

Inductive Specification

Exercise 1.1

1. $\{3n + 2 \mid n \in N\}$

Definition (top-down) *A natural number n is in S if and only if*

1. $n = 2$, or
2. $n - 3 \in S$.

Definition (bottom-up) *Define the set S to be the smallest set contained in N and satisfying the following two properties:*

1. $2 \in S$, and
2. if $n \in S$, then $n + 3 \in S$.

Definition (rules of inference)

$$2 \in S$$

$$\frac{n \in S}{n + 3 \in S}$$

We generate some elements of S .

1. $2 \in S$.
- 2.

$$\frac{2 \in S}{(2 + 3) \in S}$$

- 3.

$$\frac{5 \in S}{(5 + 3) \in S}$$

2. $\{2n + 3m + 1 \mid n, m \in N\}$

Definition (top-down) *A natural number n is in S if and only if*

1. $n = 1$, or
2. $n - 2 \in S$, or
3. $n - 3 \in S$.

Definition (bottom-up) *Define the set S to be the smallest set contained in N and satisfying the following three properties:*

1. $1 \in S$, and
2. if $n \in S$, then $n + 2 \in S$, and
3. if $n \in S$, then $n + 3 \in S$.

Definition (rules of inference)

$$1 \in S$$

$$\frac{n \in S}{(n + 2) \in S}$$

$$\frac{n \in S}{(n + 3) \in S}$$

We generate some elements of S .

1. $1 \in S$.
- 2.

$$\frac{1 \in S}{(1 + 2) \in S}$$

- 3.

$$\frac{3 \in S}{(3 + 2) \in S}$$

4.

$$\frac{5 \in S}{(5 + 2) \in S}$$

We could have derived $7 \in S$ differently using only property 3.

$$\frac{\frac{1 \in S}{(1 + 3) \in S}}{(4 + 3) \in S}$$

3. $\{(n, 2n + 1) | n \in N\}$

Definition (top-down) *A pair of natural numbers (n, m) is in the set S if and only if*

1. *the pair is $(0, 1)$, or*
2. *$(n - 1, 2(n - 1) + 1) \in S$.*

Definition (bottom-up) *We define S to be the smallest set of pairs whose elements are contained in N and satisfying the following properties:*

1. *$(0, 1) \in S$, and*
2. *if $(n, m) \in S$, then $(n + 1, 2(n + 1) + 1) \in S$.*

Definition (rules of inference)

$$(0, 1) \in S$$

$$\frac{(n, m) \in S}{(n + 1, 2(n + 1) + 1) \in S}$$

We generate some elements of S .

1. $(0, 1) \in S$.

2.

$$\frac{(0,1) \in S}{(0+1, 2(0+1)+1) \in S}$$

3.

$$\frac{(1,3) \in S}{(1+1, 2(1+1)+1) \in S}$$

4. $\{(n, n^2) | n \in N\}$

Definition (top-down) A pair of natural numbers (n, m) is in the set S if and only if

1. the pair is $(0, 0)$, or
2. $(n-1, n(n-2)+1) \in S$.

Definition (bottom-up) We define S to be the smallest set of pairs of natural numbers and satisfying the following properties:

1. $(0, 0) \in S$, and
2. if $(n, m) \in S$, then $(n+1, n(n+2)+1) \in S$.

Definition (rules of inference)

$$(0,0) \in S$$

$$\frac{(n,m) \in S}{(n+1, n(n+2)+1) \in S}$$

We generate some elements of S .

1. $(0, 0) \in S$.
- 2.

$$\frac{(0,0) \in S}{(1,1) \in S}$$

3.

$$\frac{(1,1) \in S}{(2,4) \in S}$$

Exercise 1.2

1.

$$\frac{\frac{(0,1) \in S}{(1,8) \in S}}{(2,15) \in S}$$

$$\frac{(2,15) \in S}{(3,22) \in S}$$

We find a pattern and determine the set is $\{(n, 7n + 1) | n \in N\}$.

2.

$$\frac{(0,1) \in S}{(1,2) \in S}$$

$$\frac{(1,2) \in S}{(2,4) \in S}$$

$$\frac{(2,4) \in S}{(3,8) \in S}$$

We find a pattern and determine the set is $\{(n, 2^n) | n \in N\}$.

3.

$$\frac{(0,0,1) \in S}{(1,1,1) \in S}$$

$$\frac{(1,1,1) \in S}{(2,1,2) \in S}$$

$$\frac{(2,1,2) \in S}{(3,2,3) \in S}$$

$$\frac{(3,2,3) \in S}{(4,3,5) \in S}$$

$$\frac{(4,3,5) \in S}{(5,5,8) \in S}$$

This is a reasonable number of derivations to find the Fibonacci pattern. We

determine the set is $\{(n, fib(n), fib(n + 1)) | n \in N\}$.

4.

$$\begin{array}{c}
 (0, 1, 0) \in S \\
 \hline
 (1, 3, 1) \in S \\
 \hline
 (2, 5, 4) \in S \\
 \hline
 (3, 7, 9) \in S \\
 \hline
 (4, 9, 16) \in S \\
 \hline
 (5, 11, 25) \in S
 \end{array}$$

It is clear at this point what the pattern is. We determine the set is $\{(n, 2(n + 1) + 1, n^2) | n \in N\}$.