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Page 1

HW #3

1) $\cancel{Accum} = In - out + \cancel{gen}$

$$k\frac{d^2T}{dx^2} = 0 \rightarrow \frac{dT}{dx} = C_1 \rightarrow T = C_1x + C_2$$

$$\frac{dT}{dt} = \frac{-q''}{k} \rightarrow q'' = \overbrace{h(T_2 - T_\infty)}^{conv} = \frac{-k}{L}(T_2 - T_1) = k\frac{dT}{dx}$$

$$T_1 = \frac{Lh}{+k}(T_2 - T_\infty) + T_2 \qquad T(0) = T_1 = C_2$$

$$\therefore T = C_1x + \frac{Lh}{k}(T_2 - T_\infty) + T_2$$

$$T(L) = \cancel{T_2} = C_1L + \frac{Lh}{k}(T_2 - T_\infty) + \cancel{T_2}$$

$$C_1 = -\frac{h}{k}(T_2 - T_\infty) \quad \therefore \boxed{T(x) = \frac{-h}{k}(T_2 - T_\infty)x + \frac{Lh}{k}(T_2 - T_\infty) + T_2}$$

b) $T(0) = 0 + \dfrac{0.2\,m \cdot 95\,\frac{W}{m^2K}}{25\,W/mK}(50-20)K + 50°C \;\boxed{= 72.8\,°C = T_1}$

c) $q'' = -k\dfrac{dT}{dx} \rightarrow q'' = -k \cdot \dfrac{-h}{k}(T_2 - T_\infty) \;\boxed{= 2850\,\frac{W}{m^2}}$

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HW #3

2) Accum = In − out + gen

$$-k\frac{d^2T}{dz^2} = 0 \rightarrow \frac{dT}{dz} = C_1$$

$$\frac{q''}{-k} = \frac{dT}{dz} \quad \therefore C_1 = \frac{dT}{dz} \rightarrow \frac{dT}{dz} = \frac{q''}{-k} \rightarrow T = \frac{q''}{-k}z + C_2$$

$$T(L) = T_2 = \frac{q''}{-k}L + C_2 \rightarrow C_2 = T_2 + \frac{q''}{k}L$$

$$\therefore T(z) = -\frac{q''}{k}z + T_2 + \frac{q''}{k}L \boxed{= \frac{q''}{k}(L-z) + T_2}$$

b) @ $z = 0$, $T_1 = ?$

$$T(0) = \frac{900\frac{W}{m^2}}{15\frac{W}{mK}}(0.15-0) + 45°C \boxed{= 54°C}$$

c) $T(z) = \frac{q''}{k}(L-z) + T_2 \cdot \quad T(0.1) = 48$

$$q'' = \frac{(T(0.1) - T_2)k}{L-z} = \frac{(48-45)\cdot 15}{0.05} \boxed{= 900\frac{W}{m^2}}$$

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HW#3

3) $-k\frac{dT}{dx^2}=\dot{q} \rightarrow \frac{dT}{dx}=\frac{-\dot{q}}{k}x+C_1$

$T=\int\frac{-\dot{q}}{k}x+C_1\,dx \rightarrow T(x)=\frac{-\dot{q}}{2k}x^2+C_1x+C_2$

$T(0)=T_1=0+0+C_2 \rightarrow C_2=T_1$

$T(L)=\frac{-\dot{q}}{2k}L^2+C_1L+T_1=T_2 \quad \therefore C_1=\frac{T_2-T_1+\frac{\dot{q}}{2k}L^2}{L}$

$\boxed{T(x)=\frac{-\dot{q}}{2k}x^2+x\left[\frac{T_2-T_1+\frac{\dot{q}}{2k}L^2}{L}\right]+T_1}$

$T(L)=T_2=\frac{-\dot{q}}{2k}L^2+\frac{L}{\cancel{L}}\left(T_2-T_1+\frac{\dot{q}L^2}{2k}\right)+T_1$

b) $\boxed{T_1=72.5}$

c) $q''=-k\nabla T=-k\frac{dT}{dx}\boxed{=-k\left(\frac{-\dot{q}x}{k}+\frac{T_2-T_1}{L}+\frac{\dot{q}L}{2k}\right)}$

d) $q''=-15\left(\frac{50000\cdot0.15}{15}+\frac{35-72.5}{0.15}+\frac{50000\cdot0.15}{2\cdot15}\right)\boxed{=7500\frac{W}{m^2}}$

$Acc\cancel{um}=In-out+gen \rightarrow gen=out \rightarrow \dot{q}\cdot L\boxed{=7500\frac{W}{m^2}}$

Both methods yield same answer for q''

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HW #3

4) $\cancel{\text{Accum}} = \text{In} - \text{out} + \cancel{\text{gen}} \rightarrow \text{In} = \text{out}$

b) $q''_x = q''_{x+\Delta x} \rightarrow \dfrac{q''_x - q''_{x+\Delta x} = 0}{\Delta x} = 0 \xrightarrow{\Delta x \to 0} \dfrac{d q''_x}{dx} = 0$

