1 Zug und Druck in Stäben

1.1 Spannung

$$\underbrace{\sigma}_{\text{Spannung}\left[\frac{N}{mm^2}\right]} = \frac{\overbrace{N}^{\text{Normalspannung}[N]}}{\underbrace{A}_{\text{Fläche}[mm^2]}} \tag{1.1}$$

$$\underbrace{\sigma}_{\text{Spannung}\left[\frac{N}{mm^2}\right]} = \frac{\overbrace{F}^{\text{Kraft}[N]}}{\underbrace{A}_{\text{Fläche}[mm^2]}} \tag{1.2}$$

$$\sigma = \frac{\overbrace{\sigma_0 = F/A}^{\substack{\text{Normalspannung}\\ \text{in einem Schnitt}\\ \text{Senkrecht zur Stabachse}}}}{2}(1+\cos 2\varphi), \tau = \frac{\sigma_0}{2}(\sin 2\varphi) \tag{1.3}$$

$$\sigma(x) = \frac{N(x)}{A(x)} \tag{1.4}$$

$$A_{\text{erf}} = \frac{|N|}{\sigma_{\text{zul}}} \tag{1.5}$$

1.2 Dehnung

$$\underbrace{\varepsilon}_{\text{Dehnung}[1]} = \frac{\overbrace{\Delta\ell}^{\text{Verlängerung}[m]}}{\underbrace{\ell_0}_{\substack{\text{Ursprüngliche}\\ \text{Länge } [m]}}} = \frac{\ell-\ell_0}{\ell_0} \tag{1.6}$$

Örtliche (lokale Dehnung)

$$\varepsilon(x) = \frac{du}{dx}, \quad |\varepsilon| \ll 1 \tag{1.7}$$

1.3 Stoffgesetz

Hooke'sches Gesetz

$$\underbrace{E}_{\substack{\text{Elastizitätsmodul}\\ \left[\frac{N}{mm^2}\right]}} = \frac{\overbrace{\sigma}^{\text{Spannung}\left[\frac{N}{mm^2}\right]}}{\underbrace{\varepsilon}_{\text{Dehnung}[1]}} \tag{1.8}$$

Umgestellt nach Sigma, übliche Form:

$$\sigma = E\varepsilon = \frac{\Delta\ell}{\ell_0}E \qquad \text{(andere Form)}$$

$$\underbrace{\varepsilon}_{\text{Dehnung}[1]} = \frac{\sigma}{E} \tag{1.9}$$

$$\underbrace{\varepsilon_T}_{\text{Wärmedehnung}[1]} = \underbrace{\alpha}_{\substack{\text{Thermischer Aus-}\\ \text{dehnungskoeffizient}\\ \text{(Wärmeausdehnungs}\\ \text{koeffizient) } [1/^\circ C]}} \cdot \underbrace{\Delta T}_{\substack{\text{Temperatur}\\ \text{änderung}\\ [^\circ C]}} \tag{1.10}$$

$$\varepsilon = \frac{\sigma}{E} + \alpha_T \Delta T \tag{1.11}$$

$$\sigma = E(\varepsilon - \alpha_T \Delta T) \tag{1.12}$$

1.4 Einzelstab

$$\frac{dN}{dx} + \underbrace{n}_{\text{Linienkraft}} = 0 \tag{1.13}$$

$$\frac{du}{dx} = \frac{N}{EA} + \alpha_T \Delta T \tag{1.14}$$

$$\Delta\ell = u(l) - u(0) = \int_0^\ell \varepsilon dx \tag{1.15}$$

$$\Delta\ell = \int_0^\ell \left(\frac{N}{EA} + \alpha_T \Delta T\right) dx \tag{1.16}$$

$$\Delta\ell = \frac{F\ell}{EA} + \alpha_T \Delta T \ell \tag{1.17}$$

Für $\Delta T = 0$

$$\Delta\ell = \frac{F\ell}{EA} \tag{1.18}$$

Oder $F = 0$

$$\Delta\ell = \alpha_T \Delta T \ell \tag{1.19}$$

$$(EAu')' = -n + (EA\alpha_t \Delta T)' \tag{1.20a}$$

Sei in 1.20a $EA = const$ und $\Delta T = const$

$$EAu'' = -n \tag{1.20b}$$

1.5 Statisch bestimmte Stabsysteme

$$\begin{aligned} u &= |\Delta\ell_1| = \frac{F\ell}{EA}\frac{1}{\tan\alpha}, \\ v &= \frac{\Delta\ell_2}{\sin\alpha} + \frac{u}{\tan\alpha} = \frac{F\ell}{EA}\frac{1+\cos^3\alpha}{\sin^2\alpha\cos\alpha} \end{aligned} \tag{1.21}$$

