AE602A - Mathematics for Aerospace Engineers

Assignment - 2

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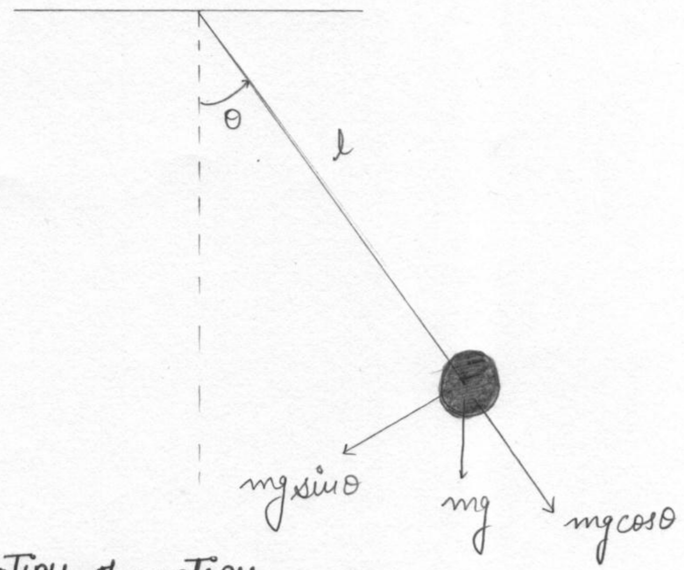

The governing equation of motion is given by

$$\frac{d^2\theta}{dt^2} + \frac{g\sin\theta}{l} + \frac{b}{m}\dot{\theta} = 0 \quad \text{——— (1)}$$

where, m = mass of system, kg
l = length of rod, m
θ = angle made by rod w.r.t vertical
b = friction coefficient, Ns/m

The term $\frac{b}{m}\dot{\theta}$ is due to the damping which is proportional to $\dot{\theta}$

Q 1.

Ans $l = 0.5m$
$m = 0.5 kg$

No damping, the term $\frac{b}{m}\dot{\theta} \to 0$

Hence the equation is

$$\frac{d^2\theta}{dt^2} + \frac{g\sin\theta}{l} = 0$$

AE602A-Mathematics for Aerospace Engineers

Assignment-2

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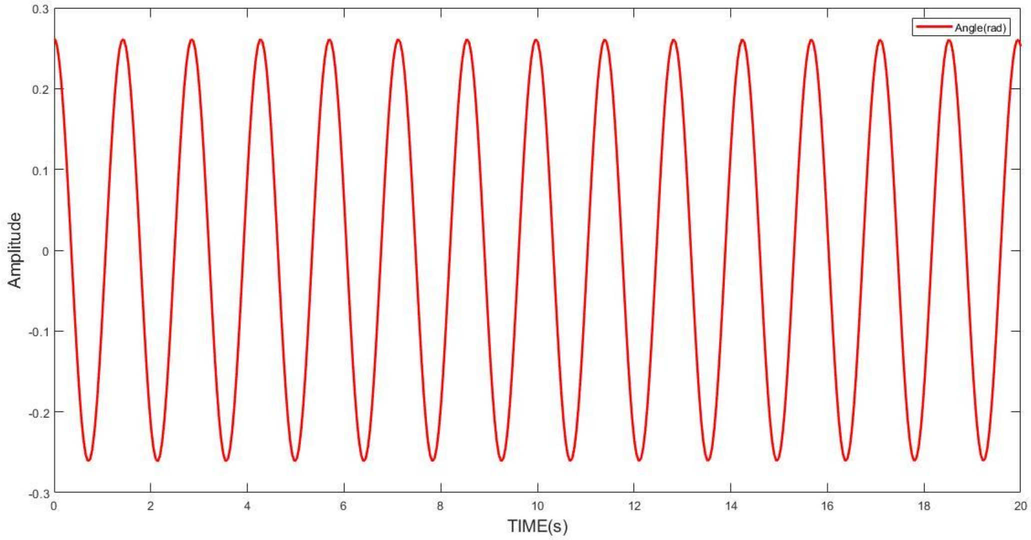
Q1. Assume the length of rod is 0.5 meter, mass of the pendulum is 0.5kg, plot the solutions of the given system is MATLAB with NO damping. Report the above procedure for b=0.12Ns/m. Assume the initial angle and angular rates: 15degree and 0 deg/s.

