AE618 Assignment 1

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A.

A. Given, $v_r = 0$, $v_\theta = \frac{5}{r}$

1) $\vec{r} = r\hat{r} \Rightarrow \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$\Rightarrow \dot{r} = 0$ and $r\dot{\theta} = \frac{5}{r}$

$\Rightarrow r = c_1$ and $\theta = \frac{5t}{c_1^2} + c_2$

using initial condition

at $t=0$, $r=1$, $\theta = 0$

$\Rightarrow c_1 = 1$ and $c_2 = 0$

$\Rightarrow \boxed{r=1}$ and $\boxed{\theta = 5t}$

2) Sample calculation for euler method,

$\Delta t = 1.0$

$\dot{\theta} = \frac{5}{r^2} = 5$

$\Rightarrow \frac{\theta_2 - \theta_1}{t_2 - t_1} = 5 \Rightarrow \frac{\theta_2 - 0}{1.0} = 5$

$\Rightarrow \theta_2 = 5$ rad.

using this step several times we can get discrete values of θ of particle at discrete particular intervals.

3) Sample calculation for improved euler method.

$$\dot{\theta} = 5$$

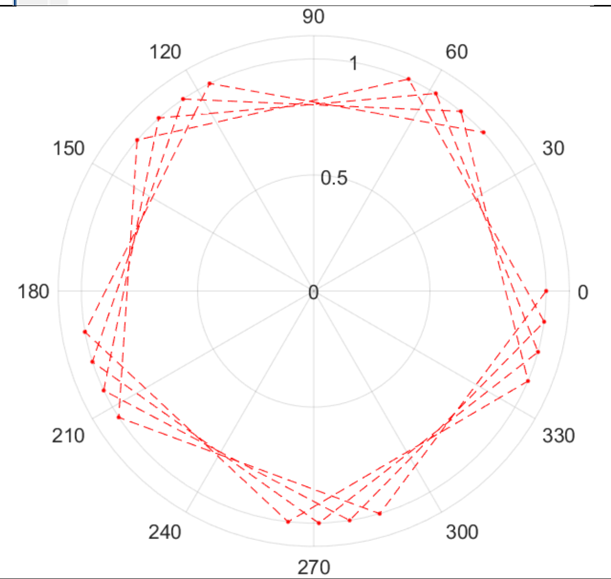
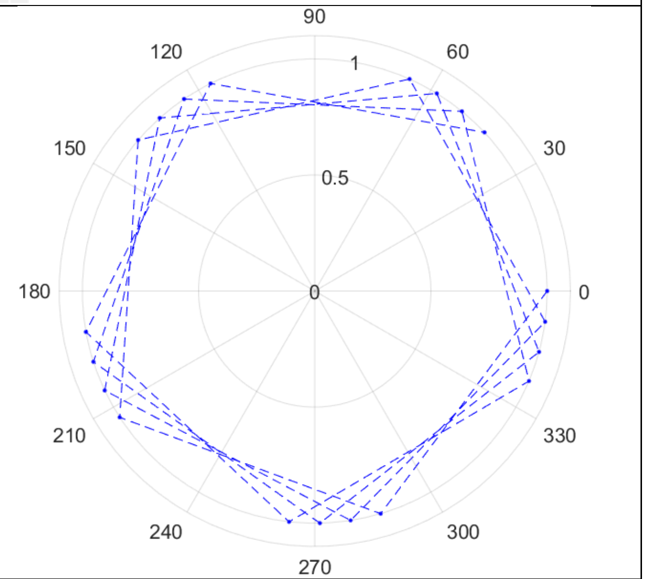
$$\frac{\theta_2' - \theta_1}{\Delta t} = 5 \Rightarrow \theta_2' = 5$$

$$\frac{\theta_2 - \theta_1}{\Delta t} = \frac{1}{2}(5+5) \Rightarrow 5$$

in short, improved euler methods and basic euler methods will give same results

	Basic Euler	Improved Euler

	Editor - E:\joshi\Downloads\polar.m*
	polar.m*
	`tf = 6*pi;`
	`dt = 1;`
	`t = 0:dt:tf;`
	`theta = 0;`
	`r = 1;`
	
	`for i = 1 : length(t)`
	`    theta(i+1) = theta(i) + 5*dt;`
	`    r(i+1) = 1;`
	`end`
	
	`polarplot(theta,r,'r--.')`
	`rlim([0 1.1])`
		Polar.m
		`tf = 6*pi;`
		`dt = 1;`
		`t = 0:dt:tf;`
		`theta = 0;`
		`r = 1;`
		
