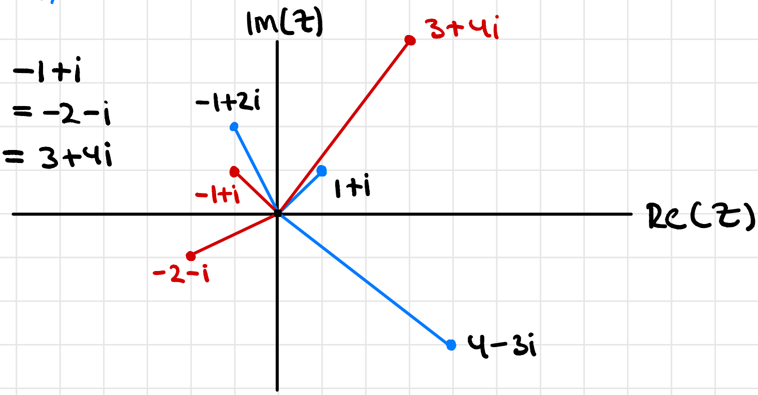
Øving 7

13.1) 2, 3, 14, 16

2) $(1+i)i = -1+i$

$(-1+2i)i = -2-i$

$(4-3i)i = 3+4i$




ser at vinkelen mellom z og iz er <u>90° i hvert tilfelle.</u>

3) fra 7 i 13.1)

$$= \frac{x_1+iy_1}{x_2+iy_2} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\,\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$$

vi har $\frac{(26-18i)}{(6-2i)}$, $x_1 = 26$, $x_2 = 6$, $y_1 = -18$, $y_2 = -2$

Formelen gir: $\underline{\underline{-\frac{24}{5} - \frac{7}{5}i}}$

14) $\bar{z}_1/\bar{z}_2$ når $z_1$ er $-2+5i$ $\rightarrow$ $\bar{z}_1 = -2-5i$

$z_2$ er $3-i$ $\quad$ $\bar{z}_2 = 3+i$

$$(\bar{z}_1/\bar{z}_2 = (\overline{z_1/z_2}))$$

$\frac{-2-5i}{3+i}$ bruker formel:

$$\left(\frac{(-2\times3)+(-5\times1)}{3^2+1^2} - i\,\frac{-15-(-2)}{3^2+1^2}\right) = \underline{\underline{-\frac{11}{10} - \frac{13}{10}i}}$$

16) $z = x+iy$, finne $\mathrm{Im}(1/z)$ og $\mathrm{Im}(1/z^2)$ hvor

$$z^2 = (x+iy)^2 = (x+iy)(x+iy)$$
$$= x^2 + 2ixy - y^2$$

$$\mathrm{Im}\left(\frac{1}{x+iy}\right) =$$ forenkler inni

$$\frac{1}{x+iy} \cdot \frac{(x-iy)}{(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

$$\mathrm{Im}\left(\frac{1}{z}\right) = -\frac{y}{x^2+y^2}$$

$$\mathrm{Im}\left(\frac{1}{z^2}\right) = \mathrm{Im}\left(\frac{1}{(x+iy)^2}\right)$$ forenkler nevner

$$\frac{1\,(x-iy)^2}{(x+iy)^2(x-iy)^2} = \frac{x^2 - 2ixy - y^2}{(x^2+y^2)^2}$$

ser at imaginærdelen må være lik:

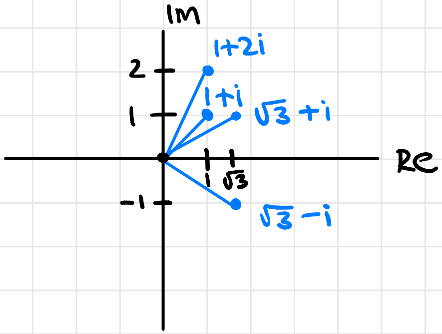
$$\mathrm{Im}\left(\frac{1}{z^2}\right) = -\frac{2xy}{(x^2+y^2)^2}$$

13.2) $\cancel{1}, \cancel{8}, \cancel{11}, \cancel{21}, \cancel{25}$

Regner ut 8) først: $\frac{7+4i}{3-2i}$ bruker formel fra 13.1:

$$= 1 + 2i$$

1), 8), 11) i det komplekse plan:



11) Principial value til et komplekstall er en vinkel mellom $-\pi$ og $\pi$.

for $\sqrt{3} \pm i$ : $\tan\theta = \frac{1}{\sqrt{3}}$ og $-\frac{1}{\sqrt{3}}$ som gir $\theta = \pm\frac{\pi}{6}$

principial value: $\underline{\underline{\frac{\pi}{6}}}$

21) røttene til $\sqrt[3]{1-i} = z$

$z^3 = 1-i$, $\arg(1-i) = \sqrt{2}$ $(1^2+(-1)^2)^{1/2} = r$, $\arctan(-1) = -\frac{\pi}{4}$

