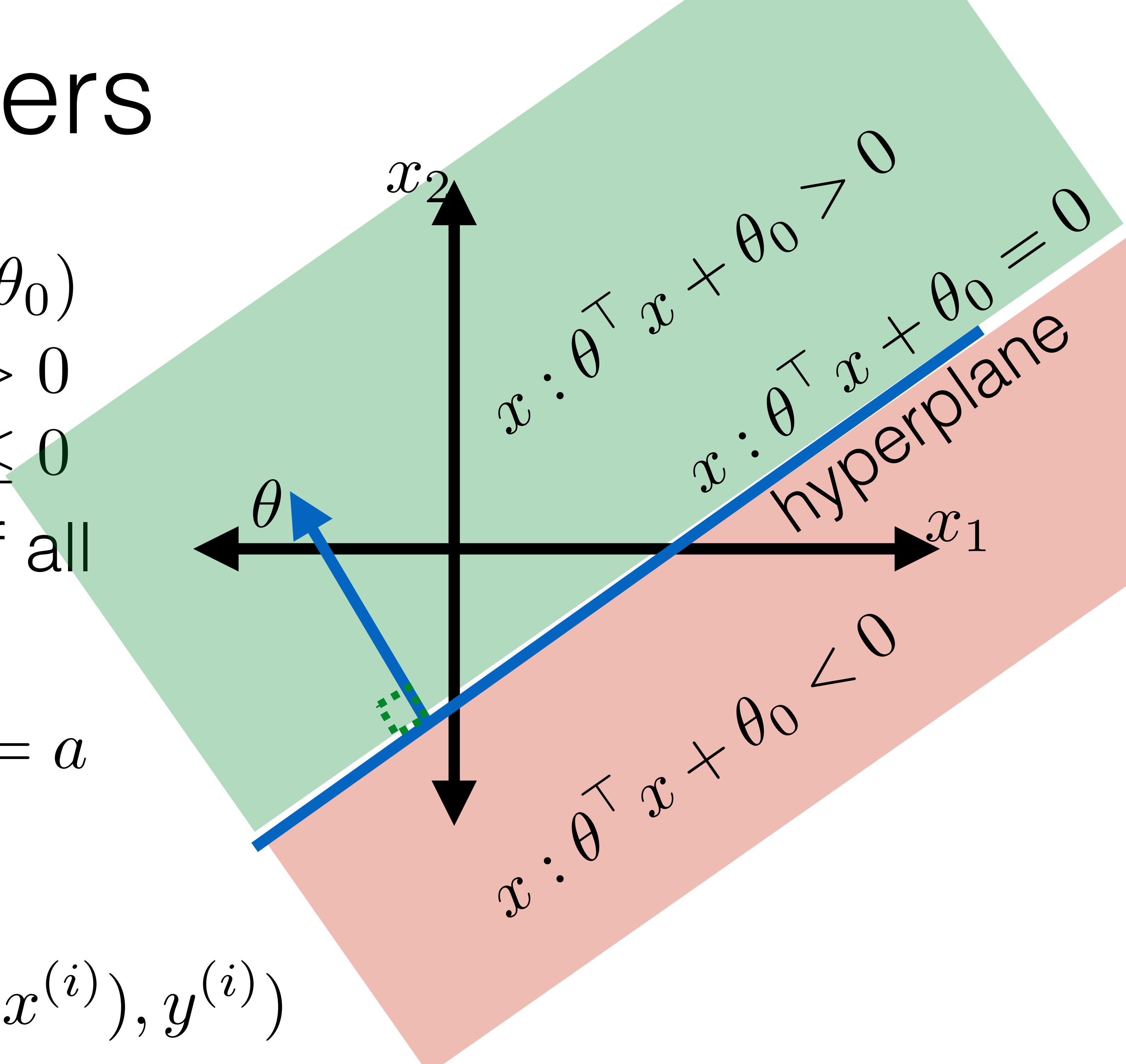
Recall: Classifiers

- A linear classifier:

$$h(x;\theta,\theta_0) = \text{sign}(\theta^\top x + \theta_0)$$
$$= \begin{cases} +1 \text{ if } \theta^\top x + \theta_0 > 0 \\ -1 \text{ if } \theta^\top x + \theta_0 \leq 0 \end{cases}$$



