Innlevering 1, Matematikk 3

1a) $(\sqrt{3}+i)(1-i)$ $z(a,b) = (\sqrt{3}, 1)$
$\quad\; z \qquad\; w \qquad w(a,b) = (1, 1)$

$r_z = |z| = \sqrt{(\sqrt{3})^2+1^2} = 2, \; \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$z = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) = 2e^{i\frac{\pi}{6}}$

$w = (r_w = \sqrt{2}), \; \theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$w = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}e^{i\frac{\pi}{4}}$

$z \cdot w = 2\sqrt{2}e^{i(\frac{\pi}{6}+\frac{\pi}{4})} = \underline{\underline{2\sqrt{2}e^{i\frac{5\pi}{12}}}}$

b) $\frac{z}{w} = \frac{2e^{i\frac{\pi}{6}}}{\sqrt{2}e^{i\frac{\pi}{4}}} = \frac{2}{\sqrt{2}}e^{i(\frac{\pi}{6}-\frac{\pi}{4})} = \underline{\underline{\sqrt{2}e^{-i\frac{\pi}{12}}}}$

2a) $z = \frac{3\pi}{4}i \qquad w = -\frac{3\pi}{4}i$

$e^z - e^w = e^{\frac{3\pi}{4}i} - e^{-\frac{3\pi}{4}i} =$ a for $z = r\cos(\theta) = 1\cdot\cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$
b for $z = r\sin(\theta) = \sin(\theta) = \frac{\sqrt{2}}{2}$

$z = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ $\qquad$ a for $w = \cos(-\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$
b for $w = \sin(-\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$

$w = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

~~$z - w = a - c + (b+d)i \to -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2})i$~~

~~$= \sqrt{2} + \sqrt{2}i$~~

~~$\sqrt{2}i$ i polar form: $r = \sqrt{2}, \theta = \tan^{-1}\left(\frac{0}{\sqrt{2}}\right) = 0$~~

$z - w = a - c + (b+d)i \to (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) + (\sqrt{2})i$

$= \sqrt{2}i$

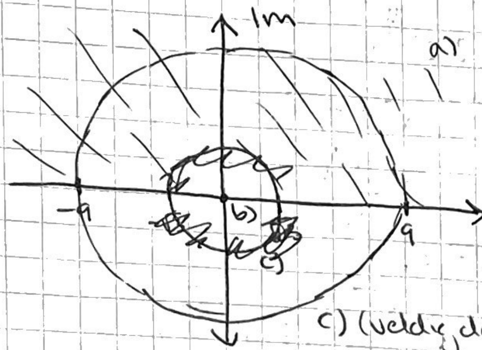
$r = \sqrt{(\sqrt{2})^2 + 0^2} = \sqrt{2}, \; \theta = \arctan\left(\frac{\sqrt{2}}{0}\right) = \frac{\pi}{2}$

$= \underline{\underline{\sqrt{2}(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})}}$

b) $\frac{e^z}{e^w} \rightarrow \frac{1}{1} e^{i\left(\frac{3\pi}{4} - \left(-\frac{3\pi}{4}\right)\right)} = \underline{\underline{e^{i\frac{3\pi}{2}}}}$

$= \underline{\underline{\cos\left(\frac{3\pi}{2}\right) + i\sin\frac{3\pi}{2}}}$

3)



a) $\text{Im}\, z > 0$ gjelder for alle punkter der $\text{Im}\, z$ er positiv, altså hele øvre halvdel av koordinat-systemet.

c) (veldig dårlig sirkel)

b) $z^2 = 0$, $a+bi = 0$ vil bety at a og b er begge null

om $z = re^{i\theta} \rightarrow z^2 = r^2 e^{2i\theta} = 0$

her må r være 0 for at likningen skal gjelde, og løsningen ligger kun i origo.

c) $z \cdot \bar{z} = 9$, $z \cdot \bar{z} = |z|^2$

$\sqrt{a^2+b^2} = 9 \rightarrow$ $a^2 + b^2 = 3^2$

Sirkel m/ radius 3 og sentrum i origo

d) $z^6 = -1 + \sqrt{3}i$, skriver om på polar form

$r = 2$, $\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$2e^{i\frac{\pi}{3}} = z^6 = r^6 e^{i\theta 6}$