		`for i = 1 : length(t)`
		`    theta(i+1) = theta(i) + (1/2)*dt*(5+5);`
		`    r(i+1) = 1;`
		`end`
		
		`polarplot(theta,r,'b--.')`
		`rlim([0 1.1])`

dt = 1		

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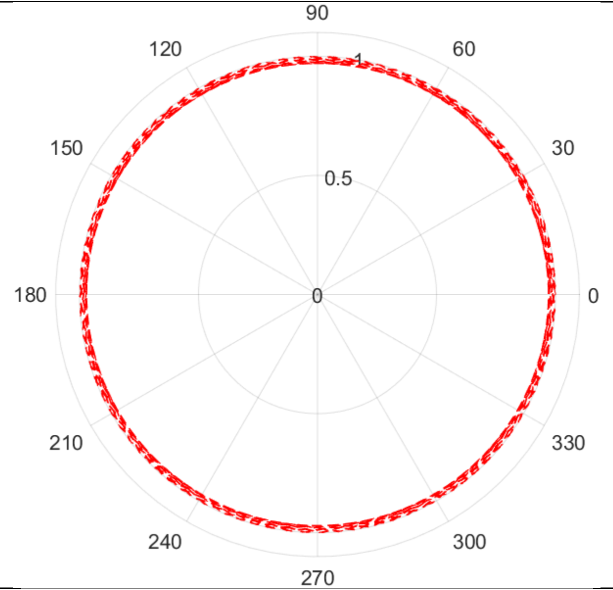

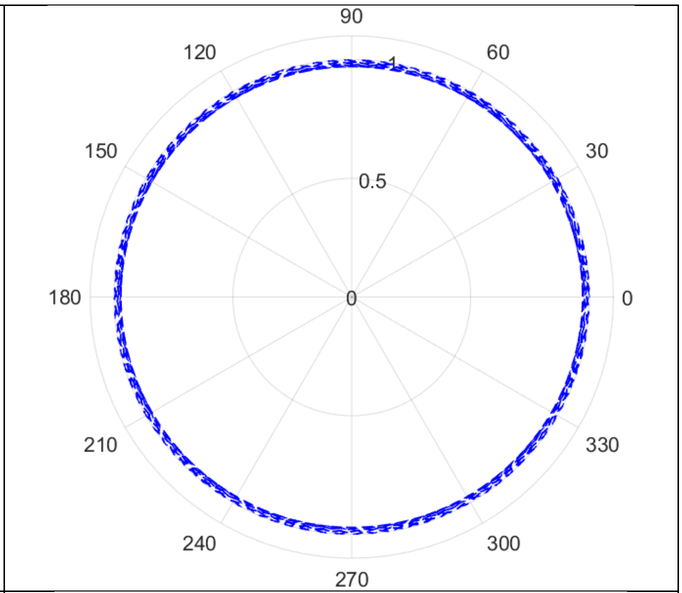

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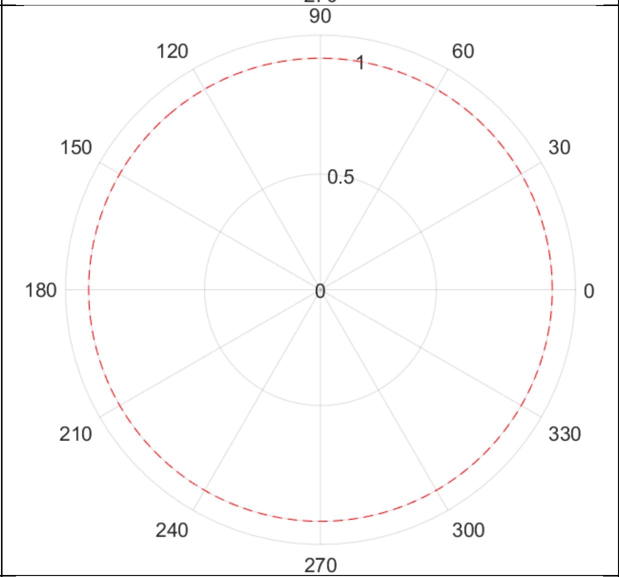