- Hypothesis class $\mathcal{H}$ of all linear classifiers
- 0-1 Loss

$$L(g,a) = \begin{cases} 0 \text{ if } g = a \\ 1 \text{ else} \end{cases}$$

- Training error

$$\mathcal{E}_n(h) = \frac{1}{n}\sum_{i=1}^{n} L(h(x^{(i)}), y^{(i)})$$

- Example learning algorithm (given hypotheses $h^{(j)}$)

```
Ex_learning_alg( 𝒟_n ; k )
    Set j* = argmin_{j∈{1,...,k}} ℰ_n(h^(j))
    Return h^(j*)
```

[demo]

Perceptron Algorithm

```
Perceptron( 𝒟_n ; τ )
  Initialize  θ = [0 0 … 0]^⊤
  Initialize  θ_0 = 0
  for t = 1 to τ
    changed = False
    for i = 1 to n
      if y^(i)(θ^⊤ x^(i) + θ_0) ≤ 0
        Set  θ = θ + y^(i) x^(i)
        Set  θ_0 = θ_0 + y^(i)
        changed = True
    if not changed
      break
  Return  θ, θ_0
```

[How many 0s?]

[i.e. True if either:

A. point is not on the line & prediction is wrong
B. point is on the line
C. initial step]

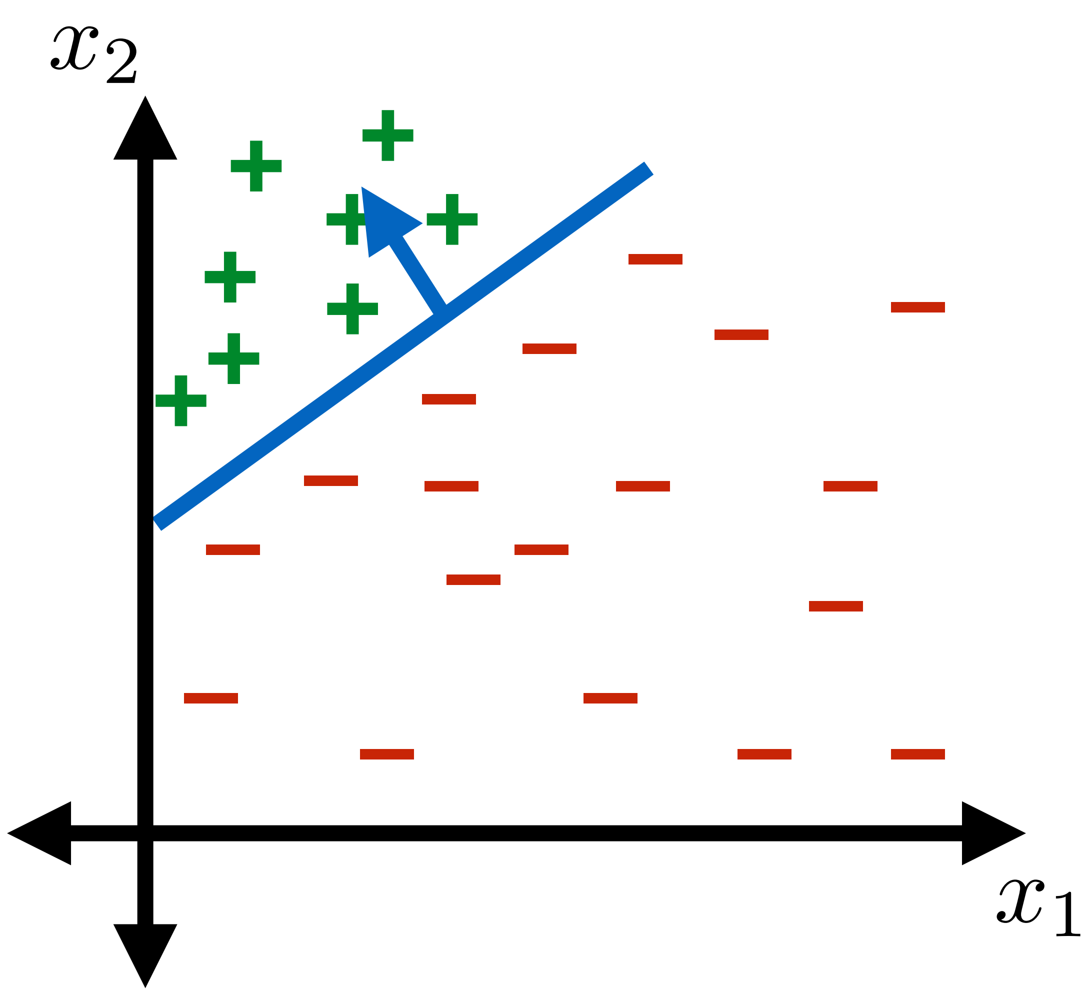
What does an update do?