ANSWER. MATLAB CODE:

NO Damping we have considered the value of b as zero.

```
%% GIVEN VALUE
m=0.5;
l=0.5;
b=0;
g=9.81;
p=g/l;
q=b/m;
%% Pendulum ODE
f=@(t,x)[x(2);-q*x(2)-p*sin(x(1))];
%intital
int =[15*pi/180;0];
%solve ODE, the time interval is set for 0 to 20 secound
[t,x]=ode45(f,[0 20],int);
%% plot the solution
plot(t,x(:,1));
xlabel('TIME(s)');
ylabel('Amplitude');
legend({'Angle(rad)'});
hold on;
```

RESULTS:

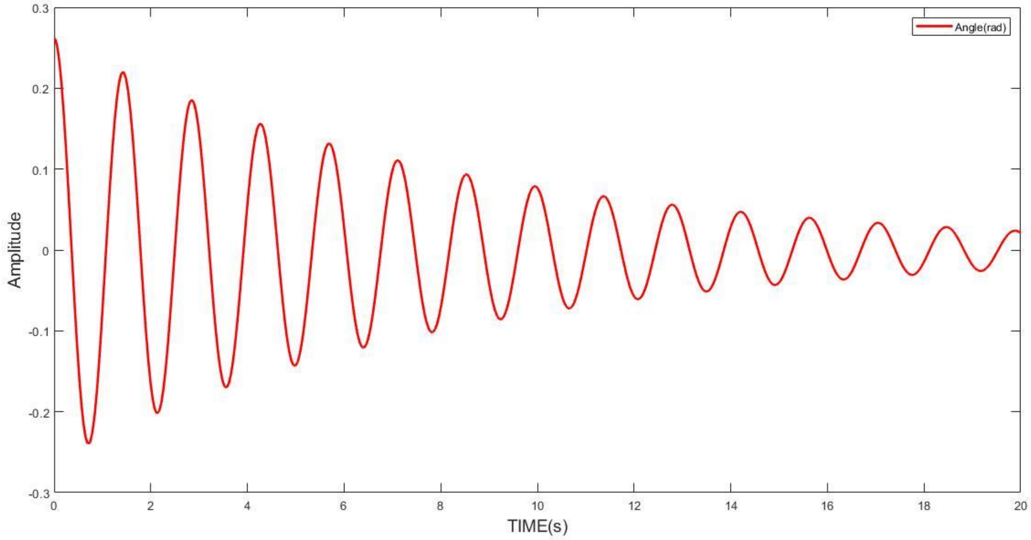


Graph of time v/s Amplitude for no damping (b=0).

Damping constant is non zero.

```
m=0.5;
l=0.5;
b=0.12;
g=9.81;
p=g/l;
q=b/m;
%% Pendulum ODE
f=@(t,x)[x(2);-q*x(2)-p*sin(x(1))];
%intital
int =[15*pi/180;0];
%solve ODE, the time interval is set for 0 to 20 secound
[t,x]=ode45(f,[0 20],int);
%% plot the solution
plot(t,x(:,1),'r','linewidth',2);
xlabel('TIME(s)','fontsize',14);
ylabel('Amplitude','fontsize',14);
legend({'Angle(rad)'});
hold on;
```

RESULTS:



Graph of time v/s Amplitude for damping (d=0.12Ns/m)

Q 2.

Ans The governing equation of motion is

$$\frac{d^2\theta}{dt^2} + g\frac{\sin\theta}{l} + \frac{b}{m}\dot{\theta} = 0$$

we, can convert the above diff equation which is 2^{nd} order eq^n to two first order equation. In similar way we did for spring mass damper system.

