1.
a) We prove the following statement by induction on T, the root of a tree.

STATEMENT $S(T)$: Calling `preorder` on a tree T prints the labels of T in preorder.

BASIS. The basis is where T is a single node. Then line (1) prints the label of T, line (2) gets the leftmost child which is `NULL`, and thus line (3) fails, stopping execution of `preorder`.

INDUCTION. Suppose T is not a leaf. Then T has at least one child. Assume by the inductive hypothesis that `preorder` prints the labels of the children of T in preorder. Clearly the label of the root of T is printed by line (1). This proves the inductive step. We conclude that $S(T)$ is true for all labeled trees T. ✦

b) We prove the following statement by induction on T, the root of a tree.

STATEMENT $S(T)$: Calling `postorder` on a tree T prints the labels of T in postorder.

BASIS. Consider when T is a leaf. Line (1) assigns `c` and line (2) fails. All that is left is for line (5) to print the label of T.

INDUCTION. Suppose T is not a leaf. Then T has at least one child. Assume by the inductive hypothesis that `postorder` prints the labels of the children of T in postorder. After the labels of the children of T have been printed, then lastly on line (5), the label of the root of T is printed. This is the correct behavior for postorder, and proves the inductive step. Therefore $S(T)$ holds for all labeled trees T. ✦

2. We prove the following statement by induction on n, the number of nodes a tree has.

STATEMENT $S(n)$: If a tree T has n nodes with each having a branching factor b, then there are $1 + (b-1)n$ `NULL` pointers among its nodes.

BASIS. Suppose $n = 1$. Then T is a leaf. Thus there are $1 + (b-1)1 = b$ `NULL` pointers in the root of T.

INDUCTION. Suppose $n \geq 1$. Consider that T has $n+1$ nodes. Thus T has at least one child. Assume by the inductive hypothesis that all but one leaf of T, that being n nodes, together have $1 + (b-1)n$ `NULL` pointers among them. Therefore all the nodes of T have a total of $1 + (b-1)(n+1) = b(n+1) - n$ `NULL` pointers. This proves the inductive step. We conclude that $S(n)$ holds. ✦

3. We prove the following statement by induction on T, the root of a tree.

STATEMENT $S(T)$: For any tree T, the number of nodes is 1 more than the sum of the degrees of the nodes.

BASIS. Suppose T has only one node, the root. The degree of the root is 0. Thus the number of nodes in T is 1.

INDUCTION. Suppose the root of T has children. Assume by the inductive hypothesis that the number of nodes of the children of T is 1 more than the sum of the degrees of the children. Thus the number of nodes that T has is the degree of the root plus 1 including the root.