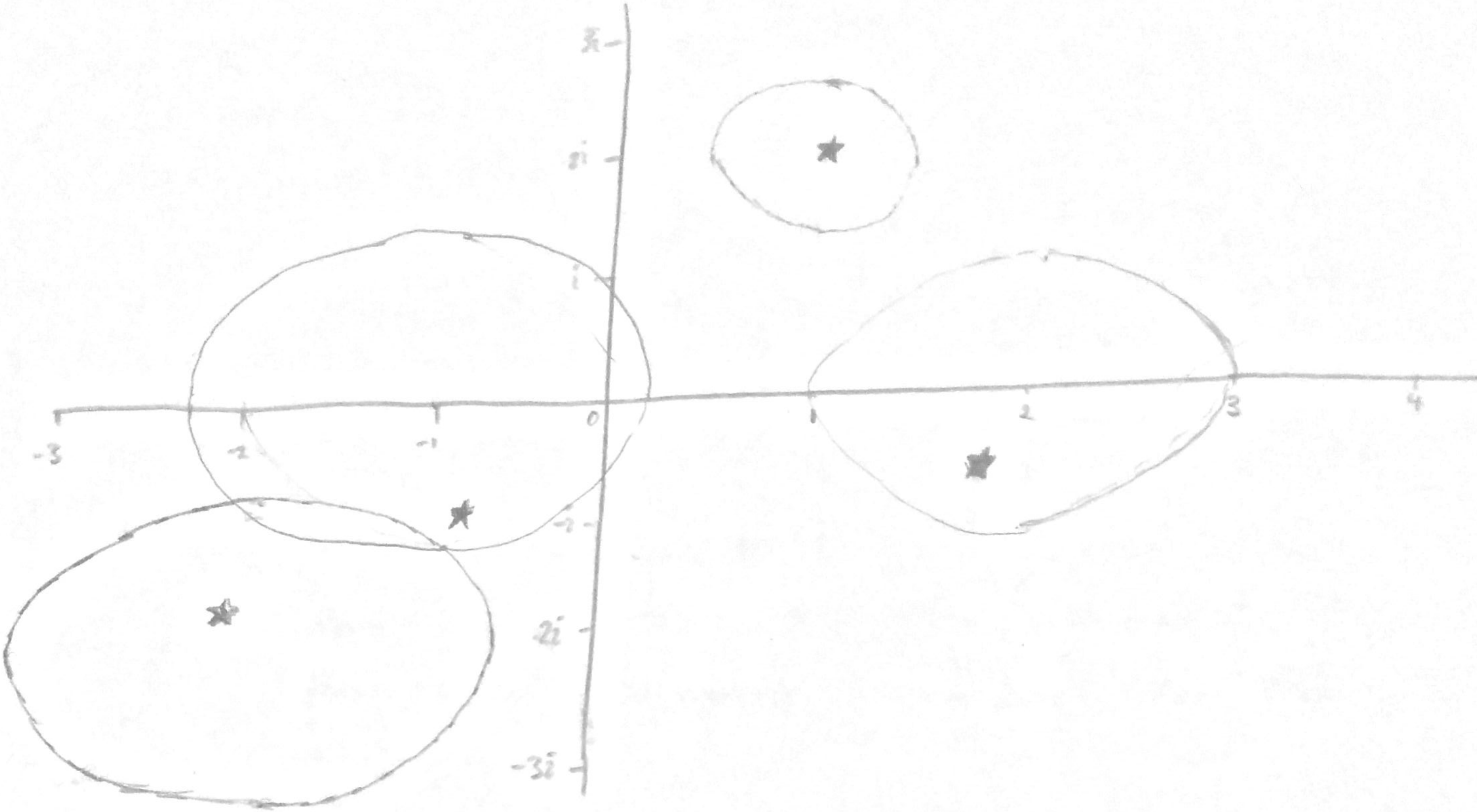
1. $D(2,1),\ D(1+2i, \frac{1}{2}),\ D(-1, \frac{5}{4}),\ D(-2-2i, \frac{5}{4})$ *



$D(a,b)$ indicates a disc at a, with radius b.

$Sp(A) = \{1.93 - .045i,\ 1.01 + 2.07i,\ -0.91 - 0.09i,\ -2.03 - 1.94i\}$

2 The algorithm I used to determine diagonalizability of a matrix is as follows:

```
diagonalize(A)
    similarity-matrix = [ ]
    for eigenvalue e of A
        if the dimension of the eigenspace of e is
            less than the multiplicity of e, then
            return false

        else:
            Add a column to similarity-matrix for
            each dimension of the eigenspace, where
            the new columns are a basis for the
            eigenspace.

    return true.
```

The algorithm can be found at
www.sosmath.com/matrix/diagonal/diagonal.html

I have implemented the algorithm in diagonalize.m, and the results of applying the algorithm to B_1 and B_2 are:

B_1 is diagonalizable, with $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4.83 & 0 \\ 0 & 0 & 0.83 \end{bmatrix}$ as the diagonal mat

and $\begin{bmatrix} 0 & 0.18 & -0.73 \\ 0 & 0.88 & 0.61 \\ 1 & 0.44 & 0.30 \end{bmatrix}$ as the similarity matrix

However, B_2 is NOT diagonalizable.

3] We start with calculating $\sigma(C_a)$, this is

$|C_a - \lambda I| = \lambda^2 + (-a-1)\lambda + a \Rightarrow \sigma(A) = \{a, 1\}$.

For eigenvalue $\lambda = a$, the eigenspace is the nullspace of $\begin{bmatrix} -a & 1 \\ -a & 1 \end{bmatrix}$

This is the space of vectors of form $(x_1, a x_1)$. This has dimension 1, which is $\geq$ multiplicity$(a) = 1$ so long as $a \neq 1$

For eigenvalue $\lambda = 1$, the eigenspace is the null space of

$\begin{bmatrix} -1 & 1 \\ -a & 1-a \end{bmatrix}$. I'll spare the working through the algebraic manipulations. This space is the set of vectors of form (x_1, x_1). This is 1-dimensional, without any assumptions on a. So C_a is diagonalizable iff $a \neq 1$. When it is diagonalizable, it can be diagonalized as

$$\begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix}^{-1} C_a \begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix}$$

4

$$J_{B_\lambda} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix} \qquad J_{C_a} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$$

---

5 In my implimentation of is_irreducible_matrix(A), I used Lemma 8.4.1:

Let $A \in M_n$. Then A is irreducible iff

$$(I + |A|)^{n-1} > 0$$

And For primitivity, I used Theorem 6.2.23

A nonnegative matrix $A \in M_n$ is primitive if it is irreducible and has only one eigenvalue of maximum modulus.

It turns out that D_1 is irreducible but not primitive

D_3 is irreducible and primitive, and

D_2 is reducible, and is already in reduced form.