1.6 Statisch unbestimmte Stabsysteme

2 Spannungszustand

2.1 Spannungvektor und Spannungtensor

$$\boldsymbol{t} = \lim_{\Delta A \to 0} \frac{\Delta \boldsymbol{F}}{\Delta A} = \frac{d\boldsymbol{F}}{dA} \tag{2.1}$$

$$\boldsymbol{t} = \tau_{yx}\boldsymbol{e_x} + \sigma_y \boldsymbol{e_y} + \tau_{yz}\boldsymbol{e_z} \tag{2.2}$$

$$\tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx}, \tau_{yz} = \tau_{zy} \tag{2.3}$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \tag{2.4}$$

2.2 Ebener Spannungszustand

2.2.1 Koordinatentransformation

$$\begin{aligned} \sigma_\xi &= \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi + 2\tau_{xy}\sin\varphi\cos\varphi \\ \tau_{\xi\eta} &= -(\sigma_x - \sigma_y)\sin\varphi\cos\varphi + \tau_{xy}(\cos^2\varphi - \sin^2\varphi) \end{aligned} \tag{2.5a}$$

$$\sigma_\eta = \sigma_x \sin^2\varphi + \sigma_y \cos^2\varphi - 2\tau_{xy}\cos\varphi\sin\varphi \tag{2.5b}$$

$$\begin{aligned} \sigma_\xi &= \frac{1}{2}(\sigma_x+\sigma_y) + &&\frac{1}{2}(\sigma_x-\sigma_y)\cos 2\varphi + \tau_{xy}\sin 2\varphi, \\ \sigma_\eta &= \frac{1}{2}(\sigma_x+\sigma_y) - &&\frac{1}{2}(\sigma_x-\sigma_y)\cos 2\varphi + \tau_{xy}\sin 2\varphi, \\ \tau_{\xi\eta} &= - &&\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\varphi + \tau_{xy}\cos 2\varphi, \end{aligned} \tag{2.6}$$

$$\sigma_\xi + \sigma_\eta = \sigma_x + \sigma_y \tag{2.7}$$

2.2.2 Hauptspannungen

$$\tan 2\varphi^* = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{2.8}$$

$$\begin{aligned} \cos 2\varphi^* &= \frac{1}{\sqrt{1+\tan^2 2\varphi^*}} &&= \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}} \\ \sin 2\varphi^* &= \frac{\tan 2\varphi^*}{\sqrt{1+\tan^2 2\varphi^*}} &&= \frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}} \end{aligned} \tag{2.9}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{2.10}$$

$$\tan 2\varphi^{**} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \tag{2.11}$$

$$\tau_{\max} = \pm\sqrt{(\frac{\sigma_x-\sigma_y}{2})^2 + \tau_{xy}^2} \tag{2.12a}$$

$$\tau_{\max} = \pm\frac{1}{2}(\sigma_1 - \sigma_2) \tag{2.12b}$$

$$\sigma_M = \frac{1}{2}(\sigma_x+\sigma_y) = \frac{1}{2}(\sigma_1+\sigma_2) \tag{2.13}$$

2.3 Mohrscher Spannungskreis

$$\begin{aligned} \sigma_\xi - \frac{1}{2}(\sigma_x+\sigma_y) &= \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\varphi + \tau_{xy}\cos 2\varphi \\ \tau_{\xi\eta} &= -\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\varphi + \tau_{xy}\cos 2\varphi \end{aligned} \tag{2.14}$$

$$\left[\sigma_\xi - \frac{1}{2}(\sigma_x+\sigma_y)\right]^2 + \tau_{\xi\eta}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2 \tag{2.15}$$

$$(\sigma - \sigma_M)^2 + \tau^2 = r^2 \tag{2.16}$$

$$r^2 = \frac{1}{4}\left[(\sigma_x+\sigma_y)^2 - 4(\sigma_x\sigma_y - \tau_{xy}^2)\right] \tag{2.17}$$

