1. We "pull out" each of the operands of $E = (u + v) + ((w + (x + y)) + z)$. We perform this arbitrarily from left to right.

By the associative law, E can be transformed into $u + (v + ((w + (x + y)) + z))$. Thus we have $E = u + E_1$ where $E_1 = v + ((w + (x + y)) + z)$. We trivially pull out v from E_1 to get an expression of the form $v + E_2$ where $E_2 = (w + (x + y)) + z$. With the associative law we transform E_2 into an expression of the form $w + E_3$ where $E_3 = (x + y) + z$. Similarly we transform E_3 into an expression of the form $x + E_4$ where $E_4 = y + z$. We transform E_4 into an expression of the form $y + E_5$ where $E_5 = z$. The sequence of transformations is

$$(u + v) + ((w + (x + y)) + z)$$
$$u + (v + ((w + (x + y)) + z))$$
$$u + (v + (w + ((x + y) + z)))$$
$$u + (v + (w + (x + (y + z)))).$$

2.

a) We transform $E = w + (x + (y + z))$ into $F = ((w + x) + y) + z$. We do so by "pulling out" one operand a from both expressions, which are equivalent, and then repeating with the next operand until none are left.

We first choose to "pull out" w from both expressions first. This is already so for E. For F we follow the sequence

$$((w + x) + y) + z \to (w + (x + y)) + z \to w + ((x + y) + z). \tag{1}$$

Now to transform $E_1 = x + (y + z)$ into $F_1 = (x + y) + z$. We "pull out" x which is already accomplished for E_1. For F_1 we perform the transformation

$$(x + y) + z \to x + (y + z). \tag{2}$$

We "pull out" y next from $E_2 = y + z$ and $F_2 = y + z$. This is done so trivially.

We now transform what is left of the expressions E_2 and F_2 without y. Consider the expressions $E_3 = z$ and $F_3 = z$. E_3 naturally transforms into F_3. Furthermore, $E_2 = y + E_3$ can transform into $F_2 = y + F_3$, and $E_1 = x + E_2$ can transform into $F_1 = x + F_2$. Finally, $E = w + E_1$ can transform into $F = w + F_1$, and we are done. The sequence of transformations is

$w + (x + (y + z))$	Expression E
$w + ((x + y) + z)$	(2) in reverse
$(w + (x + y)) + z$	Middle of (1) in reverse
$((w + x) + y) + z$	Expression F, beginning of (1) in reverse

b) We transform $E = (v + w) + ((x + y) + z)$ into $F = ((y + w) + (v + z)) + x$.

We "pull out" v first from both expressions. The sequences of transformations for E and F respectively are

$$(v + w) + ((x + y) + z) \to v + (w + ((x + y) + z)) \tag{3}$$

and

$$((y + w) + (v + z)) + x \to (((y + w) + v) + z) + x$$
$$\to ((v + (y + w)) + z) + x \to (v + ((y + w) + z)) + x \qquad (4)$$
$$\to v + (((y + w) + z) + x).$$

We "pull out" w from the subexpressions $w + ((x+y)+z)$ and $((y+w)+z)+x$:

$$((y+w)+z)+x \to ((w+y)+z)+x \to (w+(y+z))+x \to w+((y+z)+x). \qquad (5)$$

We shall "pull out" x from the subexpressions $(x + y) + z$ and $(y + z) + x$:

$$(x + y) + z \to x + (y + z) \qquad (6)$$

and

$$(y + z) + x \to x + (y + z). \qquad (7)$$

We then "pull out" y from the subexpressions $y + z$ and $y + z$. We are then left with the operand z in both expressions, which means we can transform one expression into the other. Thus $y + A_1$ can transform into $y + B_1$ if we consider $A_1 = z = B_1$. By successively letting the subexpressions of E and F (starting with z) being added to y, x, w, v in order, we transform E into F. The sequence of transformations is

$(v + w) + ((x + y) + z)$	Expression E
$v + (w + ((x + y) + z))$	(3)
$v + (w + (x + (y + z)))$	(6)
$v + (w + ((y + z) + x))$	(7) in reverse
$v + ((w + (y + z)) + x)$	Middle-right of (5) in reverse
$v + (((w + y) + z) + x)$	Middle-left of (5) in reverse
$v + (((y + w) + z) + x)$	Beginning of (5) in reverse
$(v + ((y + w) + z)) + x$	Middle-right of (4) in reverse
$((v + (y + w)) + z) + x$	Middle of (4) in reverse
$(((y + w) + v) + z) + x$	Middle-left of (4) in reverse
$((y + w) + (v + z)) + x$	Expression F, beginning of (4) in reverse

3. We shall prove the following statement by complete induction on n, the number of occurrences of operators in an expression.

STATEMENT $S(n)$: Let E be an expression with operators $+, -, *$, and $/$. If E has n operator occurrences, then E has $n + 1$ operands.

BASIS. Let $n = 1$. Then E has $1 + 1 = 2$ operands since the operator is binary.

INDUCTION. Assume $n \geq 1$ and $S(n)$ is true for any $n \geq 1$. We shall prove $S(n + 1)$. That is, we assume that E has at least two operators. Suppose that E is an expression involving E_1 which has at least one operator, and the operand a. Then E has some operator with E_1 and a as operands. By the inductive hypothesis, E_1 has $n + 1$ operands. Together with a, the expression E has $n + 2$ operands.