

1. We prove that the two definitions given of lexicographic order are the same. Recall the definitions. The recursive definition:

BASIS.

1. $\epsilon < w$ for any string w other than ϵ itself.
2. If $c < d$, where c and d are characters, then for any strings w and x , we have $cw < dx$.

INDUCTION. If $w < x$ for strings w and x , then for any character c we have $cw < cx$.

The iterative definition. Let $C = c_1c_2 \cdots c_k$ and $D = d_1d_2 \cdots d_m$ be two strings. We say $C < D$ if either of the following holds:

1. That $k < m$ and for $i = 1, 2, \dots, k$ we have $c_i = d_i$.
2. For some value of $i > 0$, the first $i - 1$ characters of the two strings agree, but the i th character of the first string is less than the i th character of the second string.

We prove first that the recursive definition is the same as the iterative definition by complete induction on the number of times the recursive rule is applied to the string.

STATEMENT $S(n)$: If string w is defined to be less than string x after n applications of the recursive rule,