COSI 101A PS3
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16.5 The Surprise Candy Company makes candy in two flavors: 70% are strawberry flavor and 30% are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves along the production line, a machine randomly selects a certain percentage to be trimmed into a square; then, each piece is wrapped in a wrapper whose color is chosen randomly to be red or brown. 80% of the strawberry candies are round and 80% have a red wrapper, while 90% of the anchovy candies are square and 90% have a brown wrapper. All candies are sold individually in sealed, identical, black boxes. Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box. Consider the three Bayes nets:

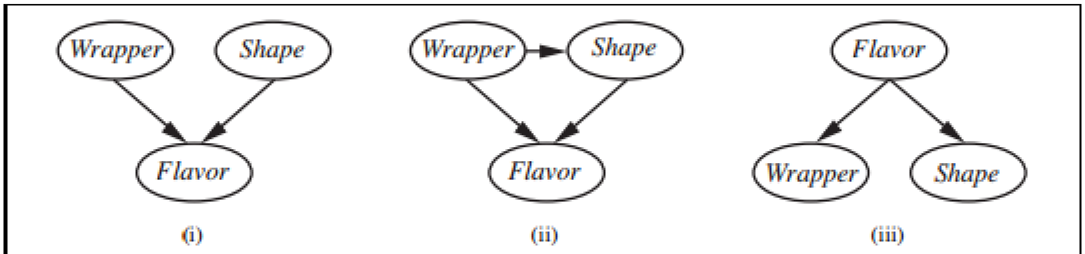


Figure 16.11 Three proposed Bayes nets for the Surprise Candy problem, Exercise 16.5.

a. Which network(s) can correctly represent P(Flavor, Wrapper, Shape)?

Network (ii) and (iii) correctly represent P(Flavor, Wrapper, Shape).

(ii) can't be wrong because it included all possible dependences that can happen under the circumstances described in the question.

(iii) is correct because we have all flavor, wrapper, and shape correlated, which fits the description of the question.

(i) is wrong because wrapper and shape are independent. However, as we know "80% of the strawberry candies are round and 80% have a red wrapper, while 90% of the anchovy candies are square and 90% have a brown wrapper", both events are dependent.

b. Which network is the best representation for this problem?

Network (iii) best represents the problem. Both (ii) and (iii) are correct, and (ii) is a simpler network compare to network (ii).

c. Does network (i) assert that P(Wrapper| Shape) = P(Wrapper)?

Yes, it asserts wrapper and shape are independent. Therefore, by Bayes rules, we can ignore the condition Shape. The assertion that P(Wrapper| Shape) = P(Wrapper) is true under the circumstances.

d. What is the probability that your candy has a red wrapper?

Let W be the abbreviation for Wrapper. Let R be the event that the wrapper is red.

Let F be the abbreviation for Flavor. Let St be the event that the flavor is strawberry. Let An be the event that the flavor is anchovy.

$$P(W = R) = P(F = \mathrm{St})P(W = R|F = St) + P(F = \mathrm{An})P(W = R|F = An) =$$
$$(0.70)(0.80) + (0.30)(1 - 0.90) = 0.59$$

e. In the box is a round candy with a red wrapper. What is the probability that its flavor is strawberry?

From the description of the problem, we need to find $P(Flavor = Strawberry|Wrapper = Red, Shape = Round)$. By Bayes rule:

$$P(Strawberry|Red, Round) = \frac{P(Strawberry, Red, Round)}{P(Red, Round)}$$
$$= \frac{P(Strawberry, Red, Round)}{P(Strawberry, Red, Round) + P(\text{anchovy}, Red, Round)}$$

Since flavor and shape are independent given flavor, we have:

$$= \frac{(0.70)(0.80)(0.80)}{(0.70)(0.80)(0.80) + (0.30)(0.10)(0.10)} = 0.993$$

f. A unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the value of an unopened candy box.

We need to find the EMV of the box of candies:

Assuming there are n number of candies in a box, then we have (for a single candy):

$$EMV = \frac{7s + 3a}{10}$$

(for multiple candies):

$$EMV = \sum_{i=1}^{n} \frac{7s + 3a}{10} = \frac{7sn + 3an}{10}$$

g. A new law prohibits trading of unwrapped candies, but it is still legal to trade wrapped candies (out of the box). Is an unopened candy box now worth more than less than, or the same as before?

It should be the same, because the EMV and EU of a wrapped candy are not changed even if the candy itself is unwrapped. The expected value, disregard if the candy is unwrapped or not, will still be $EMV = \frac{7s+3a}{10}$.