$z^3$ i polarform : $r^3 e^{3i\theta}$

$1-i$ i polarform : $\sqrt{2}e^{-i\frac{\pi}{4}}$

$r^3 = \sqrt{2}$ og $e^{-i\frac{\pi}{4}} = e^{3i\theta}$ gir en $r = \sqrt[6]{2}$

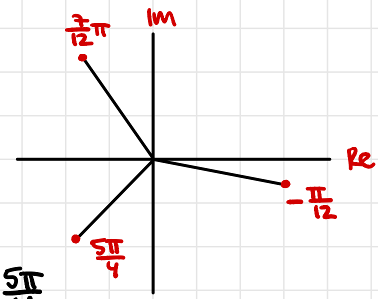
$-i\frac{\pi}{4} + 2\pi n i = 3i\theta$ gir $\theta = -\frac{\pi}{12} + \frac{2}{3}\pi n$

$n=0$, $\theta = -\frac{\pi}{12}$

$n=1$, $\theta = 7/12\pi$

$n=2$, $\theta = 5/4\pi$

løsningen for $\theta$ m/ $n \in (0,1,2)$ blir de tre røttene vi er ute etter, m/ $r = \sqrt[6]{2}$




$= \underline{\underline{\sqrt[6]{2}e^{-i\frac{\pi}{12}}, \sqrt[6]{2}e^{i\frac{7}{12}\pi}, \sqrt[6]{2}e^{i\frac{5\pi}{4}}}}$

25) $\sqrt[4]{i}$, $z^4 = i \rightarrow r^4 e^{4i\theta} = e^{i\frac{\pi}{2}}$ hvor $r = 1$

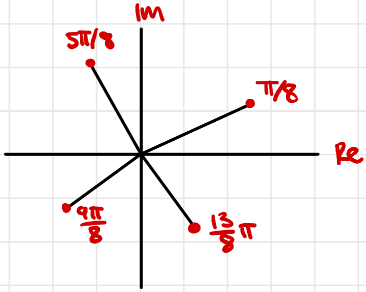
$4i\theta = i\frac{\pi}{2} + 2\pi n i$ gir $\theta = \frac{\pi}{8} + \frac{\pi n}{2}$

$4\theta = \frac{\pi}{2} + 2\pi n$

$n=0$, $\theta = \pi/8$ $\quad$ $n=2$, $\theta = 9/8\pi$

$n=1$, $\theta = 5\pi/8$ $\quad$ $n=3$, $\theta = 13/8\pi$

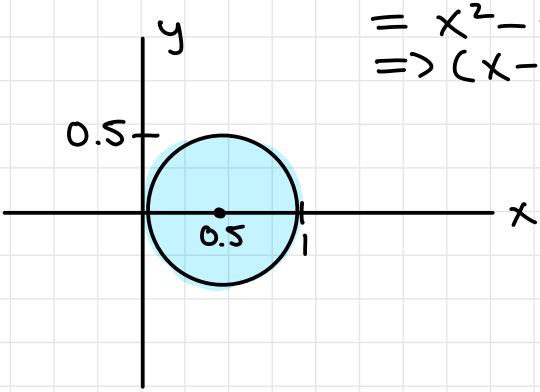
$= \underline{\underline{e^{i\frac{\pi}{8}}, e^{i\frac{5\pi}{8}}, e^{i\frac{9\pi}{8}}, e^{i\frac{13}{8}\pi}}}$




13.3) ~~6~~, ~~15~~, ~~16~~, ~~18~~

6) $\mathrm{Re}(1/z) < 1 \longrightarrow \dfrac{x}{x^2+y^2} < 1$

$$\longrightarrow \quad x < x^2 + y^2$$
$$= x^2 - x + y^2 > 0$$
$$\Rightarrow (x-0.5)^2 + y^2 - \frac{1}{4} > 0$$




15) $|z|^2 \cdot \mathrm{Im}(1/z)$

$$\cancel{(x^2+y^2)}\left(-y/\cancel{x^2+y^2}\right) = -y$$
$$= -r\sin\theta \longrightarrow 0$$

f(z) er kontinuerlig fordi $r = 0$ når $z = 0$.

16) $(\mathrm{Im}\, z^2) = 2xy$ fra en tidligere oppgave.

$$= \frac{2xy}{x^2+y^2} \quad \text{i polarkoordinater:}$$

$$= \frac{2r^2 \cos\theta \sin\theta}{r^2(\cos^2\theta + \sin^2\theta)} = \underline{\underline{2\cos\theta \sin\theta}}$$

f(z) er ikke kontinuerlig i $z=0$ siden uttrykket er avhengig av $\theta$ og ikke r.
$f(0) \neq 0$.

18) $$\frac{d}{dz}\left(\frac{z-i}{z+i}\right) = \frac{(i+z)\dot{}' - (z-i)\dot{}'}{(z+i)^2} = \underline{\underline{\frac{2i}{(z+i)^2}}}$$

at i: $$\frac{2i}{(2i)^2} = \underline{\underline{-\frac{1}{2}i}}$$