3: Runtime Analysis

a)
```
void f1(int n) {
    create int
    while (i < n) {
        something Θ(1)
        i = i * j
    }
}
```

$\sum_{i}^{n} \Theta(\frac{1}{i}) = \Theta(\log n)$

b)
```
void f2(int n)
    for (i: 1 → n)
        if (i % √n == 0)
            for (k: 0 → i^3)
                runs 1 Θ(1) something
```
triggers $\sqrt{n}$ times; $\Theta(i^3)$

$\sum_{i}^{n} [\Theta(1) + O(\sum_{k}^{i^3} \Theta(1))]$

$n = 16$

i	i % √n = 4
1	1
2	2
3	3
4	0
5	1
6	2
7	3
8	0
12	0
16	0

$n = 2$: $\sqrt{n} = 1$; $1 \% 2 = 1$; $2 \% 2 = 0$

$n = 9$

i	i % 3 =
1	1
2	2
3	0
4	1
5	2
6	0
7	1
8	2
9	0

$\frac{n}{\sqrt{n}} = \sqrt{n}$

$i = \sqrt{n}, 2\sqrt{n}, 3\sqrt{n}$

$i = \sqrt{n} \cdot j$

$\sum_{i}^{n} [\Theta(1) + O(\sum_{k}^{i^3} \Theta(1))]$

$= \sum_{i}^{n} [\Theta(1)] + \sum_{i} \sum_{k}^{i^3} \Theta(1)$

$= \Theta(n) + \sum_{j}^{\sqrt{n}} \sum_{k}^{j^3 n^{3/2}} \Theta(1)$

$= \Theta(n) + \sum_{j}^{\sqrt{n}} \Theta(j^3 n^{3/2})$

$= \Theta(n) + \Theta(n^{3/2} (\sqrt{n})^4)$

$= \Theta(n^{7/2})$

$i = 1$

c)
```
for (i: 1 → n)
    for (k: 1 → n)
        if (A[k] == i)
            for (m: 1 → n, m *= 2)
                O(1)
```
can happen at most n times

$A = \{1, 1, \ldots, 1\}$ $\quad A = \{1, 2, 3, \ldots, n\}$

$m=1 \quad m=2 \quad m=4 \quad \ldots \quad m=n$

$O(1) + O(1) + O(1) + \ldots O(1) = \sum_{1}^{\log n} O(1)$

$= O(\log n)$

$\sum_{i}^{n} \sum_{k}^{n} [\Theta(1) + O(\Theta(\log n))]$

$= \sum_{i}^{n} [\Theta(n)] + \sum_{k}^{n} \Theta(\log n) = \Theta(n^2) + \Theta(n \log n)$

$= \Theta(n^2)$

d) f(n)

```
new int a[10]
int size = 10
for ( i:=0 -> n )
    if ( i == size)
        newsize = 3/2 size
        b = new int[newsize]
        for ( j = 0 -> size)
            b[j] = a[j]        Θ(1)
        delete [] a
        a = b
        size = newsize
    a[i] = i^2
```

10	1
15	2

triggers

$10(\frac{3}{2})^k = n$

$k = \log_{\frac{3}{2}}(\frac{n}{10})$ times

$\sum_j^{size} \Theta(1) = \Theta(size)$

$$\sum_i^n [\Theta(1) + \sum_i \Theta(size)]$$

$$= \sum_i^n [\Theta(1)] + \sum_k^{\log_{\frac{3}{2}}(\frac{n}{10})} \Theta(10(\tfrac{3}{2})^k)$$

$$= \Theta(n) + \Theta(\tfrac{3}{2}^{\log_{\frac{3}{2}}(\frac{n}{10})})$$

$$= \Theta(n) + \Theta(n)$$

$$= \Theta(n)$$