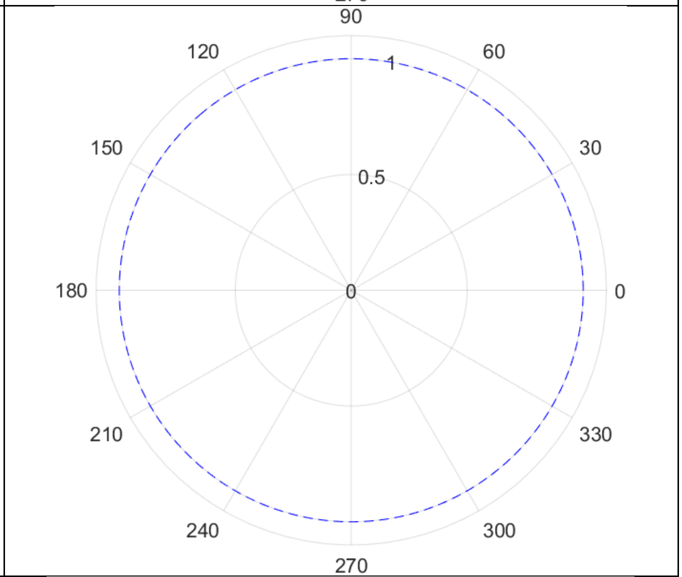

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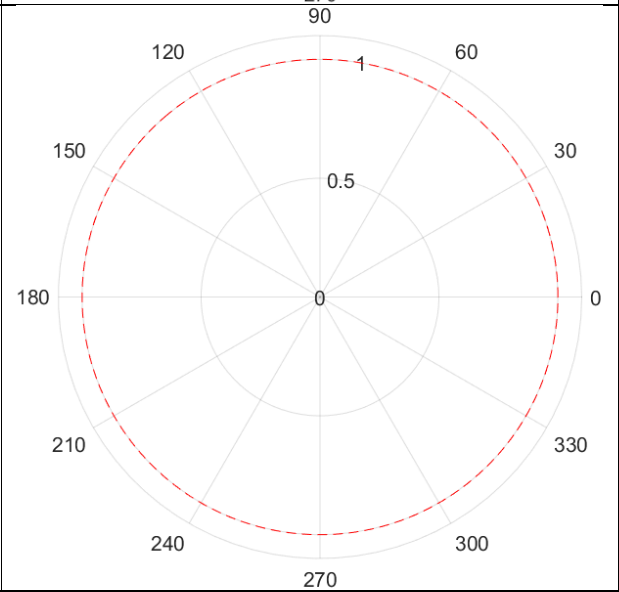
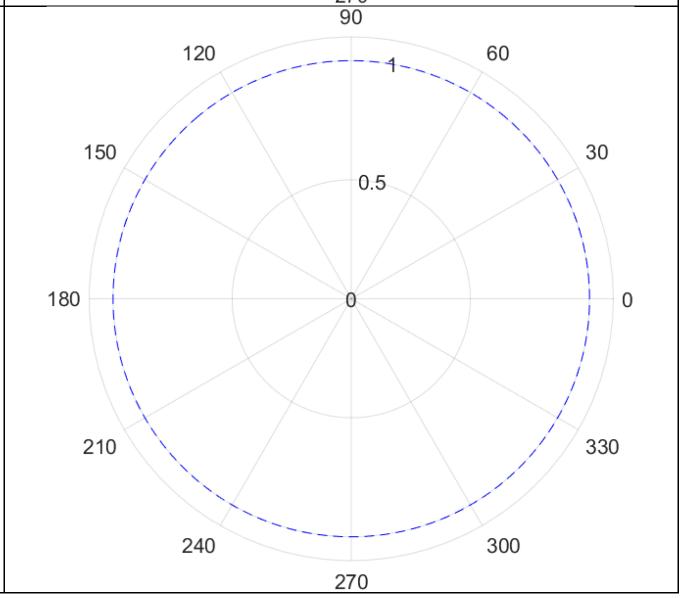




Comments:

1) Solution from Basic and Improved Euler method are same as angular position is linear in time.
2) As time step decreases accuracy of solution increases.

