Tutorial 2: First-Order ODEs (Part II)

[Attempt all questions before tutorial session]

A) **Bernoulli's Equation**

Solve

$$2x(\ln x)y' - y = -9x^3 y^3 \ln x.$$

B) **Exact DE**

Determine if the following equations are exact. If so, find its general solution

a) $\left(3x^2 y - 2y^2\right) dx + \left(x^3 - 4xy + 6y^2\right) dy = 0,$

b) $(\sin(xy) + xy\cos(xy) + 2x)dx + (x^2\cos(xy) + 2y)dy = 0,$

c) $x^2 y\, dx - (xy^2 + y^3)dy = 0.$

C) Determine an integrating factor for the equation

$$y\, dx - (2x + y^4)dy = 0,$$

and hence find the general solution.

D) Determine whether the given function $I(x, y) = \cos(xy)$, is an integrating factor for the DE

$$\left[\tan(xy) + xy\right]dx + x^2 dy = 0.$$

If so, find its general solution.

E) **Homogeneous DE**

Solve

$$y' - x^{-1}y = x^{-1}\sqrt{x^2 - y^2}, \quad x > 0.$$

F) Solve the following equations by (i) Bernoulli's technique or (ii) finding a suitable integrating factor and then converting it to an exact equation:

$$\frac{dy}{dx} = -\frac{8x^5 + 3y^4}{4xy^3}.$$