Innlevering 4
TMA4115, våren 2022

Oppgaver til kapittel 9

1 a) $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ og $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $\rightarrow (2\cdot 1) + (-5)(2) + (1)(0)$

$= 2 - 10 + 0 = \underline{\underline{-8 \neq 0}}$

Vektorene er $\underline{\underline{\text{ikke}}}$ ortogonale.

1 b) $\underset{u_1}{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}$, $\underset{u_2}{\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}}$ og $\underset{u_3}{\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}}$

$u_1 \cdot u_2 = (1 \cdot 1) + (0 \cdot \sqrt{2}) + (-1) \cdot (1)$
$1 + (-1) = \underline{\underline{0}}$

$u_2 \cdot u_3 = (1 \cdot 1) + (\sqrt{2} \cdot -\sqrt{2}) + (1 \cdot 1)$
$1 + 2 + 1 \neq \underline{\underline{0}}$

$u_1 \cdot u_3 = (1 \cdot 1) + (-1 \cdot 1) = 1 - 1 = \underline{\underline{0}}$

u_1 er ortogonal m/ både u_2 og u_3, men u_2 og u_3 er ikke ortogonale m/ hverandre.

2 a) $\underset{w}{\begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix}}$ og $\underset{v}{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}$ projiserer w på v $= P_v(w)$

$= \dfrac{v \cdot w}{v \cdot v} \cdot v$

$$\frac{\begin{bmatrix}2\\-5\\0\end{bmatrix}\cdot\begin{bmatrix}1\\2\\0\end{bmatrix}}{\begin{bmatrix}1\\2\\0\end{bmatrix}\cdot\begin{bmatrix}1\\2\\0\end{bmatrix}}\begin{bmatrix}1\\2\\0\end{bmatrix} = \frac{-8}{5}\begin{bmatrix}1\\2\\0\end{bmatrix} = P_v(w)$$

w projisert på v

$$\begin{bmatrix}2\\-5\\0\end{bmatrix} - \frac{-8}{5}\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}18/5\\-9/5\\0\end{bmatrix}$$

$$\underline{\underline{\begin{bmatrix}18/5\\-9/5\\0\end{bmatrix} \text{ og } \begin{bmatrix}1\\2\\0\end{bmatrix}}}$$ danner en ortogonal basis.

b) $$\underline{\underline{\begin{bmatrix}1\\0\\-1\end{bmatrix} \text{ og } \begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}}}$$ danner en ortogonal basis.

3 a) $w = Sp\left(\begin{bmatrix}2\\-5\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}\right)$, $\vec{v} = \begin{bmatrix}1\\2\\3\end{bmatrix}$

$$P_w(v) = \frac{\begin{bmatrix}2\\-5\\1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\end{bmatrix}}{\begin{bmatrix}2\\-5\\1\end{bmatrix}\cdot\begin{bmatrix}2\\-5\\1\end{bmatrix}}\cdot\begin{bmatrix}2\\-5\\1\end{bmatrix} + \frac{\begin{bmatrix}1\\2\\0\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\end{bmatrix}}{\begin{bmatrix}1\\2\\0\end{bmatrix}\cdot\begin{bmatrix}1\\2\\0\end{bmatrix}}\begin{bmatrix}1\\2\\0\end{bmatrix}$$

$$= \frac{(2+(-10)+3)}{4+(25)+1} \rightarrow \frac{-5}{30} \rightarrow -\frac{1}{6}\begin{bmatrix}2\\-5\\1\end{bmatrix}$$

$$+ \frac{1+4+0}{1+4+0}\begin{bmatrix}1\\2\\0\end{bmatrix}$$

$$= -\frac{1}{6}\begin{bmatrix}2\\-5\\1\end{bmatrix} + \begin{bmatrix}1\\2\\0\end{bmatrix}$$

b) $$W = Sp\left(\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix} \text{ og } \begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix}\right)$$

$$\vec{v} = \begin{bmatrix}2\\\sqrt{2}\\2\end{bmatrix}$$

$$P_W(v) =$$

$$u_1 = \frac{\begin{bmatrix}1\\0\\-1\end{bmatrix}\cdot\begin{bmatrix}2\\\sqrt{2}\\2\end{bmatrix}}{\begin{bmatrix}1\\0\\-1\end{bmatrix}\cdot\begin{bmatrix}1\\0\\-1\end{bmatrix}} \cdot \begin{bmatrix}1\\0\\-1\end{bmatrix} = \frac{2+(-2)}{1+1} \to \underline{\underline{0}}$$

