Not having a good time with the mathpartir package here.

Exercise 4.1

Calls to `g` declare a reference `counter`, set its contents, then return its contents. The information between calls is lost because `counter` is different on each call. That is, the reference data structure represents a different location. The first program was made such that the environment of the closure had access to `counter`, and thus was able to reference that variable.

Exercise 4.2

$$\frac{\texttt{(value-of}\ \ exp_1\ \ \rho\ \ \sigma_0\texttt{)} = \texttt{(}val_1\texttt{,}\ \sigma_1\texttt{)}}{\begin{array}{c}\texttt{(value-of (zero?-exp}\ \ exp_1\texttt{)}\ \ \rho\ \ \sigma_0\texttt{)} = \\ \texttt{((bool-val \#t),}\ \sigma_1\texttt{)}\ \ \text{if}\ \ \lfloor val_1 \rfloor = 0 \\ \texttt{((bool-val \#f),}\ \sigma_1\texttt{)}\ \ \text{if}\ \ \lfloor val_1 \rfloor \neq 0\end{array}}$$

Exercise 4.3

$$\frac{\begin{array}{c}\texttt{(value-of}\ \ exp_1\ \ \rho\ \ \sigma_0\texttt{)} = \texttt{(}val_1\texttt{,}\ \sigma_1\texttt{)} \\ \texttt{(value-of}\ \ exp_2\ \ \rho\ \ \sigma_1\texttt{)} = \texttt{(}val_2\texttt{,}\ \sigma_2\texttt{)}\end{array}}{\begin{array}{c}\texttt{(value-of (call-exp}\ \ exp_1\ \ exp_2\texttt{)}\ \ \rho\ \ \sigma_0\texttt{)} \\ = \texttt{(apply-procedure (expval->proc}\ \ val_1\texttt{)}\ \ val_2\ \ \sigma_2\texttt{)}\end{array}}$$

$$\frac{val_1 = \texttt{(procedure}\ \ var\ \ body\ \ \rho\texttt{)}}{\begin{array}{c}\texttt{(apply-procedure}\ \ val_1\ \ val_2\ \ \rho_0\texttt{)} \\ = \texttt{(value-of}\ \ body\ \ [var = val_2]\rho\ \ \sigma_0\texttt{)}\end{array}}$$

Exercise 4.4

$$\frac{\begin{array}{c}(\texttt{value-of}\ \ exp_1\ \ \rho\ \ \sigma_0) = (val_1, \sigma_1) \\ \vdots \\ (\texttt{value-of}\ \ exp_n\ \ \rho\ \ \sigma_{n-1}) = (val_n, \sigma_n)\end{array}}{(\texttt{value-of}\ \ (\texttt{begin}\ \ exp_1 \ldots exp_n)\ \ \rho\ \ \sigma_0) = (val_n, \sigma_n)}$$

Exercise 4.5

$$\frac{\begin{array}{c}(\texttt{value-of}\ \ exp_1\ \ \rho\ \ \sigma_0) = (val_1, \sigma_1) \\ \vdots \\ (\texttt{value-of}\ \ exp_n\ \ \rho\ \ \sigma_{n-1}) = (val_n, \sigma_n)\end{array}}{\begin{array}{c}(\texttt{value-of}\ \ (\texttt{list-exp}\ \ exp_1 \ldots exp_n)\ \ \rho\ \ \sigma_0) \\ = ((\texttt{pair-val}\ \ val_1\ \ (\ldots (\texttt{pair-val}\ \ val_n\ \ (\texttt{emptylist-val}))\ \ldots)), \sigma_n)\end{array}}$$

Exercise 4.6

$$\frac{\begin{array}{c}(\texttt{value-of}\ \ exp_1\ \ \rho\ \ \sigma_0) = (l, \sigma_1) \\ (\texttt{value-of}\ \ exp_2\ \ \rho\ \ \sigma_1) = (val, \sigma_2)\end{array}}{(\texttt{value-of}\ \ (\texttt{setref-exp}\ \ exp_1\ \ exp_2)\ \ \rho\ \ \sigma_0) = (val, [l = val]\sigma_2)}$$

Exercise 4.7

$$\frac{\begin{array}{c}(\texttt{value-of}\ \ exp_1\ \ \rho\ \ \sigma_0) = (l, \sigma_1) \\ (\texttt{value-of}\ \ exp_2\ \ \rho\ \ \sigma_1) = (val, \sigma_2)\end{array}}{(\texttt{value-of}\ \ (\texttt{setref-exp}\ \ exp_1\ \ exp_2)\ \ \rho\ \ \sigma_0) = (\sigma_0(l), [l = val]\sigma_2)}$$

Exercise 4.8

`newref`, `deref`, and `setref!` take linear time.