Rewriting the equation,

$$\frac{d^2\theta}{dt^2} + \frac{b}{m}\dot{\theta} + g\frac{\sin\theta}{l} = 0 \quad \text{—— (1)}$$

$$\ddot{\theta} + \frac{b}{m}\dot{\theta} + g\frac{\sin\theta}{l} = 0 \quad \text{—— (1.a)}$$

where $\ddot{\theta}$ denotes $\frac{d^2\theta}{dt^2}$

$\dot{\theta}$ denotes $\frac{d\theta}{dt}$

let take $\theta = \theta_1$

and on differeting the expression we will get

$$\frac{d\theta}{dt} = \frac{d\theta_1}{dt} \Rightarrow \quad \dot{\theta} = \dot{\theta}_1$$

lets take $\dot{\theta}_1 = \theta_2 = \dot{\theta}$

$\Rightarrow \boxed{\dot{\theta}_1 = \theta_2}$ By putting these value to ~~our~~ our eq^n (1.a) we will get

$$\Rightarrow \frac{d}{dt}(\dot{\theta}_1) + \frac{b}{m}\dot{\theta}_1 + g\frac{\sin\theta_1}{l} = 0$$

$$\Rightarrow \frac{d}{dt}(\theta_2) + \frac{b}{m}(\theta_2) + g\frac{\sin\theta_1}{l} = 0$$

$$\Rightarrow \boxed{\dot{\theta}_2 + \frac{b}{m}\theta_2 + g\frac{\sin\theta_1}{l} = 0}$$

Hence the two first order ODEs are

$$\dot{\theta}_1 = \theta_2 \quad \text{—— (11)}$$

$$\dot{\theta}_2 + \frac{b}{m}\theta_2 + g\frac{\sin\theta_1}{l} = 0 \quad \text{—— (111)}$$

We can determine the equilibrium points by putting $\dot{\theta}_1 = 0$ and $\dot{\theta}_2 = 0$

$$\left.\begin{aligned} \frac{d\theta_1}{dt} &= 0 \\ \frac{d\theta_2}{dt} &= 0 \end{aligned}\right\} \text{ we can find the equilibrium point}$$

By (1),

$$\dot{\theta}_1 = \theta_2 = 0$$

$$\boxed{\theta_2 = 0}$$

By (2),

$$\dot{\theta}_2 + \frac{b}{m}\theta_2 + g\frac{\sin\theta_1}{l} = 0$$

$$\dot{\theta}_2 = -\frac{b}{m}\theta_2 - \frac{g}{l}\sin\theta_1 = 0$$

$$-\frac{b}{m}\theta_2 - \frac{g}{l}\sin\theta_1 = 0 \quad , \quad \therefore \theta_2 = 0$$

on putting $\theta_2 = 0$ we will get.

$$-\frac{g}{l}\sin\theta_1 = 0$$

$$\sin\theta_1 = 0$$

$$\theta_1 = 0, \pi, 2\pi \ldots n\pi$$

$$\boxed{\theta_1 = 0, \pi}$$

The equilibrium points will be $\longrightarrow$ $(0,0)$ and $(\pi, 0)$

Q 3

Ans Lets consider, $\theta(t) = \theta_e(t) + \underbrace{\Delta\theta(t)}_{\text{Perturbation in } \theta}$

On diff the eqn to find the time derivative

$$\dot{\theta}(t) = \dot{\theta}_e(t) + \Delta\dot{\theta}(t)$$

$$\dot{\theta}(t) = f(\theta_e(t) + \Delta\dot{\theta}(t))$$

We can expand the above function $f(x+h)$ as the taylor series.

By the help of taylor series

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \ldots$$

$$\Rightarrow \dot{\theta}(t) = f(\theta_e) + \left.\frac{df}{d\theta}\right|_{\theta=\theta_e}(\theta - \theta_e) + \frac{1}{2!}\left.\frac{d^2 f}{\partial\theta^2}\right|_{\theta=\theta_e}(\theta-\theta_e)^2 + \ldots$$

we can consider $\theta - \theta_e$ as small quantity

$$\Rightarrow \dot{\theta}(t) = f(\theta_e) + \left.\frac{\partial f}{\partial \theta}\right|_{\theta=\theta_e} (\theta - \theta_e)$$

$$\Rightarrow \dot{\theta}(t) - f(\theta_e) = \left.\frac{\partial f}{\partial \theta}\right|_{\theta=\theta_e} (\dot{\theta} - \theta_e)$$

$$\Delta\dot{\theta} = \frac{\partial f}{\partial \theta} . \Delta\theta \quad \Rightarrow \quad \boxed{\dot{\theta} = A . \theta}$$

we can write the matrix A as a jacobian matrix.