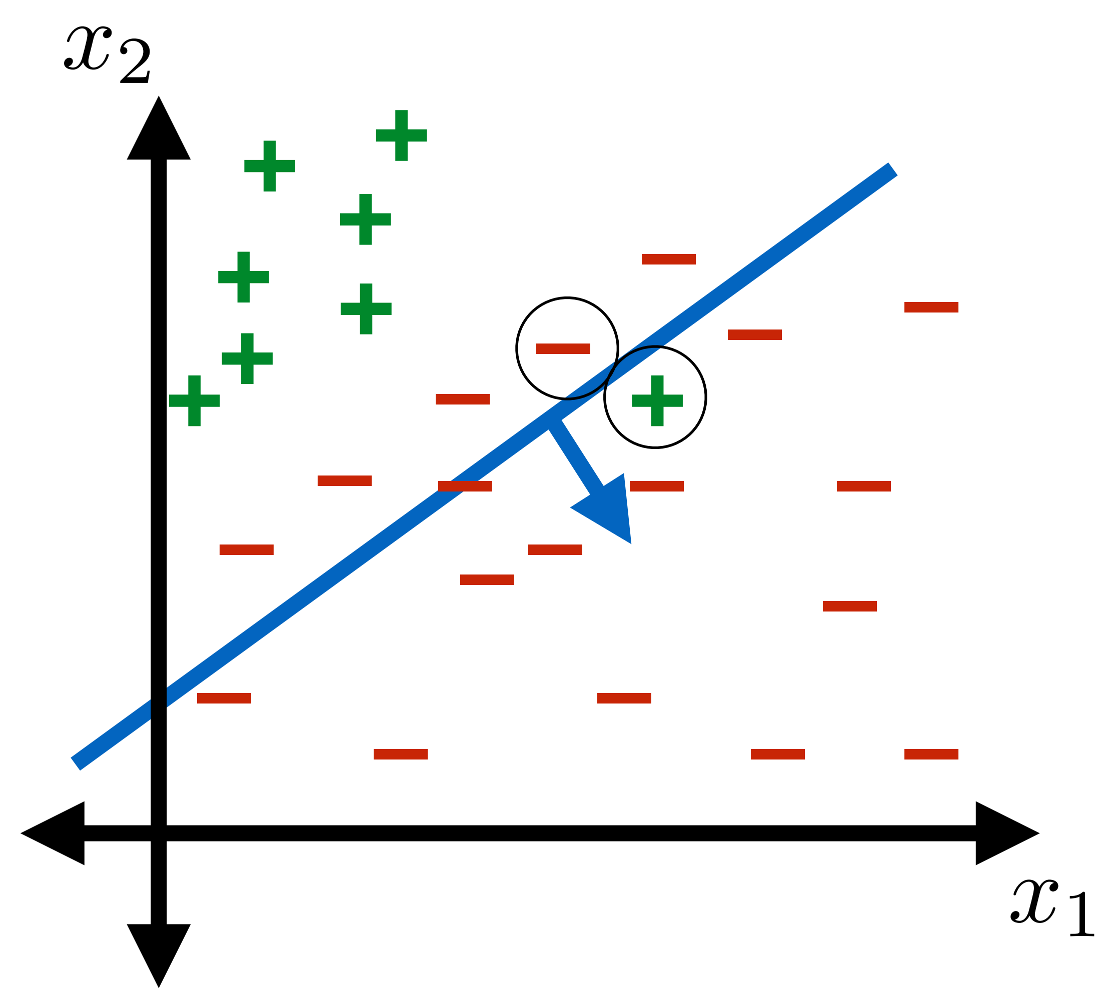
$$y^{(i)}\left((\theta + y^{(i)}x^{(i)})^\top x^{(i)} + (\theta_0 + y^{(i)})\right)$$
$$= y^{(i)}(\theta^\top x^{(i)} + \theta_0) + (y^{(i)})^2(x^{(i)\top}x^{(i)} + 1)$$
$$= y^{(i)}(\theta^\top x^{(i)} + \theta_0) + (\|x^{(i)}\|^2 + 1)$$

