Øving 9

13.7 – 22

22) Prinsipiellverdien til $(2i)^{2i}$, $\mathrm{Ln}(z) = \ln|z| + i\,\mathrm{Arg}(z)$

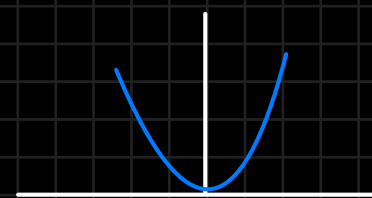
$$z^c = e^{c\ln(z)} \rightarrow z^c = e^{2i\ln|2i|} = e^{2i(\ln 2 + \ln|i|)}$$

$$= e^{2i(\ln 2 + \frac{\pi}{2}i \pm 2\pi n i)} = \underline{\underline{e^{2i\ln(2) - \frac{\pi}{2}}}}$$

14.1 – 3, 11, 20, 22, 26, 29

3) $z(t) = t + 4t^2 i$, $0 < t < 1$

$$\left.\begin{aligned} \mathrm{Re}(z) &= t = x \\ \mathrm{Im}(z) &= 4t^2 = y \end{aligned}\right\} \text{ gir } y = 4x^2:$$



11) veien mellom $(-1,2)$ og $(1,4)$

krysser y-aksen i $y=3$ og har stigningstall 1:

$$y = x + 3$$

kan parametriseres som $\underline{\underline{(t, t+3)}}$

$z(t) = \underline{\underline{t + (t+3)i}}$ (komplekes?)

20) $4(x-2)^2 + 5(y+1)^2 = 20$

kan skrives som $\dfrac{(x-2)^2}{5} + \dfrac{(y+1)^2}{4} = 1$

som er en ellipse m/ senter i $(2, -1)$ og halvakselengder $2\sqrt{5}$ og 4.

skriver $\begin{aligned} x(t) &= a\cos(t) \\ y(t) &= b\sin(t) \end{aligned}$ her: $\underline{\underline{\begin{aligned} x(t) &= \sqrt{5}\cos(t) + 2 \\ y(t) &= 2\sin(t) - 1 \end{aligned}}}$

22) $\int_C \mathrm{Re}(z)\,dz$, $C := y = 1 + \frac{1}{2}(x-1)^2$ mellom $1+i$ og $3+3i$

$$x(t) = t + 1$$
$$y(t) = 1 + \frac{1}{2}t^2$$

$$z(t) = \underbrace{t+1}_{x} + (\underbrace{1 + \tfrac{1}{2}t^2}_{y})i, \quad t \in [0, 2]$$

$$\int_0^2 (1+t)(1+ti)\,dt = \int_0^2 1+ti+t+t^2 i\,dt$$

$$= \left[t + \frac{t^2}{2}i + \frac{t^2}{2} + \frac{t^3}{3}i\right]_0^2$$

$$= 2 + \frac{2^2}{2}i + \frac{2^2}{2} + \frac{2^3}{3}i$$

$$= \frac{8i}{3} + 2i + 4 = \underline{\underline{\frac{14}{3}i + 4}}$$

26) $\int_C z + z^{-1}\,dz$, $C=$ enhetssirkelen mot klokka

$$\int_0^{2\pi} z + \frac{1}{z}\,dz =$$

$$z(t) = \cos(t) + \sin(t)\cdot i$$

$$\int_0^{2\pi} \left(\cos(t) + i\sin(t) + \frac{1}{\cos(t)+i\sin(t)}\right)\cdot -\sin(t) + \cos(t)i$$

$$\left.\begin{aligned}&\cos(t)(-\sin(t)+\cos(t)i)\\ &-\sin(t)\cos(t) + \cos^2(t)i\\ &i\sin(t)(-\sin(t)+\cos(t)i)\\ &-i\sin^2(t) - \cos(t)\sin(t)\end{aligned}\right\} \begin{aligned}&-2\cos(t)\sin(t)\\ &+\cos^2(t)i - \sin^2(t)i\end{aligned}$$

$$= \sin(2t) + \cos^2(t)i - \sin^2(t)i$$

$$= -\sin(2t) + \cos(2t)i$$

$$\frac{-\sin(t)+\cos(t)i}{\cos(t)+\sin(t)i}\cdot\left(\frac{\cos(t)-\sin(t)i}{\cos(t)-\sin(t)i}\right)$$