$$J\left(\frac{f_1, f_2}{\theta_1, \theta_2}\right) = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} \end{bmatrix}$$

$$\dot{\theta}_1 = f_1(\theta_1, \theta_2) = \theta_2$$

$$\dot{\theta}_2 = f_2(\theta_1, \theta_2) = -\frac{b}{m}\theta_2 - \frac{g}{l}\sin\theta_1$$

Putting these values to determine the jacobian matrix.

$$\frac{\partial f_1}{\partial \theta_1} = 0 \quad , \quad \frac{\partial f_1}{\partial \theta_2} = 1$$

$$\frac{\partial f_2}{\partial \theta_1} = -\frac{g}{l}\cos\theta_1 \quad \frac{\partial f_2}{\partial \theta_2} = -\frac{b}{m}$$

$$\Rightarrow J = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos\theta_1 & -\frac{b}{m} \end{bmatrix}$$

From here we can calulate two systemmatrix at equlibrium point J_1 at $(0,0)$ and J_2 at $(\pi, 0)$

$$\Rightarrow J_1 = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos\theta_1, & -\frac{b}{m} \end{bmatrix}_{(0,0)} \qquad \Rightarrow J_2 = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos\theta_1, & -\frac{b}{m} \end{bmatrix}_{(\pi,0)}$$

$$\Rightarrow J_1 = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{b}{m} \end{bmatrix} \qquad \Rightarrow J_2 = \begin{bmatrix} 0 & 1 \\ g/l & -b/m \end{bmatrix}$$

Q4

Ans Determining the eigenvalues and eigenvector by the char. eq^n

For point (0,0)

we have the matrix, $J_1 = \begin{bmatrix} 0 & 1 \\ -g/l & -b/m \end{bmatrix}$

Lets take $J_1 = A_1$

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{-g}{l} & -\frac{b}{m} \end{bmatrix}$$

Char. eq^n $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 0 & 1 \\ \frac{-g}{l} & -\frac{b}{m} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -\frac{g}{l} & -\frac{b}{m} - \lambda \end{vmatrix} = 0 \Rightarrow -\lambda\left(-\frac{b}{m} - \lambda\right) + \frac{g}{l} = 0$$

$$\Rightarrow \frac{b}{m}\lambda + \lambda^2 + \frac{g}{l} = 0$$

$$\Rightarrow \lambda^2 + \frac{b}{m}\lambda + \frac{g}{l} = 0$$

On solving the quadratic eq^n

$$\lambda_1, \lambda_2 \Rightarrow \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4\left(\frac{g}{l}\right)}}{2} \quad \text{(Eigen values)}$$

If we do consider the values of the parameters given

$l = 0.5m$

$m = 0.5kg$

$b = 0.12$ Ns/m

$g = 9.81$ m/s^2

we will get eigenvalues as

$$\lambda_1, \lambda_2 = \frac{-\frac{0.12}{0.5} \pm \sqrt{\left(\frac{0.12}{0.5}\right)^2 - \frac{4(9.81)}{0.5}}}{2}$$

$$\lambda_1, \lambda_2 = -\frac{0.24}{2} \pm \frac{\sqrt{0.0576 - 78.48}}{2}$$

$$\boxed{\lambda_1, \lambda_2 = -0.12 \pm 4.4278\, i}$$

(ii) The eigenvector corresponding to the eigenvalue $\lambda_{1,2} = -0.12 \pm 4.4278\,i$

$\lambda_1 = -0.12 + 4.4278\,i$

$$\Rightarrow \begin{bmatrix} -(-0.12+4.4278i) & 1 \\ -\frac{g}{l} & -\frac{b}{m}-(-0.12+4.4278i) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0.12-4.4278i & 1 \\ -19.62 & -0.24+0.12-4.4278i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0.12-4.4278i & 1 \\ -19.62 & -0.12-4.4278i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{-(0.12+4.4278i)} = \frac{x_2}{19.62} = k$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.12+4.4278i \\ 19.62 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 0.12 \pm 4.4278i \\ 19.62 \end{bmatrix}$$

$\lambda_2 = -0.12 - 4.4278\,i$

$$\Rightarrow \begin{bmatrix} -(-0.12-4.4278i) & 1 \\ -\frac{g}{l} & -\frac{b}{m}-(-0.12-4.278i) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.12+4.4278i & 1 \\ -19.62 & -0.24+0.12+4.4278i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.12+4.4278i & 1 \\ -19.62 & -0.12+4.4278i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{-0.12+4.4278i} = \frac{x_2}{19.62} = k$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.12+4.4278i \\ 19.62 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -0.12+4.4278i \\ 19.62 \end{bmatrix}$$

The eigenvalues are complex and have -ve real part than the phase portrait is spiral to eqb^m point and converges to eqb^m point. (stable).