2.3.1 Dünnwandiger Kessel

$$\sigma_x = \frac{1}{2}p\frac{r}{t} \tag{2.18}$$

$$\sigma_\varphi = p\frac{r}{t} \tag{2.19}$$

$$\sigma_t = \sigma_\varphi = \frac{1}{2}p\frac{r}{t} \tag{2.20}$$

2.4 Gleichgewichtsbedingungen

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + f_x = 0 \tag{2.21a}$$

$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + f_y = 0 \tag{2.21b}$$

$$\begin{aligned} \frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} + f_x &= 0 \\ \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + f_y &= 0 \\ \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} + f_z &= 0 \end{aligned} \tag{2.22}$$

3 Verzerrungszustand, Elastizitätsgesetze

3.1 Verzerrungszustand

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} \tag{3.1}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{3.2}$$

$$\tan 2\varphi^* = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \tag{3.4}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right) + \left(\frac{1}{2}\gamma_{xy}\right)} \tag{3.5}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \tag{3.6a}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \tag{3.6b}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_x & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix} \tag{3.7}$$

3.2 Elastizitätsgesetz

$$\varepsilon_y = -\nu\varepsilon_x \tag{3.8}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \tag{3.9}$$

$$\tau_{xy} = G\gamma_{xy} \tag{3.10}$$

$$G = \frac{E}{2(1+\eta)} \tag{3.11}$$

$$\begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) \\ \gamma_{xy} &= \frac{1}{G}\tau_{xy} \end{aligned} \tag{3.12a}$$

$$\begin{aligned} \sigma_x &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \\ \sigma_y &= \frac{E}{1-\nu^2}(\varepsilon_y - \nu\varepsilon_x) \\ \tau_{xy} &= G\gamma_{xy} \end{aligned} \tag{3.12b}$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2), \quad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \tag{3.13}$$

$$\begin{aligned} \varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha_T \Delta T \\ \varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] + \alpha_T \Delta T \\ \varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha_T \Delta T \\ \gamma_{xy} &= \frac{1}{G}\tau_{xy}, \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz} \end{aligned} \tag{3.14}$$

3.3 Festigkeitshypothesen

$$\sigma_V \le \sigma_{zul} \tag{3.15}$$

$$\sigma_V = \sigma_1 \tag{3.16}$$

$$\sigma_V = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \tag{3.17}$$

$$\sigma_V = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \tag{3.18}$$

4 Balkenbiegung

4.1 Einführung

$$\sigma(z) = cz \tag{4.1}$$

$$M = \int z\sigma \, dA \tag{4.2}$$

$$I = \int z^2 \, dA \tag{4.3}$$

$$\sigma = \frac{M}{I}z \tag{4.4}$$

4.2 Flächenträgheitsmomente

4.2.1 Definition

Das statische Moment ist quasi Fläche × Hebelarm bezogen auf den Schwerpunkt der Fläche:

$$\boxed{S_y = \int z\,\mathrm{d}A, \quad S_z = \int y\,\mathrm{d}A} \tag{4.5}$$

$$\boxed{I_y = \int z^2\,\mathrm{d}A, \quad I_z = \int y^2\,\mathrm{d}A} \tag{4.6a}$$

$$\boxed{I_{yz} = I_{zy} = -\int yz\,\mathrm{d}A} \tag{4.6b}$$

$$\boxed{I_p = \int r^2\,\mathrm{d}A = \int \left(z^2 + y^2\right)\mathrm{d}A = I_y + I_z} \tag{4.6c}$$

$$\boxed{i_y = \sqrt{\frac{I_y}{A}}, \quad i_z = \sqrt{\frac{I_z}{A}}, \quad i_p = \sqrt{\frac{I_p}{A}}} \tag{4.7}$$