16.8 Tickets to a lottery cost \$1. There are two possible prizes: a \$10 payoff with probability 1/50, and a \$1,000,000 payoff with probability 1/2,000,000. What is the expected monetary value of a lottery ticket? When (if ever) is it rational to buy a ticket? Be precise—show an equation involving utilities. You may assume current wealth of \$k and that $U(S_k)=0$. You may also assume that $U(S_{k+10}) = 10 \times U(S_{k+1})$, but you may not make any assumptions about $U(S_{k+1,000,000})$. Sociological studies show that people with lower income buy a disproportionate number of lottery tickets. Do you think this is because they are worse decision makers or because they have a different utility function? Consider the value of contemplating the possibility of winning the lottery versus the value of contemplating becoming an action hero while watching an adventure movie.

Answer:

First, we find the EMV of the problem:

$$EMV = \frac{1}{50}(10) + \frac{1}{2{,}000{,}000}(1{,}000{,}000) = \frac{1{,}400{,}000}{2{,}000{,}000} = 0.70$$

For the second part, we use the expected utility function:

As the problem suggested, we can assume the following:

We assume current wealth of \$k and that $U(S_k)=0$, and assume that $U(S_{k+10}) = 10 \times U(S_{k+1})$,

Then we have:

$$EU(no\ ticket) = (1.0)U(S_{k+1})$$

$$EU(ticket) = \left(\frac{1}{50}\right)U(S_{k+10}) + \left(\frac{1}{2{,}000{,}000}\right)U(S_{k+1{,}000{,}000})$$

To be rational to buy a ticket, we need to find the situation when $EU(no\ ticket) < EU(ticket)$.

So, we let:

$$(1.0)U(S_{k+1}) < \left(\frac{1}{50}\right)U(S_{k+10}) + \left(\frac{1}{2{,}000{,}000}\right)U\left(S_{k+1{,}000{,}000}\right) = \left(\frac{1}{5}\right)U(S_{k+1}) + \left(\frac{1}{2{,}000{,}000}\right)U\left(S_{k+1{,}000{,}000}\right)$$

$$\left(\frac{4}{5}\right)U(S_{k+1}) < \left(\frac{1}{2{,}000{,}000}\right)U\left(S_{k+1{,}000{,}000}\right)$$

$$\left(\frac{4\times 2{,}000{,}000}{5}\right)U(S_{k+1}) = (1{,}600{,}000)U(S_{k+1}) < U\left(S_{k+1{,}000{,}000}\right)$$

Therefore, when the utility for winning 1,000,000 dollars is 1,600,000 times higher than not getting 1 dollar, i.e. $U\left(S_{k+1{,}000{,}000}\right) > (1{,}600{,}000)U(S_{k+1})$, it is reasonable to buy a ticket.

The possible reason that lower income buy a disproportionate number of lottery tickets is that they don't have too many chances to be in contact with a dramatic number of money. Therefore, the thrill of winning 1,000,000 is much higher than just the monetary value of 1,000,000 dollars. As for the value of contemplating the possibility of winning the lottery versus the value of contemplating becoming an action hero while watching an adventure movie, the possibility of paying 1 dollar for 1,000,000 is much bigger than the possibility of becoming an action hero, so people tend to buy lotteries.

16.12 Economists often make use of an exponential utility function for money: U(x) = $-e^{-x/R}$, where R is a positive constant representing an individual's risk tolerance. Risk tolerance reflects how likely an individual is to accept a lottery with a particular expected monetary value (EMV) versus some certain payoff. As R (which is measured in the same units as x) becomes larger, the individual becomes less risk-averse.

a. Assume Mary has an exponential utility function with R = \$500. Mary is given the choice between receiving \$500 with certainty (probability 1) or participating in a lottery which has a 60% probability of winning \$5000 and a 40% probability of winning nothing. Assuming Marry acts rationally, which option would she choose? Show how you derived your answer.

First, we find the EU of \$500: $EU(no\ lottery\ ) = EU(500) = -e^{-\frac{x}{R}} = -e^{-\frac{500}{500}} = -0.37$

Second, we find EU of participate:

$$EU(lottery\ ) = \frac{3}{5}EU(5{,}000) + \frac{2}{5}EU(0) = -\frac{3}{5}e^{-\frac{5000}{500}} - \frac{2}{5}e^{-\frac{0}{500}} = -\frac{3}{5e^{10}} - \frac{2}{5} = -\frac{2}{5} = -0.40$$

We know $EU(no\ lottery\ ) = -0.37 > -0.40 = EU(lottery\ )$, so we tend to choose taking the 500 dollars.

b. Consider the choice between receiving \$100 with certainty (probability 1) or participating in a lottery, which has a 50% probability of winning \$500 and a 50% probability of winning nothing. Approximate the value of R (to 3 significant digits) in an exponential utility function that would cause an individual to be indifferent to these two alternatives. (You might find it helpful to write a short program to help you solve this problem.)

