1. We prove that the two definitions given of lexicographic order are the same. Recall the definitions. The recursive definition:

BASIS.

1. $\epsilon < w$ for any string w other than ϵ itself.

2. If $c < d$, where c and d are characters, then for any strings w and x, we have $cw < dx$.

INDUCTION. If $w < x$ for strings w and x, then for any character c we have $cw < cx$.

The iterative definition. Let $C = c_1c_2 \cdots c_k$ and $D = d_1d_2 \cdots d_m$ be two strings. We say $C < D$ if either of the following holds:

1. That $k < m$ and for $i = 1, 2, \ldots, k$ we have $c_i = d_i$.

2. For some value of $i > 0$, the first $i - 1$ characters of the two strings agree, but the ith character of the first string is less than the ith character of the second string.

We prove first that the recursive definition is the same as the iterative definition by complete induction on the number of times the recursive rule is applied to the string.

STATEMENT $S(n)$: If it is necessary to apply the recursive rule n times to show that $w < x$, then w precedes x according to the iterative definition of 'lexicographic order'.

We say that there is a necessary number of times to apply the recursive rule to the strings to show that $w < x$. There is a minimum number, which is the lowest number of applications needed until either basis case is satisfied. There is a maximum number, which corresponds to the length of w. The minimum number here is exactly what we mean by the necessary number in the statement.

We say this as opposed to "under the recursive definition, $w < x$ after n applications of the recursive rule ...". The number n cannot be arbitrary since there is a minimum and maximum. Thus we specify that we must meet this necessary number to show that $w < x$.

BASIS. The basis is $n = 0$, that is when either basis case holds trivially. Then $w < x$ by the recursive definition. Thus rule (1) of the iterative definition holds where $w = \epsilon$, and rule (2) holds where the basis (2) applies. Therefore the basis is true.

INDUCTION. Assume that $S(i)$ is true for $0 \le i \le n$. We shall prove $S(n+1)$. That is, we apply the recursive rule $n + 1$ times to show that $w < x$. By the inductive hypothesis, $w < x$. After the $n + 1$th