HW1_4

January 20, 2022

1 Problem 4

Problem 4.1 Given the relationships

$$y = ar^2 = a(x^2 + z^2) \tag{1}$$

$$w = 2r \tag{2}$$

and

$$W = \text{diameter at the top}$$

$$Y = \text{height of full tank}$$

$$R = \text{radius at the top}$$

find V as a function of W.

First, relating the differential volume to the differential height

$$dV = \pi r^2 dy$$

rearanging (1) gives

$$r^2 = \frac{y}{a}$$

therefore

$$V = \int_0^Y \frac{\pi}{a} y dy$$

integrating and evaluating from 0 to Y gives

$$V = \frac{\pi}{2a} Y^2 \tag{3}$$

combining (1) and (3) gives

$$V = \frac{\pi}{2a}(aR^2)^2$$

then substituting R for (2) and simplifying The total volume of the tank is given by

$$V = \frac{\pi a W^4}{32} \tag{4}$$

and the volume at any w is given by

$$V = \frac{\pi a w^4}{32}$$

Problem 4.2 (1) and (2) can be combined to find the height of the tank since

$$Y = aR^2$$

and

$$W = 2R$$

then the total height of the tank is given by

$$Y = \frac{a}{4}W^2$$

and the height at any w is given by

$$y = \frac{a}{4}w^2 \tag{5}$$

Problem 4.3 If a hole was cut at $\frac{W}{16}$ then (5) may be used to find the new height by evaluating at y(W) and subtracting y($\frac{W}{16}$)

$$y_{new} = \frac{a}{4}(W^2 - (\frac{W}{16})^2)$$