**1.**
a) We prove the following statement by structual induction on $T$, the root of a tree.

STATEMENT $S(T)$: Calling `preorder` on a tree $T$ prints the labels of $T$ in preorder.

BASIS. The basis is where $T$ is a single node. Then line (1) prints the label of $T$, line (2) gets the leftmost child which is `NULL`, and thus line (3) fails, stopping execution of `preorder`.

INDUCTION. Suppose $T$ is not a leaf. Then $T$ has at least one child. Assume by the inductive hypothesis that `preorder` prints the labels of the children of $T$ in preorder. Clearly the label of the root of $T$ is printed by line (1). This proves the inductive step. We conclude that $S(T)$ is true for all labeled trees $T$. ✦

b) We prove the following statement by structural induction on $T$, the root of a tree.

STATEMENT $S(T)$: Calling `postorder` on a tree $T$ prints the labels of $T$ in postorder.

BASIS. Consider when $T$ is a leaf. Line (1) assigns `c` and line (2) fails. All that is left is for line (5) to print the label of $T$.

INDUCTION. Suppose $T$ is not a leaf. Then $T$ has at least one child. Assume by the inductive hypothesis that `postorder` prints the labels of the children of $T$ in postorder. After the labels of the children of $T$ have been printed, then lastly on line (5), the label of the root of $T$ is printed. This is the correct behavior for postorder, and proves the inductive step. Therefore $S(T)$ holds for all labeled trees $T$. ✦

**2.** We prove the following statement by induction on $n$, the number of nodes a tree has.

STATEMENT $S(n)$: If a tree $T$ has $n$ nodes with each having a branching factor $b$, then there are $1 + (b-1)n$ `NULL` pointers among its nodes.

BASIS. Suppose $n = 1$. Then $T$ is a leaf. Thus there are $1 + (b-1)1 = b$ `NULL` pointers in the root of $T$.

INDUCTION. Suppose $n \geq 1$. Consider that $T$ has $n+1$ nodes. Thus $T$ has at least one child. Assume by the inductive hypothesis that all but one leaf of $T$, that being $n$ nodes, together have $1 + (b-1)n$ `NULL` pointers among them. Therefore all the nodes of $T$ have a total of $1 + (b-1)(n+1) = b(n+1) - n$ `NULL` pointers. This proves the inductive step. We conclude that $S(n)$ holds. ✦

**3.** We prove the following statement by structural induction on $T$, the root of a tree.

STATEMENT $S(T)$: The number of nodes in $T$ is 1 more than the sum of the degrees of the nodes.

BASIS. Suppose $T$ has only one node, the root. The degree of the root is 0. Thus the number of nodes in $T$ is 1.

INDUCTION. Suppose the root of $T$ has children. Let $n$ be the root of $T$. Let $c_i$ for $1 \leq i \leq k$ be the children of $n$. By the inductive hypothesis we know that the number of nodes in $T$ is 1 more than the sum of the degrees of the nodes in $T$. The number of nodes in $T$ is 1 more than the sum of the number of nodes in each $c_i$. The sum of the degrees of all the nodes in $T$ is the sum of the degrees of all $c_i$ plus the degree of $n$.