Image Processing with Linear Algebra

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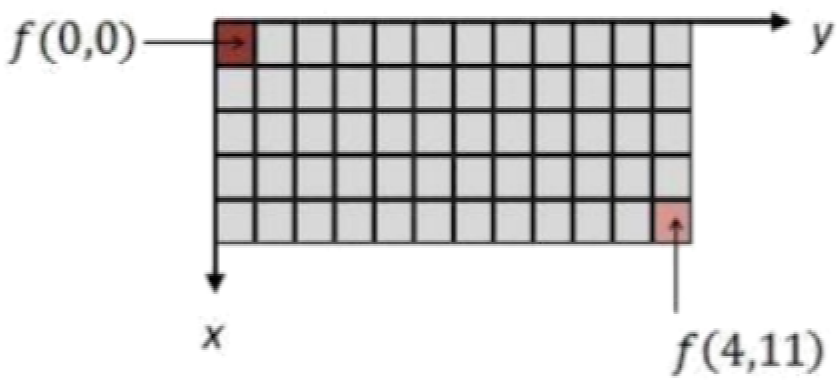
1) Importance of matrix in Image processing

A digital image is a three-dimensional matrix, where the pair of first two dimensions is called a pixel. The word pixel, derived from English "picture element", determines the smallest logical unit of visual information used to construct an image. The third dimension is always 3 units wide and stores the r, g, b values of the pixel. Thus, without the loss of generality one image can be represented by a matrix where each element (i, j) corresponds to the value of the pixel image position (i, j). The below figure is a digital image of size 5 × 12.



2) Matrix Operations

One of the most basic topics of Linear Algebra is matrix operations. In our project, with use of the following operations, we will do the required image processing!

1. Addition

Let A and B be two matrices (images) with the same dimension, i.e. with the same number of rows and columns. The matrix obtained by adding the previous matrices A and B is called addition matrix.

If we add two images, it adds the pixel values of Image A to the pixel values of Image B. Basically, this operation adds the colors of the overlay to the background causing the two images either overflow, or saturate. However, if the added colors exceed the color limits, the color will be capped (at 255) and the result will not necessarily be as you expect.

2. Subtraction

Let A and B be two matrices (images) with the same dimension. The matrix obtained by subtracting each element of A by corresponding element in B is called subtraction matrix.
The difference operation is a sequence of two operations. First, a subtraction operation is performed. Then, the absolute value of the result of the subtraction operation is taken. Hence, the minimum pixel value is 0.

3. Multiplication

Let A and B be matrices such that the number of columns of A is equal to the number of rows of B, say A is an $\{m \times p\}$ matrix and B is a $\{p \times n\}$ matrix. Then the product of the matrix A with the matrix B, called A×B is a $\{m \times n\}$ matrix whose (i ,j) element is obtained by multiplying the i-th row of A by the j-th column of b.
Multiplies the Image A pixel values by the Image B pixel values. In clip mode, for RGB images with byte values, results greater than 255 are set to 255. This method works very well if one image (either A or B) is basically black or gray, or just has a light background. If both images are color images, then you may get strange results. This technique is also perfect for overlaying line drawings, diagrams or images on an image with a very light white or colored background.

4. Transpose

Let A be a matrix(image) of size M×N, then its transpose, let's say A^T will be the image we get by exchanging the pixel values of the rows and columns.
If we take a transpose of an image, it is the same as taking the right mirror image (keeping the object on the left side of the mirror) and rotating it by 90° in an anticlockwise direction.

5. Element by element multiplication

Sometimes it is useful applying multiplication element by element, i.e., multiplying each element of A with the corresponding element of the matrix B. Thus, if A and B are two matrices of dimension $\{m \times n\}$ then the matrix created by element by element multiplication is called A •× B
This is really helpful when we want to apply a mask to the image.

3) Rotation

The rotation of an object at a positive theta angle (counter clockwise) around the origin is a geometric transformation that does not deform the object. To rotate an object it is necessary to turn all the pixels. For getting the new coordinate of the rotated matrix, we will multiply [x Y] matrix with the matrix shown below. Now, we will just copy the rgb values of the old pixel to the new pixel.

$$\begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

4) Transformation Functions

Transformation functions are a variety of functions which can be used to change the specific properties of the image. This can range from changing the brightness, taking the negative of an image, and making the image clear of any disturbance by using average and median spatial filters. The following are the examples of transformative functions:

1) **Negative of an image:** Inverts the intensity values of the pixels (i.e. sends black to white, white to black and inverts the gray scale values).
   T can be defined as:
   T(u) = 255 - u
2) **Gamma Transformation:** Either maps a narrow range of dark input values into a wider range of output values, or maps a wide range of input values into a narrower range of output values.
   T can be defined as:
   T(u) = c(u +α)^γ
   Where c, α, γ are appropriate parameters.
3) **Average Spatial Transformation:** In this function, we take the average of the pixel values surrounding the specific pixel.
   T can be defined as:
   $$T(i,j) = \left(\sum_{a=-1}^{1}\sum_{b=-1}^{1} T(i+a,\ j+b)\right)/9$$

4) **Median Spatial Transformation:** In this function, we take the median of the pixel values surrounding the specific pixel.
   T can be defined as:
   $T(i,j) = MED(T(i-1, j-1), T(i-1, j), T(i-1, j+1), T(i, j-1) \ldots\ldots T(i+1, j+1))$

Conclusion:

This project was one of the most interesting projects every one of us has worked on. Everyone worked on it with passion, determination and the urge to learn. Everyone contributed equally and was present in every meet we did to work on this project.