$\left(2e^{i\frac{\pi}{3}}\right)^{\frac{1}{6}} = z \rightarrow 2^{1/6} e^{i\frac{\pi}{18} + 2\pi \cdot k/18} = z$

løsninger: $\sqrt[6]{2}e^{i\frac{\pi}{18}}$, $\sqrt[6]{2}e^{i\frac{7\pi}{18}}$, $\sqrt[6]{2}e^{i\frac{13\pi}{18}}$, $\sqrt[6]{2}e^{i-\frac{17\pi}{18}}$, $\sqrt[6]{2}e^{i-\frac{11}{18}\pi}$

$\theta = \pi/18$, $\underline{\underline{k=0}}$; $\theta = \frac{7\pi}{18}$ $\underline{\underline{k=1}}$; $\theta = \frac{13\pi}{18}$ $\underline{\underline{k=2}}$; $\theta = -\frac{17}{18}\pi$ $k=3$ trekke fra -2π; $\theta = -\frac{11}{18}\pi$ $k=4$ trekke fra 2π

$6\theta = \frac{\pi}{3} + 2\pi k$

$\theta = \left(\frac{\pi}{3} + 2\pi k\right)\frac{1}{6}$

$= \frac{\pi}{18} + \frac{2\pi k}{6}$

3e) $z - (\overline{z - 2i}) = 0$

$z - (\bar{z} - \overline{2i}) = 0$

$z - \bar{z} = \overline{2i} \rightarrow z - \bar{z} = 2bi$ som er det samme

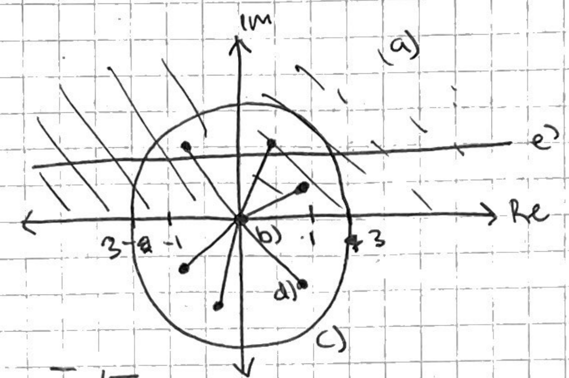
som $2bi - \overline{0 + 2i} = 0$

$2bi - 2i = 0$

$2i(b - 1) = 0 \quad | \div 2i$

$b - 1 = 0$

$\underline{\underline{b = 1}}$, og a kan være hva som helst



a) $\text{Im } z > 0$
b) origo
c) sirkel m/ $r = 3$ og origo som sentrum
d) forrige side
e) $b = 1$

4a) $\overline{z / w} = \bar{z} / \bar{w}$

$z = a + bi$ og $w = c + di \rightarrow \bar{z} = a - bi$, $\bar{w} = c - di$

finner først $\left(\overline{\frac{z}{w}}\right) \rightarrow \frac{a + bi \cdot (c - di)}{c + di \cdot (c - di)} = \frac{ac - adi + bci - bdi^2}{c^2 - \cancel{cdi} + \cancel{cdi} + d^2}$

$= \frac{ac - (ad - bc)i + bd}{c^2 + d^2} = \frac{ac + db}{c^2 + d^2} - \frac{i(ad - bc)}{c^2 + d^2}$

kompleks konjugert av denne $= \frac{ac + db}{c^2 + d^2} + \frac{i(ad - bc)}{c^2 + d^2}$ (*)

finner så $\frac{\bar{z}}{\bar{w}} \rightarrow \frac{a - bi}{c - di} = \frac{ac + (-b)(-d) + (-bc + ad)i}{c^2 + d^2}$

$= \frac{ac + bd}{c^2 + d^2} + \frac{i(-bc + ad)}{c^2 + d^2}$ (β)

Ser at * og β er like.