$q'' = -k \dfrac{dT}{dx} \quad \therefore \quad 0 = \dfrac{d}{dx}\left(-k \dfrac{dT}{dx}\right) = -k \dfrac{d^2T}{dx^2}$

$$\boxed{0 = -aT \frac{d^2T}{dx^2}}$$

a) $\text{Accum} = \text{In} - \text{out} + \text{gen} \rightarrow \rho C_p \dfrac{dT}{dt} \Delta x = q''_x - q''_{x+\Delta x} + \dot{q}\Delta x$

$\rho C_p \dfrac{dT}{dt} = \dfrac{q''_x - q''_{x+\Delta x}}{\Delta x} + \dot{q} \xrightarrow{\Delta x \to 0} \rho C_p \dfrac{dT}{dt} = \dfrac{d q''_x}{dx} + \dot{q}$

$q''_x = -k \dfrac{dT}{dx} \quad \therefore \boxed{\rho C_p \dfrac{dT}{dt} = -aT \dfrac{d^2T}{dx^2} + \dot{q}} \rightarrow 0 = -aT \dfrac{d^2T}{dx^2} + 0$

c) $0 = -aT \dfrac{d^2T}{dx^2} \rightarrow \dfrac{d^2T}{dx^2} = 0 \rightarrow \dfrac{dT}{dx} = C_1 = \dfrac{q''_x}{-k}$

$\therefore \cancel{\dfrac{dT}{dx} = \dfrac{q''}{k}} \qquad \int dT = \int C_1\,dx \Rightarrow T = C_1 x + C_2$

$T(0) = T_1 = 0 + C_2 \quad \therefore \quad C_2 = T_1$

$T(L) = C_1 L + T_1 = T_2 \quad \therefore \quad C_1 = \dfrac{T_2 - T_1}{L}$

$$\boxed{T(x) = \frac{T_2 - T_1}{L} x + T_1}$$

d) $T(0.03) = \dfrac{600 - 300}{0.1} \cdot 0.03 + 300 \; \boxed{= 390\ \text{K}}$

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ChEn 3453

HW#3

5) Accum = In − out + gen

$$\rho C_p \frac{\partial T}{\partial t}\Delta r \cdot r\Delta\phi \cdot \Delta z = (q_r'' - q_{r+\Delta r}'')r\Delta\phi \cdot \Delta z + (q_\phi'' - q_{\Delta\phi})\Delta r\Delta z$$

$$+ (q_z'' - q_{\Delta z}'')\Delta r \cdot r\Delta\phi + \dot q\,\Delta r \cdot r\Delta\phi \cdot \Delta z$$

$$\rho C_p \frac{\partial T}{\partial t} = \frac{q_r'' - q_{\Delta r}''}{\Delta r} + \frac{q_\phi'' - q_{\Delta\phi}''}{r\Delta\phi} + \frac{q_z'' - q_{\Delta z}''}{\Delta z} + \dot q$$

$$\lim_{\Delta r, \Delta\theta, \Delta z \to 0} \rho C_p \frac{\partial T}{\partial t} = \frac{\partial q_r''}{\partial r} + \frac{\partial q_\phi''}{r\partial\phi} + \frac{\partial q_z''}{\partial z} + \dot q$$

$$q_r'' = k\frac{\partial T}{\partial r}, \quad q_\phi'' = k\frac{\partial T}{r\partial\phi}, \quad q_z'' = k\frac{\partial T}{\partial z}$$

$$\therefore \boxed{\cancel{\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial r^2} + k\frac{\partial^2 T}{(r\partial\phi)^2} + k\frac{\partial^2 T}{\partial z^2} + \dot q}}$$

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot q$$

HW #3

6) Accum = In − out + gen

no $\Delta\phi$ or Δz $\therefore$ $0 = \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \rightarrow r\frac{dT}{dr} = C_1$

$\frac{dT}{dr} = \frac{q''}{-k}$ $\therefore$ $\frac{C_1}{r} = \frac{q''}{-k} \rightarrow C_1 = \frac{rq''}{-k}$

$\int dT = \int \frac{C_1}{r}\,dr \Rightarrow T = C_1 \ln(r) + C_2 = \frac{rq''}{-k}\ln(r) + C_2$

$T(r_1) = \frac{r_1 q''}{-k}\ln(r_1) + C_2 \rightarrow C_2 = T_1 + \frac{r_1 q''}{k}\ln(r_1)$

a) $\boxed{\therefore T(r) = \frac{rq''}{-k}\ln(r) + T_1 + \frac{r_1 q''}{k}\ln(r_1)}$

b) $T(0.1) = T_2 = \frac{0.1 \cdot 1200}{-1}\ln(0.1) + 450 + \frac{0.05 \cdot 1200}{1} \cdot \ln(0.05)$