Let's Talk About Classifier Quality

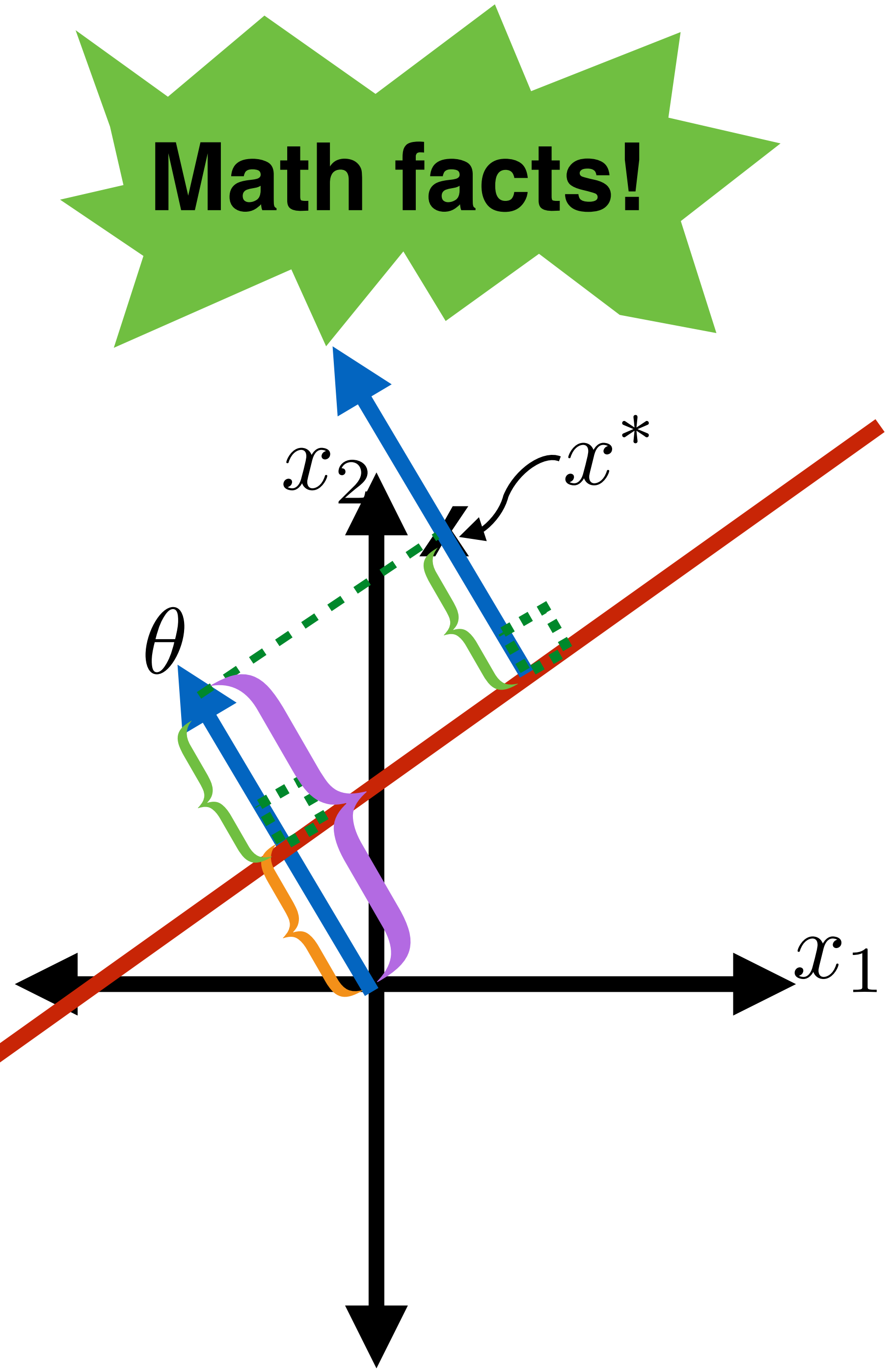
- *Definition*: A training set $\mathcal{D}_n$ is **linearly separable** if there exist θ, θ_0 such that, for every point index $i \in \{1, \ldots, n\}$, we have

$$y^{(i)}(\theta^\top x^{(i)} + \theta_0) > 0$$





Let's Talk About Classifier Quality



- The signed distance from a hyperplane defined by θ, θ_0 to a point x^* is:

