

Auxiliary Procedures and Context Arguments

Exercise 1.14

Theorem Given the assumption $0 \leq n < \text{length}(v)$, `partial-vector-sum` computes $\sum_{i=0}^{i=n} v_i$.

Proof: The proof is by induction on n . We take $IH(n)$ to be that the procedure `partial-vector-sum` satisfies its contract given that $0 \leq n < \text{length}(v)$, where v is a vector.

Where $n = 0$, we have $(\text{partial-vector-sum } v \ 0) = \sum_{i=0}^{i=0} v_0$.

Let n be an integer such that $IH(n - 1)$ holds. We need to show that $IH(n - 1)$ implies $IH(n)$.

We have that $IH(n - 1)$ states that $(\text{partial-vector-sum } v \ (- \ n \ 1))$ computes $\sum_{i=0}^{i=n-1} v_i$. When we add v_n to this summation, we get

$$\sum_{i=0}^{i=n-1} v_i + v_n = \sum_{i=0}^{i=n} v_i,$$

which is $IH(n)$. This completes the proof of the claim that $IH(n)$ holds and therefore completes the induction. \square