Instrumentation Report One

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1 Questions

1.1 Wheatstone Bridges

Capacitance can be measured using a Schering Bridge.

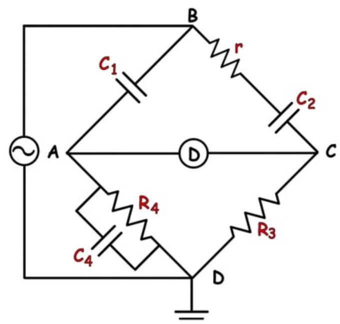

Figure 1: Schering Bridge Circuit

Assuming the bridge is balanced ($Z_{AB}Z_{CD} = Z_{AD}Z_{BC}$):

$$[\frac{1}{j\omega C_1}][R_3] = [R_4 || \frac{1}{j\omega C_4}][r + \frac{1}{j\omega C_2}] = [\frac{R_4}{1 + j\omega C_4 R_4}][r + \frac{1}{j\omega C_2}] \tag{1}$$

$$-j\frac{R_3}{\omega C_1} + \frac{R_3 R_4 C_4}{C_1} = rR_4 - j\frac{R_4}{\omega C_2} \tag{2}$$

Equating Real And Imaginary Parts results in the design equations for the Schering Bridge:

$$C_2 = \frac{R_4 C_1}{R_3} \quad \text{and} \quad r = \frac{R_3 C_4}{C_1} \tag{3}$$

1.2 Conservation Of Energy

Using Hooke's Law $F = kx$ where F is force, k is spring constant and x is displacement, the spring constant for the sensor can be calculated:

$$k = \frac{F}{x} = \frac{100}{10 \times 10^{-9}} = 10 \times 10^{9}\text{N/m} \tag{4}$$

The elastic potential energy at this displacement is then calculated:

$$E = \frac{1}{2}kx^2 = \frac{1}{2}(10 \times 10^{9})(10 \times 10^{-9})^2 = 500\text{nJ} \tag{5}$$

Since energy is conserved this must be equal to the energy stored in the sensor's capacitance, using $E = \frac{1}{2}CV^2$:

$$C = \frac{2E}{V^2} = \frac{2 \times 500 \times 10^{-9}}{10^2} = 10\text{nF} \tag{6}$$

This is the capacitance of the sensor.
At low frequencies ($f \ll 6$THz) the Johnson noise power spectral density is given by

$$V^2 = 4k_b TR \tag{7}$$

The RMS noise voltage is then given by:

$$V = \sqrt{V^2 B} = \sqrt{\frac{4k_b TR}{2\pi RC}} = \sqrt{\frac{2k_b T}{\pi C}} \tag{8}$$

1.3 The Quantum Hall Effect

The von Klitzing constant in conjunction with the Quantum Hall Effect are used to establish the SI standard for resistance.

When $R_L = 0$ this implies that the longitudinal voltage V_L also goes to zero, thus longitudinally the conductor becomes perfect. This implies that the charge will be uniformly distributed longitudinally inside the conductor since no electric field exists.

From the graph of the quantum hall plateau, it can be seen that the hall resistivity is constant and the longitudinal resistivity is zero on the plateux. Using the conductivity tensor the longditudinal conductivity is:

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{9}$$

From this equation it can be seen that if ρ_{xy} is non-zero as on the plateux and $\rho_{xx} = 0$, then σ_{xx} is also 0, thus longditudinally the material is a perfect conductor and insulator.

1.4 Quantum Technologies

Graphene Graphene is an allotrope of Carbon made of a single layer of atoms forming a hexagonal lattice. The theory of its electrical properties were first explored by P. R. Wallace in 1947. In 2004 Andre Geim and Kostya Novoselov at the University of Manchester found a way to easily extract layers of Graphene from Graphite.