4.2.2 Parallelverschiebung der Bezugsachsen

$$\boxed{\begin{aligned} I_{\bar{y}} &= I_y + \bar{z}_s^2 A \\ I_{\bar{z}} &= I_z + \bar{y}_s^2 A \\ I_{\bar{y}\bar{z}} &= I_{yz} - \bar{y}_s \bar{z}_s A \end{aligned}} \tag{4.13}$$

I_y ist das Flächenträgheitsmoment der zusammengesetzten Fläche, $I_{\bar{y}}$ das von jeder einzelnen, $\bar{z}_s$ ist der Abstand des Schwerpunktes der Einzelflächen zur GesamtAchse y und A ist der jewilige Flächeninhalt.

$$\boxed{I_y = \sum I_{\bar{y}} + \bar{z}_s^2 A} \qquad \text{(Satz v. Steiner)}$$

4.2.3 Drehung des Bezugssystems, Hauptträgheitsmomente

$$\boxed{\begin{aligned} I_\eta &= \frac{1}{2}\left(I_y + I_z\right) && + \frac{1}{2}\left(I_y - I_z\right)\cos 2\varphi + I_{yz}\sin 2\varphi \\ I_\zeta &= \frac{1}{2}\left(I_y - I_z\right) && - \frac{1}{2}\left(I_y - I_z\right)\cos 2\varphi - I_{yz}\sin 2\varphi \\ I_{\eta\zeta} &= && -\frac{1}{2}\left(I_y - I_z\right)\sin 2\varphi + I_{yz}\cos 2\varphi \end{aligned}} \tag{4.14}$$

$$\boxed{I_\eta + I_\zeta = I_y + I_z = I_p} \tag{4.15}$$

$$\boxed{\tan 2\varphi^* = \frac{2 I_{yz}}{I_y - I_z}} \tag{4.16}$$

$$\boxed{I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}} \tag{4.17}$$

4.3 Grundgleichungen der geraden Biegung

$$\boxed{\frac{\mathrm{d}Q}{\mathrm{d}x} = -q, \quad \frac{\mathrm{d}M}{\mathrm{d}x} = Q} \tag{4.18}$$

$$\boxed{M = \int z\sigma\,\mathrm{d}A} \tag{4.19a}$$

$$\boxed{Q = \int \tau\,\mathrm{d}A} \tag{4.19b}$$

$$\boxed{N = \int \sigma\,\mathrm{d}A} \tag{4.19c}$$

$$\boxed{\varepsilon = \frac{\partial u}{\partial x}} \tag{4.20}$$

$$\boxed{\sigma = E\,\varepsilon, \quad \tau = G\,\gamma} \tag{4.21}$$

$$\boxed{w = w(x)} \tag{4.22a}$$

$$\boxed{u(x,z) = \psi(x) z} \tag{4.22b}$$

$$\boxed{\sigma = E\frac{\partial u}{\partial x} = E\psi' z} \tag{4.23a}$$

$$\boxed{\tau = G\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = G(w' + \psi)} \tag{4.23b}$$

$$\boxed{M = EI\psi'} \tag{4.24}$$

$$\boxed{Q = \varkappa GA(w' + \psi)} \tag{4.25}$$

4.4 Normalspannungen

$$\boxed{\sigma = \frac{M}{I} z} \tag{4.26}$$

$$\boxed{W = \frac{I}{|z|_{\max}}} \tag{4.27}$$

Aber hier mit subscript, also $W_{\text{Achse}} = \frac{I_{\text{Achse}}}{|\text{andere Achse}|_{\max}}$

$$\boxed{\sigma_{\max} = \frac{|M|}{W}} \tag{4.28}$$

4.5 Biegelinie

4.5.1 Differentialgleichung der Biegelinie

$$\boxed{w' + \psi = 0} \tag{4.29}$$

$$\boxed{Q' = -q, \quad M' = Q, \quad \psi' = \frac{M}{EI}, \quad w' = -\psi} \tag{4.30}$$

$$\boxed{w'' = -\frac{M}{EI}} \tag{4.31}$$

$$\boxed{\varkappa_B = \frac{w''}{(1 + w'^2)^{3/2}}} \tag{4.32a}$$

$$\boxed{\varkappa_B \approx w''} \tag{4.32b}$$

$$\boxed{Q = -(EIw'')'} \tag{4.33}$$

$$\boxed{(EIw'')'' = q} \tag{4.34a}$$

$$\boxed{EIw^{IV} = q} \tag{4.34b}$$

4.5.2 Einfeldbalken

4.5.3 Balken mit mehreren Feldern

Balken mit zwei Feldern. Eingespannt rechts und links, krafteinwirkung dazwischen, bei a.

