For structural induction on trees, we assume the inductive hypothesis holds for the children of the root of a tree T. Then we relate the children to the root somehow, and that is what proof is meant to involve. We can assume properties of the children that apply recursively downward. From that, we can assume that values of whatever type we wish percolate upward to the children of the root of T and go from there.

1.

a) We prove the following statement by structual induction on T, the root of a tree.

STATEMENT $S(T)$: Calling `preorder` on a tree T prints the labels of T in preorder.

BASIS. The basis is where T is a single node. Then line (1) prints the label of T, line (2) gets the leftmost child which is `NULL`, and thus line (3) fails, stopping execution of `preorder`.

INDUCTION. Suppose T is not a leaf. Then T has at least one child. Assume by the inductive hypothesis that `preorder` prints the labels of the children of T in preorder. Clearly the label of the root of T is printed by line (1). This proves the inductive step. We conclude that $S(T)$ is true for all labeled trees T. ✦

b) We prove the following statement by structural induction on T, the root of a tree.

STATEMENT $S(T)$: Calling `postorder` on a tree T prints the labels of T in postorder.

BASIS. Consider when T is a leaf. Line (1) assigns `c` and line (2) fails. All that is left is for line (5) to print the label of T.

INDUCTION. Suppose T is not a leaf. Then T has at least one child. Assume by the inductive hypothesis that `postorder` prints the labels of the children of T in postorder. After the labels of the children of T have been printed, then lastly on line (5), the label of the root of T is printed. This is the correct behavior for postorder, and proves the inductive step. Therefore $S(T)$ holds for all labeled trees T. ✦

2. We prove the following statement by induction on n, the number of nodes a tree has.

STATEMENT $S(n)$: If a tree T has n nodes with each having a branching factor b, then there are $1 + (b-1)n$ `NULL` pointers among its nodes.

BASIS. Suppose $n = 1$. Then T is a leaf. Thus there are $1 + (b-1)1 = b$ `NULL` pointers in the root of T.

INDUCTION. Suppose $n \geq 1$. Consider that T has $n+1$ nodes. Thus T has at least one child. Assume by the inductive hypothesis that all but one leaf of T, that being n nodes, together have $1 + (b-1)n$ `NULL` pointers among them. Therefore all the nodes of T have a total of $1 + (b-1)(n+1) = b(n+1) - n$ `NULL` pointers. This proves the inductive step. We conclude that $S(n)$ holds. ✦

3. We prove the following statement by structural induction on T, the root of a tree.

STATEMENT $S(T)$: The number of nodes in T is 1 more than the sum of the degrees of the nodes.

BASIS. Suppose T has only one node, the root. The degree of the root is 0. Thus the number of nodes in T is 1.

What we will essentially prove is an equality between the number of nodes in a tree with a number relative to the sum of the degrees of the nodes.

INDUCTION. Suppose the root of T has children. Let n be the root of T. Let c_i for $1 \leq i \leq k$ be the children of n. Let $nodes(c_i)$ be the number of nodes in the subtree rooted at c_i. Let $degree(c_i)$ be the sum of the degrees of the nodes in the subtree rooted at c_i. By the inductive hypothesis we know that $nodes(c_i) = 1 + degree(c_i)$ for $1 \leq i \leq k$. The number of nodes in T is $1 + \sum_{i=1}^{k} nodes(c_i)$. The sum of the degrees of the nodes in T is $k + \sum_{i=1}^{k} degree(c_i)$. Therefore, by expanding these numbers that we have declared, we have

$$1 + \sum i = 1^k nodes(c_i) = 1 + k + \sum_{i=1}^{k} degree(c_i).$$