The conduction and valance bands in Graphene meet at Dirac points making it a zero-gap semiconductor. The area around the Dirac points is special as the energy dispersion is linear. In most metals and semiconductors, the energy is quadratic in momentum. This means the charge carriers in Graphene are described by the Dirac equation instead of the Schrodinger equation. They act like particles of light in a vacuum rather than being affected by the material. They tunnel through the barrier without being reflected.

Graphene can be used to make extremely fast switching transistors that work for high frequencies and are unaffected by temperature. Bistable Graphene transistors have been made out of two sheets of Graphene and an insulating layer that involves quantum tunnelling electrons and exhibit negative differential conductance.

Tunnel Diodes The tunnel diode was invented in 1957 by Leo Esaki, Yiko Kurose, and Takashi Suzuki when they were working at Tokyo Tsushin Kogyo. Tunnel diodes are a diode in which the current decreases as a voltage flows through it. They consist of a heavily doped PN junction with doping concentrations so high that the depletion region very thin. In reverse bias, the filled states on the P side align with the empty states on the N side allowing electrons to quantum tunnel easily. When connected in forward bias electrons tunnel through the junction due to its narrow depletion region and fill electron states on the conduction band of the N side which align with empty valance band holes on the P side. As voltage increases, the states become maligned reducing the amount of electrons able to tunnel across the gap reducing the current. At a transition voltage, the electrons stop transitioning via tunnelling and cross using conductance like a normal diode.

Under reverse bias, the reverse breakdown voltage becomes zero and the diode becomes highly conductive. It becomes a back diode that can be used as a fast rectifier, or switch. Other uses include oscillators and amplifiers due to their negative differential resistance low capacitance that allows them be used at microwave frequencies. The resonant-tunnelling diode can even be used at terahertz frequencies.

2 Design Outline

2.1 Design Overview

The design of the complex impedance meter will be based on the Auto-Balancing Bridge method. This will allow the voltage and current vectors in the device under test to be measured, and thus the impedance determined.

2.2 Operating Principle

The Auto-Balancing Bridge operates on the principle of applying a voltage V_x to the device under test (DUT) and measuring the current flowing through it I_x. The current is measured using a current to voltage converter

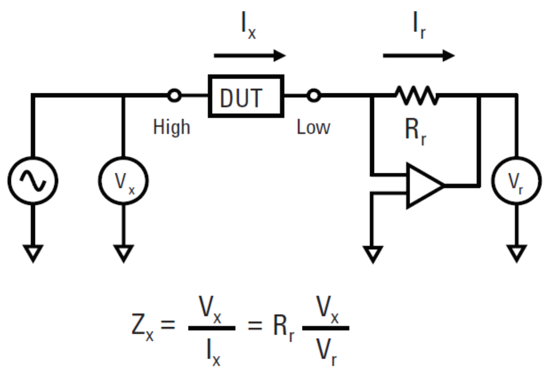

Figure 2: Auto-Balancing Bridge Schematic [1]

(transimpedance amplifier), this enables the 'Low' terminal of the DUT to be maintained at virtual ground. By measuring the amplitude and phases of the voltages V_x and V_r this will enable the impedance of the DUT to be calculated.

The complex impedance will be measured by taking the magnitude and phase of these voltages. The magnitude measurement will be performed by finding the envelope of the signals using an envelope detector. This will allow a direct measurement to be performed by the MCU ADC.

The phase difference between V_x and V_r will be measured by using a comparator to find the zero crossing of these signals and then timing the difference between the output of the comparator using timers on the MCU. Alternatively the real and imaginary components of the voltages could be found by multiplying by the excitation signal and the quadrature of this, and the amplitudes measured. These could then be used to calculate the complex value of impedence. Another option is that since the frequencies of the signals are known, the Goertzel algorithm could also be used to find the DFT of only those frequencies and then the phase calculated. This would be less computationally complex than running an FFT on the microcontroller. These methods will be evaluated and the most effective chosen.

2.3 Modes of Operation

The device will operate in the apply V, measure I mode. In order to encompass the full impedance range the feedback resistor on the transimpedance amplifier will be switchable by the MCU, using analog switches.

