MTMW14: Numerical Modelling of Atmosphere and Oceans

Project 1:
The ocean recharge oscillator, a reduced coupled model for ENSO illustrating the effects of nonlinearity and forcing, based on the model of Fei-Fei Jin (1997).

Aim

Learn about oscillatory behaviour, stability, and how the choice of **a time scheme appropriate for this specific problem** will influence the realism of your results.

Problem Description

El Niño Southern Oscillation (ENSO) is the most prominent mode of coupled ocean-atmosphere variability. One essential mechanism to create the oscillation is the interaction between surface wind stress and thermocline depth along the equator. Bjerknes (1969) first recognised that a gradient in sea surface temperature (SST) increasing from east to west along the equator drives easterly winds across the Pacific. These easterlies cause the thermocline (the mixed layer on top of the ocean) to deepen in the west Pacific and become shallower in the east, acting to enhance the SST gradient and forming a positive feedback. Similarly, warm anomalies in the east Pacific are enhanced through positive feedback via anomalous westerlies. A second mechanism is required to enable the system to flip from the -ve to +ve SST gradient and back. Dijkstra (2005) discusses a number of theories that have been proposed and points to the paper of Jin (1997a) as providing the most plausible simple model that can explain the observations. In Jin's model the cold SST phase in the East Pacific is brought to an end by *recharge* of the zonal average ocean heat content as the enhanced easterly wind stress drives convergence of Sverdrup transport in the ocean. The physical justification for the model ((Jin, 1997b)) will not be discussed further here. Instead we will focus on using the model to explore the effects of nonlinearity and forcing on coupled systems.

Jin's recharge oscillator is described by two ordinary differential equations:

$$\frac{dT}{dt} = RT + \gamma h - \epsilon(h + bT)^3 + \gamma\xi \qquad (1)$$

$$\frac{dh}{dt} = -rh - \alpha bT - \alpha\xi \qquad (2)$$

where the prognostic variables are T (east Pacific SST anomaly) and h (west Pacific ocean thermocline depth). The model is **non-dimensionalised** using:

SST anomaly-scale	Thermocline depth-scale	Time-scale
$[T] = 7.5\,\mathrm{K}$,	$[h] = 150\,\mathrm{m}$	$[t] = 2\,\mathrm{months}$

Parameters

The list below gives default (non-dimensional) values for the model parameters, together with their physical interpretation:

$b = b_0\mu$	relates a stronger thermocline gradient to stronger easterly wind stress caused by anomalously negative East Pacific SST. μ is the *coupling coefficient*.
$b_0 = 2.5$	is a high-end value of the coupling parameter.
$\gamma = 0.75$	specifies the feedback of the thermocline gradient on the SST gradient.
$c = 1$	is the damping rate of SST anomalies.
$R = \gamma b - c$	collectively describes the Bjerknes positive feedback process.
$r = 0.25$	represents damping of the upper ocean heat content.
$\alpha = 0.125$	relates enhanced easterly wind stress to the recharge of ocean heat content.
ϵ	varies the degree of nonlinearity.
ξ	represents additional wind stress forcing of the system.

Tasks

Task A

First devise a finite difference numerical model to simulate the recharge oscillator without forcing ($\xi = 0$) or nonlinearity ($\epsilon = 0$).

Write down the equations for your finite difference scheme and then implement them in a program using variable names for all the parameters.

Set the values of model parameters as given above and set $\mu = 2/3$. Jin (1997a) has shown that this setting produces a stable oscillation with frequency $\omega_c = \sqrt{3/32}$ and therefore period $\tau_c = 2\pi/\omega_c$ ($\approx$41 months when dimensionalised).

Run your model for exactly one period from the initial conditions: $T = 1.125\,\mathrm{K}$ and $h = 0$. Produce a plot with T and h as the axes (re-dimensionalise by multiplying values by $[T]$ and $[h]$) showing the trajectory of the solution (see Fig. 3 of Jin (1997a) for an example - your solution should be similar but not exactly the same). Is your numerical method stable? Explain how you deduce this. If it is not stable, you might check your scheme for errors, or try a different scheme.

Task B

Re-run your model for 5 periods with a value of $\mu > 2/3$ and $\mu < 2/3$. Plot the trajectory in $T - h$ coordinates as before and also plot T versus time (dimensionalised). What happens to the trajectory? Can you explain why?

Task C

Turn on nonlinearity by setting $\epsilon = 0.1$. Run the model with $\mu = 2/3$. Compare the trajectory to the case without nonlinearity.

Try increasing μ and compare what happens to the case with $\epsilon = 0$ (Task B).

Task D

Galanti and Tziperman (2000) introduced the annual frequency to the problem by allowing the coupling parameter to vary on an annual cycle using:

$$\mu = \mu_0 \left(1 + \mu_{ann} \cos\left(\frac{2\pi t}{\tau} - \frac{5\pi}{6}\right)\right). \tag{3}$$

Modify your model to include the annual cycle in coupling parameter, setting $\epsilon = 0.1$, $\mu_0 = 0.75$ and $\mu_{ann} = 0.2$. τ is 12 months (remember that you must divide this by 2 months to non-dimensionalise). What happens to the time series of T and the trajectory?