$$\boxed{M(x) = \begin{cases} F\frac{b}{\ell}x & \text{für } 0 \le x \le a \\ F\frac{a}{\ell}(\ell - x) & \text{für } a \le x \le l \end{cases}} \qquad \text{(Momentenverlauf)}$$

4.5.4 Superposition

4.6 Einfluss des Schubes

4.6.1 Schubspannungen

$$\boxed{\frac{\partial\sigma}{\partial x} = \frac{Q}{I}\zeta} \tag{4.35}$$

$$\boxed{S(z) = \int_{A^*} \zeta\,\mathrm{d}A} \tag{4.36}$$

$$\boxed{\underbrace{\tau(z)}_{N/mm^2} = \frac{\overbrace{Q}^{[N]}}{\underbrace{I}_{mm^4}}\,\frac{\overbrace{S(z)}^{mm^3}}{\underbrace{b(z)}_{mm}}} \tag{4.37}$$

4.6.2 Durchbiegung infolge Schub

$$\boxed{w' + \psi = \frac{Q}{GA_S}} \tag{4.40}$$

$$\boxed{w_s' = \frac{Q}{GA_S}} \tag{4.41}$$

$$\boxed{w' = w_B' + w_S'} \tag{4.42}$$

$$\boxed{w = w_B + w_S} \tag{4.43}$$

$$\boxed{w_S = \frac{F}{GA_S}x} \tag{4.44}$$

4.7 Schiefe Biegung

$$\boxed{\sigma = \frac{M_y}{I_y}z - \frac{M_z}{I_z}y} \tag{4.45}$$

$$\boxed{w'' = \frac{M_y}{EI_y}, \quad v'' = \frac{M_z}{EI_z}} \tag{4.46}$$

$$\boxed{\begin{aligned} \frac{\mathrm{d}Q_z}{\mathrm{d}x} &= -q_z, & \frac{\mathrm{d}Q_y}{\mathrm{d}x} &= -q_y \\ \frac{\mathrm{d}M_y}{\mathrm{d}x} &= Q_z, & \frac{\mathrm{d}M_z}{\mathrm{d}x} &= -Q_y \end{aligned}} \tag{4.47}$$

$$\boxed{\varepsilon = -(w''z + v''y)} \tag{4.48}$$

$$\boxed{\sigma = -E\,(w''z + v''y)} \tag{4.49}$$

$$\boxed{M_y = \int z\sigma\,\mathrm{d}A, \quad M_z = -\int y\sigma\,\mathrm{d}A} \tag{4.50}$$

4.8 Biegung und Zug/Druck

4.9 Kern des Querschnitts

4.10 Temperaturbelastung

5 Torsion

5.1 Einführung

5.2 Die kreiszylindrische Welle

$$\boxed{r\,\mathrm{d}\vartheta = \gamma\,\mathrm{d}x \rightarrow \gamma = r\frac{\mathrm{d}\vartheta}{\mathrm{d}x}} \tag{5.1}$$

Man nennt die Verdrehung pro Längeneinheit $\mathrm{d}\vartheta = \mathrm{d}x$ manchmal auch Verwindung $\varkappa_T$.

$$\boxed{\tau = Gr\frac{\mathrm{d}\vartheta}{\mathrm{d}x} = Gr\vartheta'} \tag{5.2}$$

$$\boxed{M_T = \int r\vartheta\,\mathrm{d}A} \tag{5.3}$$

$$\boxed{M_T = G\vartheta' \int r^2\,\mathrm{d}A = G\vartheta' I_p} \tag{5.4}$$

$$\boxed{GI_T\vartheta' = M_T} \tag{5.5}$$

Die Größe GI_T heißt Torsionssteifigkeit.

