Innlevering 6
TMA4115, våren 2022

Oppgaver til kapittel 12

1a) $\left[\begin{array}{cc|c} 2 & 1 & -1 \\ -3 & 1 & -2 \\ -1 & 1 & 1 \end{array}\right]$

$$A^T = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -2 \\ -2 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 14 & -2 & 3 \\ -2 & 3 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1/7 & 3/14 \\ -2 & 3 & -2 \end{array}\right]$$

$$2R_1 + R_2 \sim \left[\begin{array}{cc|c} 1 & -1/7 & 3/14 \\ 0 & 19/7 & -11/7 \end{array}\right] \sim \frac{7}{19} R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & -1/7 & 3/14 \\ 0 & 1 & -11/19 \end{array}\right] \sim \frac{1}{7} R_2 + R_1$$

$$\sim \underline{\underline{\left[\begin{array}{cc|c} 1 & 0 & 5/38 \\ 0 & 1 & -11/19 \end{array}\right]}}$$

b) $\left[\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{array}\right]$

$$A^T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$1 - 1 - 0 + 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 4 & 1 & -2 \\ 1 & 1 & 3 & 1 \end{array}\right] \underset{\frac{1}{2}R_3}{\sim -R_1 + R_3 \sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 4 & 1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\underset{\frac{1}{3}R_2}{-R_1 + R_2} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 1 \end{array}\right] \sim -R_3 + R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim -R_2+R_1 \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 1 \end{array}\right] = \frac{1}{3}\begin{bmatrix} -5 \\ -1 \\ 3 \end{bmatrix}$$

2) a) $ax^4 + bx^3 + cx^2 + d$

$$\begin{aligned} d &= 1 \\ a + b + c + d &= 2 \\ 16a + 8b + 4c + d &= 3 \\ 81a + 27b + 9c + d &= 5 \\ 256a + 64b + 16c + d &= 7 \end{aligned}$$

glemte x !

← kolonne for x som ble skrevet inn senere.

$$\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 \\ 16 & 8 & 4 & 2 & 1 & 3 \\ 81 & 27 & 9 & 3 & 1 & 5 \\ 256 & 64 & 16 & 4 & 1 & 7 \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 16 & 8 & 4 & 2 & 1 & 3 \\ 81 & 27 & 9 & 3 & 1 & 5 \\ 256 & 64 & 16 & 4 & 1 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$-256R_1 + R_4$
$-81R_1 + R_3$
$-16R_1 + R_2$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & -8 & -12 & -14 & -15 & -29 \\ 0 & -54 & -72 & -78 & -80 & -157 \\ 0 & -192 & -240 & -252 & -255 & -505 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Fortsetter sånn:

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & -1/12 \\ 0 & 1 & 0 & 0 & 0 & 2/3 \\ 0 & 0 & 1 & 0 & 0 & -17/12 \\ 0 & 0 & 0 & 1 & 0 & 11/6 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Tilsvarer polynomet: $-\frac{1}{12}x^4 + \frac{2}{3}x^3 - \frac{17}{12}x^2 + \frac{11}{16}x + 1$

b) $p(x) = ax^2 + bx + c$

koeffisientmatrise:

punkter: $(0,1)$ $\quad c = 1$

$(1,2)$ $\quad a + b + c = 2$

$(2,3)$ $\quad 4a + 2b + c = 3$

$(3,5)$ $\quad 9a + 3b + c = 5$

$(4,7)$ $\quad 16a + 4b + c = 7$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 0 & 1 & 4 & 9 & 16 \\ 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 354 & 100 & 30 \\ 100 & 30 & 10 \\ 30 & 10 & 5 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 0 & 1 & 4 & 9 & 16 \\ 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 171 \\ 51 \\ 18 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 354 & 100 & 30 & 171 \\ 100 & 30 & 10 & 51 \\ 30 & 10 & 5 & 18 \end{array}\right] \sim \frac{1}{354} R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{100}{354} & \frac{30}{354} & \frac{171}{354} \\ 100 & 30 & 10 & 51 \\ 30 & 10 & 5 & 18 \end{array}\right] \sim -100R_1 + R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{100}{354} & \frac{30}{354} & 171/354 \\ 0 & \frac{310}{77} & \frac{90}{59} & 159/59 \\ 30 & 10 & 5 & 18 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/14 \\ 0 & 1 & 0 & 9/14 \\ 0 & 0 & 1 & 36/35 \end{array}\right] \rightarrow \underline{\underline{p(x) = \frac{3}{14}x^2 + \frac{9}{14}x + \frac{36}{35}}}$$

3) a) Likevektsvektor → finne egenvektoren til $\lambda = 1$

$$\begin{bmatrix} -1/5 & 1/2 \\ 1/5 & -1/2 \end{bmatrix} \sim \begin{bmatrix} -1/5 & 1/2 \\ 0 & 0 \end{bmatrix} \sim -5R_1$$

$\sim \begin{bmatrix} 1 & -5/2 \\ 0 & 0 \end{bmatrix}$ x_2 er fri variabel

$$\begin{aligned} x_1 &= \tfrac{5}{2} x_2 \\ x_2 &= x_2 \end{aligned} \rightarrow \underline{\underline{t \cdot \begin{bmatrix} 5/2 \\ 1 \end{bmatrix}}}$$

b) $\begin{bmatrix} -0.3 & 0.2 & 0.2 \\ 0 & -0.8 & 0.4 \\ 0.3 & 0.6 & -0.6 \end{bmatrix} \sim \begin{bmatrix} -0.3 & 0.2 & 0.2 \\ 0 & -0.8 & 0.4 \\ 0 & 0.8 & -0.4 \end{bmatrix}$

$\sim \begin{bmatrix} -0.3 & 0.2 & 0.2 \\ 0 & -0.8 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{2}{3} & -\frac{2}{3} \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \sim$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$ x_3 er fri variabel

$$\left.\begin{aligned} x_1 &= x_3 \\ x_2 &= 1/2 x_3 \\ x_3 &= x_3 \end{aligned}\right. \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix} \cdot t}}$$

4) $M = \begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix}$

For $\lambda = 1 \rightarrow \begin{bmatrix} -a & b \\ a & -b \end{bmatrix} \sim \begin{bmatrix} -a & b \\ 0 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -b/a \\ 0 & 0 \end{bmatrix}$ x_2 er fri

$$\left.\begin{aligned} x_1 &= \frac{b}{a} x_2 \\ x_2 &= x_2 \end{aligned}\right\} \underline{\underline{\begin{bmatrix} \frac{b}{a} \\ 1 \end{bmatrix} \cdot t}}$$