For point $(\pi, 0)$

$$J_2 = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & -\frac{b}{m} \end{bmatrix}$$

lets take $J_2 = A_2 = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & -\frac{b}{m} \end{bmatrix}$

Char eq^n $|A_2 - \lambda I| = 0$ $\quad \begin{vmatrix} 0-\lambda & 1 \\ \frac{g}{l} & -\frac{b}{m}-\lambda \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 \\ \frac{g}{l} & -\frac{b}{m}-\lambda \end{vmatrix} = 0 \qquad \Rightarrow -\lambda\left(-\frac{b}{m}-\lambda\right) - \frac{g}{l} = 0$$

$$\Rightarrow \frac{b}{m}\lambda + \lambda^2 - \frac{g}{l} = 0$$

$$\Rightarrow \lambda^2 + \frac{b}{m}\lambda - \frac{g}{l} = 0$$

on solving

$$\Rightarrow \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 + 4(g/l)}}{2}$$

for given values

$$\lambda_1, \lambda_2 \Rightarrow \frac{-\frac{0.12}{0.5} \pm \sqrt{0.0576 + 78.48}}{2}$$

$$\lambda_1, \lambda_2 \Rightarrow -0.12 \pm 4.4310$$

$\lambda_1, \lambda_2 \Rightarrow 4.311 \ , \ -4.551$ $\therefore$ Eigenvalues are real one +ve and one -ve.

For $\lambda_1 = 4.311$

$$\Rightarrow \begin{bmatrix} -4.311 & 1 \\ 19.62 & -0.24-4.311 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4.311 & 1 \\ 19.62 & -4.551 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{x_1}{-4.551} = \frac{x_2}{-19.62} = k$$

$$\Rightarrow v_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4.551 \\ 19.62 \end{bmatrix}$$

$\lambda_2 = -4.551$

$$\Rightarrow \begin{bmatrix} 4.551 & 1 \\ 19.62 & -0.24+4.551 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4.551 & 1 \\ 19.62 & 4.311 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{x_1}{4.311} = \frac{x_2}{-19.62} = k$$

$$\Rightarrow v_2 = \begin{bmatrix} 4.311 \\ -19.62 \end{bmatrix}$$

Here the eigenvalues are real one +ve and one -ve so the phase portrait will be saddle point at the eqb^m $(\pi, 0)$.

Q 5.

Ans At equlibrium point (0,0) we have maxtrix

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{-g}{l} & \frac{-b}{m} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -19.6 & -0.24 \end{bmatrix}$$

We got the eigenvalues and eigenvectors as follow,

$\lambda_1 = -0.12 + 4.4278i$ $\lambda_2 = -0.12 - 4.4278i$

$v_1 = \begin{bmatrix} -0.12 - 4.4278i \\ 19.62 \end{bmatrix}$ $v_2 = \begin{bmatrix} -0.12 + 4.4278i \\ 19.62 \end{bmatrix}$

We can convert the equation in form of Z and we the write the equation as

$$\dot{Z} = DZ$$

$\dot{Z} = P^{-1}APZ$ where P represents the matrix containg eigenvector as in column

$$P = \begin{bmatrix} -0.12 - 4.4278i & -0.12+4.4278i \\ 19.62 & 19.62 \end{bmatrix}$$

By the help of P we can determine P^{-1}

$$P^{-1} = \begin{bmatrix} 0.11292289 & 0.025484+0.00069i \\ -0.11292289 & 0.02548-0.00069065i \end{bmatrix}$$

$$\dot{Z} = P^{-1}APZ = \begin{bmatrix} -0.12+4.4278i & 0 \\ 0 & -0.12-4.4278i \end{bmatrix} Z$$

$$\dot{Z} = \frac{d}{dt}\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} -0.12+4.4278i & 0 \\ 0 & -0.12-4.4278i \end{bmatrix}\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