$$\boxed{M_T = M_x} \tag{5.6}$$

$$\boxed{\vartheta_\ell = \frac{M_T \ell}{GI_T}} \tag{5.7}$$

$$\boxed{\tau = \frac{M_T}{I_T}r} \tag{5.8}$$

Der Größtwert tritt am Rand $r = R$ auf: $\tau_{\max} = (M_T/I_T)\,R$. Um die Analogie zur Biegung herzustellen, führen wir ein *Torsionswiderstandsmoment* W_T ein:

$$\boxed{\tau_{\max} = \frac{M_T}{W_T}} \tag{5.9}$$

$$\boxed{I_t = I_P = \frac{\pi}{2}R^4, \quad W_T = \frac{\pi}{2}R^3} \tag{5.10}$$

$$\boxed{I_T = \frac{\pi}{2}\left(R_a^4 - R_i^4\right), \quad W_T = \frac{\pi}{2}\frac{R_a^4 - R_i^4}{R_a}} \tag{5.11}$$

$$\boxed{I_T \approx 2\pi R_m^3 t \quad W_T \approx 2\pi R_m^2 t} \tag{5.12}$$

$$\boxed{\frac{\mathrm{d}M_T}{\mathrm{d}x} = M_T' = -m_T} \tag{5.13}$$

$$\boxed{(GI_T\vartheta')' = -m_T} \tag{5.14}$$

5.3 Dünnwandige geschlossene Profile

$$\boxed{T = \tau t} \tag{5.15}$$

$$\boxed{T = \tau t = \text{const}} \tag{5.16}$$

$$\boxed{M_T = \oint \mathrm{d}M_T = T\oint r_\perp\,\mathrm{d}s} \tag{5.17}$$

$$\boxed{\oint r_\perp\,\mathrm{d}s = 2A_m} \tag{5.18}$$

$$\boxed{M_T = 2A_m T} \tag{5.19}$$

1. Bredtsche Formel

$$\boxed{\tau = \frac{T}{t} = \frac{M_T}{2A_m t}} \tag{5.20}$$

$$\boxed{\tau_{\max} = \frac{M_T}{W_T} \quad \text{mit} \quad W_T = 2A_m t_{\min}} \tag{5.21}$$

$$\boxed{\mathrm{d}v = r_\perp\,\mathrm{d}\vartheta} \tag{5.22}$$

$$\boxed{\frac{T}{Gt} = r_\perp\vartheta' + \frac{\partial u}{\partial s}} \tag{5.23}$$

$$\boxed{\vartheta' = \frac{M_T}{GI_T}} \tag{5.24}$$

$$\boxed{I_T = \frac{(2A_m)^2}{\oint \frac{\mathrm{d}s}{t}}} \tag{5.25}$$

5.4 Dünnwandige offene Profile

7 Knickung

7.1 Verzweigung einer Gleichgewichtslage

$$\boxed{\Pi' = \frac{\mathrm{d}\Pi}{\mathrm{d}\varphi} = 0 \rightarrow -F\ell\sin\varphi + c_T\varphi = 0} \tag{7.1}$$

$$\boxed{\varphi_1 = 0} \tag{7.2}$$

$$\boxed{\frac{\varphi_2}{\sin\varphi_2} = \frac{F\ell}{c_T}} \tag{7.3}$$

$$\boxed{\Pi'' = \frac{\mathrm{d}^2\Pi}{\mathrm{d}\varphi^2} = -F\ell\cos\varphi + c_T} \tag{7.4}$$

$$\boxed{F_{\text{krit}} = \frac{c_T}{\ell}} \tag{7.5}$$

7.2 Der Euler-Stab

$$\boxed{M = Fw} \tag{7.6}$$

$$\boxed{EIw'' + Fw = 0} \tag{7.7a}$$

$$\boxed{\lambda = \frac{\pi}{\ell_k}, \quad \lambda^2 = {}^{F}\!/_{EI}} \qquad \text{(abkürzung)}$$

$$\boxed{w'' + \lambda^2 w = 0} \tag{7.7b}$$

$$\boxed{w = A\cos\lambda x + B\sin\lambda x} \tag{7.8}$$

$$\boxed{\sin\lambda\ell = 0 \rightarrow \lambda_n\ell = n\pi \quad \text{mit} \quad n = 1,2,3,...} \tag{7.9}$$