Task E

Finally add "wind stress forcing" to your model by setting:

$$\xi = f_{ann} \cos\left(\frac{2\pi t}{\tau}\right) + f_{ran} W \frac{\tau_{cor}}{\Delta t} \tag{4}$$

where W is a number between -1 and +1 picked at random (assuming uniform probability for the number lying at any point across the range) after every interval τ_{cor}. This represents a white noise process. Use a random number function to generate the random numbers (and check that you re-scale the result to the correct range between -1 and +1).

Run the model with the parameter settings: $\epsilon = 0.1$, $\mu_0 = 0.75$, $\mu_{ann} = 0.2$, $f_{ann} = 0.02$, $f_{ran} = 0.2$ and $\tau_{cor} = 1\,\text{day}$. Set the model time-step $\Delta t = 1\,\text{day}$ for simplicity. Describe the new time series of T and the trajectory. What are the major differences from the earlier runs? What are the effects of annual forcing and the random forcing?

Task F

The irregular nature of the trajectories indicates that the model is likely to be sensitive to initial conditions. A consequence is that an ensemble should be used to indicate the spread of possible ENSO forecasts, given uncertainty in the initial state of the system and the random nature of the forcing.

Design a forecast ensemble using this model by introducing an outer loop over ensemble members and perturbing the initial T and h used to start each forecast. Ensure that you pick a sensible range over which to vary the initial conditions (for example within the amplitude of the oscillation).

Produce “plume diagrams” showing the time series of SST from each ensemble forecast on the same plot.

Use your ensemble system to explore the ingredients necessary to produce sensitivity to initial conditions. Is the model chaotic? Can you produce an SST time series that resembles the character of the observed ENSO signal? If not, how does it differ?

Milestones

1. Week 2: complete reading the full problem set and the two papers; design the code structure on paper, then implement a small library of functions, including one for parameters
2. Week 3: implement full set of equations and run for each case
3. Week 4: complete the ensemble definition work and run the ensemble experiment AFTER you have fully interpreted all the results in Tasks A to E
4. Week 5: work on the physical interpretation of each Task; wrap-up as an overall conclusion on the realism of your model

Practical matters

Project 1 is **worth 35%** of the mark for this module. Assessment is via a short scientific report describing what you have found, very much like a scientific paper. The report should be word-processed and:

1. preferentially in the form of a Python notebook (for 3 bonus points). Please note that, while recommended, this is entirely your choice, it is NOT compulsory.
2. the scientific discussion should not exceed **six sides of A4 including figures**
3. your code will also be assessed (upload it by 9 February). It should be highly structured, so that the main program should be at most 1 page. Everything else is to be written as functions, called by the main program (those should be included as an Appendix, and do not count towards the page limit, but concise code will earn full points)
4. variable names and units should be declared at the top of the main program and of each function
5. all figures and equations must be labelled, numbered and captioned

Start by stating the problem and the fundamental equations, but do not include lengthy background material or a literature review. The emphasis is on the scientific justification of your method to solve this problem numerically and the accuracy and interpretation of the results. Follow the structure that we discussed in Lecture 1: formulation, implementation, evaluation.

The marks will be distributed as follows:

- implementation of the model (including your program's accuracy and legibility and a detailed description of the numerical scheme) [10/35]
- using the model to answer Tasks A to E [10/35]
- creating ensemble forecasts and interpreting results (Task F) [7/35]
- the scientific presentation and interpretation of the results, including a coherent and concise (scientific!) writing style (remember to label sections, graph axes, units etc.) [8/35]

The deadline for submission of the report is **12:00, Friday 9 February 2018**. Late submission will result in loss of marks. Submit your reports to the BlackBoard as a single *tar.gz* archive.

Planning your work

Project 1 is fairly complex and requires careful planning, as well as regular work.

1. start by reading the paper and tasks and set realistic aims for each week
2. aim to understand the overall problem, and each task, by referring to the original papers, before you attempt to code things up
3. communicate progress regularly to the instructor and demonstrators, so that we may identify problems and/or delays as early as possible
4. **design your plan, algorithms etc. on paper**, including equations, units etc., and only then start coding
5. draw your diagrams on paper and compare with the ones produced by your programs
6. use modern coding design, so that your code is easy to maintain
7. use the GitHub repository for backup etc.

References

Bjerknes, J. (1969). Atmospheric teleconnections from the equatorial pacific. *Mon. Weather Rev.*, **97**, 163–172.

Dijkstra, H. (2005). *Nonlinear Physical Oceanography*. Springer. 532 pp.

Galanti, E. and Tziperman, E. (2000). ENSO's phase locking to the seasonal cycle in the fast-SST, fast-wave and mixed-mode regimes. *J. Atmos. Sci.*, **57**, 2936–2950.

Jin, F.-F. (1997a). An equatorial ocean recharge paradigm for ENSO: Part I: Conceptual model. *J. Atmos. Sci.*, **54**, 811–829.

Jin, F.-F. (1997b). An equatorial ocean recharge paradigm for ENSO: Part II: A stripped-down coupled model. *J. Atmos. Sci.*, **54**, 830–847.