Exercise 4.9

`newref` uses `new-store-longer-by-one` which takes linear time because I could not find a built-in procedure for copying vectors. `deref` takes as much time as `vector-ref`. `setref!` takes as much time as `vector-length`.

```
(define empty-store
  (lambda ()
    (make-vector 0)))

;; initialize-store! : () -> Sto
;; usage: (initialize-store!) sets the-store to the empty-store
(define initialize-store!
  (lambda ()
    (set! the-store (empty-store))))

;; reference? : SchemeVal -> Bool
(define reference?
  (lambda (v)
    (integer? v)))

;; new-store-longer-by-one : Sto -> Sto
(define new-store-longer-by-one
  (lambda (store)
    (let ((new-store (make-vector (+ 1 (vector-length store)))))
      (letrec ((inner
                (lambda (current-index stop)
                  (if (= current-index stop)
                      new-store
                      (begin (vector-set!
                              new-store
                              current-index
```

```
                                    (vector-ref store current-index))
                                   (inner (+ current-index 1) stop))))))
          (inner 0 (vector-length store))))))

;; newref : ExpVal -> Ref
(define newref
  (lambda (val)
    (let* ((next-ref (vector-length the-store))
           (new-store (new-store-longer-by-one the-store)))
      (vector-set! new-store next-ref val)
      (set! the-store new-store)
      (when (instrument-newref)
        (eopl:printf
         "newref: allocating location ~s with initial contents ~s~
         next-ref val))
      next-ref)))

;; deref : Ref -> ExpVal
(define deref
  (lambda (ref)
    (vector-ref the-store ref)))

;; setref! : Ref * ExpVal -> Unspecified
(define setref!
  (lambda (ref val)
    (if (> (vector-length the-store) ref)
        (vector-set! the-store ref val)
        (report-invalid-reference ref the-store))))
```

Exercise 4.10

```
;; the-grammar
(expression
 ("begin" expression (arbno ";" expression) "end")
```

```
 begin-exp)

;; value-of
(begin-exp (exp1 rest-exps)
           (letrec ((begin-inner
                     (lambda (exps final-val)
                       (if (null? exps)
                           final-val
                           (begin-inner (cdr exps)
                                        (value-of (car exps)
                                                  env))))))
             (begin-inner rest-exps (value-of exp1 env))))
```

Exercise 4.11

```
;; the-grammar
(expression
 ("list" "(" (separated-list expression ",") ")")
 list-exp)

;; value-of
(list-exp (exps)
          (letrec ((list-inner
                    (lambda (exps)
                      (if (null? exps)
                          (emptylist-val)
                          (pair-val (value-of (car exps) env)
                                    (list-inner (cdr exps)))))))
            (list-inner exps)))

(define-datatype expval expval?
  (num-val
   (value number?))
  (bool-val
```

```
   (boolean boolean?))
  (proc-val
   (proc proc?))
  (ref-val
   (ref reference?))
  (pair-val
   (val1 expval?)
   (val2 expval?))
  (emptylist-val))
```