$$\boxed{F_{\text{krit}} = \lambda_1^2 EI = \pi^2\frac{EI}{\ell^2}} \tag{7.10}$$

$$\boxed{N = const = -F} \tag{7.11}$$

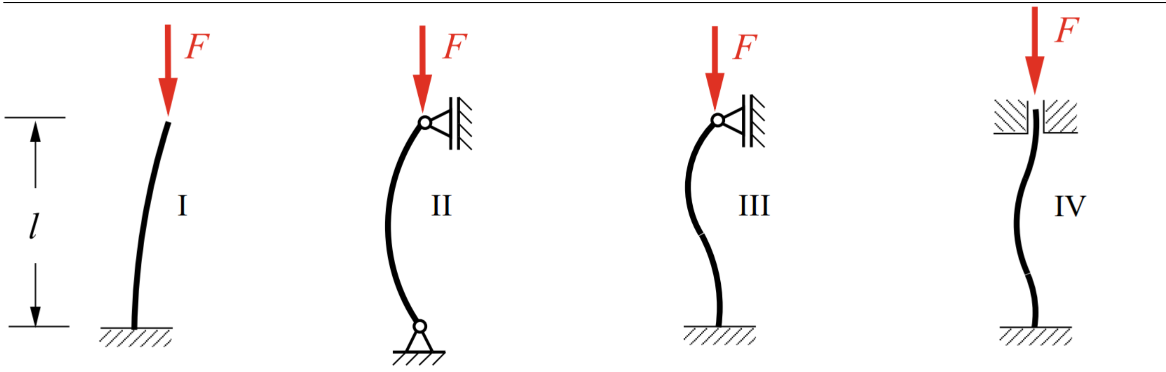
$$\boxed{(EIw'')'' + Fw'' = 0} \tag{7.12}$$

$$\boxed{w^{IV} + \lambda^2 w'' = 0} \tag{7.13}$$

$$\boxed{w = A\cos\lambda x + B\sin\lambda x + C\lambda x + D} \tag{7.14}$$

$\lambda_k = \ell_k \cdot \sqrt{\frac{A}{I}} = \frac{\ell_k}{i}$

$$\boxed{F_{\text{krit}} = \pi^2\frac{EI_{\min}}{\ell_k^2}} \tag{7.19}$$



$F_{\text{krit}} = \frac{\pi^2}{4}\frac{EI}{l^2}$	$\pi^2\frac{EI}{l^2}$	$(1{,}43)^2\,\pi^2\frac{EI}{l^2}$	$4\pi^2\frac{EI}{l^2}$
$l_k = 2\,l$	l	$l/1{,}43$	$l/2$

Fläche	I_y	I_z	I_{yz}	I_p	$I_{\bar{y}}$
Rechteck	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$	0	$\frac{bh}{12}\left(h^2+b^2\right)$	$\frac{bh^3}{3}$
Dreieck <u>BILD</u>	$\frac{bh^3}{36}$	$\frac{bh}{36}\left(b^2-ba+a^2\right)$	$-\frac{bh^2}{72}(b-2a)$	$\frac{bh}{36}\left(h^2+b^2-ba+a^2\right)$	$\frac{bh^3}{12}$
Kreis	$\frac{\pi R^4}{4}$	$\frac{\pi R^4}{4}$	0	$\frac{\pi R^4}{2}$	$\frac{5\pi}{4}R^4$
Dünner Kreisring $t \ll R_m$	$\pi R_m^3 t$	$\pi R_m^3 t$	0	$2\pi R_m^3 t$	$3\pi R_m^3 t$
Halbkreis	$\frac{R^4}{72\pi}\left(9\pi^2-64\right)$	$\frac{\pi R^4}{8}$	0	$\frac{R^4}{36\pi}\left(9\pi^2-32\right)$	$\frac{\pi R^4}{8}$
Ellipse	$\frac{\pi}{4}ab^3$	$\frac{\pi}{4}ba^3$	0	$\frac{\pi ab}{4}\left(a^2+b^2\right)$	$\frac{5\pi}{4}ab^3$