B.

c. $\vec{v} = v_r \hat{e}_r + v_\theta \hat{e}_\theta = v_r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + v_\theta(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$= v_x \hat{i} + v_y \hat{j}$

$$\Rightarrow \quad \dot{x} = \cancel{v_r}\cos\theta - v_\theta \sin\theta = -\frac{5}{r}\sin\theta = -\frac{5y}{r^2} \quad \text{— (1)}$$

$$\dot{y} = \cancel{v_r}\sin\theta + v_\theta\cos\theta = \frac{5}{r}\cos\theta = \frac{5x}{r^2} \quad \text{— (2)}$$

(with $v_r = 0$)

$$(2)/(1) \Rightarrow \frac{dy}{dx} = \frac{-x}{y} \Rightarrow x\,dx + y\,dy = 0 \Rightarrow x^2 + y^2 = c$$

at $t=0$, $x=1$ and $y=0$

$$\Rightarrow \underline{c = 1} \Rightarrow x^2 + y^2 = 1$$

using $r = \sqrt{x^2+y^2}$

$$\Rightarrow \boxed{\dot{x} = -5y} \text{ and } \boxed{\dot{y} = 5x}$$

sample calculation
using Basic Euler method

$$\frac{dx}{dt} = -5y \Rightarrow \frac{x_2 - x_1}{\Delta t} = -5y_1$$

$$\Rightarrow \frac{x_2 - 1}{1} = -5(0)$$

$$\Rightarrow x_2 = 1$$

sample calculation using Improved Euler method

$$\frac{dx}{dt} = -5y \qquad \frac{dy}{dt} = 5x$$

$$\frac{x_2' - x_1}{\Delta t} = -5y_1 \qquad \frac{y_2' - y_1}{\Delta t} = 5x$$

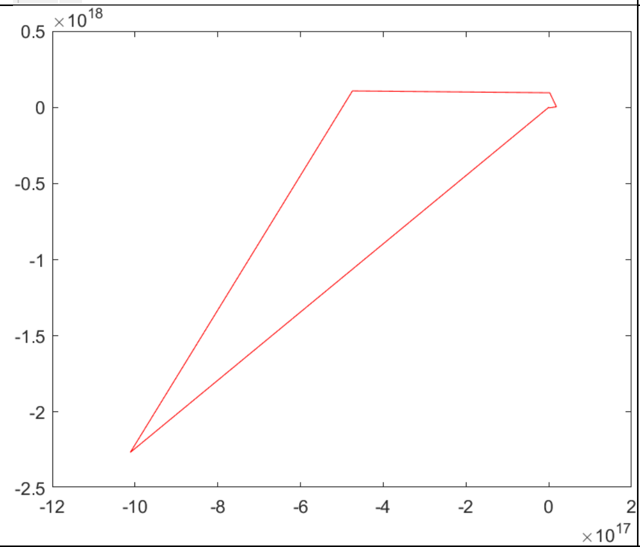
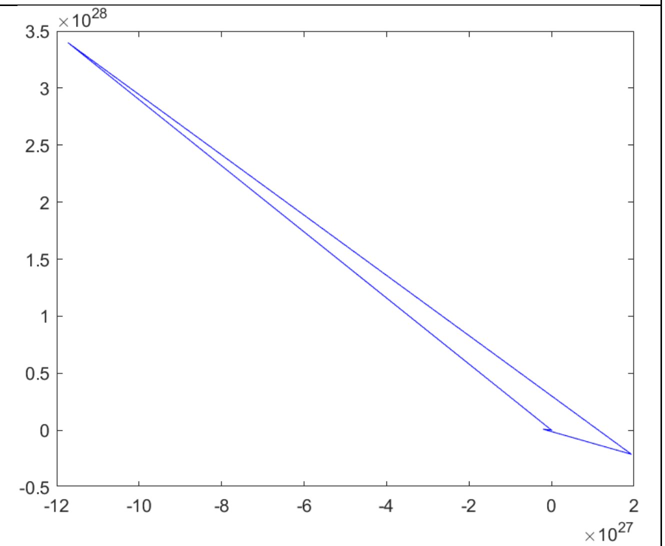
$$x_2' = 1 \qquad y_2' = 5(1)\times 1 + 0 = 5$$

$$\frac{x_2 - x_1}{\Delta t} = -5\left(\frac{y_1 + y_2'}{2}\right) \qquad \frac{y_2 - y_1}{\Delta t} = 5\left(\frac{x_1 + x_2'}{2}\right)$$

$$\Rightarrow x_2 = -11.5 \qquad \Rightarrow y_2 = 5$$

	Basic Euler	Improved Euler

	Editor - E:\joshi\Downloads\Cartesian.m
	Cartesian.m
	tf = 25;
	dt = 1;
	t = 0:dt:tf;
	x(1) = 1;
	y(1) = 0;
	
	for i = 1 : length(t)
	    x(i+1) = x(i) - dt*5*y(i);
	    y(i+1) = y(i) + dt*5*x(i);
	end
	
