Lab 1

1.) a) $\frac{dx}{dt} = t^n$, $n \neq -1$, $x(1) = 1$ $\quad \int \frac{dx}{\cancel{dt}} \cancel{dt} = \int t^n \, dt$

$x(t) = \frac{t^{n+1}}{n+1} + C \rightarrow x(1) = 1 = \frac{1^{n+1}}{n+1} + C \rightarrow 1 = \frac{1}{n+1} + C \rightarrow \cancel{C = \frac{1}{2}}$

$\boxed{\therefore x(t) = \frac{t^{n+1}}{n+1} + 1 - \frac{1}{n+1} \quad n \neq -1}$

b.) $\frac{dx}{dt} = \frac{1}{t}$, $t > 0$ $\quad \int \frac{dx}{\cancel{dt}} = \int \frac{1}{t} dt$

$\boxed{x = \ln|t| + C, \quad t > 0}$

c) $f(t) = \frac{dx}{dt} = te^{-t}$, $x(0) = 1$ $\quad (\int uv' = uv - \int v\,du)$

$\int dx = \int te^{-t} dt \rightarrow$ $u = t$, $v' = e^{-t}$; $u' = 1$, $v = -e^{-t}$

$x = -te^{-t} - \int -e^{-t} = -te^{-t} - e^{-t} + C = e^{-t}(-t-1) + C$

$x(0) = 1 = e^{0}(0-1) + C = 1(-1) + C \rightarrow 1 = -1 + C \rightarrow C = 2$

$\therefore \boxed{x = e^{-t}(-t-1) + 2}$

Lab 1

1) d) $\frac{dx}{dt} = \frac{1}{(t+1)(t+2)}$, $x(0) = 0$ $\qquad \frac{1}{(t+1)(t+2)} = \frac{A}{(t+1)} + \frac{B}{(t+2)}$

$1 = A(t+2) + B(t+1)$

@ $t = -1$: $1 = A(2-1) + 0 \rightarrow A = 1$

@ $t = -2$: $1 = 0 + B(-1) \rightarrow B = -1$

$$\underbrace{\frac{dx}{dt} = \frac{1}{t+1} + \frac{-1}{t+2}}$$

$$\int \frac{dx}{\cancel{dt}} = \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt \rightarrow x = \int \frac{1}{u} du - \int \frac{1}{w} dw$$

$u = t+1$, $du = dt$ $\qquad w = t+2$, $dw = dt$

$$x = \ln|u| - \ln|w| + c = \ln|t+1| - \ln|t+2| + c$$

$$x(0) = 0 = \ln(1) - \ln(2) + c \rightarrow c = \ln(2) - \ln(1)$$

$$\therefore \boxed{x(t) = \ln|t+1| - \ln|t+2| + \ln(2) - \ln(1)}$$

Lab 1

2. a) $x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 0$, $y(x) = x^m$, $y' = mx^{(m-1)}$
$y'' = m(m-1)x^{m-2}$

$x^2 \cdot m(m-1)x^{(m-2)} - 5x\,mx^{(m-1)} + 8x^m = 0$

$= x^2(m^2-m)x^{(m-2)} - 5x\,mx^{(m-1)} + 8x^m = 0$

$= \cancel{x^2}(m^2-m)x^m \cdot \cancel{x^{-2}} - 5\cancel{x}\,mx^m \cdot \cancel{x^{-1}} + 8x^m = 0$

$= x^m(m^2 - m - 5m + 8) = 0$

$\therefore m^2 - 6m + 8 = 0 \rightarrow (m-4)(m-2) = 0 \therefore \boxed{\begin{matrix} m_1 = 4 \\ m_2 = 2 \end{matrix}}$

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2. b) $x^2 \cdot y'' - 5x\,y' + 8y = 0$; $y(x) = C_1x^{m_1} + C_2x^{m_2}$

$y'(x) = 4C_1x^3 + 2C_2x$, $y'' = 12C_1x^2 + 2C_2$

$\therefore x^2(12C_1x^2 + 2C_2) - 5x(4C_1x^3 + 2C_2x) + 8(C_1x^4 + C_2x^2) = 0$

$\rightarrow \cancel{12C_1x^4} + 2C_2x^2 \cancel{- 20C_1x^4} - 10C_2x^2 \cancel{+ 8C_1x^4} + 8C_2x^2 = 0$

$\rightarrow \boxed{\cancel{2C_2x^2}} \boxed{\cancel{-10C_2x^2}} \boxed{\cancel{+8C_2x^2}} = 0 \therefore 0 = 0$ ✓

$\boxed{y(x) = C_1x^4 + C_2x^2 \text{ is a solution to the DE}}$

Lab 1

3.) $v(t) = \begin{cases} 0, & t \in [0,1) & (a) \\ 2t-2, & t \in [1,3) & (b) \\ 4, & t \in [3,5) & (c) \\ 14-2t & t \in [5,7] & (d) \end{cases}$

a.) $\underline{x(t) = 0}, \ t \in [0,1)$

b.) $x(t) = \int (2t-2)dt = t^2 - 2t + c \rightarrow x(1) = 0 = 1^2 - 2(1) + c \rightarrow c = 1$

$\therefore \underline{x(t) = t^2 - 2t + 1}, \ x(3) = 9 - 6 + 1 = \underline{4}$

c.) $x(t) = \int 4 \, dt = 4t + c \rightarrow x(3) = 4 = 4(3) + c \rightarrow c = -8$

$\therefore \underline{x(t) = 4t - 8} \qquad x(5) = 20 - 8 = 12$

d.) $x(t) = \int (14-2t)dt = 14t - t^2 + c \rightarrow x(5) = 12 = 14(5) - 5^2 + c$

$\rightarrow 12 = 70 - 25 + c$

$12 = 45 + c$

$c = -33$

$\therefore x(t) = 14t - t^2 - 33$

$$\boxed{\therefore x(t) = \begin{cases} 0, & t \in [0,1) \\ t^2 - 2t + 1, & t \in [1,3) \\ 4t - 8, & t \in [3,5) \\ 14t - t^2 - 33 & t \in [5,7] \end{cases}}$$

$x(7) = 14 \cdot 7 - 7^2 - 33 = \boxed{16 = x(7)}$