

1. We prove that the two definitions given of lexicographic order are the same. Recall the definitions. The recursive definition:

BASIS.

1. $\epsilon < w$ for any string w other than ϵ itself.
2. If $c < d$, where c and d are characters, then for any strings w and x , we have $cw < dx$.

INDUCTION. If $w < x$ for strings w and x , then for any character c we have $cw < cx$.

The iterative definition. Let $C = c_1c_2 \cdots c_k$ and $D = d_1d_2 \cdots d_m$ be two strings. We say $C < D$ if either of the following holds:

1. That $k < m$ and for $i = 1, 2, \dots, k$ we have $c_i = d_i$.
2. For some value of $i > 0$, the first $i - 1$ characters of the two strings agree, but the i th character of the first string is less than the i th character of the second string.

We prove first that the recursive definition is the same as the iterative definition by complete induction on the number of times the recursive rule is applied to the string.

STATEMENT $S(n)$: If it is necessary to apply the recursive rule n times to show that $w < x$, then w precedes x according to the definition of 'lexicographic order' in Section 2.2.

We say that there is a necessary number of times to apply the recursive rule to the strings to show that $w < x$. There is a minimum number, which is the lowest number of applications needed until either basis case is satisfied. There is a maximum number, which corresponds to the length of w . The minimum number here is exactly what we mean by the necessary number in the statement.