Josh Whitehead
ChEn 3453

Hw 8

1) a) 450 kg/hr $\rightarrow$ 27° $\downarrow q' = a \cdot x = 20x$

Accum = In − out + ~~gen~~ → In = out

$$\dot m C_p (T_x - T_{in}) + q'\Delta x = \dot m C_p (T_{x+\Delta x} - T_{in})$$

$$\rightarrow \dot m C_p (T_x - T_{x+\Delta x}) = q'\Delta x \rightarrow \frac{dT}{dx} = \frac{20x}{\dot m C_p}$$

$$\rightarrow T(x) = \frac{20x^2}{2\dot m C_p} + C = \frac{10x^2}{\dot m C_p} + C$$

$$T(0) = T_{in} = C \therefore \boxed{T(x) = \frac{10x^2}{\dot m C_p} + T_{in}}$$

b) $$T(30) = \frac{10 \cdot 30^2}{\frac{450}{3600} \cdot 4816} + 27 = \boxed{42.0\,°C}$$

---

2) a) $$\frac{T_s - T_{mo}}{T_s - T_{mi}} = \exp\left(\frac{-PL\bar h}{\dot m C_p}\right)$$

@ 505 K: $k = 0.637$, $\mu = 116 \times 10^{-6}$, $\rho = 998$, $Pr = 0.855$

$$Re = \frac{\rho V D}{\mu} = \frac{\rho \cdot \frac{\dot m}{\rho A} \cdot D}{\mu} = \frac{998 \cdot \frac{0.1}{998 \cdot \pi (0.05)^2} \cdot 0.1}{116 \times 10^{-6}}$$

$= 11023 \therefore$ Turbulent

$$Nu = 0.023 Re^{4/5} \cdot Pr^{0.4} = 37.01$$

$$h = \frac{Nu\, k}{D} = 235.6$$

$$T_{mo} = -(T_s - T_{mi}) \exp\left(\frac{-PLh}{\dot m C_p}\right) + T_s \boxed{= 506\,K}$$

Josh Whitehead
ChEn 3453

HW 8

2) b) @ 305 K: $C_p = 4178$
$\mu = 769 \times 10^{-6}$
$k = 0.62$
$Pr = 5.2$
$\rho = 998$

$$T_{mo} = T_s - (T_s - T_{mi}) \exp\left(\frac{-PLh}{C_p \cdot \dot{m}}\right)$$

$Re = 1656 < 2300$ ∴ Laminar

$$Nu = 3.66 \rightarrow h = \frac{Nu\, k}{D} = 22.7$$

$$T_{mo} = 3\phi0 - 10 \cdot \exp\left(\frac{-2\pi \cdot 0.05 \cdot 6 \cdot 22.7}{4178 \cdot 0.1}\right)$$

$$\boxed{= 30\phi\ K}$$

---

3) $$Re = \frac{\rho V D}{\mu} = \frac{\rho \cdot \frac{\dot{m}}{A\rho} \cdot D}{\mu}$$ $\dot{m} = \frac{36}{3600} = 0.01 \frac{kg}{sec}$

$$Re = \frac{1079 \cdot 0.118 \cdot 0.01}{0.0034} = 374.5 < 2300$$ ∴ Laminar

$$Nu = 3.66 \rightarrow h = \frac{Nu\, k}{D} = \frac{3.66 \cdot 0.261}{0.01} = 95.5$$

$$T_{mo} = 25 - (25 - 85) \exp\left(\frac{-2\pi \cdot 0.005 \cdot 1 \cdot 95.5}{2637 \cdot 0.01}\right)$$

$$\boxed{= 78.5^\circ C}$$

Josh Whitehead
ChEn 3453

HW 8

4.) for 380.65 K : $\rho = 0.919 \frac{kg}{m^3}$ $Pr = 0.694$

a.)