$\frac{d}{dt} z_1 = -0.12 + 4.4278i \cdot z_1$, $\frac{dz_2}{dt} = (-0.12 - 4.4278i)\, z_2$

$\ln z_1 = c_1(-0.12+4.4278i)t$ $\ln z_2 = c_2(-0.12 - 4.4278i)t$

$\Rightarrow z_1 = c_1 e^{(-0.12+4.4278i)t}$, $\Rightarrow z_2 = c_2 e^{(-0.12-4.4278i)t}$

$X = PZ$ Fundamental system can be writen as,

$\Theta = P \cdot Z$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -0.12 - 4.4278i & -0.12+4.4278i \\ 19.62 & 19.62 \end{bmatrix}}_{\text{This matrix is fundamental system matrix.}} \begin{bmatrix} c_1 e^{(-0.12+4.4278i)t} \\ c_2 e^{(-0.12-4.4278i)t} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} (-0.12-4.4278i)\, c_1 e^{(-0.12+4.4278i)t} + c_2(-0.12+4.4278i)\, e^{(-0.12-4.4278i)t} \\ 19.62\, c_1 e^{(-0.12+4.4278i)t} + c_2\, 19.62\, e^{(-0.12-4.4278i)t} \end{bmatrix}$$

So we have the initial value's as,

at $t=0$ $\theta_1 = 15$ deg

at $\dot{t}=0$ $\theta_2 = 0$ deg/s

$$\Rightarrow 15^\circ = c_1(-0.12-4.4278i)\, e^{-0.12t}\left[\cos(4.4278t) + i\sin(4.4278t)\right] + c_2(-0.12+4.4278i)\, e^{-0.12t}\left[\cos(4.4278t) - i\sin(4.4278t)\right]$$

$$\Rightarrow 15^\circ = e^{-0.12t}\left[c_1(-0.12-4.4278i)\left[\cos(4.4278t)+i\sin(4.4278t)\right] + c_2(-0.12+4.4278i)\left[\cos(4.4278t) - i\sin(4.4278t)\right]\right]$$

at $t \to 0$

$$15^\circ = \left[c_1(-0.12-4.4278i)(1+0) + c_2(-0.12+4.4278i)\right]$$

$$-0.12c_1 - 4.4278ic_1 - 0.12c_2 + 4.4278ic_2 = 15^\circ$$

$$\Rightarrow -0.12(c_1+c_2) - 4.4278i(c_1-c_2) = 15^\circ \times \boxed{\frac{\pi}{180}} \quad \text{(11)}$$

$$\theta_2 = 19.62\, c_1 e^{(-0.12+4.4278i)t} + c_2\, 19.62\, e^{(-0.12-4.4278i)t}$$

at $t \to 0$

$$\therefore 0 = 19.62\, c_1 + 19.62\, c_2 \Rightarrow \boxed{c_1 = -c_2} \;(1)$$

in eq^n (11)

$$\Rightarrow \cancel{(c_1+c_2)} - 4.4278i(c_1-c_2) = \frac{15\pi}{180}$$

$$\Rightarrow -4.4278i(c_1-c_2) = \frac{15\pi}{180}$$

$$\Rightarrow 2c_2(4.4278)i = \frac{15\pi}{180}$$

$$\Rightarrow c_2 = -0.02956i = -c_1$$

General sol^n $$\theta(t) = 0.02956i\, e^{(-0.12+4.4278i)t}\begin{bmatrix} -0.12-4.4278i \\ 19.62 \end{bmatrix} - 0.02956i\, e^{(-0.12-4.4278i)t} \times \begin{bmatrix} -0.12+4.4278i \\ 19.62 \end{bmatrix}$$

For equilibrium point $(\pi, 0)$ we have matrix as,

$$A_2 = \begin{bmatrix} 0 & 1 \\ g/l & -b/m \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 \\ 19.6 & -0.24 \end{bmatrix}$$

$$\lambda_1 = 4.311, \quad \lambda_2 = -4.551$$

$$v_1 = \begin{bmatrix} 4.551 \\ 19.62 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4.311 \\ -19.62 \end{bmatrix}$$

we can convert the equation in form of z and we

$\Rightarrow \dot{z} = Dz$ the write the equation as.

