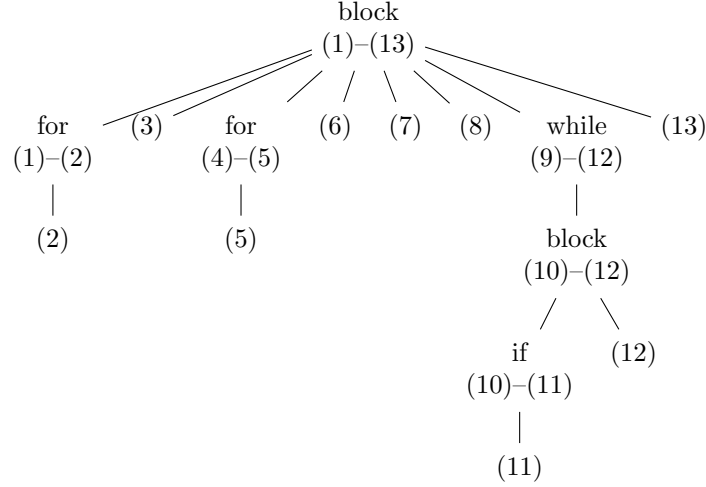


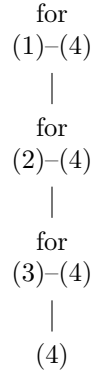
1. The running time of the body of the for-loop on lines (1) and (2) is 0. The loop goes around at most 100 times. Hence the for-loop takes  $O(100)$  time, which is  $O(1)$ . Line (3) takes  $O(1)$  time. Lines (4) and (5) take  $O(n)$  time. Lines (6) to (8) take  $O(1)$  time each. The while-loop on lines (9) to (12) goes around  $n - i$  times. But we know from line (8) that  $i = 1$ . Thus it goes around  $n - 1$  times. The body of the loop is a block. Line (10) is an if-statement that takes  $O(1)$  time with no else-part. The if-part takes  $O(1)$  time. Line (12) also takes  $O(1)$  time. Thus the while-loop takes  $O(1 + (3 + 1)(n - 1))$  time which is  $O(n)$ . Line (13) takes  $O(1)$  time. Using the summation rule, we determine that the program takes  $O(n)$  time.



2.

a) Line (3) goes around  $n - i$  times. Line (2) goes around  $n - i - 1$  times. Line (4) takes  $O(1)$  time. Thus lines (2) to (4) take  $O((n - i)(n - i - 1))$  time. We can neglect the lower order terms and say it is  $O((n - i)^2)$ .

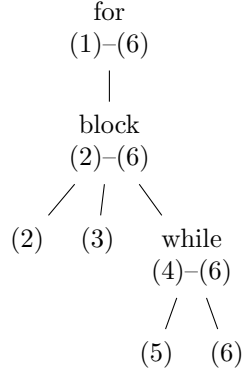
b) The loop starting on line (1) goes around  $n - 1$  times. Multiply this with  $(n - i)^2$  to get  $O(n^3)$  after dropping the lower order terms.



3.

a) The while-loop goes around  $\log i$  times. The running time of the body of the while-loop is  $O(1)$ . Hence the while-loop takes  $O(\log i)$  time.

b) The for-loop runs  $((n+1) - 1)$  times and the body takes  $O(\log i)$  time. But the upper bound of the for-loop has the while loop take  $O(\log n)$  time. Therefore the entire program takes  $O(n \log n)$  time.



4. Lines (1), (4), (5), and (6) are  $O(1)$ . Line (3) is  $O(1)$  and both the if-part and else-part are  $O(1)$  hence the entire if-statement is  $O(1)$ , which is the body of the loop. The loop stops when  $i^2 > n$ , or  $i > \sqrt{n}$ . Hence the number of times the loop goes around has a limit of  $\sqrt{n}$ . Thus the function takes  $O(\sqrt{n})$  time.