$\boxed{= 547.\ K}$

Josh Whitehead
ChEn 3453
Page 7

HW #3

7) Accum = In − out + gen

$$0 = \frac{k}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) \rightarrow r^2\frac{dT}{dr} = C_1$$

$$\int dT = \frac{-C_1}{r} + C_2 = T$$

$$T_1 = \frac{-C_1}{r_1} + C_2 \qquad T_2 = \frac{-C_1}{r_2} + C_2$$

$$C_2 = T_1 + \frac{C_1}{r_1}$$

$$T_2 = \frac{-C_1}{r_2} + T_1 + \frac{C_1}{r_1} \rightarrow (T_2 - T_1) = \frac{C_1(r_2 - r_1)}{r_2 r_1}$$

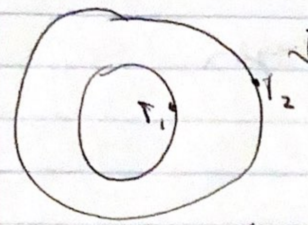
$$C_1 = \frac{r_2 r_1 (T_2 - T_1)}{(r_2 - r_1)}$$

$$T_1 = \frac{-r_2 \cancel{r_1}(T_2 - T_1)}{(r_2 - r_1)\cancel{r_1}} + C_2 \rightarrow C_2 = T_1 + \frac{r_2(T_2 - T_1)}{(r_2 - r_1)}$$

$$\therefore \boxed{T = \frac{r_2 r_1 (T_2 - T)}{(r_2 - r_1) r} + T_1 + \frac{r_2(T_2 - T_1)}{(r_2 - r_1)}}$$

Josh Whitehead
Ch En 3453
Page 8

HW #3

8)  ~~Accum~~ = ~~In~~ - out + gen → gen = out

a) T_f) $-\dot{q} = \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \rightarrow \int d\left(r\frac{dT}{dr}\right) = \int \frac{-\dot{q}r}{k_f}\,dr$

$r\frac{dT}{dr} = \frac{-\dot{q}r^2}{2k_f} + C_1 \rightarrow \int dT = \int \frac{-\dot{q}r}{2k_f} + \frac{C_1}{r}\,dr$

$T_f = \frac{-\dot{q}r^2}{4k_f} + C_1\ln(r) + C_2$ ~~Assume uniform T in fuel~~

$T_f(0) \Rightarrow \frac{dT_f}{dr} = 0 \;@\; r=0 \rightarrow \frac{dT_f}{dr} = \frac{-\dot{q}r}{2k_f} + \frac{C_1}{r} = 0$

$C_1 = 0 \rightarrow T_f = \frac{-\dot{q}r^2}{4k_f} + C_2$

T_c) $0 = \frac{k}{r}\frac{d}{dr}\left(r k_c\frac{dT}{dr}\right) \rightarrow T_c = \frac{C_3}{k_c}\ln r + C_4$

$T_f(r_1) = T_c(r_1) \rightarrow \frac{-\dot{q}r_1^2}{4k_f} + C_2 = \frac{C_3}{k_c}\ln(r_1) + C_4$

$q''_f = q''_c \;\therefore\; \cancel{k_f}\left(\frac{\cancel{-}\dot{q}r_1}{2\cancel{k_f}}\right) = -k_c\left(\frac{C_3}{k_c r_1}\right) \rightarrow \frac{\dot{q}r_1}{2} = \frac{-C_3}{r_1}$

$C_3 = \frac{-\dot{q}r_1^2}{2}$

$@\; r = r_2 \quad q''_c = q''_{conv} \rightarrow -k_c\left(\frac{C_3}{k_c r_1}\right) = h\,(T_2 - T_\infty)$

$T_2 = T_c(r_2) = \frac{-\dot{q}r_1^2}{2k_c}\ln(r_2) + C_4$

$\therefore -k_c\frac{\left(\frac{-\dot{q}r_1^2}{2}\right)}{k_c r_2} = h\left(\frac{-\dot{q}r_1^2}{2k_c}\ln(r_2) + C_4 - T_\infty\right)$

Josh Whitehead
ChEn 3453

HW#3

$$C_4 = \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty$$

$$\therefore T_c = \frac{-\dot{q} r_1^2}{2 k_c} \ln(r) + \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty$$

$$T_f = \frac{-\dot{q} r^2}{4 k_f} + C_2 \rightarrow T_f(r_1) = T_c(r_1)$$

$$\frac{-\dot{q} r_1^2}{4 k_f} + C_2 = \frac{-\dot{q} r_1^2}{2 k_c} \ln(r_1) + \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty$$

$$C_2 = \frac{\dot{q} r_1^2}{4 k_f} - \frac{\dot{q} r_1^2}{2 k_c} \ln(r_1) + \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty$$

$$\therefore \boxed{T_f = \frac{-\dot{q} r^2}{4 k_f} + \frac{\dot{q} r_1^2}{4 k_f} - \frac{\dot{q} r_1^2}{2 k_c} \ln(r_1) + \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty}$$

b) hottest in middle: $r = 0$

$$T_f(0) = 0 + \frac{\dot{q} r_1^2}{4 k_f} - \frac{\dot{q} r_1^2}{2 k_c} \ln(r_1) + \frac{\dot{q} r_1^2}{2 h r_2} + \frac{\dot{q} r_1^2}{2 k_c} \ln(r_2) + T_\infty$$

$$\boxed{= 1.46 \times 10^3 \text{ K}}$$