$C_p = 1012 \frac{J}{kg\,K}$

$\mu = 221.6 \times 10^{-7}$

$k = 32.3 \times 10^{-3}$

$Re = \frac{\rho v D}{\mu}$

$v = \frac{\dot{m}}{A\rho} = \frac{0.003}{\pi (0.003)^2 \cdot 0.919} = 115.5$

$Re_1 = 28728 \therefore$ Turb

$\overline{Nu} = Re^{4/5} \cdot 0.023 \cdot Pr^{0.3} = 75.99$

$h = \frac{Nu\,k}{D} = \frac{75.99 \cdot 32.3 \times 10^{-3}}{0.006} \boxed{= 409.1 \frac{W}{m^2 K}}$

b.) ~~Nu=~~ $Re = \frac{0.919 \cdot 5 \cdot 0.006}{221.6 \times 10^{-7}} = 1244$

$Nu = C Re^m Pr^{1/3} = 0.683 \cdot 1244^{0.466} \cdot 0.694^{1/3}$

$= 16.7 \rightarrow h = \frac{Nu\,k}{D} = \frac{16.7 \cdot 32.3 \times 10^{-3}}{0.006} \boxed{= 90.1 \frac{W}{m^2 K}}$

c.) $UA = \frac{1}{R_{tot}}$

$R_{tot} = \frac{1}{h_o A_o} + \frac{1}{h_i A_i}$

$UA = \frac{1}{R_{tot}} \longrightarrow U = \left(\frac{1}{h_1} + \frac{1}{h_2}\right)^{-1} = \left(\frac{1}{409.1} + \frac{1}{90.1}\right)^{-1}$

$\boxed{= 73.8 \frac{W}{m^2 K}}$

Josh Whitehead
ChEn 3453

Hw 8

5) a) $h = \frac{Nu\,k}{D} = \frac{7.6 \cdot 0.0263}{0.05} = 3.998 \frac{W}{m^2 K}$

$T_{m,o} = T_s - (T_s - T_{m,i}) \exp\left(\frac{-P \cdot L \cdot h}{\dot{m} c_p}\right)$

$P = 12 \cdot 2 \cdot \pi \cdot r = 24\pi \cdot 0.025 = 0.157\ m$

$\therefore T_{m,o} = 100 - (100-20) \exp\left(\frac{-0.157 \cdot 100 \cdot 3.998}{2 \cdot 1007}\right)$

$\boxed{= 44.97\ °C}$

b) $\frac{1}{R_{tot}} = \frac{1}{\ }$

$q = hA\,LMTD = 3.998 \cdot 2\pi \cdot 0.025 \cdot 100 \left(\cancel{\frac{100-45}{\ln(100/45)}}\right)$

$= 4191\ W \cdot 12$

$\boxed{= 50289\ W}$

$LMTD = \frac{(100-45) - (100-20)}{\ln\left(\frac{100-45}{100-20}\right)}$

Josh Whitehead
Ch En 3453

HW 8

6.) $Nu = \left(0.6 + \frac{0.387 Ra^{1/6}}{\left(1 + (0.559/Pr)^{9/16}\right)^{8/27}}\right)^2$

$Ra = Gr \cdot Pr$

$Gr = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}$

~~β @ 18°C = 0.0001832 / 186.3×10⁻⁶ 1/°K~~ $\nu =$

$\beta @ 310K = 361.9 \times 10^{-6} \frac{1}{°C}$ $\nu = \frac{695 \times 10^{-6}}{998} = 6.96 \times 10^{-7}$

$Pr = 4.62$ $k = 0.628$

$Gr = \frac{9.8 \cdot 361.9 \times 10^{-6} (56-18)\, 0.005^3}{6.96 \times 10^{-7}} = 48377$

$Ra = 48377 \cdot 4.62 = 160488$

$Nu = \left(0.6 + \frac{0.387 Ra^{1/6}}{\left(1 + (0.559/Pr)^{9/16}\right)^{8/27}}\right)^2 = 10.48$

