1. We prove the following statement by induction on i, the length of the list passed as an argument.

STATEMENT $S(i)$: If `list` is a list of length i when `PrintList` is called, then `PrintList` prints the elements of `list`.

BASIS. When `list` is of length 0, then `list` is `NULL`. Therefore there are no elements to print, so we do not call `printf`, and this is the desired behavior.

INDUCTION. Assume $S(i)$ is true for $i \geq 0$. Let us consider what happens when `PrintList` is called on a list of length $i+1$. Then `list` has at least one element. This element gets printed. By the inductive hypothesis, the rest of `list`, which has i elements, has all its elements printed under `PrintList`. We have proved the inductive step, therefore `PrintList` works as intended. We conclude $S(i)$ for $i \geq 0$. ✦

2. We prove the following statement by induction on i, the length of the list passed as an argument.

STATEMENT $S(i)$: If `L` is a list of length i when `sum` is called, then `sum` returns the sum of the elements of `L`.

BASIS. If $i = 0$, then `sum` returns the sum of no elements, and that is 0.

INDUCTION. Assume $S(i)$ is true for $i \geq 0$. We shall prove $S(i+1)$. Consider a list of length $i+1$. Then `list` is of length at least 1. Line (2) returns the sum of the first element plus the `sum` of the rest of `list`. Since the rest of `list` has i elements, then by the inductive hypothesis, the number returned to be added is the sum of the remaining i elements. Therefore line (2) returns the sum of all the elements of `list`. This proves the inductive step. Therefore $S(i)$ holds for $i \geq 0$. ✦

3. We prove the following statement by induction on i, the length of the list.

STATEMENT $S(i)$: If `L` is a list of length i when `find0` is called, then `find0` returns whether 0 is an element of `L`.

BASIS. If $i = 0$, then there are no elements hence 0 is not an element of `L`. Thus returning `FALSE` is the correct value.

INDUCTION. Assume $S(i)$ is true for $i \geq 0$. Suppose `L` is of length $i+1$. Then `L` is of length at least 1. Line (2) returns `TRUE` if the first element of `L` is 0. Otherwise, on line (3), the value of the `find0` of the rest of `L` is returned. Since the rest of `L` has length i, then the inductive hypothesis applies. This proves the inductive step. Therefore $S(i)$ holds for $i \geq 0$. ✦

4. We prove the following statement by induction on i, the sum of the lengths of both lists passed as arguments. Although the basis is not dependent on i, we write the basis in terms of i. It would be better to state only whether either list is `NULL`.

STATEMENT $S(i)$: If `list1` and `list2` are sorted lists such that sum of their lengths is i, then `merge` returns the sorted list having all the elements of `list1` and `list2`.

BASIS. If $i = k$ where k is exactly the length of `list1` or `list2`, then one of the two lists must be of length 0. If `list1` is of length 0, then line (1) returns `list2`. If `list2` is of length 0, then line (2) returns `list1`. Line (1) returns `list2` when both lists are empty as well. In either case, a sorted list having the elements of both lists is returned.

INDUCTION. Assume $S(i)$ is true for $i \geq k$. We shall prove $S(i+1)$. That is, the sum of the lengths of the lists is $i+1$. We assume neither list is of length 0. Then each list has a first element. The lowest of the first two elements becomes the first element of the merger of the two lists. This preserves the sorted order. The rest of the merger of the lists is of length i, since only one of either `list1` or `list2` has their length decremented by 1. Thus the inductive hypothesis applies, and `merge` returns the merged sorted list. This proves the inductive step and we conclude that $S(i)$ is true for $i \geq k$. ✦

We prove the following statement by induction on i, the length of the list.

STATEMENT $S(i)$: If `list` is a list of length i, then `split(list)` returns the list of all the even-numbered cells of `list`.

BASIS. We take the basis to be both $S(0)$ and $S(1)$. When `list` is of length 0, then `split` returns `NULL`. When `list` is of length 1, then `split` returns `NULL`. There are no even-numbered cells in an empty list or in a list with only one element. This proves the basis.

INDUCTION. Assume $S(j)$ is true for $0 \leq j \leq i$ and $i \geq 1$. We shall prove $S(i+1)$. Suppose `list` is of length $i+1$. `split` has a side-effect, but we are not interested in that. We can write *list* as $(a, (b, M))$ where a, b are elements and M is a list. `pSecondCell` gets (b, M) and assigns as the rest of `pSecondCell` the `split` of M. Since M is at most as long as $i-1$, because the size of the argument is reduced by 2, then the inductive hypothesis applies to M. Therefore calls to `split` gather every second cell of `list` and returns the list of all even-numbered cells. This proves the inductive step and we conclude $S(i)$ for $i \geq 0$. ✦

6. We prove the following statement by induction on r, the remainder of i divided by j.

STATEMENT $S(r)$: If $r = i \mod j$, then `gcd(i, j)` returns the GCD of i and j.

BASIS. The basis is $r = 0$. Then `gcd` returns `j`.

INDUCTION. Assume $S(k)$ is true for $0 \leq k \leq r$ and $r \geq 0$. We shall prove $S(r+1)$. Since $r \geq 0$, then $r+1 \geq 1$. Thus we reach line (2).

We return `gcd(j, r + 1)`. Then the remainder under this call is $j \mod (r+1)$. We can reason that $j \mod (r+1) < r$ for $r > 0$. Hence subsequent calls to `gcd` reduce the induction parameter. Therefore the subsequent remainders are less than r, and thus the inductive hypothesis applies. We have proved the inductive step and conclude $S(r)$ for $r \geq 0$. ✦