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HW3 Answer template

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Problem 1

$u(r) = C(R^2 - r^2)$

$\frac{dA}{dr} = 2\pi r \therefore$

$dA = 2\pi r\, dr$

$\int (u(A) \cdot n)\, dA = Q$

$\therefore Q = \int C(R^2 - r^2)\, dr = C\int R^2 - r^2 = C\left(R^2 r - \frac{1}{3}r^3\right)$

$Q = \int C(R^2 - r^2)\, dA = \int_0^R C(R^2 - r^2)\, 2\pi r\, dr$

$= 2C\pi \int_0^R r(R^2 - r^2)\, dr = 2C\pi \left(\int_0^R R^2 r\, dr - \int_0^R r^3 dr\right)$

$= 2C\pi \left(\frac{1}{2}R^2 r^2 - \frac{1}{4}r^4\right)\Big]_0^R$

$= 2C\pi \left(\frac{1}{2}R^2 R^2 - \frac{1}{4}R^4\right) - 0 = 2C\pi \left(\frac{1}{4}R^4\right) = \frac{1}{2}C\pi R^4$

$Q = \frac{1}{2}C\pi R^4$

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Problem 2

$$Q_1 = Q_2 \quad \therefore \int_0^R u_0 2\pi r\,dr = \int_0^R u_m\left(1-\frac{r}{R}\right)^{1/7} 2\pi r\,dr$$

a) $Q_1 = u_0 \pi r^2 \Big]_0^R = u_0 \pi R^2$

b) $Q_2 = u_m 2\pi \int_0^R \left(\frac{R-r}{R}\right)^{1/7} r\,dr = \frac{u_m 2\pi}{R^{1/7}} \int (R-r)^{1/7} r\,dr \qquad u = R-r \quad du = dr$

$= \frac{u_m 2\pi}{R^{1/7}} \int u^{1/7}(R-u)\,du = \frac{u_m 2\pi}{R^{1/7}} \left[\int Ru^{1/7} - \int u^{8/7}\right] = \frac{u_m 2\pi}{R^{1/7}} \left(\frac{7}{8}Ru^{8/7} - \frac{7}{15}u^{15/7}\right)$

$= \frac{u_m 2\pi}{R^{1/7}} \left(\frac{7}{8}R(R-r)^{8/7} - \frac{7}{15}(R-r)^{15/7}\right)_0^R = 0 - \frac{u_m 2\pi}{R^{1/7}} \left(\frac{7}{8}R\cdot R^{8/7} - \frac{7}{15}R^{15/7}\right)$

$= -\frac{u_m 2\pi}{R^{1/7}} \left(\frac{7}{8}R^{15/7} - \frac{7}{15}R^{15/7}\right) = -\frac{u_m 2\pi}{R^{1/7}} \left(\frac{105}{120}R^{15/7} - \frac{56}{120}R^{15/7}\right)$

$= -\frac{u_m 2\pi}{R^{1/7}} \cdot \frac{49}{120} R^{15/7} = \frac{-u_m \cdot 2 \cdot 49 \cdot \pi \cdot R^{15/7}}{120 \cdot R^{1/7}}$

$= \frac{-u_m \cdot 98 \cdot \pi \cdot R^2}{120} = \frac{-u_m \cdot 49 \pi R^2}{60} = u_0 \pi R^2$

$\therefore \frac{u_0}{u_m} = \frac{49}{60}$

$$\boxed{\frac{u_0}{u_m} = \frac{49}{60}}$$

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Problem 3

$r_1 = 0.375 \text{ in} = 0.9525 \text{ cm}$

$r_2 = 0.015 \text{ in} = 0.0381 \text{ cm}$

(a)