$$u_2 = \frac{\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}\cdot\begin{bmatrix}2\\\sqrt{2}\\2\end{bmatrix}}{\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}\cdot\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}} \begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix} = \frac{2+2+2}{1+2+1}\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}$$

$$\underline{\frac{3}{2}\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}}$$

$$u_3 = \frac{\begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix}\begin{bmatrix}2\\\sqrt{2}\\2\end{bmatrix}}{\begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix}\begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix}} \begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix} = \frac{\cancel{2-}2+2}{1+2+1} = \underline{\frac{1}{2}\begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix}}$$

$$P_W(v) = \frac{3}{2}\begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\-\sqrt{2}\\1\end{bmatrix} = \begin{bmatrix}3/2\\3/2\sqrt{2}\\3/2\end{bmatrix} + \begin{bmatrix}1/2\\-\sqrt{2}/2\\1/2\end{bmatrix}$$

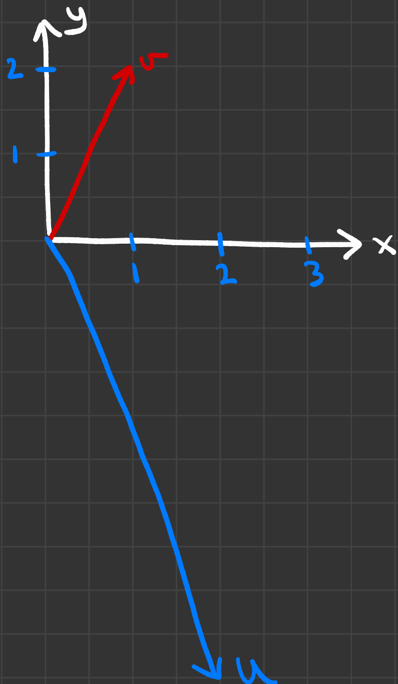
$$\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$$

$$\sqrt{2}\left(\frac{3}{2} - \frac{1}{2} = \frac{2}{2}\right)$$

$$= \underline{\underline{\begin{bmatrix}2\\\sqrt{2}\\2\end{bmatrix}}}$$

4) $u = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$P_v(u) = \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -5 \end{bmatrix}}{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{\underline{\frac{2+(-10)}{1+2^2}}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{\underline{-\frac{8}{5}}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



$$\cos\theta = \frac{\langle v, u \rangle}{\|v\| \, \|u\|}$$

$$\cos\theta = \frac{-8}{\sqrt{2^2+(-5)^2} \cdot \sqrt{1^2+2^2}}$$

$$\underline{\underline{\theta = \arccos\left(-\frac{8}{\sqrt{29} \cdot \sqrt{5}}\right)}}$$

5 a) $u = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$$P_u(v) = \frac{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \underline{\underline{\frac{11}{9} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}}$$

$$v - P_u(v) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \frac{11}{9} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 22/9 \\ 22/9 \\ 11/9 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5/9 \\ -4/9 \\ -2/9 \end{bmatrix}}}$$

b) $\dfrac{\begin{bmatrix}-1\\2\\-1\end{bmatrix}\cdot\begin{bmatrix}1\\3\\1\end{bmatrix}}{\begin{bmatrix}-1\\2\\-1\end{bmatrix}\cdot\begin{bmatrix}-1\\2\\-1\end{bmatrix}}\cdot\begin{bmatrix}-1\\2\\-1\end{bmatrix} = \dfrac{-1+6-1}{1+4+1}\begin{bmatrix}-1\\2\\-1\end{bmatrix} = \dfrac{2}{3}\begin{bmatrix}-1\\2\\-1\end{bmatrix}$

$= \begin{bmatrix}-2/3\\4/3\\-2/3\end{bmatrix}$

$v - P_u(v) = \begin{bmatrix}1\\3\\1\end{bmatrix} - \begin{bmatrix}-2/3\\4/3\\-2/3\end{bmatrix} = \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}$

c) $P_u(v)\cdot(v - P_u(v))$

– a) $\begin{bmatrix}22/9\\22/9\\11/9\end{bmatrix}\cdot\begin{bmatrix}5/9\\-4/9\\-2/9\end{bmatrix} = \begin{bmatrix}110/81\\-88/81\\-44/81\end{bmatrix}$

– b) $\begin{bmatrix}-2/3\\4/3\\-2/3\end{bmatrix}\cdot\begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix} = \begin{bmatrix}-10/9\\20/9\\-10/9\end{bmatrix}$