  = projection of x^* on θ

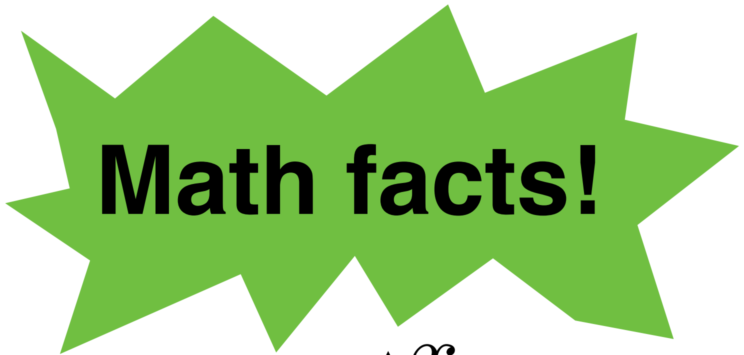
  – signed distance of line to origin

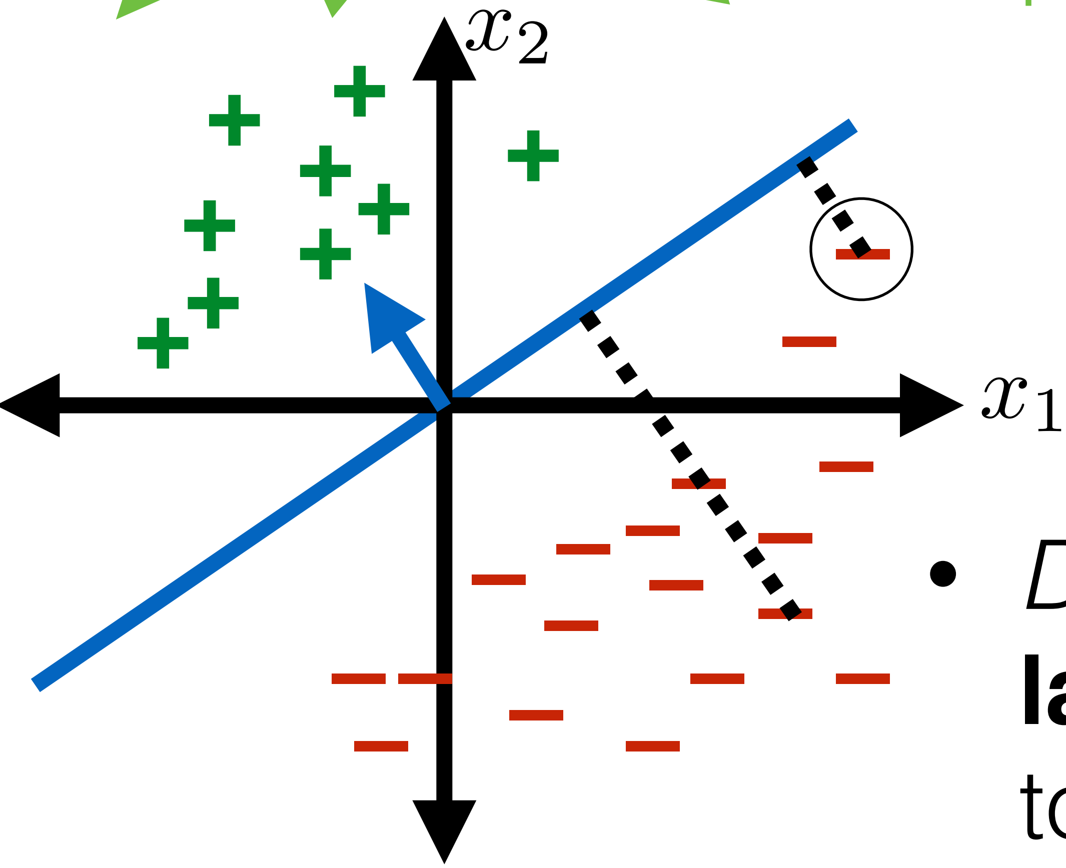
  $$= \frac{\theta^\top x^*}{\|\theta\|} - \frac{-\theta_0}{\|\theta\|} = \frac{\theta^\top x^* + \theta_0}{\|\theta\|}$$