Exercise 4.12

The fragment of the interpreter given in the book does not have enough context. The meaning of `apply-store` is unclear and `deref` takes only one argument which makes the program look like the store is still a global variable.

```
;; value-of-program : Program -> ExpVal
(define value-of-program
  (lambda (pgm)
    (cases program pgm
      (a-program (exp1)
                 (cases answer (value-of exp1
                                         (init-env)
                                         (init-store))
                   (an-answer (val store)
                              val))))))

;; value-of : Exp * Env -> Answer
(define value-of
  (lambda (exp env store)
    (cases expression exp
      (const-exp (num) (an-answer (num-val num) store))
      (var-exp (var)
               (an-answer (apply-env env var)
```

```
                    store))
(diff-exp (exp1 exp2)
          (cases answer (value-of exp1 env store)
            (an-answer (v1 store1)
                       (cases answer (value-of exp2
                                               env
                                               store1)
                         (an-answer (v2 store2)
                                    (an-answer
                                     (num-val
                                      (- (expval->num v1)
                                         (expval->num v2)))
                                     store2))))))
(zero?-exp (exp1)
           (cases answer (value-of exp1 env store)
             (an-answer (val1 store1)
                        (an-answer
                         (if (zero? (expval->num val1))
                             (bool-val #t)
                             (bool-val #f))
                         store1))))
(if-exp (exp1 exp2 exp3)
        (cases answer (value-of exp1 env store)
          (an-answer (val1 store1)
                     (if (expval->bool val1)
                         (value-of exp2 env store1)
                         (value-of exp3 env store1)))))
(let-exp (var exp1 body)
         (cases answer (value-of exp1 env store)
           (an-answer (val1 store1)
                      (value-of body
                                (extend-env var val1 env)
```

```
                                    store1))))
(proc-exp (var body)
          (an-answer (proc-val (procedure var body env))
                     store))
(call-exp (rator rand)
          (cases answer (value-of rator env store)
            (an-answer (proc-val store1)
                       (cases answer (value-of rand
                                               env
                                               store1)
                         (an-answer (arg store2)
                                    (apply-procedure
                                     (expval->proc proc-val)
                                     arg
                                     store2))))))
(letrec-exp (p-names b-vars p-bodies letrec-body)
            (value-of letrec-body
                      (extend-env-rec* p-names
                                       b-vars
                                       p-bodies
                                       env)
                      store))
(begin-exp (exp1 exps)
           (letrec
               ((begin-inner
                  (lambda (e1 es store)
                    (cases answer (value-of e1 env store)
                      (an-answer (v1 store1)
                                 (if (null? es)
                                     (an-answer v1 store1)
                                     (begin-inner
                                       (car es)
```

```
                                        (cdr es)
                                        store1)))))))
              (begin-inner exp1 exps store)))
(newref-exp (exp1)
            (cases answer (value-of exp1 env store)
              (an-answer (val1 store1)
                         (let ((newref-pair
                                (newref val1 store1)))
                           (let ((ref (car newref-pair))
                                 (store2 (cdr newref-pair)))
                             (an-answer (ref-val ref)
                                        store2))))))
(deref-exp (exp1)
           (cases answer (value-of exp1 env store)
             (an-answer (val1 store1)
                        (let ((ref1 (expval->ref val1)))
                          (an-answer (deref store1 ref1)
                                     store1)))))
(setref-exp (exp1 exp2)
            (cases answer (value-of exp1 env store)
              (an-answer (val1 store1)
                         (cases answer (value-of exp2
                                                 env
                                                 store1)
                           (an-answer (val2 store2)
                                      (an-answer
                                       (num-val 23)
                                       (setref
                                        (expval->ref val1)
                                        val2
                                        store2)))))))))
```

```
;; apply-procedure : Proc * ExpVal * Sto -> ExpVal

;; uninstrumented version
(define apply-procedure
  (lambda (proc1 arg store)
    (cases proc proc1
      (procedure (bvar body saved-env)
                 (value-of body
                           (extend-env bvar arg saved-env)
                           store)))))

;; instrumented version
(define apply-procedure
  (lambda (proc1 arg store)
    (cases proc proc1
      (procedure (var body saved-env)
                 (let ((r arg))
                   (let ((new-env (extend-env var r saved-env)))
                     (when (instrument-let)
                       (begin
                         (eopl:printf
                          "entering body of proc ~s with env =~%"
                          var)
                         (pretty-print (env->list new-env))
                         (eopl:printf "store =~%")
                         (pretty-print
                          (store->readable
                           (get-store-as-list store)))
                         (eopl:printf "~%")))
                     (value-of body new-env store)))))))

;; empty-store : () -> Sto
```

```
(define empty-store
  (lambda () '()))

;; init-store : () -> Sto
(define init-store
  (lambda ()
    (empty-store)))

;; newref : ExpVal * Sto -> Cons(Ref,Sto)
(define newref
  (lambda (val store)
    (let ((next-ref (length store)))
      (when (instrument-newref)
        (eopl:printf
         "newref: allocating location ~s with initial contents ~s~
         next-ref val))
      (cons next-ref (append store (list val))))))

;; deref : Ref * Sto -> ExpVal
(define deref
  (lambda (store ref)
    (list-ref store ref)))

;; setref : Ref * ExpVal * Sto -> Sto
(define setref
  (lambda (ref val store)
    (letrec ((setref-inner
              (lambda (r s)
                (cond ((null? s)
                       (report-invalid-reference r store))
                      ((zero? r)
                       (cons val (cdr s)))
```

```
                        (else
                         (cons
                          (car s)
                          (setref-inner (- r 1) (cdr s))))))))
      (setref-inner ref store))))

(define report-invalid-reference
  (lambda (ref the-store)
    (eopl:error 'setref
                "illegal reference ~s in store ~s"
                ref the-store)))

;; get-store-as-list : Sto -> Listof(List(Ref,Expval))
(define get-store-as-list
  (lambda (store)
    (letrec
        ((inner-loop
          (lambda (sto n)
            (if (null? sto)
                '()
                (cons
                 (list n (car sto))
                 (inner-loop (cdr sto) (+ n 1)))))))
      (inner-loop store 0))))

(define-datatype answer answer?
  (an-answer
   (val expval?)
   (store store?)))

;; store? : Schemeval -> Bool
(define store?
```

```
  (lambda (scmval)
    ((list-of expval?) scmval)))
```