The signal source used to generate the test signal applied to the unknown device will consist of a digital direct synthesizer, this enables generation of precise frequencies. The synthesizer should then feed into appropriate signal conditioning, which will allow the user to control both the amplitude and DC offset of the signal sent to the DUT, as both of these parameters will affect the measured impedance.

The software will incorporate the means to resolve the measured complex impedance into a series or parallel RC circuit, controllable by the user.

2.4 Accuracy Estimates

Since the device will use separate the complex impedance measurement into a magnitude and phase measurement, the accuracy estimate can be split into magnitude accuracy and phase accuracy.

2.4.1 Magnitude Accuracy

ADC Resolution The maximum overall accuracy of the system is determined by the MCU ADC resolution. Since the MCU has a 12-bit ADC this will correspond to maximum resolution of 1.22mV per bit (assuming a 5V supply is used and this is divided down to 0-2.5V for the full range of the ADC).

[1] *Agilent Impedance Measurement Handbook.* Aglient Technologies, 4th Edition.

Test Frequency	degrees per timer tick
1kHz	0.002
10kHz	0.02
100kHz	0.2
1MHz	2

Table 1: Phase error at different test frequencies.

Op-Amp offsets/bias The op amps used in the signal conditioning prior to the signal conditioning may also introduce errors, due to their input bias/offsets. This error however can be compensated using an open/short calibration procedure. Therefore the op-amp errors can be reduced to errors due to finite gain bandwidth and error due to $\frac{1}{f}$ noise. For a typical non auto zero op amp such as the OP177 this error can be around 9ppm at room temperature and lower error can be achieved using chopper stabilised op amps.

Envelope Detector Offset Since a conventional envelope detector makes use of a diode, this will introduce an offset to the signal of 0.7V, however this can be combatted using a precision envelope detector circuit, which prevents this offset by making use of an op amp.

RFI rectification Any offsets due to DC rectification of RF signals can be negated by applying RF filter circuits at the input of instrumentation and op amps used in the project. RF can also be minimised through the PCB layout by making use of ground planes and separating digital and analog supplies and grounds.

2.4.2 Phase Accuracy

The accuracy of the phase measurement will be dependant on the resolution of the timers used. Since the timers in the MCU are 32 bit, they need to be able to time the signal to +180 degrees. If the timer is used without a prescaler the timer tick value corresponds to 5.56ns. This means the maximum value of the timer before a rollover is approximately 23s. Therefore for all test frequencies the timer can be operated without a prescaler. The value of one timer tick in degrees for each test frequency is the approximate error. This is shown in table 1. This shows using the timer the full spec can be achieved at frequencies lower than 1MHz.

The signal conditioning circuitry used on the current and voltage waveforms will also introduce errors, however if the signal conditioning for both is suitably matched, then this will effectively cancel out.

2.5 System Block Diagram

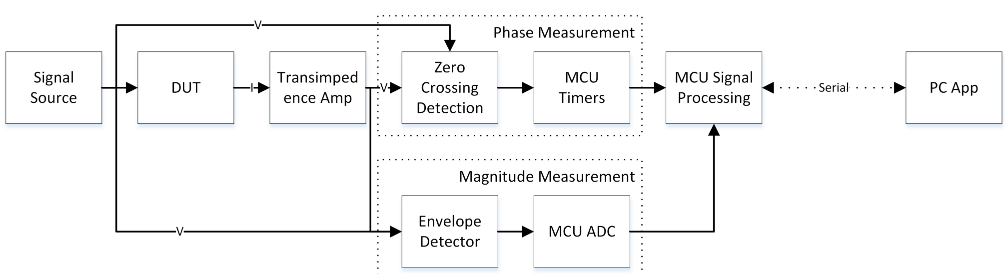

Figure 3: Block Diagram of Complex Impedance Meter

2.6 Schematic

A general schematic for the auto balancing bridge circuit is shown in figure 2.