First, we find the EU of \$500: $EU(no\ lottery\ ) = EU(100) = -e^{-\frac{100}{R}}$

Second, we find the EMV for participating the lottery:

$$EU(lottery\ ) = \frac{1}{2}EU(500) + \frac{1}{2}EU(0) = -\frac{1}{2}e^{-\frac{500}{R}} - \frac{1}{2}e^{-\frac{0}{R}} = -\frac{1}{2}e^{-\frac{500}{R}} - \frac{1}{2}$$

So, we need to find $-e^{-\frac{100}{R}} = -\frac{1}{2}e^{-\frac{500}{R}} - \frac{1}{2}$, or $e^{-\frac{100}{R}} - \frac{1}{2}e^{-\frac{500}{R}} = \frac{1}{2}$, and we only need to solve that equation for R. By a program we know that $e^{-\frac{100}{R}} = 0.51879$,

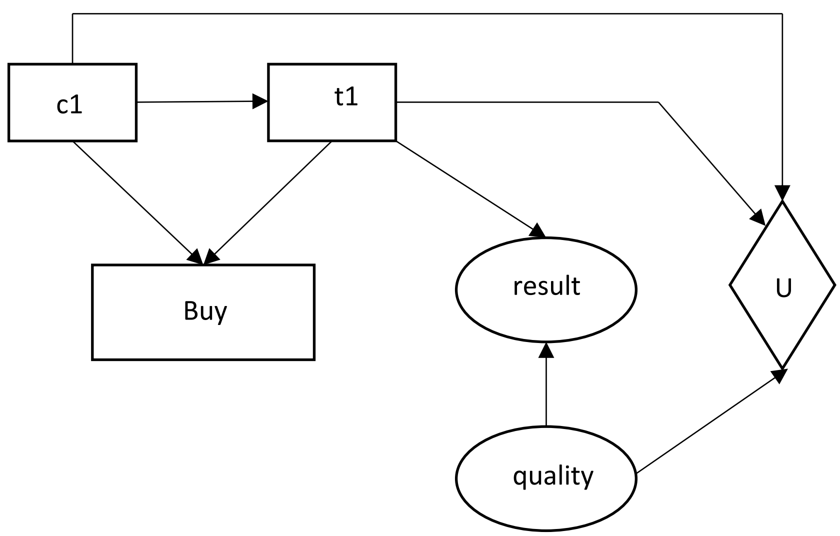
and $R = -\frac{100}{\ln(0.51879)} = 152.380$, so when R equals 152.380, the decisions are indifferent.

16.17 (Adapted from Pearl (1988).) A used-car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the tests, decide which car to buy. We will assume that the buyer is deciding whether to buy car c1, that there is time to carry out at most one test, and that t1 is the test of c1 and costs $50.

A car can be in good shape (quality q+) or bad shape (quality q−), and the tests might help indicate what shape the car is in. Car c1 costs $1,500, and its market value is $2,000 if it is in good shape; if not, $700 in repairs will be needed to make it in good shape. The buyer's estimate is that c1 has a 70% chance of being in good shape.

a. Draw the decision network that represents this problem.

c1 means the car 1 and how much the use-car buyer likes it; t1 means the test, which together with the quality of the car, produces the result. The buyer can also buy the car with or without test 1.



b. Calculate the expected net gain from buying c1, given no test.

We find the EMV of the used car:

$$EMV(buy) = \frac{7}{10}(500) + \frac{3}{10}(-200) = 290$$

c. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

P(pass(c1, t1)|q+(c1)) = 0.8

P(pass(c1, t1)|q−(c1)) = 0.35

Use Bayes' theorem to calculate the probability that the car will pass (or fail) its test and hence the probability that it is in good (or bad) shape given each possible test outcome.

