1. We know that for any function f that $f(n)$ is $O(f(n))$. Thus we need not consider the $O(f_i(n))$ for $f_i(n)$.

For f_1, choose witnesses $c = 1$ and $n_0 = 0$. We know that $f_1(n) \leq cf_2(n)$, $f_1(n) \leq cf_3(n)$, and $f_1(n) \leq cf_4(n)$ for $n \geq n_0$. Therefore $f_1(n)$ is $O(f_2(n))$, $O(f_3(n))$, and $O(f_4(n))$.

For f_3, $n^2 \leq cn^2$ if n is odd. In the case of even n, it is trivially true. For f_4, $n^2 \leq cn^2$ if n is prime. The qualification of n being prime does not change the simple inequality. Where n is composite, $n^2 \leq cn^3$ is clearly true.

f_2 is not $O(f_1(n))$, $O(f_3(n))$, or $O(f_4(n))$.

Suppose there are witnesses n_0 and c such that $f_2(n) \leq cf_1(n)$ for all $n \geq n_0$. Pick n_1 to be at least $2c$ or n_0. Since $n_1 \geq n_0$ then the inequality holds, because it holds for all $n \geq n_0$. If we divide both sides by n_1^2, we have $n_1 \leq c$. But n_1 is at least $2c$, and n_1 cannot be less than c and greater than $2c$. Therefore witnesses that show $f_2(n)$ to be $O(f_1(n))$ do not exist.

Suppose there are witnesses n_0 and c such that $f_2(n) \leq cf_3(n)$ for all $n \geq n_0$. Pick n_1 to be at least $2c$ or n_0. We know that $f_2(n_1) \leq cf_3(n_1)$ because $n_1 \geq n_0$. But when n is odd, we must show that $f_2(n) \leq cf_3(n)$.

$f_2(n) \leq f_3(n)$ is false.

$f_2(n) \leq f_4(n)$ is false.

For f_3. $f_3(n) \leq f_1(n)$ is false.

$f_3(n) \leq f_2(n)$ is true.

$f_3(n) \leq f_4(n)$ is true for $n_0 > 2$.

For f_4. $f_4(n) \leq f_1(n)$ is false.

$f_4(n) \leq f_2(n)$ is true.

$f_4(n) \leq f_3(n)$ is false because of the odd composite n case.