1. We have the recurrence relation in terms of big-oh expressions.

BASIS. $T(0) = O(1)$.

INDUCTION. $T(n) = O(1) + T(n-1)$, for $n > 0$.

Here is the recurrence relation now in terms of unknown constants.

BASIS. $T(0) = a$.

INDUCTION. $T(n) = b + T(n-1)$, for $n > 0$.

We shall now solve this recurrence relation. We determine the sequence

$$\begin{aligned}
T(n) &= b + T(n-1) \\
T(n-1) &= b + T(n-2) \\
T(n-2) &= b + T(n-3) \\
&\cdots \\
T(1) &= b + T(0).
\end{aligned}$$

Now we can substitute simply. We express $T(n)$ in terms of the value of $T(n-1)$ and get

$$T(n) = b + (b + T(n-2)) = 2b + T(n-2).$$

And again, $T(n)$ expressed in terms of the value of $T(n-2)$ to get

$$T(n) = 2b + (b + T(n-3)) = 3b + T(n-3).$$

This should suffice to capture the pattern. We see that, in terms of i, we have

$$T(n) = ib + T(n-i).$$

Where $i = n$, we will have expressed $T(n)$ in terms of the basis $T(0)$. This is

$$T(n) = nb + T(0) = nb + a.$$

Now we express $T(n)$ back in terms of big-oh expressions. The term nb is n proportional to some constant and a is only proportional to a constant. Thus the terms are $O(n) + O(1)$. Therefore the running time of `sum` is $O(n)$.

2. A suitable size measure is the length of the list input.

BASIS. $T(0) = T(1) = O(1)$.

INDUCTION. $T(n) = O(1) + T(n-1)$, for $n > 1$.

Rewriting this in terms of unknown constants we have

BASIS. $T(0) = T(1) = a$.

INDUCTION. $T(n) = b + T(n-1)$, for $n > 1$.

Let us find a pattern. We have

$$T(n) = b + (b + T(n-2)) = 2b + T(n-2).$$

That is good enough. We determine that

$$T(n) = ib + T(n-i).$$

To express $T(n)$ in terms of $T(1)$, substitute i for $n+1$. Then we have

$$T(n) = (n+1)b + T(n-(n+1)) = (n+1)b + T(1) = (n+1)b + a.$$

We have a function proportional to n and a function proportional to a constant. The two terms expressed as big-oh expressions are $O(n) + (1)$, and $O(n)$ is the running time of `find0`.

3. A suitable size measure is $m = n - i$, the number of elements still unsorted.

BASIS. $T(1) = O(1)$.

INDUCTION. $T(m) = O(m) + T(m-1)$, for $m > 1$.

We substitute for constants.

BASIS. $T(1) = a$.

INDUCTION. $T(m) = bm + T(m-1)$, for $m > 1$.

By substitution we try to discover a pattern

$$\begin{aligned}
T(m) &= bm + T(m-1) \\
&= bm + b(m-1) + T(m-2) = b(2m-1) + T(m-2) \\
&= b(2m-1) + b(m-2) + T(m-3) = b(3m-3) + T(m-3) \\
&= 3b(m-1) + b(m-3) + T(m-4) = b(4m-6) + T(m-4) \\
&= b(4m-6) + b(m-4) + T(m-5) = b(5m-10) + T(m-5).
\end{aligned}$$

Now we see the pattern. It is

$$T(m) = b\left(km - \sum_{j=1}^{k-1} j\right) + T(m-k) = b(km - k(k-2)/2) + T(m-k).$$

Let $k = m-1$. Then we have $T(n)$ expressed in terms of $T(1)$ and also with k in terms of m, which is

$$\begin{aligned}
T(m) &= b\left((m-1)m - \sum_{j=1}^{m-2} j\right) + T(1) \\
&= b((m-1)m - (m-2)(m-2+1)/2) + a \\
&= b((m-1)m - (m-2)(m-1)/2) + a \\
&= b((m-1)(m-(m-2)/2)) + a \\
&= b((m-1)((2m-m+2)/2)) + a \\
&= b((m-1)((m+2)/2)) + a \\
&= b(m-1)(m+2)/2 + a.
\end{aligned}$$