

1.

a) STATEMENT $S(n) : \sum_{i=1}^n i = \frac{n(n+1)}{2}$.

BASIS. The basis is $S(1)$. That is $\sum_{i=1}^1 i = 1(1+1)/2 = 1$. This is indeed true and thus the basis of $S(n)$ holds.

INDUCTION. Let $n \geq 1$. We must prove that $S(n)$ implies $S(n+1)$. To prove $S(n+1)$, write

$$\sum_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}. \quad (1)$$

The left side of Equation (1) is defined in terms of the inductive hypothesis $S(n)$. That is, we have

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^n i + n + 1. \quad (2)$$

By the inductive hypothesis, the right side of Equation (2) is $n(n+1)/2 + n + 1$, which is equal to the right side of (1). We have thus proved Equation (1), which is $S(n+1)$, in terms of $S(n)$. Therefore $S(n)$ is true for $n \geq 1$. ♦

b) STATEMENT $S(n) : \sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$.

BASIS. The basis is $S(1)$. We substitute $n = 1$ and find

$$\sum_{i=1}^1 i^2 = \frac{1(1+1)(2+1)}{6}. \quad (3)$$

The summation on the left side of Equation (3) is equal to 1, and the right side of (3) is also 1. Thus we have proved the basis of $S(n)$.

INDUCTION. We must prove that $S(n)$ implies $S(n+1)$. Let $n \geq 1$ and write

$$\sum_{i=1}^{n+1} i^2 = \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}. \quad (4)$$

Since $S(n+1)$ is defined in terms of $S(n)$, then we can write

$$\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^n i^2 + (n+1)^2, \quad (5)$$

where the term $(n+1)^2$ is added to the summation. By the inductive hypothesis the right side of Equation (5) is equal to the right side of (4). Thus we have proven that $S(n)$ implies $S(n+1)$. Therefore $S(n)$ holds for $n \geq 1$. ♦

c) STATEMENT $S(n) : \sum_{i=1}^n i^3 = n^2(n+1)^2/4$.

BASIS. The basis is $S(1)$. That is

$$\sum_{i=1}^1 i^3 = 1.$$

The summation has value 1 and is equal to the right side of the basis. Hence the basis of $S(n)$ is proven.

INDUCTION. We must prove the statement $S(n+1)$. Let $n \geq 1$. Write

$$\sum_{i=1}^{n+1} i^3 = \frac{(n+1)^2((n+1)+1)^2}{4}. \quad (6)$$

By the inductive hypothesis, we decompose the summation on the left side of Equation (6) and get

$$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^n i^3 + (n+1)^3. \quad (7)$$

The right side of Equation (7) is equal to the right side of Equation (6). Hence $S(n)$ implies $S(n+1)$. Therefore $S(n)$ holds for $n \geq 1$. ♦