1.

a) Choose $n_0 = 0$ and $c = 1$. Because $a \leq b$, then $n^a \leq cn^b$. Therefore n^a is $O(n^b)$.

b) Suppose there are witnesses n_0 and $c > 0$ such that n^a is $O(n^b)$ if $a > b$. Then $n^a \leq cn^b$ for all $n \geq n_0$. But n^a/n^b becomes arbitrarily large as n increases since $a > b$. Thus $n^a/n^b \leq c$ contradicts c being constant.

c) Choose $n_0 = 0$ and $c = 1$. Since $1 < a \leq b$, then $a^n \leq cb^n$. Therefore a^n is $O(b^n)$.

d) Suppose there are witnesses n_0 and $c > 0$ such that $a^n \leq cb^n$ if $1 < b < a$ for all $n \geq n_0$. From the big-oh inquality we must have $1 \leq cb^n/a^n$. But b^n/a^n approaches 0 as n increases. For large n we would have $1 \leq 0$, a contradiction.
e) Exponentials grow faster than polynomials. We can likely choose $c = 1$, but the general n_0 would be too difficult to find, however, n_0 must be large. Therefore n^a is $O(b^n)$ for any a and for any $b > 1$.
f)