4b) $(\bar{z})^n = \overline{z^n}$

Teorem 1.4 sier at $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
I dette tilfellet, om $n=2$ er:

$\bar{z} \cdot \bar{z} = \overline{z^2}$ som følge av teoremet.

Om $z = re^{i\theta}$ vil $\bar{z} = re^{-i\theta}$

$(re^{-i\theta})^n = r^n e^{-i\theta n}$ (α)

$\overline{(re^{i\theta})^n} = \overline{r^n e^{i\theta n}}$ (β)

For β vet man at for ~~k~~ komplekse tall i polar form skifter man fortegn til eksponent for å finne konjugatet.

$\beta = \overline{r^n e^{i\theta n}} = r^n e^{-i\theta n}$ som er likt α.

4c) $|z+w|^2 + |z-w|^2 = 2|z|^2 + 2|w|^2$

$z = a+bi$, $w = c+di$

$|z| = \sqrt{(a^2+b^2)}$ $|w| = \sqrt{(c^2+d^2)}$

Teorem 1.5 sier at $|z+w| \leq |z| + |w|$

$\cancel{(|z|+|w|)^2} = (a+bi+c+di)^2 + (a+bi-(c+di))^2$

$= (a+c)^2 + (b+d)^2 + (a-c)^2 + (b-d)^2$

$= a^2 + \cancel{2ac} + c^2 + b^2 + \cancel{2bd} + d^2 + a^2 - \cancel{2ac} + c^2 + b^2 - \cancel{2bd} + d^2$

$= a^2 + c^2 + b^2 + d^2 + a^2 + c^2 + b^2 + d^2$

$= 2(a^2+b^2) + 2(c^2+d^2) = 2|z|^2 + 2|w|^2$

5a) $iz = \bar{z}$, $i = e^{i\frac{\pi}{2}}$

$$e^{i\frac{\pi}{2}} \cdot re^{i\theta} = re^{-i\theta} \quad | \div re^{i\theta}$$

$$e^{i\frac{\pi}{2}} = \frac{e^{-i\theta}}{e^{i\theta}} = e^{-2i\theta} = e^{i\frac{\pi}{2}}$$

$$-2i\theta = i\frac{\pi}{2} \quad | \div i$$

$$-2\theta = \frac{\pi}{2} \rightarrow \theta = -\frac{\pi}{4}, \quad r = 1$$

$$\theta = -\frac{\pi}{4} + 2\pi k$$

Løsninger: $k = 0 \rightarrow \underline{e^{-i\frac{\pi}{4}}}$

$k = 1 \rightarrow \underline{\underline{e^{i\frac{7\pi}{4}}}}$

5b) $z^4 = (z-1)^4$

$z^2 = \pm (z-1)^2$

$\underbrace{z = \text{enten } z-1}_{\text{kan forkaste denne løsningen, } 0 \neq -1}$ eller $-z+1 \quad | -z$

$z = -z+1$ eller $z^2 = -(z-1)^2$

$\downarrow \qquad\qquad \downarrow$

$2z = 1 \qquad z^2 = -z^2 + 2z - 1$

$\downarrow \qquad\qquad 2z^2 - 2z + 1 = 0$

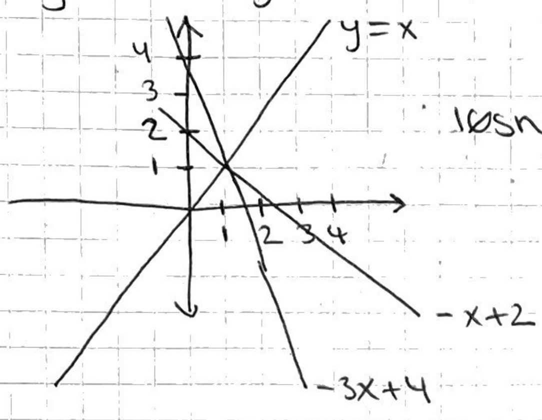
$z = \frac{1}{2} \qquad z = \frac{-(-2) \pm \sqrt{4-8}}{4} \rightarrow \frac{2+2i}{4}$ eller $\frac{2-2i}{4}$