$\dot{z} = P^{-1}APz$ when P matrix $= \begin{bmatrix} 4.551 & 4.311 \\ 19.62 & -19.62 \end{bmatrix}$

$$\dot{z} = \begin{bmatrix} 4.311 & 0 \\ 0 & -4.551 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = D\,z \Rightarrow \begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}4.311 & 0\\ 0 & -4.551\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}$$

$$\Rightarrow \dot{z}_1 = 4.31 z_1 \qquad \dot{z}_2 = -4.55 z_2$$

$$\boxed{z_1 = c_1 e^{4.31t}} \qquad \boxed{z_2 = c_2 e^{-4.551t}}$$

$$X = P \cdot z$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}4.551 & 4.31\\ 19.62 & -19.62\end{bmatrix}\begin{bmatrix}c_1 e^{4.31t}\\ c_2 e^{-4.551t}\end{bmatrix}$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} \Rightarrow \begin{bmatrix}c_1\, 4.551\, e^{4.31t} + c_2\, 4.31 e^{-4.551t}\\ c_1\, 19.62\, e^{4.31t} + c_2(-19.62)\, e^{-4.551t}\end{bmatrix}$$

So we have the initial value's

$$\theta_1 = 15 \quad @\, t = 0$$

$$\theta_2 = 0 \text{ deg/s} \quad @\, t = 0$$

at $t=0$

$$\frac{15\times\pi}{180} = 4.551 c_1 + 4.31 c_2$$

$$0 = \theta_2 = 19.62 c_1 - 19.62 c_2$$

$$c_1 - c_2 = 0$$

$$\boxed{c_1 = c_2}$$

$$4.551 c_2 + 4.31 c_2 = 15\times\frac{\pi}{180}$$

$$c_2(4.551 + 4.31) = \frac{15\pi}{180}$$

$$c_2 = \frac{15\pi}{180}\times\left(\frac{1}{8.861}\right)$$

$$\Rightarrow c_2 = 0.02954$$

$$\Rightarrow c_1 = 0.02954$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}0.02954\times 4.551 e^{4.31t} + 0.02954\times 4.31 e^{-4.551t}\\ 0.02954\times 19.62\, e^{4.31t} + 0.02954\times(-19.62)\, e^{-4.551t}\end{bmatrix}$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}0.1344 e^{4.31t} + 0.1273 e^{-4.551t}\\ 0.5795 e^{4.31t} + -0.5795 e^{-4.551t}\end{bmatrix}$$

General solⁿ

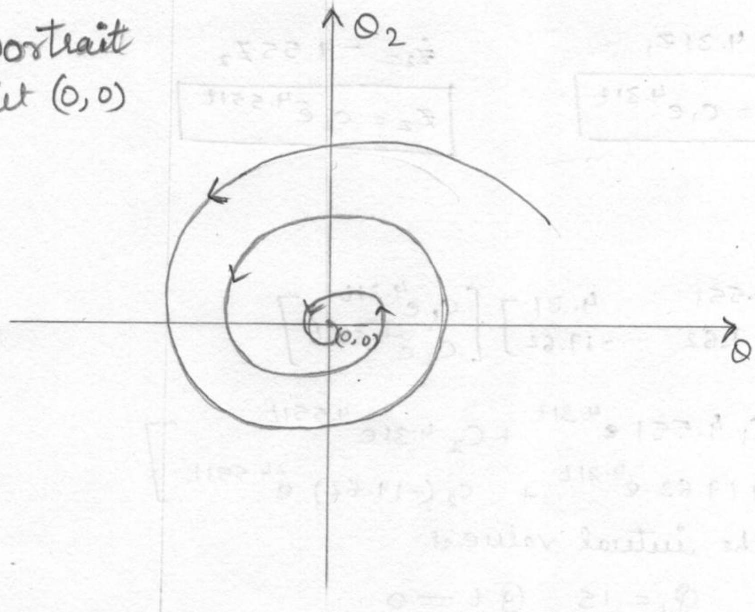
$$\Rightarrow \theta(t) = e^{4.31t}\begin{bmatrix}0.1344\\ 0.5795\end{bmatrix} + e^{-4.55t}\begin{bmatrix}0.1273\\ -0.5795\end{bmatrix}$$

$$\Rightarrow \theta(t) = 0.02954\, e^{4.31t}\begin{bmatrix}4.551\\ 19.62\end{bmatrix} + 0.02954\, e^{-4.55t}\begin{bmatrix}0.1273\\ -0.5795\end{bmatrix}$$

