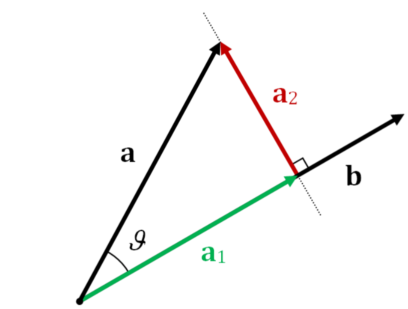
ue1:
1.Aufgabe:

$$b \in \left\{ x_1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} | x_1, x_2 \in \mathbb{R} \right\} \tag{1}$$

eine möglichst „gute“ Lösung könnte sinnvollerweise foglendes erfüllen:

$$\|Ax - b\|_2^2 \longrightarrow min$$

2.Aufgabe:



$$\begin{aligned} u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad v = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \\ v = v_\perp + v_\parallel \end{aligned} \tag{2}$$

$$\begin{aligned} < v; u > &=< v_\perp + v_\parallel; u > \\ &=< v_\perp; u > + < v_\parallel; u > \\ &= \cancelto{0}{< v_\perp; u >} + < v_\parallel; u > \\ &=< v_\parallel; u > \end{aligned} \tag{3}$$

$$v_u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{4}$$

$$u_v = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \tag{5}$$

Orthogonale Projektion:

$$\begin{aligned} u \to P_u : v \mapsto v_1 \\ v \mapsto P_u(v) = \frac{< v; u >}{< u; u >} u = \frac{1}{< u; u >} u \cdot u^T v \end{aligned} \tag{6}$$

$$
\begin{aligned}
v &\mapsto \frac{1}{\|u\|^2} u^T v \cdot u = y \\
y_i &= \frac{1}{\|u\|^2} u_i (u_1 v_1 + u_2 v_2 + u_3 v_3) \\
&\overset{!}{=} a_{i1} v_1 + a_{i2} v_2 + a_{i3} v_3 \\
a_{ij} &= \frac{1}{\|u\|^2} u_i u_j \\
\rightsquigarrow A &= \frac{1}{\|u\|^2} u \cdot u^T
\end{aligned} \tag{7}
$$

$$
Bild(P_u) = span\{u\} = \{\lambda u | \lambda \in \mathbb{R}\} \tag{8}
$$

$$
\begin{aligned}
Kern(P_u) &= Bild(P_u)^{\perp} \\
&= \{y | y \perp u\} \\
&= \{y | < u, y >= 0\}
\end{aligned} \tag{9}
$$

2b)

$$
\begin{aligned}
v \in V = \mathbb{R}^m \quad &; \quad u \in U = \mathbb{R}^n \\
v &\overset{P_u}{\mapsto} \lambda_1 u_1 + \ldots + \lambda_n u_n \\
\text{mit } \lambda_i &=< v; u_i > u_i
\end{aligned} \tag{10}
$$

$\square$

3a)

$$
\begin{aligned}
f(x) &= \|b - Ax\|_2^2 \\
&= \langle b - Ax; b - Ax \rangle \\
&= \langle b; b - Ax \rangle - \langle Ax; b - Ax \rangle \\
&= \langle b; b \rangle - \langle b; Ax \rangle - \langle Ax; b \rangle + \langle Ax; Ax \rangle \\
&= \langle b; b \rangle - 2 \langle b; Ax \rangle + \langle Ax; Ax \rangle
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
f'(x) = Jacobi(f) &= -2 \left\langle b; A \cdot \vec{1} \right\rangle + 2 \left\langle Ax; A \cdot \vec{1} \right\rangle \\
f''(x) = Hess(f) &= 2 \left\langle A \cdot \vec{1}; A \cdot \vec{1} \right\rangle
\end{aligned} \tag{12}
$$

5. Aufgabe:

$$f(t) = \alpha \cos\left(\frac{\pi}{4}t\right) + \beta \sin\left(\frac{\pi}{3}t\right)$$
$$\varphi(t, \alpha, \beta) = \alpha \cos\left(\frac{\pi}{4}t\right) + \beta \sin\left(\frac{\pi}{3}t\right)$$

$$\rightarrow A = \begin{pmatrix} \cos\left(\frac{\pi}{4}1\right) & \sin\left(\frac{\pi}{3}1\right) \\ \cos\left(\frac{\pi}{4}2\right) & \sin\left(\frac{\pi}{3}2\right) \\ \cos\left(\frac{\pi}{4}3\right) & \sin\left(\frac{\pi}{3}3\right) \end{pmatrix} \tag{13}$$
$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \quad , \quad b = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

Normalengleichung:

$$A^T A x = A^T b$$