6) $W = Sp(u, v)$

der $u = \begin{bmatrix}2\\-5\\1\end{bmatrix}$ og $v = \begin{bmatrix}4\\-4\\2\end{bmatrix}$

a) ortogonal basis

$v_1 = \begin{bmatrix}2\\-5\\1\end{bmatrix}$ $\quad v_2 = \begin{bmatrix}4\\-4\\2\end{bmatrix} - \dfrac{\begin{bmatrix}4\\-4\\2\end{bmatrix}\begin{bmatrix}2\\-5\\1\end{bmatrix}}{\begin{bmatrix}2\\-5\\1\end{bmatrix}\begin{bmatrix}2\\-5\\1\end{bmatrix}}\begin{bmatrix}2\\-5\\1\end{bmatrix} =$

$$\begin{bmatrix}4\\-4\\2\end{bmatrix} - \frac{8+20+2}{4+25+1}\begin{bmatrix}2\\-5\\1\end{bmatrix} = \begin{bmatrix}4\\-4\\2\end{bmatrix} - \begin{bmatrix}2\\-5\\1\end{bmatrix} = \underline{\begin{bmatrix}2\\1\\1\end{bmatrix}}$$

vektorene $\begin{bmatrix}2\\-5\\1\end{bmatrix}$ og $\begin{bmatrix}2\\1\\1\end{bmatrix}$ danner en ortogonal basis.

b) Standardmatrisen $[P_W]$

for $u = \begin{bmatrix}2\\-5\\1\end{bmatrix}$ og $v = \begin{bmatrix}2\\1\\1\end{bmatrix}$

$$P_u(e_1) + P_v(e_1) = \frac{\begin{bmatrix}2\\-5\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}{\begin{bmatrix}2\\-5\\1\end{bmatrix}\begin{bmatrix}2\\-5\\1\end{bmatrix}}\begin{bmatrix}2\\-5\\1\end{bmatrix} = \frac{1}{15}\begin{bmatrix}2\\-5\\1\end{bmatrix}$$

$$+ \frac{\begin{bmatrix}2\\1\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}{\begin{bmatrix}2\\1\\1\end{bmatrix}\begin{bmatrix}2\\1\\1\end{bmatrix}}\begin{bmatrix}2\\1\\1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}2/3\\1/3\\1/3\end{bmatrix}$$

$$P_u(e_1) + P_v(e_1) = \frac{1}{15}\begin{bmatrix}2\\-5\\1\end{bmatrix} + \begin{bmatrix}2/3\\1/3\\1/3\end{bmatrix} = \underline{\underline{\begin{bmatrix}4/5\\0\\2/5\end{bmatrix}}}$$

Finne $\underbrace{P_u(e_2) + P_v(e_2)}_{2)}$ og $\underbrace{P_u(e_3) + P_v(e_3)}_{3)}$

2) $\frac{\langle u, e_2 \rangle}{\langle u, u \rangle} \cdot u = \frac{\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{30} \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$

$\frac{\langle v, e_2 \rangle}{\langle v, v \rangle} \cdot v = \frac{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{6} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

2) $= \frac{1}{6}\left(\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right) = \underline{\underline{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}}$

nederste element i u

3) $\frac{1}{30}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2/5 \\ 0 \\ 1/5 \end{bmatrix}}}$

$[P_W] = \underline{\underline{\begin{bmatrix} 4/5 & 0 & 2/5 \\ 0 & 1 & 0 \\ 2/5 & 0 & 1/5 \end{bmatrix}}}$

c) $[P_W] \cdot Ax = Ax$

For å kunne multiplisere to matriser $m \times n$ og $s \times t$, må $n = s$. Her:

$3 \times \underline{2}$ og $\underline{3} \times 3 \rightarrow 2 \neq 3$.

A fins ikke.

