1.

STATEMENT $S(i)$: If $1 \leq i < n$, then

$$T(n) = T(n-i) + \sum_{j=0}^{i-1} g(n-j).$$

BASIS. The basis is $i = 1$, as given by the lower bound of the interval. Then we have the inductive definition of the recurrence hence the basis holds.

INDUCTION. If $i \geq n$ then there is nothing to prove. Suppose that $i + 1 < n$. From the inductive hypothesis we get

$$\begin{aligned} T(n) &= T(n-i-1) \sum_{j=0}^{i-1} g(n-j) + g(n-i) \\ &= T(n-i-1) + \sum_{j=0}^{i} g(n-j). \end{aligned}$$

This is the statement $S(i+1)$ and we have proved the inductive step.