))))

```

```

;; apply-procedure : Proc * ExpVal * Sto -> ExpVal

;; uninstrumented version
(define apply-procedure
  (lambda (proc1 arg store)
    (cases proc proc1
      (procedure (bvar body saved-env)
        (value-of body
                    (extend-env bvar arg saved-env)
                    store))))))

;; instrumented version
(define apply-procedure
  (lambda (proc1 arg store)
    (cases proc proc1
      (procedure (var body saved-env)
        (let ((r arg))
          (let ((new-env (extend-env var r saved-env)))
            (when (instrument-let)
              (begin
                (eopl:printf
                  "entering body of proc ~s with env =~%"
                  var)
                (pretty-print (env->list new-env))
                (eopl:printf "store =~%")
                (pretty-print
                  (store->readable
                    (get-store-as-list store))))
                (eopl:printf "~%"))
              (value-of body new-env store)))))))

;; empty-store : () -> Sto

```

```

(define empty-store
  (lambda () ' ()))

;; init-store : () -> Sto
(define init-store
  (lambda ()
    (empty-store)))

;; newref : ExpVal * Sto -> Cons(Ref,Sto)
(define newref
  (lambda (val store)
    (let ((next-ref (length store)))
      (when (instrument-newref)
        (eopl:printf
          "newref: allocating location ~s with initial contents ~s~
          next-ref val))
      (cons next-ref (append store (list val))))))

;; deref : Ref * Sto -> ExpVal
(define deref
  (lambda (store ref)
    (list-ref store ref)))

;; setref : Ref * ExpVal * Sto -> Sto
(define setref
  (lambda (ref val store)
    (letrec ((setref-inner
      (lambda (r s)
        (cond ((null? s)
              (report-invalid-reference r store))
              ((zero? r)
               (cons val (cdr s))))
      (setref-inner ref store))))))

```

```

                                (else
                                (cons
                                (car s)
                                (setref-inner (- r 1) (cdr s)))))))))
    (setref-inner ref store)))

(define report-invalid-reference
  (lambda (ref the-store)
    (eopl:error 'setref
      "illegal reference ~s in store ~s"
      ref the-store)))

;; get-store-as-list : Sto -> Listof(List(Ref,Expval))
(define get-store-as-list
  (lambda (store)
    (letrec
      ((inner-loop
        (lambda (sto n)
          (if (null? sto)
              '()
              (cons
               (list n (car sto))
               (inner-loop (cdr sto) (+ n 1)))))))
      (inner-loop store 0))))

(define-datatype answer answer?
  (an-answer
   (val expval?)
   (store store?)))

;; store? : Schemeval -> Bool
(define store?

```

```

(lambda (scmval)
  ((list-of expval?) scmval)))

```

Exercise 4.13

```

;; the-grammar
(expression
  ("proc" "(" (separated-list identifier ",") ")" expression)
  proc-exp)

(expression
  "(" expression (arbno expression) ")"
  call-exp)

(expression
  ("letrec"
    (arbno identifier "(" (separated-list identifier ",") ")"
      "=" expression)
    "in" expression)
  letrec-exp)

;; value-of
(call-exp (rator rands)
  (letrec
    ;; Listof(Exp) * Nil * Sto -> Cons(ExpVals,Sto)
    ((call-inner
      (lambda (exps vals store)
        (if (null? exps)
            (cons (reverse vals) store)
            (cases answer (value-of (car exps) env store)
              (an-answer (vall store1)
                (call-inner (cdr exps)
                           (cons vall vals)
                           store1)))))))

```

```

(cases answer (value-of rator env store)
  (an-answer (value-of-rator store1)
    (let ((args/store-pair
          (call-inner rands '() store1)))
      (let ((args (car args/store-pair))
            (new-store (cdr args/store-pair)))
        (apply-procedure
          (expval->proc value-of-rator)
          args
          new-store))))))