7 a) bruker at $\cos\theta = \frac{\langle f, g\rangle}{\|f\| \ \|g\|}$ for to funksjoner f og g.

$$\langle x, \sin(x)\rangle = \int_0^1 x \cdot \sin(x)\, dx$$

$u = x \qquad du = 1$

$dv = \sin(x) \qquad v = -\cos(x)$

$$-x\cos(x) + \int \cos(x)\, dx = \left[-x\cos(x) + \sin(x)\right]_0^1$$

$$= \sin(1) - \cos(1)$$

$$\|\sin(x)\| = \sqrt{\langle \sin(x), \sin(x)\rangle}$$

$$\int_0^1 \sin^2(x)\, dx = \int_0^1 \frac{1-\cos(2x)}{2}\, dx = \frac{1}{2}\int_0^1 1-\cos(2x)\, dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^1 = \frac{1}{2} - \frac{1}{4}\sin(2)$$

$$\|x\| = \sqrt{\langle x, x\rangle} = \int_0^1 x^2\, dx = \left[\frac{x^3}{3}\right]_0^1 = \sqrt{\frac{1}{3}}$$

$$\cos\theta = \frac{\sin(1) - \cos(1)}{\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{1}{2} - \frac{1}{4}\sin(2)}}$$

løser for θ ved å ta arccos $\rightarrow$ $\theta = \underline{\underline{0.0456}}$ radianer.

Tilsvarende for x og cos(x):

bruker at $\cos\theta = \frac{\langle f, g \rangle}{\|f\| \, \|g\|}$ for to funksjoner f og g.

$$\frac{\langle x, \cos(x) \rangle}{\|x\| \cdot \|\cos(x)\|}$$

$$\langle x, \cos(x) \rangle = \int_0^1 \overset{u}{x} \cdot \overset{dv}{\cos(x)} \, dx = uv - \int v \, du$$

$u = x \qquad du = 1$

$dv = \cos(x) \qquad v = \sin(x)$

$$= x\sin(x) - \int \sin(x) \, dx = x\sin(x) + \cos(x)$$

$$= \sin(1) + \cos(1) - 1$$

Vet at $\|x\| = \frac{1}{\sqrt{3}}$

$$\|\cos(x)\| = \sqrt{\langle \cos(x), \cos(x) \rangle} = \int_0^1 \cos^2(x) \, dx$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\frac{1}{2} \int \cos(2x) \, dx + \frac{1}{2} \int 1 \, dx$$

$$= \frac{1}{2} \left[\frac{1}{2} \sin(2x) \right]_0^1 + \left[\frac{1}{2} x \right]_0^1 = \frac{\sin(2) + 2}{4}$$

$$\theta = \arccos \left(\frac{\sin(1) + \cos(1) - 1}{\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{\sin(2) + 2}{4}}} \right) = \underline{\underline{0.6835}} \text{ radianer}$$

Ser at vinkelen mellom $\underline{\underline{x \text{ og } \cos(x)}}$ er størst.

b) Avstanden mellom to vektorer f og g er $\| f - g \|$.

$$\| x - \cos(x) \| = \sqrt{\langle x - \cos(x), x - \cos(x) \rangle}$$

$$\int_0^1 (x - \cos(x))^2 \, dx = \int_0^1 \cos^2(x) - 2x\cos(x) + x^2 \, dx$$

Fra tidligere vet vi at $\int_0^1 \cos^2(x) = \frac{\sin(2)+2}{4}$

og at $\int_0^1 x\cos(x) = \sin(1) + \cos(1) - 1$

Dvs. det fulle integralet:

$$= \frac{\sin(2)+2}{4} - 2(\sin(1) + \cos(1) - 1) + \int_0^1 x^2 \, dx$$

$$= \frac{\sin(2)+2}{4} - 2(\sin(1) + \cos(1) - 1) + \frac{1}{3}$$

$$= \sqrt{0.29711} = \underline{\underline{0.54507}}$$

For x og $\sin(x)$:

$$\int_0^1 (x - \sin(x))^2 = \int_0^1 \sin^2(x) - 2x\sin(x) + x^2 \, dx$$

Bruker verdier fra tidligere integraler:

$$\int_0^1 \sin^2(x) = \frac{1}{2} - \frac{1}{4}\sin(2)$$

$$2\int_0^1 x\sin(x) = 2(\sin(1) - \cos(1))$$

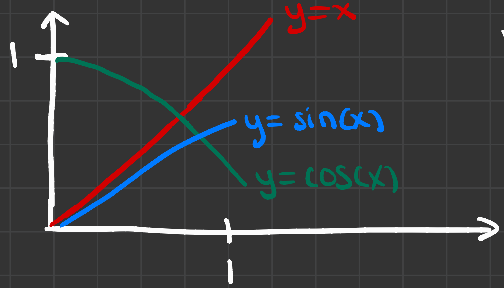
Fulle definerte integralet blir da:

$$\frac{1}{2} - \frac{1}{4}\sin(2) - 2(\sin(1) - \cos(1)) + \frac{1}{3} = 0.0036713$$

Kvadratroten av dette: $\underline{\underline{0.06059\ldots}}$

Avstanden mellom <u>x og sin(x)</u> er minst.

c)



Vi ser at x og sin(x) nesten tangerer hverandre i $x \in [0,1]$, mens cos(x) starter i $y=1$. Dette forklarer hvorfor avstanden og vinklene mellom x og sin(x) er mindre enn x og cos(x).

