104 HW #1 Final

3\. a)

k	n
0	2
1	4
2	16
3	256

Looking at the relationship between k and n, we can see that it is $2^{2^k} = n$.

$$2^{2^k} = n \to \log_2(2^{2^k}) = \log_2 n \to 2^k \log_2(2) = \log_2 n \to 2^k = \frac{\log_2 n}{\log_2(2)} \to$$

$$\to \log_2(2^k) = \log_2\left(\frac{\log_2 n}{\log_2(2)}\right) \to k\log_2(2) = \log_2\left(\frac{\log_2 n}{\log_2(2)}\right) \to k = \frac{\log_2 \frac{\log_2 n}{\log_2(2)}}{\log_2 2} \to$$

$$\to k = \log_2(\log_2(n)) = \boxed{\Theta(\log(\log(n)))}$$

b)

i	n
4	2
9	3
16	4

This relationship can be represented as $k = (i\sqrt{n})^3$ for the inner loop

The total runtime can be represented by:

Entering the for loop $\to$ $$\sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{i=\sqrt{n}} \sum_{k=0}^{k=(i\sqrt{n})^3} \Theta(1) \to \sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{i=\sqrt{n}} \Theta(i^3(\sqrt{n})^3) \to$$

$$\to \sum_{i=1}^{n} \Theta(1) + (\sqrt{n})^3 \sum_{i=1}^{i=\sqrt{n}} \Theta(i^3) = (\sqrt{n})^3\, \Theta(\sqrt{n})^4 = \boxed{\Theta(n^{7/2})}$$

c) Worst case array: [1, 2, 3, 4, ..., n], if statement executes n times

$$\sum_{i=0}^{n} \sum_{j=1}^{\log_2 n} \Theta(1) \to \sum_{i=0}^{n} \Theta(\log_2(n)) \to \Theta(\log_2(n)\cdot n) \to \Theta(n\log n) + \Theta(n^2) \to \boxed{\Theta(n^2)}$$

Relationship for inner loop: $2^m = n \to \log_2(2^m) = \log_2(n) \to m\log_2(2) = \log_2 n$

$$\to m = \frac{\log_2 n}{\log_2} \to m = \log_2(n)$$

d) $T(n) = \sum_{i=1}^{n} \Theta(1) + \sum_{i}^{n} \sum_{j=0}^{size} \Theta(1)$

Assume the if statement is true for k times, and when true, size will update with the corresponding pattern

$i == size == 10 \cdot (\frac{3}{2})^0 : size \rightarrow 10 \cdot (\frac{3}{2})^1$

$i == size == 10 \cdot (\frac{3}{2})^1 : size \rightarrow 10 \cdot (\frac{3}{2})^2$

$\vdots$

$i == size == 10 \cdot (\frac{3}{2})^{k-1} : size \rightarrow 10 \cdot (\frac{3}{2})^k$

The stopping condition is $i < n$ so we have $10 \cdot (\frac{3}{2})^k < n$ or $k < \log_{3/2}(n/10)$

k is the number of times the if statement executes given n

$$\sum_{i=1}^{\log_{3/2}(\frac{n}{10})} \sum_{j=1}^{size} \Theta(1) = 10 + 10 \cdot \frac{3}{2} + 10 \cdot (\frac{3}{2})^2 + \ldots + 10 \cdot (\frac{3}{2})^k$$

$$10 \cdot \sum_{i=0}^{k} (\frac{3}{2})^i = 10 \cdot \Theta((\frac{3}{2})^k), \text{ where } k = (\log_{3/2})^{n/10}$$

$$= 10 \cdot \Theta((\frac{3}{2})^{(\log_{3/2})^{n/10}}) = 10 \cdot \Theta(\frac{n}{10}) = \boxed{\Theta(n)}$$