$\underline{\underline{z \text{ er enten } \frac{1}{2}, \ \frac{1}{2} + \frac{1}{2}i \text{ eller } \frac{1}{2} - \frac{1}{2}i.}}$

[redacted]

Oppgaver kapittel 2:

1) $x+y=2 \rightarrow y=2-x=-x+2$
$x-y=0 \rightarrow y=x$
$3x+y=4 \rightarrow y=-3x+4$



løsningen er $\underline{\underline{(1,1)=(x,y)}}$

2) vet at $x=1 \rightarrow 1=2-y \rightarrow y=1$

$$1+1+z=3$$
$$z=3-2=1$$

Likningssystemene er ekvivalente.

3) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right)$ og $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right)$

De er ikke radekvivalente, siden ingen radoperasjoner kan gjøre dem like uten at de har forskjellige løsninger.

4a) Nei, $ax+c$ er en rett linje som aldri kan treffe alle tre punktene.

b) Setter inn de ulike verdiene for x og y:

$a(-1)^2+b(-1)+c=3 \quad = a-b+c=3$
$a(1)^2+b(1)+c=3 \quad = a+b+c=3$
$a(2)^2+b(2)+c=6 \quad = 4a+2b+c=6$

Siden både $+b$ og $-b$ i de to første likningene leder til samme svar må $b=0$

$a+c=3 \rightarrow 4a+(3-a)=6$
$c=3-a \qquad 4a+3-a=6$
$3a+3=6$
$c=3-1=2 \qquad 3a=3$
$a=1$

Andregradspolynomet som passer er $\underline{\underline{x^2+2}}$

5) a) $\left(\begin{array}{cc|c} a & b & m \\ c & d & n \end{array}\right)$, $ad \neq bc$

Er bare én løsning, siden determinanten ikke er lik 0. $(ad \neq bc)$

5 b) $\left(\begin{array}{cc|c} a & b & m \\ c & d & n \end{array}\right) \quad \frac{c}{a}R_1 - R_2 \sim \left(\begin{array}{cc|c} a & b & m \\ 0 & d-\frac{bc}{a} & n \end{array}\right)$

$$\left(d - \frac{bc}{a}\right)y = n \rightarrow \underline{\underline{y = \frac{n}{\left(d - \frac{bc}{a}\right)}}}$$

$$ax + by = m$$

$$ax + b\left(\frac{n}{d - \frac{bc}{a}}\right) = m \rightarrow \underline{\underline{x = \frac{m - b\left(\frac{n}{d - \frac{bc}{a}}\right)}{a}}}$$

6a)

$$\left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 4 & -3 & 8 & -26 \\ -2 & 4 & -2 & 17 \end{array}\right) \sim R_1 + R_3 \sim \left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 4 & -3 & 8 & -26 \\ 0 & 0 & 7 & -21 \end{array}\right)$$

$$\sim 2R_1 - R_2 \sim \left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 0 & -5 & 10 & -50 \\ 0 & 0 & 7 & -21 \end{array}\right) \sim \frac{1}{7}R_3 \sim \left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 0 & -5 & 10 & -50 \\ 0 & 0 & \cancel{7}\,1 & -3 \end{array}\right)$$

$$\sim -10R_3 + R_2 \sim \left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 0 & -5 & 0 & -20 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim -\frac{1}{5}R_2 \sim$$

$$\left(\begin{array}{ccc|c} 2 & -4 & 9 & -38 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim$$

$$\sim \frac{1}{2}R_1 \sim \left(\begin{array}{ccc|c} 1 & -2 & \frac{9}{2} & -19 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim 2R_2 + R_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{9}{2} & -11 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim -\frac{9}{2}R_3 + R_1$$

$$\sim \underline{\underline{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right)}}$$

6b) $\left(\begin{array}{ccc|c} 1 & 3 & 6 & 4 \\ 2 & 8 & 16 & 8 \end{array}\right) \sim -2R_1 + R_2$

$\sim \left(\begin{array}{ccc|c} 1 & 3 & 6 & 4 \\ 0 & 2 & 4 & 0 \end{array}\right) \sim \frac{1}{2}R_2 \sim \left(\begin{array}{ccc|c} 1 & 3 & 6 & 4 \\ 0 & 1 & 2 & 0 \end{array}\right)$

$\sim -3R_2 + R_1 \sim \underline{\underline{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 \end{array}\right)}}$

Fri variabel: $x = \underline{\underline{4}}$

$y + 2z = 0$, $z = \underline{\underline{t}}$, $y = \underline{\underline{-2t}}$

t kan være hva som helst.