8) $V = sp(1, e^x)$ av $C[0,1]$ med indreprodukt

$$\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx$$

$u_1 = 1 \qquad v_2 = e^x$

$$u_2 = e^x - \frac{\langle 1, e^x \rangle}{\langle 1, 1 \rangle} 1 = e^x - \frac{1-e}{1} 1$$

$$u_2 = e^x - e + 1$$

En ortogonal basis for V blir:

$$\underline{\underline{(1,\ e^x - e + 1)}}$$

a) $V = \mathrm{sp}(1, x, e^x)$ av $C[0,1]$ m/ indreprodukt

$$\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx$$

a) $v_1 = 1$

$$v_2 = x - \frac{\langle 1, x \rangle}{\langle 1, 1 \rangle} 1 = \underline{\underline{x - \frac{1}{2}}}$$

$$v_3 = \underbrace{e^x - \frac{\langle 1, e^x \rangle}{\langle 1, 1 \rangle} 1}_{= e^x - e - 1} - \underbrace{\frac{\langle x - \frac{1}{2}, e^x \rangle}{\langle x - \frac{1}{2}, x - \frac{1}{2} \rangle}}_{*)} (x - \tfrac{1}{2})$$

*) $$\langle x - \tfrac{1}{2}, e^x \rangle = \int_0^1 (x - \tfrac{1}{2})(e^x)\, dx$$

$$= \int_0^1 x e^x - \tfrac{1}{2} e^x \, dx$$

$$= \left[e^x (x-1) - \tfrac{1}{2} e^x \right]_0^1$$

$$= -\frac{e-3}{2}$$

$$\langle x - \tfrac{1}{2}, x - \tfrac{1}{2} \rangle = \int_0^1 (x - \tfrac{1}{2})^2 \, dx$$

$$= \left. \frac{(x - \frac{1}{2})^3}{3} \right|_0^1 = \frac{1}{12}$$

$$v_3 = e^x - e - 1 - \left(-(6e - 18) \right)(x - \tfrac{1}{2})$$

$$\underline{\underline{v_3 = e^x - e - 1 + (6e - 18)(x - \tfrac{1}{2})}}$$

v_1, v_2 og v_3 danner en ortogonal basis.

b) $h(x) = x^2$

$$p(x^2) = \overbrace{\frac{\langle v_1, x^2 \rangle}{\langle v_1, v_1 \rangle} v_1}^{\alpha)} + \overbrace{\frac{\langle v_2, x^2 \rangle}{\langle v_2, v_2 \rangle} v_2}^{\beta)} + \overbrace{\frac{\langle v_3, x^2 \rangle}{\langle v_3, v_3 \rangle} v_3}^{\Omega)}$$

hvor $v_1 = 1$, $v_2 = x - \frac{1}{2}$ og

$$v_3 = e^x - e - 1 + (6e - 18)(x - \tfrac{1}{2})$$

$\alpha)$ $$\frac{\langle 1, x^2 \rangle}{\langle 1, 1 \rangle} 1 = \frac{1}{3}$$

$\beta)$ $$\frac{\langle x - \frac{1}{2}, x^2 \rangle}{\langle x - \frac{1}{2}, x - \frac{1}{2} \rangle}(x - \tfrac{1}{2}) = \frac{\cancel{1/12}}{\cancel{1/12}}(x - \tfrac{1}{2})$$

$\Omega)$ $$\frac{\langle e^x - e - 1 - (6e-18)(x - \frac{1}{2}), x^2 \rangle}{\langle e^x - e - 1 - (6e-18)(x - \frac{1}{2}), e^x - e - 1 - (6e-18)(x - \frac{1}{2}) \rangle} \cdot e^x - e - 1 - (6e-18)(x - \tfrac{1}{2})$$

$$= \frac{\left(\frac{7e - 23}{6}\right)}{\left(-\frac{7e^2 - 40e + 49}{2}\right)} \cdot e^x - e - 1 - (6e-18)(x - \tfrac{1}{2})$$

Summen av $\underline{\underline{\alpha + \beta + \Omega}}$ blir $\text{proj}_V\, h(x)$.