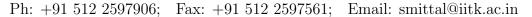
AE 618A: Finite Element Method for Fluid Dynamics

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## Assignment - 2

A. Consider the boundary value problem discussed in the class:

$$u_{,xx}(x) + f(x) = 0,$$
 on  $]0.1[,$  (1)

$$u(1) = g, (2)$$

$$-u_{,x}(0) = h$$
 (2)

Assume f(x) = qx where q is a constant and q = h = 0.

- 1. Find the exact solution.
- 2. Employing the linear finite element space with equally spaced nodes, set up and solve the Galerkin-Finite Element equations for n=1, 2, 3, 4, i.e. h=1, 1/2, 1/3, 1/4. In each case, you will need to calculate the stiffness matrix (K) and vector (F), and then carry out a solve for  $\mathbf{Kd} = \mathbf{F}$ .
- 3. Is the stiffness matrix banded? What is the consequence of boundary terms (g and h) on the bandedness?
- 4. Let  $re_{,x} = |u_{,x}^h u_{,x}|/(q/2)$  denote the relative error in  $u_{,x}$ . Compute  $re_{,x}$  at the midpoints of the 4 elements. They should be all equal.
- 5. Employing the data for h = 1, h = 1/2, h = 1/3 and h = 1/4 plot ln(re,x) versus ln(h).
- 6. What is the significance of the slope and the v-intercept?
- B. For the same problem, as described in part A, write a computer program to assemble the element level stiffness matrices and force vectors. Solve the matrix equation system using a linear equation solver (you can use a library or any other available program, for solving **A**  $\mathbf{x}=\mathbf{b}$ ). Plot the solution and the slope. Compute the relative error for n=10, n=50 and n=100. Calculate the slope as was done in the previous part. Comment on the results.