MATH 318 Homework 0

Joshua Lim (26928218)

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Problem 2

Part A

To calculate the number of arrangements we can make with this word, we observe that `ABRACADABRA` is 11 letters long and consists of 5 A's, 2 B's, 1 C, 1 D, and 2 R's. Thus, we can do the following calculation below:

$$\frac{11!}{5! \cdot 2! \cdot 2! \cdot 1! \cdot 1!} = 83160$$

There are **83160** different arrangements you can make with `ABRACADABRA`.

Part B

To calculate the number of arrangements with all 5 A's at the front of the word, we can assume the first 5 letters of the word will always be A. Therefore, the last 6 letters of the word will vary in order so that's the only part that matters. Focusing on the last 6 letters, we observe that there are 2 B's, 1 C, 1 D, and 2 R's. So, we can do a similar calculation as we did with Part A:

$$\frac{6!}{2! \cdot 2! \cdot 1! \cdot 1!} = 180$$

There are **180** different arrangements you can make with all five A's at the start.

Part C

To calculate the number of arrangements with all 5 A's together, we can make an arbitrary letter 'X' that will represent 'AAAAA', resulting in the word `XBRCDBR` for example. This new version of the word has 7 letters consisting of 1 X, 2 B's, 1 C, 1 D, and 2 R's. Again, we can do a similar calculation from before:

$$\frac{7!}{2! \cdot 2! \cdot 1! \cdot 1!} = 1260$$

There are **1260** different arrangements you can make with all five A's at the start. This makes sense because this answer should be 7 times greater than Part B's answer of 180 arrangements due to Part B being 1 out of 7 cases for Part C.

Part D

To calculate the number of arrangements with no 2 consecutive A's, let's break this problem up into two parts: the word without the A's and only the A's.
Looking at the other letters, we can create `BRCDBR` and calculate the number of arrangements:

$$\frac{6!}{2! \cdot 2! \cdot 1! \cdot 1!} = 180$$

Then, to look at the A's, let's space out `BRCDBR` to see where each A can fit.

_B_R_C_D_B_R_

Seeing that there are 7 slots for 5 A's, we can calculate the number of arrangements for the A's:

$$\binom{7}{5} = \frac{7!}{5! \cdot (7-5)!} = \frac{7!}{5! \cdot 2!} = 21$$

Finally, multiply both parts to get the total arrangements for the entire word:

$$\frac{6!}{2! \cdot 2! \cdot 1! \cdot 1!} \cdot \binom{7}{5} = 180 \cdot 21 = 3780$$

There are **3780** arrangements you can make with no two consecutive A's.

Problem 3

```
import numpy as np

def problem3(string):
    numerator = np.math.factorial(len(string))
    denominator = 1

    s = set(string)

    for i in s:
        count = 0
        for char in string:
            if(i == char):
                count += 1

        denominator *= np.math.factorial(count)

    return (numerator/denominator)
```