1.1 Spannung

$$\boxed{\underbrace{\sigma}_{\text{Spannung}\left[\frac{N}{mm^2}\right]} = \frac{\overbrace{N}^{\text{Normalspannung}[N]}}{\underbrace{A}_{\text{Fläche}[mm^2]}}} \tag{1.1}$$

$$\boxed{\underbrace{\sigma}_{\text{Spannung}\left[\frac{N}{mm^2}\right]} = \frac{\overbrace{F}^{\text{Kraft}[N]}}{\underbrace{A}_{\text{Fläche}[mm^2]}}} \tag{1.2}$$

$$\boxed{\sigma = \frac{\overbrace{\sigma_0}^{\substack{\text{Normalspannung}\\ \text{in einem Schnitt}\\ \text{Senkrecht zur Stabachse}}}}{2}\left(1+\cos 2\phi\right), \tau = \frac{\sigma_0}{2}\left(\sin 2\phi\right)} \tag{1.3}$$

$$\boxed{\sigma(x) = \frac{N(x)}{A(x)}} \tag{1.4}$$

$$\boxed{A_{\text{erf}} = \frac{|N|}{\sigma_{\text{zul}}}} \tag{1.5}$$

1.2 Dehnung

$$\boxed{\underbrace{\varepsilon}_{\text{Dehnung}[1]} = \frac{\overbrace{\Delta\ell}^{\text{Verlängerung}[m]}}{\underbrace{\ell_0}_{\substack{\text{Ursprüngliche}\\ \text{Länge }[m]}}} = \frac{\ell-\ell_0}{\ell_0}} \tag{1.6}$$

Örtliche (lokale Dehnung)

$$\boxed{\varepsilon(x) = \frac{\mathrm{d}u}{\mathrm{d}x}} \tag{1.7}$$

1.3 Stoffgesetz

Hooke'sches Gesetz

$$\boxed{\underbrace{E}_{\substack{\text{Elastizitätsmodul}\\ \left[\frac{N}{mm^2}\right]}} = \frac{\overbrace{\sigma}^{\text{Spannung}\left[\frac{N}{mm^2}\right]}}{\underbrace{\varepsilon}_{\text{Dehnung[1]}}}} \tag{1.8}$$

Umgestellt nach Sigma, übliche Form:

$$\sigma = E\varepsilon = \frac{\Delta\ell}{\ell_0}E$$

$$\boxed{\underbrace{\varepsilon}_{\text{Dehnung[1]}} = \frac{\sigma}{E}} \tag{1.9}$$

$$\boxed{\underbrace{\varepsilon_T}_{\text{Wärmedehnung[1]}} = \underbrace{\alpha}_{\substack{\text{Thermischer}\\ \text{Ausdehnungskoeffizient}\\ \text{(Wärmeausdehnugnskoeffizient)}\\ [1/{}^\circ\text{C}]}} \cdot \underbrace{\Delta T}_{\text{Temperaturänderung}[{}^\circ\text{C}]}} \tag{1.10}$$

$$\boxed{\varepsilon = \frac{\sigma}{E} + \alpha_T \Delta T} \tag{1.11}$$

$$\boxed{\sigma = E\left(\varepsilon - \alpha_T \Delta T\right)} \tag{1.12}$$

1.4 Einzelstab

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}x} + \underbrace{n}_{\text{Linienkraft}} = 0} \tag{1.13}$$

$$\boxed{\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{N}{EA} + \alpha_T \Delta T} \tag{1.14}$$

$$\boxed{\Delta\ell = u(l) - u(0) = \int_0^\ell \varepsilon \mathrm{d}x} \tag{1.15}$$

$$\boxed{\Delta\ell = \int_0^\ell \left(\frac{N}{EA} + \alpha_T \Delta T\right) \mathrm{d}x} \tag{1.16}$$

$$\boxed{\Delta\ell = \frac{F\ell}{EA} + \alpha_T \Delta T \ell} \tag{1.17}$$

Für $\Delta T = 0$

$$\boxed{\Delta\ell = \frac{F\ell}{EA}} \tag{1.18}$$

Oder $F = 0$

$$\boxed{\Delta\ell = \alpha_T \Delta T \ell} \tag{1.19}$$

$$\boxed{\left(EAu'\right)' = -n + \left(EA\alpha_t \Delta T\right)'} \tag{1.20a}$$