;; apply-procedure : Proc * Listof(ExpVal) * Store -> ExpVal
;; uninstrumented version
(define apply-procedure
  (lambda (proc1 args store)
    (cases proc proc1
      (procedure (bvars body saved-env)
        (value-of body
          (extend-env* bvars args saved-env)
          store))))))

(define-datatype environment environment?
  (empty-env)
  (extend-env
    (bvar symbol?)
    (bval expval?)
    (saved-env environment?))
  (extend-env*
    (bvars (list-of symbol?))
    (bvals (list-of expval?))
    (saved-env environment?))
  (extend-env-rec*

```

```

(proc-names (list-of symbol?))
(b-vars (list-of (list-of symbol?)))
(proc-bodies (list-of expression?))
(saved-env environment?))

(define apply-env
  (lambda (env search-sym)
    (cases environment env
      (empty-env ()
        (eopl:error 'apply-env
          "No binding for ~s"
          search-sym))
      (extend-env (bvar bval saved-env)
        (if (eqv? search-sym bvar)
          bval
          (apply-env saved-env search-sym)))
      (extend-env* (bvars bvals saved-env)
        (cond ((location search-sym bvars)
          => (lambda (n)
            (list-ref bvals n)))
          (else (apply-env saved-env search-sym))))
      (extend-env-rec* (p-names b-vars p-bodies saved-env)
        (cond
          ((location search-sym p-names)
            => (lambda (n)
              (proc-val
                (procedure
                  (list-ref b-vars n)
                  (list-ref p-bodies n)
                  env))))
            (else (apply-env saved-env search-sym))))))

```

Exercise 4.14

$$\frac{(\text{value-of } exp_1 \ \rho \ \sigma_0) = (val_1, \sigma_1) \quad l \notin dom(\sigma_1)}{(\text{value-of } (\text{let-exp } var \ exp_1 \ body) \ \rho \ \sigma_0) = (\text{value-of } body \ [var = l]\rho \ [l = val_1]\sigma_1)}$$

Exercise 4.15

Variables in the environment are bound to references which are plain integers.

Exercise 4.16

Assume the initial environment p is empty.

```
(value-of <let times 4 = 0 in begin set...> p)
store = ()

(value-of <begin set...> [times4=0]p)
store = ((0 (num-val 0)))

(value-of <(times4 3)> [times4=0]p)
store = ((0 (proc-val (<proc (x)...> [times4=0]p))))

(apply-procedure (proc-val <proc (x)...>) (num-val 3))
store = ((0 (proc-val (<proc (x)...> [times4=0]p)))
          (1 (num-val 3)))

(value-of <if zero(x)...> [x=1][times4=0]p)
store = ((0 (proc-val (<proc (x)...> [times4=0]p)))
          (1 (num-val 3)))

(value-of <-((times4 -(x,1)), -4)> [x=1][times4=0]p)
store = ((0 (proc-val (<proc (x)...> [times4=0]p)))
          (1 (num-val 3)))
```



```
(apply-procedure (proc-val <proc (x)..>) (num-val 2))
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3)))
```

```
(value-of <if zero(x)..> [x=2][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3))
          (2 (num-val 2)))
```

```
(value-of <-((times4 (-x,1)), -4)>> [x=2][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3))
          (2 (num-val 2)))
```

```
(apply-procedure (proc-val <proc (x)..>) (num-val 1))
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3))
          (2 (num-val 2)))
```

```
(value-of <if zero(x)..> [x=3][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3))
          (2 (num-val 2))
          (3 (num-val 1)))
```

```
(value-of <-((times4 -(x,1)), -4)>> [x=3][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
          (1 (num-val 3))
          (2 (num-val 2))
          (3 (num-val 1)))
```

```
(apply-procedure (proc-val <proc (x)..>) (num-val 0))
```

```
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1)))
```

```
(value-of <if zero(x)..> [x=4][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1))
         (4 (num-val 0)))
```

