AE 618A: Finite Element Method for Fluid Dynamics
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Assignment - 2

A. Consider the boundary value problem discussed in the class:

$$u_{,xx}(x) + f(x) = 0, \qquad on \qquad]0.1[, \tag{1}$$

$$u(1) = g, \tag{2}$$

$$-u_{,x}(0) = h \qquad . \tag{3}$$

Assume $f(x) = qx$ where q is a constant and $g = h = 0$.

1. Find the exact solution.
2. Employing the linear finite element space with equally spaced nodes, set up and solve the Galerkin-Finite Element equations for $n = 1,\ 2,\ 3,\ 4$, i.e. $h = 1,\ 1/2,\ 1/3,\ 1/4$. In each case, you will need to calculate the stiffness matrix ($\mathbf{K}$) and vector ($\mathbf{F}$), and then carry out a solve for $\mathbf{Kd} = \mathbf{F}$.
3. Is the stiffness matrix banded? What is the consequence of boundary terms (g and h) on the bandedness?
4. Let $re_{,x} = |u_{,x}^h - u_{,x}|/(q/2)$ denote the relative error in $u_{,x}$. Compute $re_{,x}$ at the midpoints of the 4 elements. They should be all equal.
5. Employing the data for $h = 1$, $h = 1/2$, $h = 1/3$ and $h = 1/4$ plot $ln(re_{,x})$ versus $ln(h)$.
6. What is the significance of the slope and the the y-intercept?

B. For the same problem, as described in part A, write a computer program to assemble the element level stiffness matrices and force vectors. Solve the matrix equation system using a linear equation solver (you can use a library or any other available program, for solving **A x=b**). Plot the solution and the slope. Compute the relative error for $n = 10$, $n = 50$ and $n = 100$. Calculate the slope as was done in the previous part. Comment on the results.