- *Definition*: The **margin of the labelled point** (x^*, y^*) with respect to the hyperplane defined by θ, θ_0 is:

  $$y^* \left(\frac{\theta^\top x^* + \theta_0}{\|\theta\|} \right)$$

Let's Talk About Classifier Quality





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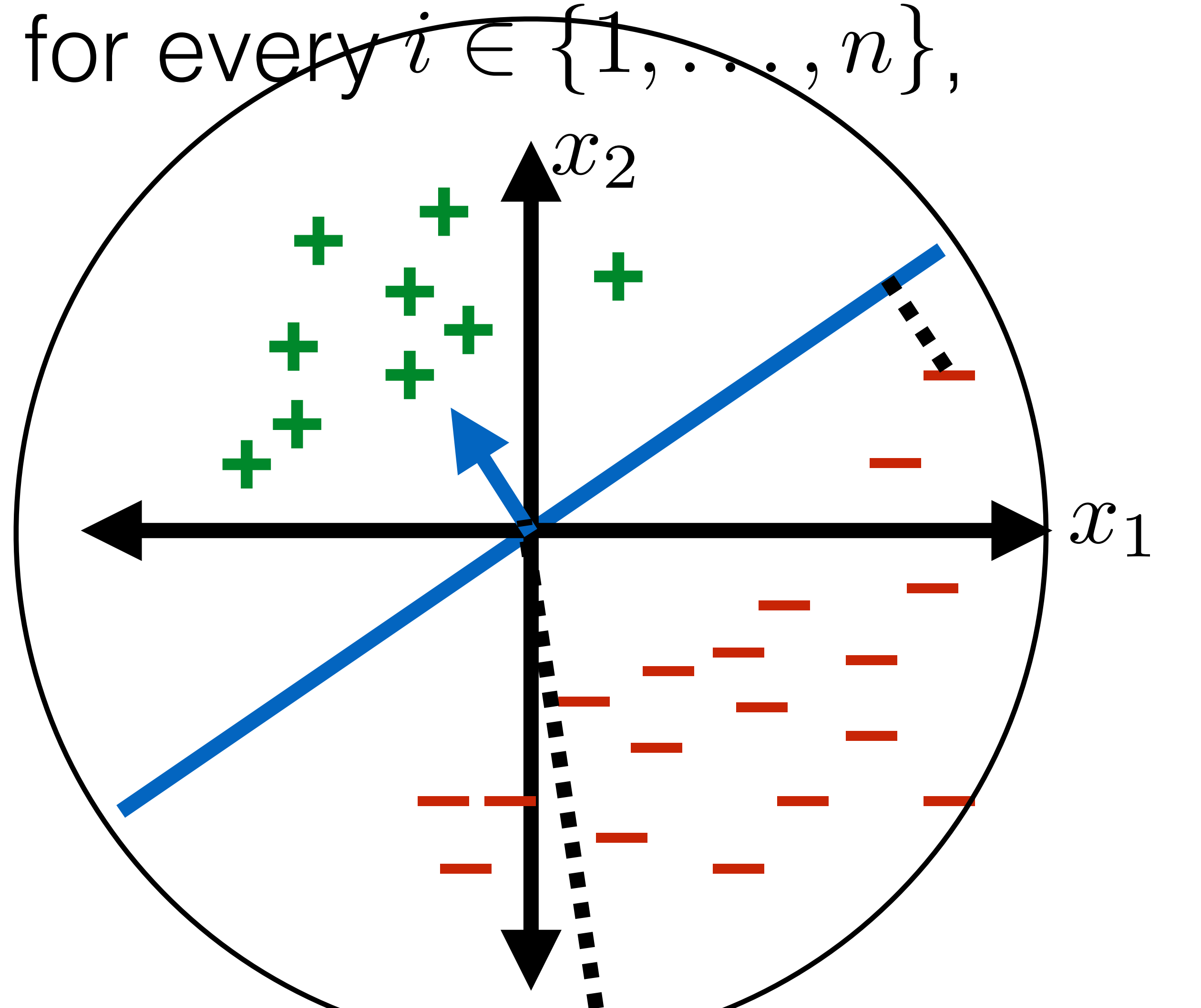
- *Definition*: The **margin of the labelled point** (x^*, y^*) with respect to the hyperplane defined by θ, θ_0 is:

$$y^* \left(\frac{\theta^\top x^* + \theta_0}{\|\theta\|} \right)$$

- *Definition*: The **margin of the training set** $\mathcal{D}_n$ with respect to the hyperplane defined by θ, θ_0 is: $$\min_{i \in \{1,\ldots,n\}} y^{(i)} \left(\frac{\theta^\top x^{(i)} + \theta_0}{\|\theta\|} \right)$$

Theorem: Perceptron Performance

- **Assumptions**:
  A. Our hypothesis class = classifiers with separating hyperplanes that pass through the origin (i.e. $\theta_0 = 0$)
  B. There exist θ^* and γ such that $\gamma > 0$ and, for every $i \in \{1, \ldots, n\}$, we have $y^{(i)} \left(\frac{\theta^{*\top} x^{(i)}}{\|\theta\|} \right) > \gamma$
  C. There exists R such that, for every $i \in \{1, \ldots, n\}$, we have $\|x^{(i)}\| \leq R$
- **Conclusion**: Then the perceptron algorithm will make at most $(R/\gamma)^2$ updates to θ. Once it goes through a pass of *i* without changes, the training error of its hypothesis will be 0.