```
-(-(- (0,-4) ,-4) ,-4)
```

```
-(- (4,-4) ,-4)
```

```
-(8,-4)
```

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Exercise 4.17

$$\begin{array}{c}
(\text{value-of } \text{exp}_1 \ \rho \ \sigma_0) = (\text{val}_1, \sigma_1) \\
\vdots \\
(\text{value-of } \text{exp}_n \ \rho \ \sigma_{n-1}) = (\text{val}_n, \sigma_n) \\
l_1, \dots, l_n \notin \text{dom}(\sigma_n)
\end{array}
\hline
\begin{array}{l}
(\text{value-of } (\text{let-exp } \text{var}_1 \dots \text{var}_n \ \text{exp}_1 \dots \text{exp}_n \ \text{body}) \ \rho \ \sigma_0) \\
= (\text{value-of } \text{body} \ [\text{var}_1 = l_1] \dots [\text{var}_n = l_n] \rho \ [l = \text{val}_1] \dots [l_n = \text{val}_n] \sigma_n)
\end{array}$$

$$\begin{array}{c}
(\text{value-of } \text{exp}_1 \ \rho \ \sigma_0) = (\text{val}_1, \sigma_1) \\
(\text{value-of } \text{exp}_2 \ \rho \ \sigma_1) = (\text{val}_2, \sigma_2) \\
\vdots \\
(\text{value-of } \text{exp}_n \ \rho \ \sigma_{n-1}) = (\text{val}_n, \sigma_n) \\
\hline
(\text{value-of } (\text{call-exp } \text{exp}_1 \ \text{exp}_2 \dots \text{exp}_n) \ \rho \ \sigma_0) \\
= (\text{apply-procedure } (\text{expval} \rightarrow \text{proc } \text{val}_1) \ \text{val}_2 \dots \text{val}_n \ \sigma_n)
\end{array}$$

$(\text{apply-procedure } (\text{procedure } \text{var}_1 \dots \text{var}_n \ \text{body} \ \rho) \ \text{val}_1 \dots \text{val}_n \ \sigma)$
 $= (\text{value-of } \text{body} \ [\text{var}_1 = l_1] \dots [\text{var}_n = l_n] \rho \ [l_1 = \text{val}_1] \dots [l_n = \text{val}_n] \sigma)$

where $l_1, \dots, l_n \notin \text{dom}(\sigma)$.

;; the-grammar

(expression

 ("let" (arbno identifier "=" expression) "in" expression)
 let-exp)

(expression

 ("proc" "(" (arbno identifier) ")" expression)
 proc-exp)

(expression

 ("(" expression (arbno expression) ")")
 call-exp)

;; value-of

(let-exp (vars exps body)

 (let ((vals (map (lambda (exp)
 (value-of exp env))

```

                                exps)))
      (value-of body
        (extend-env* vars
                      (map newref vals)
                      env))))
(proc-exp (vars body)
  (proc-val (procedure vars body env)))
(call-exp (rator rands)
  (let ((proc (expval->proc (value-of rator env)))
        (args (map (lambda (rand)
                      (value-of rand env))
                    rands)))
    (apply-procedure proc args)))

;; apply-procedure : Proc * Listof(ExpVal) -> ExpVal
(define apply-procedure
  (lambda (procl vals)
    (cases proc procl
      (procedure (vars body saved-env)
        (value-of
          body
          (extend-env* vars
                      (map newref vals)
                      saved-env))))))

(define-datatype proc proc?
  (procedure
    (bvar (list-of symbol?))
    (body expression?)
    (env environment?)))

;; environment datatype

```

```
(extend-env*  
  (bvars (list-of symbol?))  
  (bvals (list-of reference?))  
  (saved-env environment?))  
  
;; apply-env  
(extend-env* (bvars bvals saved-env)  
  (let ((n (location search-var bvars)))  
    (if n  
      (list-ref bvals n)  
      (apply-env saved-env search-var))))
```