MTMW14: Numerical Modelling of Atmosphere and Oceans

Notes on the time scheme for Project 1: practical considerations on Runge-Kutta implementation.

The equations

Jin's recharge oscillator is described by two ordinary differential equations:

$$\frac{dT}{dt} = RT + \gamma h - \epsilon(h + bT)^3 + \gamma\xi \quad (1)$$

$$\frac{dh}{dt} = -rh - \alpha bT - \alpha\xi \quad (2)$$

where the prognostic variables are T (east Pacific SST anomaly) and h (west Pacific ocean thermocline depth).

This set of equations can be thought of like this:

$$\frac{d}{dt}\begin{bmatrix} T \\ h \end{bmatrix} = \begin{bmatrix} RT + \gamma h - \epsilon(h + bT)^3 + \gamma\xi \\ -rh - \alpha bT - \alpha\xi \end{bmatrix} = F(\begin{bmatrix} T \\ h \end{bmatrix}, t, \cdots) \quad (3)$$

In order to implement the RK scheme, interpret the q in the lecture notes as a vector with dimension 2.

You can then do this:

$$\begin{bmatrix} k_1 \\ l_1 \end{bmatrix} = F(\begin{bmatrix} T \\ h \end{bmatrix}, t, \Delta t, \mu, \cdots) \quad (4)$$

$$\begin{bmatrix} k_2 \\ l_2 \end{bmatrix} = F(\begin{bmatrix} T + \frac{k_1\Delta t}{2} \\ h + \frac{l_1\Delta t}{2} \end{bmatrix}, t + \frac{\Delta t}{2}, \Delta t, \mu, \cdots) \quad (5)$$

$$\begin{bmatrix} k_3 \\ l_3 \end{bmatrix} = F(\begin{bmatrix} T + \frac{k_2\Delta t}{2} \\ h + \frac{l_2\Delta t}{2} \end{bmatrix}, t + \frac{\Delta t}{2}, \Delta t, \mu, \cdots) \quad (6)$$

$$\begin{bmatrix} k_4 \\ l_4 \end{bmatrix} = F(\begin{bmatrix} T + k_3\Delta t \\ h + l_3\Delta t \end{bmatrix}, t + \Delta t, \Delta t, \mu, \cdots) \quad (7)$$

While the scheme will run fine if you do not update the [k,l] pair concurrently through each step (1 to 4), the scheme will lose accuracy (and revert to first order) if you do so. Please refer to Durran's book (2nd edition), pages 49-58 for further details. I may be able to re-discover the demonstration in a week or so.

References

Galanti, E. and Tziperman, E. (2000). ENSO's phase locking to the seasonal cycle in the fast-SST, fast-wave and mixed-mode regimes. *J. Atmos. Sci.*, **57**, 2936–2950.

Jin, F.-F. (1997). An equatorial ocean recharge paradigm for ENSO: Part I: Conceptual model. *J. Atmos. Sci.*, **54**, 811–829.