6c) $\left(\begin{array}{l} (1+i)z - w = i \\ (1-i)z + (1+i)w = 1 \end{array}\right) \to \left(\begin{array}{cc|c} 1+i & -1 & i \\ 1-i & 1+i & 1 \end{array}\right)$

$\cancel{\frac{1}{1+i}R_1 \sim \left(\begin{array}{cc|c} 1 & -1+i & -1+i \\ 1-i & 1+i & 1 \end{array}\right)}$

$\frac{1-i}{2}R_1 \to \left(\begin{array}{cc|c} 1 & -\frac{1}{2}+\frac{1}{2}i & \frac{1}{2}+\frac{1}{2}i \\ 1-i & 1+i & 1 \end{array}\right)$

$-(1-i)R_1 + R_2 \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2}+\frac{1}{2}i & \frac{1}{2}+\frac{1}{2}i \\ 0 & 1 & 0 \end{array}\right)$

$-\left(\frac{1}{2}+\frac{1}{2}i\right)R_2 + R_1 \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2}+\frac{1}{2}i \\ 0 & 1 & 0 \end{array}\right)$

$w = \underline{\underline{0}}$ og $z = \underline{\underline{\frac{1}{2}+\frac{1}{2}i}}$

6d)
$$\left(\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 4 & 2i & 10+2i & 20+16i \\ 2i & -1 & 4+6i & 2+12i \end{array}\right)$$

$$\sim \frac{1}{2}R_1 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & \frac{5}{2}-\frac{3}{2}i & 5 \\ 4 & 2i & 10+2i & 20+16i \\ 2i & -1 & 4+6i & 2+12i \end{array}\right)$$

$$\sim -4R_1+R_2 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & \frac{5}{2}-\frac{3}{2}i & 5 \\ 0 & 0 & 8i & 6i \\ 2i & -1 & 4+6i & 2+12i \end{array}\right)$$

$$\sim -2i\cdot R_1+R_3 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & \frac{5}{2}+\left(-\frac{3}{2}\right)i & 5 \\ 0 & 0 & 8i & 6i \\ 0 & 0 & 1+i & 2+2i \end{array}\right)$$

$$\sim -\frac{1}{8}i\cdot R_2 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & \frac{5}{2}-\frac{3}{2}i & 5 \\ 0 & 0 & 1 & 3/4 \\ 0 & 0 & 1+i & 2+2i \end{array}\right)$$

$$\sim -\left(\frac{5}{2}-\frac{3}{2}i\right)R_2+R_1 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & 0 & \frac{25}{8}+\frac{9}{8}i \\ 0 & 0 & 1 & 3/4 \\ 0 & 0 & 1+i & 2+2i \end{array}\right)$$

$$\sim -(1+i)R_2+R_3 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & 0 & \frac{25}{8}+\frac{9}{8}i \\ 0 & 0 & 1 & 3/4 \\ 0 & 0 & 0 & 5/4+5/4i \end{array}\right)$$

$$\sim \frac{1}{5/4+5/4i}R_3 \sim \left(\begin{array}{ccc|c} 1 & \frac{i}{2} & 0 & 25/8+9/8i \\ 0 & 0 & 1 & 3/4 \\ 0 & 0 & 0 & \cancel{5/4+5/4i} \end{array}\right)$$

$$\sim -\left(\frac{25}{8}+\frac{9}{8}i\right)R_3+R_1 \text{ og } -\frac{3}{4}R_3+R_2 \sim$$

$$\underline{\underline{\left(\begin{array}{ccc|c} 1 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}$$