Inductive Specification

Exercise 1.1

1. $\{3n + 2 \mid n \in N\}$

Definition (top-down) *A natural number n is in S if and only if*

1. $n = 2$, or

2. $n - 3 \in S$.

Definition (bottom-up) *Define the set S to be the smallest set contained in N and satisfying the following two properties:*

1. $2 \in S$, and

2. if $n \in S$, then $n + 3 \in S$.

Definition (rules of inference)

$$2 \in S$$

$$\frac{n \in S}{n + 3 \in S}$$

We generate some elements of S.

1. $2 \in S$.

2.

$$\frac{2 \in S}{(2 + 3) \in S}$$

3.

$$\frac{5 \in S}{(5 + 3) \in S}$$

2. $\{2n + 3m + 1 \mid n, m \in N\}$

Definition (top-down) *Natural numbers n, m are in S if and only if*

1. $n + m = 1$, or

2. $n - 2 \in S$, or

3. $m - 3 \in S$.

Definition (bottom-up) *Define the set S to be the smallest set contained in N and satisfying the following three properties:*

1. $1 \in S$, and

2. if $n \in S$, then $n + 2 \in S$, and

3. if $m \in S$, then $m + 3 \in S$.

Definition (rules of inference)

$$1 \in S$$

$$\frac{n \in S}{(n+2) \in S}$$

$$\frac{m \in S}{(m+3) \in S}$$

We generate some elements of S.

1. $1 \in S$.

2.

$$\frac{1 \in S}{(1+2) \in S}$$

3.

$$\frac{3 \in S}{(3+2) \in S}$$

4.

$$\frac{5 \in S}{(5+2) \in S}$$

We could have derived $7 \in S$ in a different manner.

$$\cfrac{\cfrac{1 \in S}{(1+3) \in S}}{(4+3) \in S}$$

3. $\{(n, 2n+1) | n \in N\}$

Definition (top-down) *A pair p of natural numbers n is in the set S if and only if*

1. $p = (0, 1)$, or

2. $p = (n-1, n-2) \in S$.

Definition (bottom-up) *We define S to be the smallest set of pairs whose elements are contained in N and satisfying the following properties:*

1. $(0, 1) \in S$, and

2. if $p \in S$, then $(n+1, n+2) \in S$.

Definition (rules of inference)

$$(0, 1) \in S$$

$$\frac{p \in S}{(n+1, n+2) \in S}$$

We generate some elements of S.

1. $(0, 1) \in S$.

2.

$$\frac{(0,1) \in S}{(0+1, 1+2) \in S}$$

3.

$$\frac{(1,3) \in S}{(1+1, 3+2) \in S}$$

4. $\{(n, n^2) | n \in N\}$

Definition (top-down) *A pair p of natural numbers n is in the set S if and only if*

1. $p = (0,0)$, or

2. $(n-1, n(n-2)+1) \in S$.

Definition (bottom-up) *We define S to be the smallest set of pairs of natural numbers and satisfying the following properties:*

1. $(0,0) \in S$, and

2. if $p \in S$, then $(n+1, n(n+2)+1) \in S$.

Definition (rules of inference)

$$(0,0) \in S$$

$$\frac{p \in S}{(n+1, n(n+2)+1) \in S}$$

We generate some elements of S.

1. $(0,0) \in S$.

2.

$$\frac{(0,0) \in S}{(1,1) \in S}$$

3.

$$\frac{(1,1) \in S}{(2,4) \in S}$$