	plot(x,y,'r--.')
		Cartesian.m
		tf = 25;
		dt = 1;
		t = 0:dt:tf;
		x(1) = 1;
		y(1) = 0;
		
		for i = 1 : length(t)
		    xn(i) = x(i) - dt*5*y(i);
		    yn(i) = y(i) + dt*5*x(i);
		    x(i+1) = x(i) - (1/2)*dt*5*(y(i)+yn(i));
		    y(i+1) = y(i) + (1/2)*dt*5*(x(i)+xn(i));
		end
		
		plot(x,y,'b')

dt = 1		

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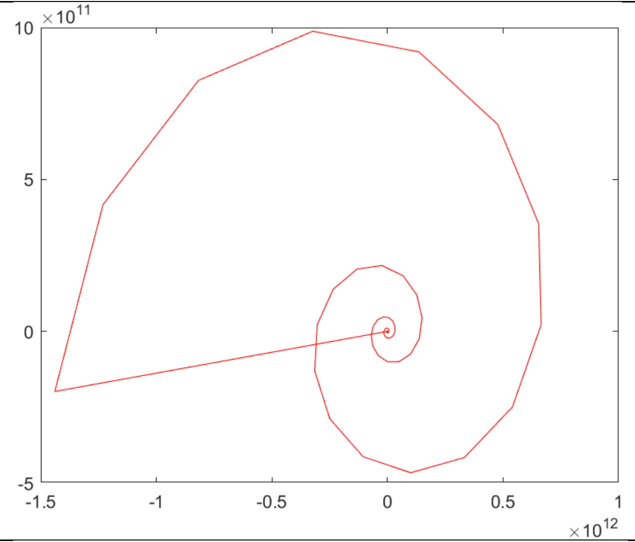
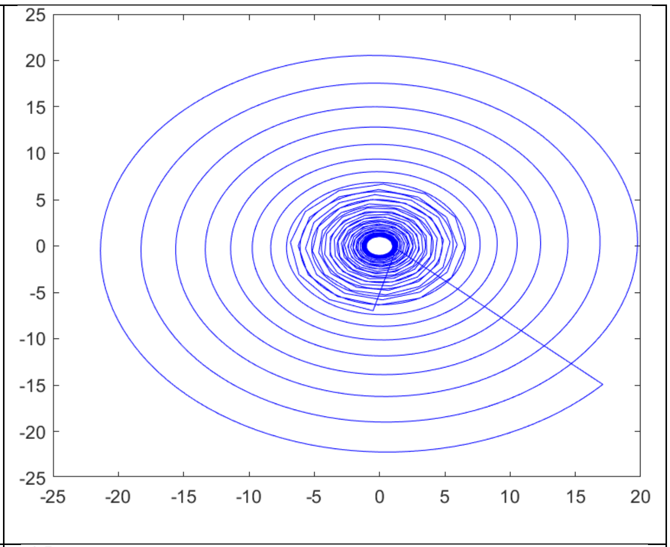
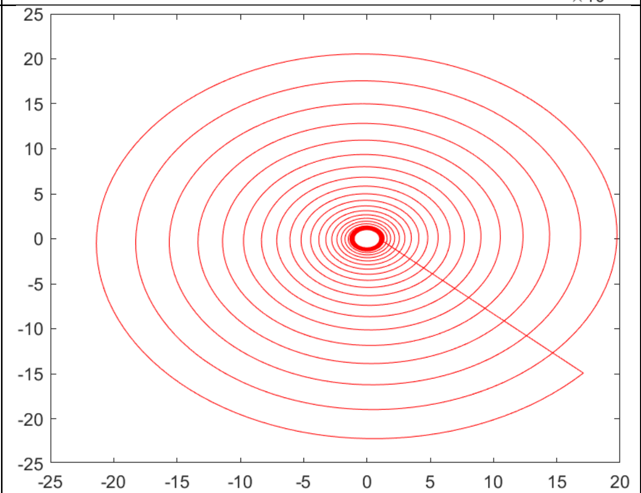
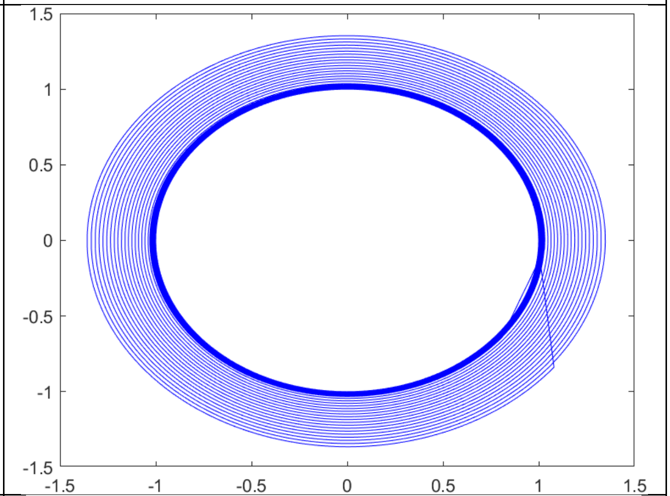


