HW 5

1. a. $b(3;8,.35)$ $x=3$ success $n=8$ trial $p=0.35$
$q=0.65$

$= \binom{8}{3} 0.35^3 \, 0.65^5 = 0.2786$

b. $b(5;8,.6) = \binom{8}{5} 0.6^5 \, 0.4^3 = 0.2787$

c. $P(3 \le X \le 5)$ $n=7$ $p=0.6$ $= b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$

$= \binom{7}{3} 0.6^3 \, 0.4^4 + \binom{7}{4} 0.6^4 \, 0.4^3 + \binom{7}{5} 0.6^5 \, 0.4^2$

$= 0.1935 + 0.2903 + 0.2613 = 0.7451$

d. $P(1 \le X) = P(X \ne 0) = 1 - P(X=0)$ $n=9$ $p=0.1$

$b(0;9,0.1) = \binom{9}{0} 0.1^0 \, 0.9^9 = 0.387 = P(X=0)$

$P(1 \le X) = 1 - 0.387 \sim 0.613$

2. a. $p = .60$ $n=10$ $x \ge 6$

$b(6;10,0.60) + \ldots b(10;10,0.60) =$

$\binom{10}{6} 6^{0.6} 4^{0.4} + \binom{10}{7} 7^{0.6} 3^{0.4} + \binom{10}{8} 8^{0.6} 2^{0.4} + \binom{10}{9} 9^{0.6} 1^{0.4} + \binom{10}{10} 10^{0.6} 0^{0.4}$

$= 0.251 + 0.215 + 0.121 + 0.040 + 0.006$

b. $\mu = 0.6 \times 10 = 6$ $\sigma = \sqrt{0.6 \cdot 0.4 \times 10} = 1.55$

$P(4.45 \le X \le 7.55) \sim P(4) + P(5) + P(6) + P(7)$

$= 0.111 + 0.200 + 0.251 + 0.215 = 0.777$

c. $n = 10 \quad p = 0.6 \quad q = 0.4$

- no more than 7 want oversize
- no more 7 want normal = no less than 3 want oversize

so, $3 \leq x \leq 7$. $P(3 \leq x \leq 7) = P(3) + P(4) \ldots + P(7)$

$= 0.777 + P(3) = 0.819$

3. revenue/vehicle $= r \quad r = 0.6 \times 1 + 0.4 \times 2.5 = 1.6$

$25 \times 1.6 = \$40$

4. a. $b(x; n, 1-p) = \binom{n}{x} x^{1-p} (n-x)^{p}$

$b(n-x; n, p) = \binom{n}{n-x} (n-x)^{p} (x^{p})$

$\binom{n}{x} \cancel{x^{1-p}} \cancel{(n-x)^{p}} = \binom{n}{n-x} \cancel{(n-x)^{p}} \cancel{x^{p}}$?

$\binom{n}{x} = \binom{n}{n-x}$?

$$\frac{n!}{x!(n-x)!} = \frac{n!}{(n-x!)(n-(n-x))!}$$

$$\frac{n!}{x!(n-x)!} = \frac{n!}{x!(n-x)!} \checkmark$$

b. $B(x; n, 1-p) = \binom{n}{x} x^{1-p} (n-x)^{p} = \frac{n!}{x!(n-x)!} x^{1-p} (n-x)^{p}$

$B(n-x-1; n, p) = \underbrace{\binom{n}{n-x-1} (n-x-1)^{p} (x+1)^{1-p}}$

$\rightarrow \frac{n!}{(n-x-1)!(x+1)!} (n-x-1)^{p} (x+1)^{1-p}$

same

substitute $x-1 \rightarrow y$: $\frac{n!}{(n-y)!\, y!} (n-y)^{p} y^{1-p}$

5. Hypergeometric

a. $P(X=x) = \frac{\binom{10}{x}\binom{20-10}{10-x}}{\binom{20}{10}}$

b. $\cancel{P(X=5) = \frac{\binom{10}{5}\binom{20-10}{10-5}}{\binom{20}{10}}}$

$20 \begin{cases} 10a \\ 10b \end{cases}$ $\frac{1}{2} \times \frac{9}{19} \times \frac{8}{18} \times \frac{7}{17} \times \frac{6}{16} = 0.1625$

p(first 5a)

$0.1625 \times 2 =$ p(first 5a) + p(first 5b) $= 0.0325$

c. $P(X=x) = \frac{\binom{n}{x}\binom{2n-n}{n-x}}{\binom{2n}{n}}$

$E(X) = \frac{2n}{2} = n$ $\quad V(X) = \frac{2np(1-p)(N-2n)}{N-1}$

6. a. $N=50 \quad M=15 \quad N-M=35$ failures

$n=10$

$$P(X=x)=\frac{\binom{15}{x}\binom{35}{10-x}}{\binom{50}{10}}$$

b. $$P(X=x)=\left(\frac{150}{500}\right)^{x}\left(\frac{350}{500}\right)^{1-x}$$

in part a

c. $E(x)=10\times\frac{15}{50}=3$

$$V(x)=\frac{N-n}{N-1}\cdot n\cdot\frac{M}{N}\cdot\left(1-\frac{M}{N}\right)$$

$$=\frac{40}{49}\cdot 10\cdot\frac{15}{50}\cdot\frac{35}{50}=1.714$$

for part b

$$E(x)=\frac{150}{500}\times 10=3$$

$$V(x)=\frac{500-10}{500-1}\cdot 10\cdot\frac{150}{500}\cdot\frac{350}{500}=2.06$$

7. a.

$$nb(x;2,0.2)=\underbrace{(1-0.2)^{x}}_{x \text{ Fails}}\underbrace{(0.2)^{2}}_{2 \text{ successes}}\binom{x+2-1}{2-1}$$

b. $nb(4-2;2,0.2)=\binom{2+2-1}{2-1}(1-0.2)^{2}(0.2^{2})$

$$=3\cdot 0.64\cdot 0.04=0.0768$$

c. $= nb(2;2,0.2) + nb(1;2,0.2) + nb(0;2,0.2)$

$= \binom{3}{1}(0.8)^2 \times (0.2)^2 + \binom{2}{1}(0.8)^1(0.2)^2 + \binom{1}{1}(0.8)^0(0.2)^2$

$= 0.0768 + 0.064 + 0.04 = 0.1808$

d. $E(x) = \frac{r(1-p)}{p} = \frac{2(1-0.2)}{0.2} = 8$ failures

total boxes = 8 fail + 2 success = 10 boxes