Why classifiers through the origin?

- If we're clever, we don't lose any flexibility
  - Classifier with offset

$$x \in \mathbb{R}^d, \theta \in \mathbb{R}^d, \theta_0 \in \mathbb{R}$$

$$x : \theta^\top x + \theta_0 \overset{<}{\underset{>}{=}} 0$$

  - Classifier without offset

$$x_{\text{new}} \in \mathbb{R}^{d+1}, \theta_{\text{new}} \in \mathbb{R}^{d+1}$$

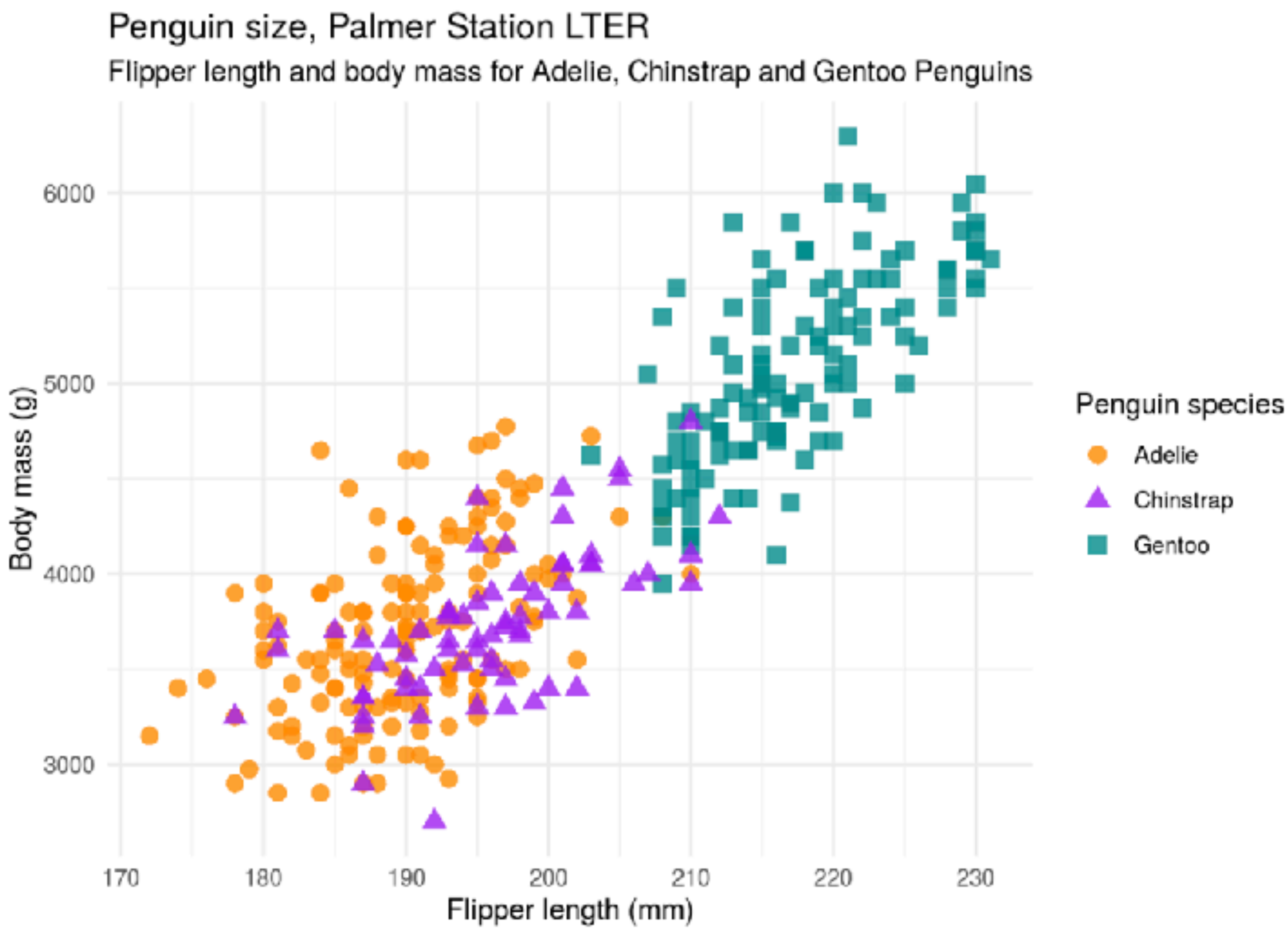
$$x_{\text{new}} = [x_1, x_2, \ldots, x_d, 1]^\top, \theta_{\text{new}} = [\theta_1, \theta_2, \ldots, \theta_d, \theta_0]^\top$$