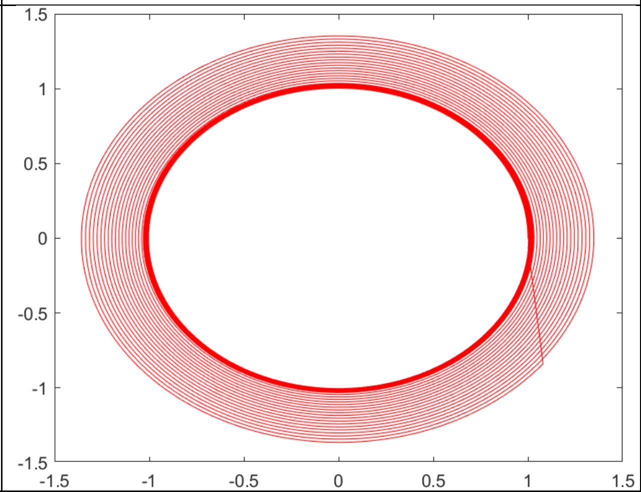


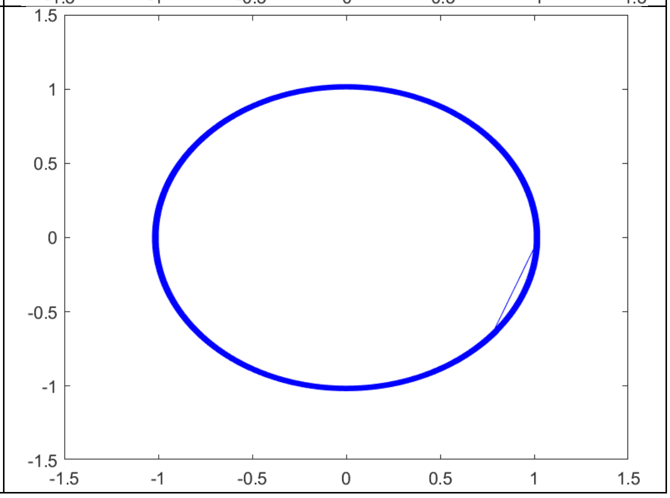
dt = 0.01





dt = 0.001





Comments:

1) Improved Euler method gives better results for fixed time step than Basic Euler method.
2) As time step decreases accuracy of solution increases and plot moves more closer to actual solution

C.

c.

In Gauss Sidel iterative method to solve equation

$$Ax = b$$

define iteration by

$$L_* x^{(k+1)} = b - Ux^{(k)}$$

where,

$$A = L_* + U \quad , \text{ and}$$

L_* is ~~strictly~~ lower and U is strictly upper triangular matrix.

So $Ax = b$ can be written as

$$L_* x = b - Ux$$

Gauss Sidel applied as

$$x^{(k+1)} = L_*^{-1} (b - Ux^{(k)})$$

as L_* is lower triangular matrix this equation can be written as

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

where $i = 1, 2, \ldots, n$

$n \to$ ~~rank~~ dim of matrix A

Example solved using Gauss Sidel

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}.$$

Iteration 1:

0.814724

0.905792

0.126987

Iteration 2:

0.061429

1.461091

-3.844363

Iteration 3:

2.922181

0.884727

-1.538909

Iteration 4:

1.769455

1.028818

-2.115273

Iteration 5:

2.057636

0.992795

-1.971182

Iteration 6:

1.985591

1.001801

-2.007205

Iteration 7:

2.003602

0.999550

-1.998199

Iteration 8:

1.999099

1.000113

-2.000450

Iteration 9:

2.000225

0.999972

-1.999887

Iteration 10:

1.999944

1.000007

-2.000028

The final answer obtained after 26 iterations is

x =

2.0000

1.0000

-2.0000