Exercise 4.13

```
;; the-grammar
(expression
 ("proc" "(" (separated-list identifier ",") ")" expression)
 proc-exp)

(expression
 ("(" expression (arbno expression) ")")
 call-exp)

(expression
 ("letrec"
  (arbno identifier "(" (separated-list identifier ",") ")"
        "=" expression)
  "in" expression)
 letrec-exp)

;; value-of
(call-exp (rator rands)
          (letrec
              ;; Listof(Exp) * Nil * Sto -> Cons(ExpVals,Sto)
              ((call-inner
                (lambda (exps vals store)
                  (if (null? exps)
                      (cons (reverse vals) store)
                      (cases answer (value-of (car exps) env store
                        (an-answer (val1 store1)
                                   (call-inner (cdr exps)
                                               (cons val1 vals)
                                               store1)))))))
```

```
              (cases answer (value-of rator env store)
                (an-answer (value-of-rator store1)
                           (let ((args/store-pair
                                  (call-inner rands '() store1)))
                             (let ((args (car args/store-pair))
                                   (new-store (cdr args/store-pair))
                               (apply-procedure
                                (expval->proc value-of-rator)
                                args
                                new-store)))))))

;; apply-procedure : Proc * Listof(ExpVal) * Store -> ExpVal
;; uninstrumented version
(define apply-procedure
  (lambda (proc1 args store)
    (cases proc proc1
      (procedure (bvars body saved-env)
                 (value-of body
                           (extend-env* bvars args saved-env)
                           store)))))

(define-datatype environment environment?
  (empty-env)
  (extend-env
   (bvar symbol?)
   (bval expval?)
   (saved-env environment?))
  (extend-env*
   (bvars (list-of symbol?))
   (bvals (list-of expval?))
   (saved-env environment?))
  (extend-env-rec*
```

```
   (proc-names (list-of symbol?))
   (b-vars (list-of (list-of symbol?)))
   (proc-bodies (list-of expression?))
   (saved-env environment?)))

(define apply-env
  (lambda (env search-sym)
    (cases environment env
      (empty-env ()
                 (eopl:error 'apply-env
                             "No binding for ~s"
                             search-sym))
      (extend-env (bvar bval saved-env)
                  (if (eqv? search-sym bvar)
                      bval
                      (apply-env saved-env search-sym)))
      (extend-env* (bvars bvals saved-env)
                   (cond ((location search-sym bvars)
                          => (lambda (n)
                               (list-ref bvals n)))
                         (else (apply-env saved-env search-sym))))
      (extend-env-rec* (p-names b-vars p-bodies saved-env)
                       (cond
                         ((location search-sym p-names)
                          => (lambda (n)
                               (proc-val
                                (procedure
                                 (list-ref b-vars n)
                                 (list-ref p-bodies n)
                                 env))))
                         (else (apply-env saved-env search-sym))))
```

Exercise 4.14

$$\frac{\texttt{(value-of}\ \ exp_1\ \ \rho\ \ \sigma_0\texttt{)} = \texttt{(}val_1\texttt{,}\ \sigma_1\texttt{)} \qquad l \notin dom(\sigma_1)}{\begin{array}{l}\texttt{(value-of (let-exp}\ \ var\ \ exp_1\ \ body\texttt{)}\ \ \rho\ \ \sigma_0\texttt{)} \\ \quad = \texttt{(value-of}\ \ body\ \ [var = l]\rho\ \ [l = val_1]\sigma_1\texttt{)}\end{array}}$$

Exercise 4.15

Variables in the environment are bound to references which are plain integers.

Exercise 4.16

Assume the initial environment p is empty.

```
(value-of <let times 4 = 0 in begin set..> p)
store = ()

(value-of <begin set..> [times4=0]p)
store = ((0 (num-val 0)))

(value-of <(times4 3)> [times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p))))

(apply-procedure (proc-val <proc (x)..>) (num-val 3))
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3)))

(value-of <if zero(x)..> [x=1][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3)))

(value-of <-((times4 -(x,1)), -4)> [x=1][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3)))
```

```
(apply-procedure (proc-val <proc (x)..>) (num-val 2))
store = ((0 (proc-val (<proc (x)..> [times4=0]p))))
         (1 (num-val 3)))

(value-of <if zero(x)..> [x=2][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2)))

(value-of <-((times4 (-x,1)), -4)>> [x=2][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2)))

(apply-procedure (proc-val <proc (x)..>) (num-val 1))
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2)))

(value-of <if zero(x)..> [x=3][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1)))

(value-of <-((times4 -(x,1)), -4)> [x=3][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1)))

(apply-procedure (proc-val <proc (x)..>) (num-val 0))
```

```
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1)))

(value-of <if zero(x)..> [x=4][times4=0]p)
store = ((0 (proc-val (<proc (x)..> [times4=0]p)))
         (1 (num-val 3))
         (2 (num-val 2))
         (3 (num-val 1))
         (4 (num-val 0)))

-(-(-(0,-4),-4),-4)

-(-(4,-4),-4)

-(8,-4)

12
```

Exercise 4.17