$$x_{\text{new},1:d} : \theta_{\text{new}}^\top x_{\text{new}} \overset{<}{\underset{>}{=}} 0$$

- Can first convert to "expanded" feature space, then apply theorem

Problem: data not linearly separable

- Typical real data sets aren't linearly separable [demo]



- What can we do? See upcoming lectures!

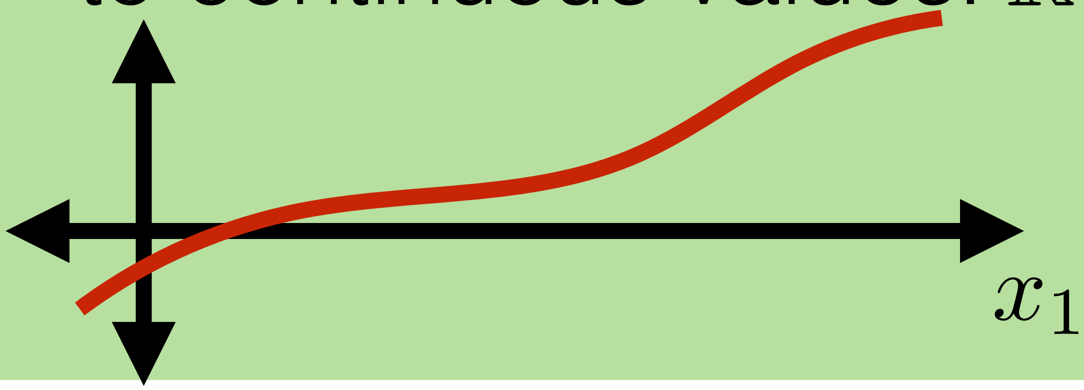
[https://towardsdatascience.com/penguins-dataset-overview-iris-alternative-9453bb8c8d95, https://github.com/allisonhorst/palmerpenguins]

Machine Learning Tasks

- **Supervised learning:** Learn a mapping from features to labels

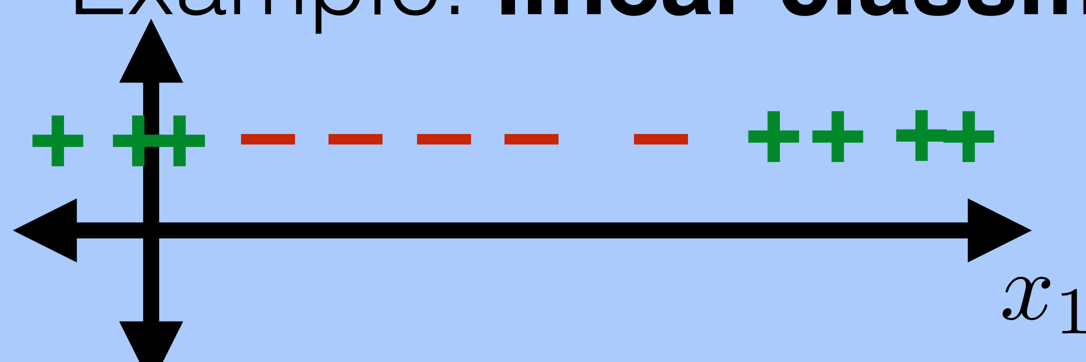
- **Unsupervised learning:** No labels; find patterns

- **Regression:** Learn a mapping to continuous values: $\mathbb{R}^d \to \mathbb{R}^k$



- **Classification:** Learn a mapping to a discrete set

- **Binary/two-class classification:** Learn a mapping: $\mathbb{R}^d \to \{-1, +1\}$
  - Example: **linear classification**



- **Multi-class classification:** > 2 label values