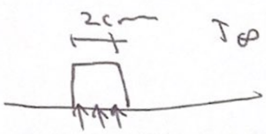
Problem 1 (25 pts)

A cube that is 2 cm on each sits on a heated surface that supplies a flux of 3,000 W/m^2 into the bottom of the cube. The cube is exposed to ambient air at T_∞ = 20°C. The heat transfer coefficient between the exposed sides of the cube and the air is h = 18 W/m^2*K. The cube is assumed to be at a uniform temperature.

After a long time, what is the temperature of the cube in °C?

The thermal properties of the cube are given below.

ρ	800 kg/m^3
C_p	1,200 J/kg*K
k	8.4 W/m*K



$T_\infty = 20°C$ $\quad h = 18 \frac{W}{m^2 K}$

$\cancel{Accum} = In - out + \cancel{gen}$

$\underline{In} = \underline{out}$

$q'' A = 5 q'' A$

Bal:

$q''_{in} A = 5A(h(T_s - T_\infty)) \rightarrow T_s - T_\infty = \frac{q''_{in}}{5h} \rightarrow T_s = \frac{q''_{in}}{5h} + T_\infty$

$\therefore T_s = \frac{3000 \, W/m^2}{5 \cdot 18 \frac{W}{m^2 K}} + 20$

$\boxed{T_s = 53.3°C}$

Problem 2

$A_s = 4\pi r^2$

$q = -k \frac{dT}{dr} A_s$

$\cancel{Acc/m} = \cancel{In} - out + gen$

$gen = out$

$T(r) = -750r^4 - 450r^3 - 100r^2 - 50r + 100$

$\frac{dT}{dr} = -3000r^3 - 1350r^2 - 200r - 50$

$\therefore q = -5\left[-3000(0.3)^3 - 1350(0.3)^2 - 200(0.3) - 50\right] \cdot 4\pi(0.3)^2$

$= 1767.14\ w \approx \boxed{1.77 \times 10^3\ w}$

Problem 3 (25 pts)

A process used to manufacture small spheres requires them to be heated for curing until the minimum temperature in the sphere is 200 °C. The spheres (4 cm in diameter) are heated by a furnace, which generates exhaust air at 500 °C. The convection coefficient from the furnace air to the surface of the spheres is h = 100 W/m²*K. The spheres also have a very thin coating on their outer surface, which has a conductive thermal resistance (for unit area) of R'' = 0.02 m²*K/W.

The thermal properties of the spheres are found in the table below:

ρ	800 kg/m³
C_p	1,200 J/kg*K
k	8.6 W/m*K

If the spheres begin the process at 20 °C, approximately how long should they be placed in the curing furnace (in s)?

$T_1 = 20°C$ $\quad r = 0.2\,m$ $\quad Bi = \dfrac{100 \cdot \frac{0.02}{3}}{8.6} = 0.077$

$T_{air} = 500°C$ $\quad T_2 = 200°C$

$h = 100 \frac{W}{m^2 K}$

Lumped capacitance

$$R_{tot} = R'' + \frac{1}{h} = 0.2 + \frac{1}{100} = 0.3 \frac{m^2 K}{W}$$

$$U = \frac{1}{R_{tot}} = 33.3 \frac{W}{m^2 K}$$

$$t = \frac{\rho V c_p}{U A_s} \ln\left(\frac{\theta_i}{\theta}\right)$$

$$= \frac{800 \frac{kg}{m^3} \cdot \frac{4}{3}\pi (0.02)^3 \cdot 1200}{33.3 \cdot 4\pi (0.02)^2} \boxed{= 90.3 \text{ sec}}$$

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a) $\cancel{\rho C_p \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \dot{q}$

$k\frac{d^2T}{dx^2} + \dot{q} = 0 \rightarrow \int d\left(\frac{dT}{dx}\right) = \int -\frac{\dot{q}}{k}\,dx \rightarrow \frac{dT}{dx} = \frac{-\dot{q}x}{k} + C_1$

$\left.\frac{dT}{dx}\right|_{x=0} = 0 \rightarrow 0 = 0 + C_1 \therefore C_1 = 0$

$\int dT = \int \frac{-\dot{q}x}{k}\,dx \rightarrow T = \frac{-\dot{q}x^2}{2k} + C_2 \rightarrow T(L) = T_{s1} = \frac{-\dot{q}L^2}{2k} + C_2$

$C_2 = T_{s1} + \frac{\dot{q}L^2}{2k} \longrightarrow \boxed{T(x) = -\frac{\dot{q}x^2}{2k} + \frac{\dot{q}L^2}{2k} + T_{s1}}$

b) $T(0.06) = \frac{-1000(0.06)^2}{2 \cdot 3.4} + \frac{1000(0.15)^2}{2 \cdot 3.4} + 300 \boxed{= 327.8\ K}$