Q6 At Eqb^m point (0,0)

Ans At Eigenvalue $-0.12 \pm 4.4278i \longrightarrow$ Spiral, stable form

Phase portrait at point (0,0)

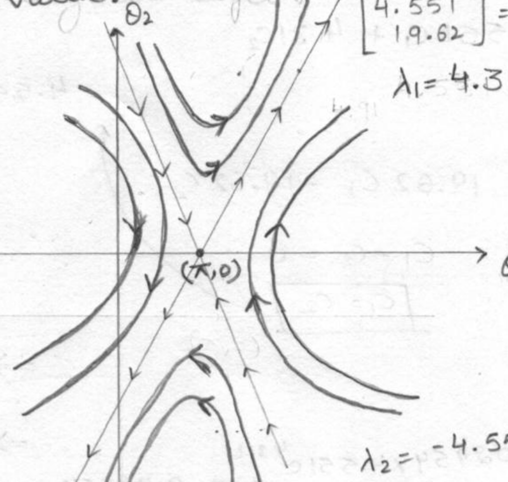


At eqb^m point $(\pi,0)$ the eigenvalues are 4.311 and -4.551 one +ve and one (−ve) eigenvalue.

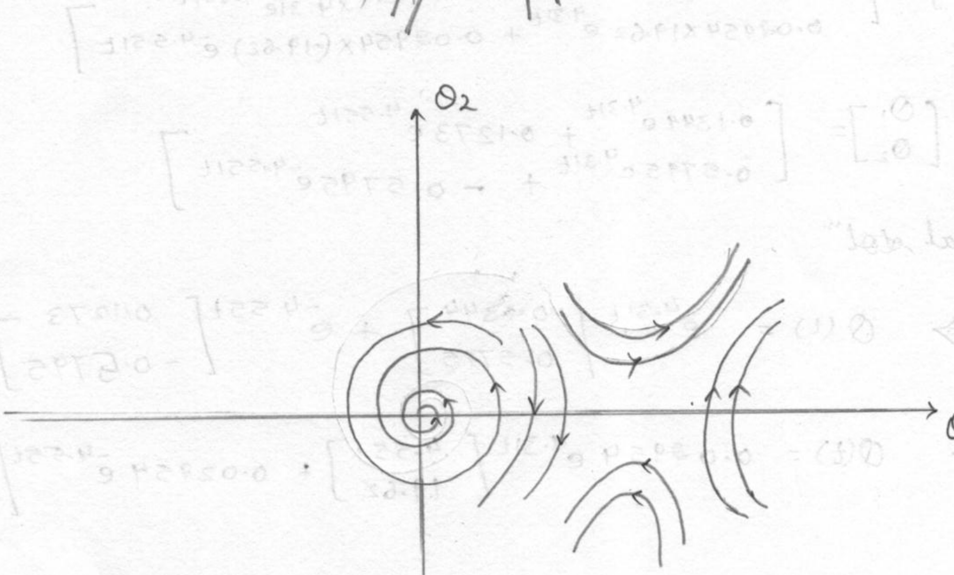
Phase portrait at point $(\pi,0)$

$\begin{bmatrix} 4.551 \\ 19.62 \end{bmatrix} = V_1$

$\lambda_1 = 4.311$



$\lambda_2 = -4.551 \quad V_2 = \begin{bmatrix} 4.311 \\ -19.62 \end{bmatrix}$



Q7.

Ans At $\theta = 180^\circ$, we have saddle point

At $\theta = 0^\circ$, we have a stable spiral, Stable converging

By the help of MATLAB we can determine the Time required to reach ball from 180° to 0°.

$t \simeq 0.8303$ sec.

Analytically solving the eqn we can solve

$$\theta_2 = \frac{d\theta_1}{dt} = 19.62\, C_1 e^{(-0.12+4.4278i)t} + C_2\, 19.62\, e^{(-0.12+-4.4278i)t}$$

$\therefore C_1 = -0.02956i = -C_2$

On integrating the above equation θ_1 from 180° to $\theta = 0$, and for Time 't' 0 to t and solving for t we will get same answer.