For q+:

$$P(pass(c1,t1)|q+(c1)) = 0.8 = \frac{P(pass(c1,t1),q+(c1))}{P(q+(c1))} = \frac{P(pass(c1,t1),q+(c1))}{0.7}$$

$$P(pass(c1,t1),q+(c1)) = (0.8)(0.7) = 0.56$$

$$P(fail(c1,t1)|q+(c1)) = 1-0.8 = 0.2 = \frac{P(fail(c1,t1),q+(c1))}{0.7}$$

$$P(fail(c1,t1),q+(c1)) = (0.2)(0.7) = 0.14$$

For q-:

$$P(pass(c1,t1)|q-(c1)) = 0.35 = \frac{P(pass(c1,t1),q-(c1))}{P(q+(c1))} = \frac{P(pass(c1,t1),q-(c1))}{0.3}$$

$$P(pass(c1,t1),q-(c1)) = (0.35)(0.3) = 0.105$$

$$P(fail(c1,t1)|q-(c1)) = 1-0.35 = 0.65 = \frac{P(fail(c1,t1),q-(c1))}{0.3}$$

$$P(fail(c1,t1),q-(c1)) = (0.65)(0.3) = 0.195$$

Therefor we given the probability that it is in good shape:

$$P\big(q+(c1)|pass(c1,t1)\big) = \frac{P\big(q+(c1),pass(c1,t1)\big)}{P\big(q+(c1),pass(c1,t1)\big)+P\big(q-(c1),pass(c1,t1)\big)} = \frac{0.56}{0.105+0.56} = 0.8421$$

$$P\big(q-(c1)|pass(c1,t1)\big) = \frac{P\big(q-(c1),pass(c1,t1)\big)}{P\big(q+(c1),pass(c1,t1)\big)+P\big(q-(c1),pass(c1,t1)\big)} = \frac{0.105}{0.105+0.56} = 0.1579$$

$$P\big(q+(c1)|fail(c1,t1)\big) = \frac{P\big(q+(c1),fail(c1,t1)\big)}{P\big(q+(c1),fail(c1,t1)\big)+P\big(q-(c1),fail(c1,t1)\big)} = \frac{0.14}{0.195+0.14} = 0.4179$$

$$P\big(q-(c1)|fail(c1,t1)\big) = \frac{P\big(q-(c1),fail(c1,t1)\big)}{P\big(q+(c1),fail(c1,t1)\big)+P\big(q-(c1),fail(c1,t1)\big)} = \frac{0.195}{0.195+0.14} = 0.5821$$

d. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

We calculate the EU without specifying it first:

For the case if the car passes the test:

$$EU(buy|pass) = (0.8421)EU(2000-1550) + (0.1579)EU(2000-1550-700)$$

$EU(buy|pass) = (0.8421)EU(450) + (0.1579)EU(-250)$

For the case if the car fails the test:

$EU(buy|not\ pass) = (0.4179)EU(2000 - 1500 - 50) + (0.5821)EU(2000 - 1500 - 50 - 700)$

$EU(buy|not\ pass) = (0.4179)EU(450) + (0.5821)EU(-250)$

For the case if we decide not to buy:

$EU(not\ buy) = EU(-50)$

In the question, no such Utility function is given. Therefore, we assume that we are described as a complete rational people and the Utility function is U(x)=x.

Then we would have:

$EU(buy|pass) = (0.8421)(450) + (0.1579)(-250) = 378.945 - 39.475 = 339.47$

$EU(buy|not\ pass) = (0.4179)(450) + (0.5821)(-250) = 188.055 - 145.525 = 42.53$

$EU(not\ buy) = -50$

e. Calculate the value of information of the test, and derive an optimal conditional plan for the buyer.

If we buy the car none the less:

$EU(buy\ anyway) = \frac{7}{10}EU(500) + \frac{3}{10}EU(-200)$

If we buy the car anyway after the test (despite the test result), then the test doesn't make any sense. In this case, we'd rather save the 50 dollars for the test. Thus, we only consider the situation where we won't buy if we didn't pass the test

$EU(buy\ w/\ test) = (0.665)(buy|pass) + (0.335)(not\ buy|not\ pass)$

$= (0.665)\big((0.8421)EU(450) + (0.1579)EU(-250)\big) + (0.335)EU(-50)$

$= (0.56)EU(450) + (0.105)EU(-250) + (0.335)EU(-50)$

Then, we only need to compare the two equations:

$(0.56)EU(450) + (0.105)EU(-250) + (0.335)EU(-50)$

with:

$(0.7)EU(500) + (0.3)EU(-200)$

and compare which one gives a bigger value.

Again, if we assume that we are described as a complete rational people and the Utility function is U(x)=x.

If we buy without test:

$$EU(buy\ anyway) = \frac{7}{10}(500) + \frac{3}{10}(-200) = 290$$

If we buy anyway after the test:

$$EU(buy\ w/\ test) = (0.665)(339.47) + (0.335)(42.53) = 225.747 + 14.248 = 239.995$$

If we use the strategy and decide base on the result:

$$EU(buy\ depends) = (0.665)(339.47) + (0.335)(-50) = 225.747 - 16.75 = 208.997$$

Therefore, if we decide the U(x)=x, the best choice is just to buy the car anyway.