$Q_1 = Q_2 \rightarrow \vec{v}_1 A_1 = \vec{v}_2 A_2$

$A = \pi r^2 \qquad Q_2 = 6 \frac{\text{cm}^3}{\text{sec}}$

$$\therefore \vec{v}_1 = \frac{Q_2}{A_1} = \frac{6 \frac{\text{cm}^3}{\text{sec}}}{\pi (0.9525)^2} = 2.105 \frac{\text{cm}}{\text{sec}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = 0.829 \frac{\text{in}}{\text{sec}}$$

$$0.829 \frac{\text{in}}{\text{sec}}$$

(b)

if 10% leak $\rightarrow Q_1 = Q_2 + Q_{\text{leak}}$

$Q_2 = 6 \frac{\text{cm}^3}{\text{sec}}$

$Q_{\text{leak}} = 0.6 \frac{\text{cm}^3}{\text{sec}}$

$$\therefore \vec{v}_1 = \frac{6.6 \frac{\text{cm}^3}{\text{sec}}}{\pi (0.9525)^2} = 2.316 \frac{\text{cm}}{\text{sec}} = 0.912 \frac{\text{in}}{\text{sec}}$$

$$0.912 \frac{\text{in}}{\text{sec}}$$

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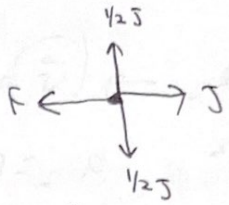
Problem 4

$\Sigma F = ma = \dot{m}\vec{v}$



$\dot{m} = v \cdot A \cdot \rho = 800\frac{cm}{sec} \cdot \pi(5cm)^2 \cdot 0.997\frac{g}{cm^3}$

$\dot{m} = 62643\frac{g}{sec}$

$F = 62643\frac{g}{sec} \cdot 800\frac{cm}{sec} = 50114686\frac{g\,cm}{sec^2} \cdot \frac{1\,Kg}{1000g} \cdot \frac{1m}{100cm}$

$= 501.N$

501. N

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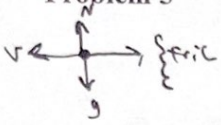
Problem 5



Starts to move when $F_v = F_{fric}$

$F_v = ma = \dot{m}v \rightarrow \dot{m} = \rho A v \quad \therefore F_v = 997 \frac{kg}{m^3} \cdot (0.045 m)^2 \pi \cdot 2gh$

$= 124.32h$

$F_{fric} = \mu \cdot W \rightarrow W = W_{water} + W_{tank} \rightarrow W_{water} = \rho V g$

$W_{water} = 997 \frac{kg}{m^3} (\pi r^2 h) g = 997 \frac{kg}{m^3} (\pi (0.5m)^2 (h)) 9.8 \frac{m}{sec^2}$

$= 7673.8h \ N$

$\therefore F_{fric} = 0.01 (7673.8h + 150) N = (7673.8h + 3142) N \cdot 0.01$

$= 76.73h + 31.42$

$F_{fric} = F_v \rightarrow 76.73h + 31.42 = 124.32h$

$31.42 = 47.58h$

$h = 0.661$

0.661 m

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Problem 6

(a)

$Q_1 = Q_2$

$v_1 A_1 = v_2 A_2$

$v_2 = \frac{v_1 A_1}{A_2}$

$v_2 = \frac{v_1 A_1}{A_2}$

(b)

$F_x = v_{avg} \cdot \dot{m} = \frac{v_2 - v_1}{2} \cdot \dot{m}$

$F_x < 0$ if $v_1 > v_2$

$v_1 A_1 = v_2 A_2 \rightarrow$ if $v_1 > v_2$, $A_1 < A_2$

~~SAOMTBC~~
larger

(c)

$F_x = \dot{m} \frac{v_2 - v_1}{2}$

$\dot{m} = \rho \cdot A_1 \cdot v_1$

$v_2 = \frac{v_1 A_1}{A_2}$

$\therefore F_x = \rho A_1 v_1 \cdot \frac{\frac{v_1 A_1}{A_2} - v_1}{2}$

$F_x = \rho A_1 u \cdot \frac{\frac{u A_1}{A_2} - u}{2}$