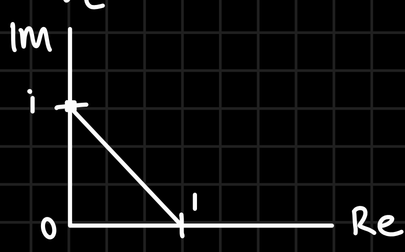
$$= -\sin(2t) + \cos(2t)i$$

$$= 2\int_0^{2\pi} -\sin(2t) + \cos(2t)i\,dt$$

$$= 2\left[\frac{\cos(2t)}{2} + \frac{\sin(2t)}{2}i\right]_0^{2\pi} = \left(\frac{1}{2}+0\right) - \left(\frac{1}{2}+0\right)\cdot 2$$

$$= \underline{\underline{0}}$$

29) $\int_C \mathrm{Im}(z^2)\,dz$



$$t + i - ti$$

$$z(t) = t + (1-t)i\ ,\ z'(t) = 1 - i$$

$$(z(t))^2 = (t + (1-t)i)^2$$

$$(t + i - ti)^2$$

$$(t + i - ti)(t + i - ti)$$

$= t^2 + 2t(1-t)i - (1-t)^2$

$2ti - 2t^2 i$

$= \cancel{2t} + 2i - 4ti + 2 - \cancel{2t}$

$= -4ti + 2i + 2$ ser at jeg deriverte z^2 istedenfor z

$\int_0^1 (2ti - 2t^2 i)(-4ti + 2i + 2)\,dt$

$\int_0^1 \cancel{8t^2} \cancel{-4t} + 4ti \cancel{-8t^3} \cancel{+4t^2} - 4t^2 i$

$\int_0^1 12t^2 - 8t^3 - 4t + 4ti - 4t^2 i \,dt$

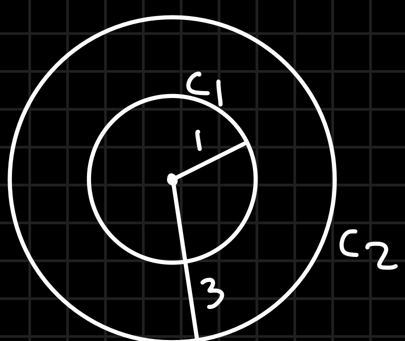
$= \left[12\frac{t^3}{3} - 8\frac{t^4}{4} - 4\frac{t^2}{2} + 4\frac{t^2}{2}i - 4\frac{t^3}{3}i\right]_0^1$

$= \frac{12}{3} - \frac{8}{4} - \frac{4}{2} + 2i - \frac{4}{3}i$

$= \left(2 - \frac{4}{3}\right)i$

14.2) ~~4~~, ~~15~~, ~~22~~, ~~23~~, ~~28~~

4)



$1 < |z| < 3$ istedenfor $1 \le |z| \le 3$ gjør at området vårt ikke er en annulus.

Om vi hadde hatt $0.5 < |z| < 3.5$, hadde det vært en annulus. Likevel er ikke funksjonen analytisk fordi

$$\oint_{C_1} f(z)\,dz \ne \oint_{C_2} f(z)\,dz$$

$$\Rightarrow 6 \ne 2$$

13) $f(z) = \frac{1}{(z^4 - 1.2)}$ mot klokka langs enhetssirkelen: (enkel lukket kurve)

f er analytisk overalt unntatt i $z = \sqrt[4]{1.2} \approx 1.05$, som ligger utenfor enhetssirkelen. Vi kan dermed bruke Cauchys integralteorem.

Det betyr at integralet blir likt 0.

22) $\oint Re(z)\,dz$ på en halv enhetssirkel:

$f(z) = Re(z) \rightarrow \left.\begin{array}{l} u(x,y) = x \\ v(x,y) = 0 \end{array}\right\}$ CR: $1 \neq 0$,

Re(z) er ikke analytisk og Cauchys teorem kan ikke brukes.

$\oint x\,dx \neq 0.$

23) $\oint_C \frac{2z-1}{z^2-z}$

$$\frac{2z-1}{z(z-1)} \rightarrow \frac{A}{z} + \frac{B}{(z-1)} = \frac{2z-1}{z(z-1)}$$

$$A(z-1) + Bz = 2z-1$$

$$Az - A + Bz = 2z-1$$

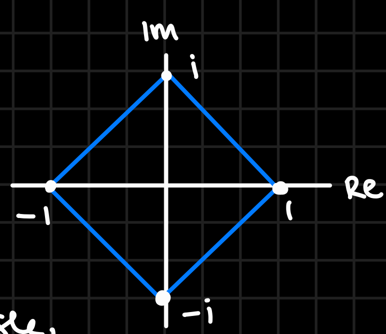
$$(A+B)z = 2z$$
$$-A = -1$$

$$A = B = 1$$

$$\oint_C \frac{1}{z} + \frac{1}{z-1}\,dz$$

f(z) er ikke analytisk i $z=0$ og $z=1$, som begge er innenfor ellipsen. Cauchys teorem kan ikke brukes.

28) $\oint \frac{\tan 1/2z}{16z^4-81}\,dz$



nevner = 0 for $z = 3/2$, $-3/2$, $-3/2i$ og $3/2i$. alle disse verdiene ligger utenfor kvadratet → f(z) er analytisk, Cauchys teorem kan brukes og <u>integralet blir 0</u>.