Sei in 1.20a $EA = const$ und $\Delta T = const$

$$\boxed{EAu'' = -n} \tag{1.20b}$$

1.5 Statisch bestimmte Stabsysteme

$$\boxed{\begin{aligned} &u = |\Delta\ell_1| = \frac{F\ell}{EA}\frac{1}{\tan\alpha}, \\ &v = \frac{\Delta\ell_2}{\sin\alpha} + \frac{u}{\tan\alpha} = \frac{F\ell}{EA}\frac{1+\cos^3\alpha}{\sin^2\alpha\cos\alpha} \end{aligned}} \tag{1.21}$$

1.6 Statisch unbestimmte Stabsysteme

1.7 Zusammenfassung

2.1 Spannungvektor und Spannungtensor

$$\boxed{\boldsymbol{t} = \lim_{\Delta A \to 0} \frac{\Delta \boldsymbol{F}}{\Delta A} = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}A}} \tag{2.1}$$

$$\boxed{\boldsymbol{t} = \tau_{yx}\boldsymbol{e_x} + \sigma_y \boldsymbol{e_y} + \tau_{yz}\boldsymbol{e_z}} \tag{2.2}$$

$$\boxed{\tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx}, \tau_{yz} = \tau_{zy}} \tag{2.3}$$

$$\boxed{\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}} \tag{2.4}$$

$$\boxed{\begin{aligned} \sigma_\xi &= \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi + 2\tau_{xy}\sin\varphi\cos\phi \\ \tau_{\xi\eta} &= -(\sigma_x - \sigma_y)\sin\varphi\cos\varphi + \tau_{xy}(\cos^2\varphi - \sin^2\varphi) \end{aligned}} \tag{2.5a}$$

$$\boxed{\sigma_\eta = \sigma_x \sin^2\varphi + \sigma_y \cos^2\varphi - 2\tau_{xy}\cos\varphi\sin\varphi} \tag{2.5b}$$

$$\boxed{\begin{aligned} \sigma_\xi &= \frac{1}{2}(\sigma_x + \sigma_y) + &&\frac{1}{2}(\sigma_x - \sigma_y)\cos 2\varphi + \tau_{xy}\sin 2\varphi, \\ \sigma_\eta &= \frac{1}{2}(\sigma_x + \sigma_y) - &&\frac{1}{2}(\sigma_x - \sigma_y)\cos 2\varphi + \tau_{xy}\sin 2\varphi, \\ \tau_{\xi\eta} &= - &&\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\varphi + \tau_{xy}\cos 2\varphi, \end{aligned}} \tag{2.6}$$

$$\boxed{\sigma_\xi + \sigma_\eta = \sigma_x + \sigma_y} \tag{2.7}$$

$$\boxed{\tan 2\varphi^* = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \tag{2.8}$$

$$\boxed{\begin{aligned} \cos 2\varphi^* &= \frac{1}{\sqrt{1 + \tan^2 2\varphi^*}} &&= \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \\ \sin 2\varphi^* &= \frac{\tan 2\varphi^*}{\sqrt{1 + \tan^2 2\varphi^*}} &&= \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \end{aligned}} \tag{2.9}$$

$$\boxed{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \tag{2.10}$$

$$\boxed{\tan 2\varphi^{**} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}} \tag{2.11}$$

$$\boxed{\tau_{\max} = \pm\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} \tag{2.12a}$$

$$\boxed{\tau_{\max} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)} \tag{2.12b}$$

$$\boxed{\sigma_M = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(\sigma_1 + \sigma_2)} \tag{2.13}$$