Innlevering 4 – blokk 2

Oppgave 1)

a) $\mu = 10$ gram, $\sigma^2 = 0.2^2$ gram^2

normalfordeling

$P(X > 10.2)$? $= 1 - P(X \leq 10.2)$, må forskyve

$$\frac{X-\mu}{\sigma} = z = \frac{10.2-10}{0.2} = 1$$

$P(Z \leq 1) = 0.8413$, $P(Z > 1) = 1 - 0.8413 = \underline{\underline{0.1587}}$

Sannsynlighet for at målt vekt avvik > 0.2 gram:

$P(X < 9.8) \cup P(X > 10.2)$ = summen av de to:

$$P(X < 9.8) = \frac{9.8-10.2}{0.2} = -1, \quad P(Z \leq -1) = \underset{\text{tabell}}{0.1587}$$

$= 2(0.1587) = \underline{\underline{0.3174}}$

Sannsynlighet for at snittet av to uavhengige målinger avvik > 0.2 gram:

$P(\bar{X} < 9.8) + P(\bar{X} > 10.2)$

sentralgrenseteoremet: $\bar{X} = \sqrt{\frac{\sigma^2}{n}}\,z + \mu = \sqrt{\frac{0.2^2}{2}}\,z + 10$

$$P\left(\sqrt{\frac{0.2^2}{2}}\,z + 10 < 9.8\right) = P(Z < -1.41421) = \underset{\text{tabell}}{0.0793}$$

$$P\left(\sqrt{\frac{0.2^2}{2}}\,z + 10 > 10.2\right) = P(Z > 1.41421) = 1 - 0.9207 = 0.0793$$

$= 2(0.0793) = \underline{\underline{0.1586}}$

b) resultat A $= X_1$, $\hat{\mu}_A$ hvor μ_A og μ_B er sann vekt
resultat B $= X_2$, $\hat{\mu}_B$

resultat A+B $= Y_1$, $\tilde{\mu}_A = 1/2(Y_1 + Y_2)$
resultat A−B $= Y_2$, $\tilde{\mu}_B = 1/2(Y_1 - Y_2)$

Skal finne $E(\hat{\mu}_A)$, $E(\hat{\mu}_B)$, $E(\tilde{\mu}_A)$, $E(\tilde{\mu}_B)$
$\mathrm{var}(\tilde{\mu}_A)$, $\mathrm{var}(\tilde{\mu}_B)$, $\mathrm{var}(\hat{\mu}_A)$, $\mathrm{var}(\hat{\mu}_B)$

$E(\hat{\mu}_A) = E(X_1) = \underline{\underline{\mu_A}}$ $\qquad E(\tilde{\mu}_A) = 1/2(E(Y_1) + E(Y_2)) = \underline{\underline{\mu_A}}$

$E(\hat{\mu}_B) = E(X_2) = \underline{\underline{\mu_B}}$ $\qquad E(\tilde{\mu}_B) = 1/2(E(Y_1) - E(Y_2)) = \underline{\underline{\mu_B}}$

$\text{var}(\hat{N}_A) = \underline{\underline{\sigma^2}} = \text{var}(\hat{N}_B)$ (gis direkte)

$\text{var}(\tilde{N}_A) = \left(\frac{1}{2}\right)^2 \text{var}(Y_1) + \text{var}(Y_2) = \frac{2\sigma^2}{4} = \underline{\underline{\frac{1}{2}\sigma^2}}$

$\text{var}(\tilde{N}_B) = \left(\frac{1}{2}\right)^2 (\text{var}(Y_1) \underset{\text{samme som +}}{-} \text{var}(Y_2)) = \underline{\underline{\frac{\sigma^2}{2}}}$

<u>Lavere varians i metode 2 gjør at den er bedre.</u>

Oppgave 2)

Z = lengde mellom to feil, uavhengig og eksponentialfordelt.

$f(z;\lambda) = \lambda e^{-\lambda z}, \quad z > 0$

$F(z;\lambda) = 1 - e^{-\lambda z}, \quad z > 0$

a) her, $\lambda = 0.05$

Ønsker $P(Z > 10) = 1 - P(Z \leq 10)$

$= 1 - (1 - e^{-0.05 \cdot 10}) = \underline{\underline{0.607}}$

$P(Z > 20 \mid Z > 10) = \frac{P(Z>20)}{P(Z>10)} = \frac{1-(1-e^{-0.05 \cdot 20})}{1-(1-e^{-0.05 \cdot 10})} = \underline{\underline{0.607}}$

20 feilfrie km gitt at de første 10 er feilfrie

b) $P(M=m) = P(m < Z < m+1)$

$P(M=1) = P(1 < Z < 2)$
mønsteret fortsetter sånn

$P(M=m+1) - P(M=m)$
bruker kumulativ fordelingsfunksjon

$F(m+1) - F(m) = 1 - e^{-\lambda(m+1)} - (1 - e^{-\lambda m})$

$= \cancel{1} - e^{-\lambda m} e^{-\lambda} \cancel{-1} + e^{-\lambda m}$

$= -e^{-\lambda m} e^{-\lambda} + e^{-\lambda m}$

$= \underline{\underline{e^{-\lambda m}(-e^{-\lambda} + 1)}}$

c) SME for $P(M=m)$:

$(1-e^{-\lambda})^n e^{-\lambda m} \cdot (1-e^{-\lambda}) e^{-\lambda(m+1)} \cdot \ldots \cdot (1-e^{-\lambda}) e^{-\lambda(m+n)}$

$(1-e^{-\lambda})^n (e^{-\lambda m} \cdot e^{-\lambda(m+1)} \cdot \ldots \cdot e^{-\lambda(m+n)})$

$(1-e^{-\lambda})^n e^{-\lambda(m_0 + m_1 + m_2 \ldots + m_n)} = L(\lambda)$

$\ln L(\lambda) = n \ln(1-e^{-\lambda}) + \underbrace{\ln(e^{-\lambda m_1} \cdot e^{-\lambda m_2} \cdot \ldots \cdot e^{-\lambda m_n})}$

$-\lambda(m + m_1 + \ldots + m_n) = \lambda \sum_{i=0}^{n} m_i$

$$= n\ln(1-e^{-\lambda}) - \lambda \sum_{i=0}^{n} m_i$$ deriverer denne

$$= \frac{ne^{-\lambda}}{1-e^{-\lambda}} - \sum_{i=0}^{n} m_i = 0$$

$$\frac{ne^{-\lambda}}{1-e^{-\lambda}} = \sum_{i=0}^{n} m_i$$ isolere λ, $| \cdot (1-e^{-\lambda})$

$$ne^{-\lambda} = \sum_{i=0}^{n} m_i - \sum_{i=0}^{n} e^{-\lambda}$$

$$e^{-\lambda}\left(n + \sum_{i=0}^{n} m_i\right) = \sum_{i=0}^{n} m_i$$

$$e^{-\lambda} = \frac{\sum_{i=0}^{n} m_i}{n + \sum_{i=0}^{n} m_i} \quad | \ln, (-1)$$

$$\underline{\underline{\lambda = -\ln\left(\sum_{i=0}^{n} m_i\right) + \ln\left(n + \sum_{i=0}^{n} m_i\right)}} = SME$$