$h = \frac{Nu\, k}{D} = \frac{10.48 \cdot 0.628}{0.005} = 1316 \frac{W}{m^2 K}$

$q'' = hA(T_s - T_\infty) \Rightarrow q' = hP(T_s - T_\infty)$

$= 1316 \cdot \pi \cdot 0.0025 \cdot 2 \cdot (56-18) = \boxed{785. \frac{W}{m}}$

Josh Whitehead
ChEn 3453

HW 8

7) $q' = hP(T_s - T_\infty) \rightarrow T_s = q'/hP + T_\infty$

$$Nu = \left(0.6 + \frac{0.387 Ra^{1/6}}{\left(1+(0.559/Pr)^{9/16}\right)^{8/27}}\right)^2$$

$Ra = Gr \cdot Pr$ $\qquad$ $Gr = \frac{g\beta(T_s - T_\infty)D^3}{\nu}$

~~RAA~~

@ $T = 53.5°C$ $\qquad$ T_s guess $= 80°C$

~~$Pr = 3.33$~~

~~$k = 646.65 \times 10^{-3}$~~

~~$\beta = 482 \times 10^{-6}$~~

~~$\nu = 5.062 \times 10^{-7}$~~

$Pr = 0.703$

$\beta = 3.4 \times 10^{-3}$

$k = 0.0283$

$\nu = 1.86 \times 10^{-5}$

$Gr = 8 \times 10^4$

$Ra = 5.63 \times 10^4$

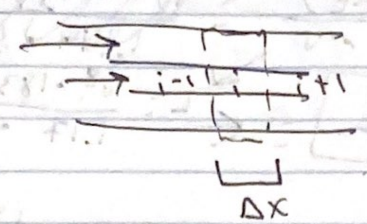
$Nu = 6.69 \rightarrow h = \frac{Nu\,k}{D} = 7.57$

$\therefore T_s = \frac{30 \cdot 0.0283}{7.57 \cdot \pi \cdot 2\left(\frac{0.025}{2}\right)}$ $\boxed{= 77.5°C}$

Josh Whitehead
ChEn 3453

Hw 8

8) a)



$A_{C_1} = \pi r_1^2$

$A_{C_2} = \pi r_2^2 - A_{C_1}$

Accum = In - out + ~~gen~~

$\rho c_p A_{C_1} \Delta x \frac{dT}{dt} = m_1 C_{p_1}(T_{(i-1)} - T_i) + U 2\pi r_1 \Delta x (T_{2i} - T_{1i})$

$\therefore \frac{dT}{dt_1} = \frac{m_1 C_1 (T_{i-1} - T_i) + U 2\pi r_1 \Delta x (T_{2i} - T_{1i})}{\rho_1 C_1 A_{C_1} \Delta x}$

$\frac{dT}{dt_2} = \frac{m_2 C_2 (T_{i-1} - T_i) + U 2\pi r_1 \Delta x (T_{2i} - T_{1i})}{\rho_2 C_2 A_{C_2}}$

b.) $\boxed{T_{w\,out} = 572.3\ K}$

c) $q = U A_s (T_2 - T_1) \rightarrow U 2\pi r_1 L \cdot LMTD$

$LMTD = \frac{(900 - 400) - (575.4 - 572.3)}{\ln(500/3.1)}$

$\boxed{\therefore q_{in} = 3.78\ MW}$

$\therefore q_{out} = 3.78\ MW$

Josh Whitehead
ChEn 3453

Hw 8

8 d) fluid 1: ~~same as~~

$$\frac{dT}{dt} = \frac{m_1 C_1 (T_{i+1} - T_i) + U 2\pi r_1 \Delta x (T_{2i} - T_{1i})}{\rho_1 C_1 A C_1 \Delta x}$$

counting backwards

fluid 2:

$$\frac{dT}{dt} = \frac{m_2 C_2 (T_{i-1} - T_i) + U 2\pi r_1 \Delta x (T_{2i} - T_{1i})}{\rho_2 C_2 A C_2 \Delta x}$$

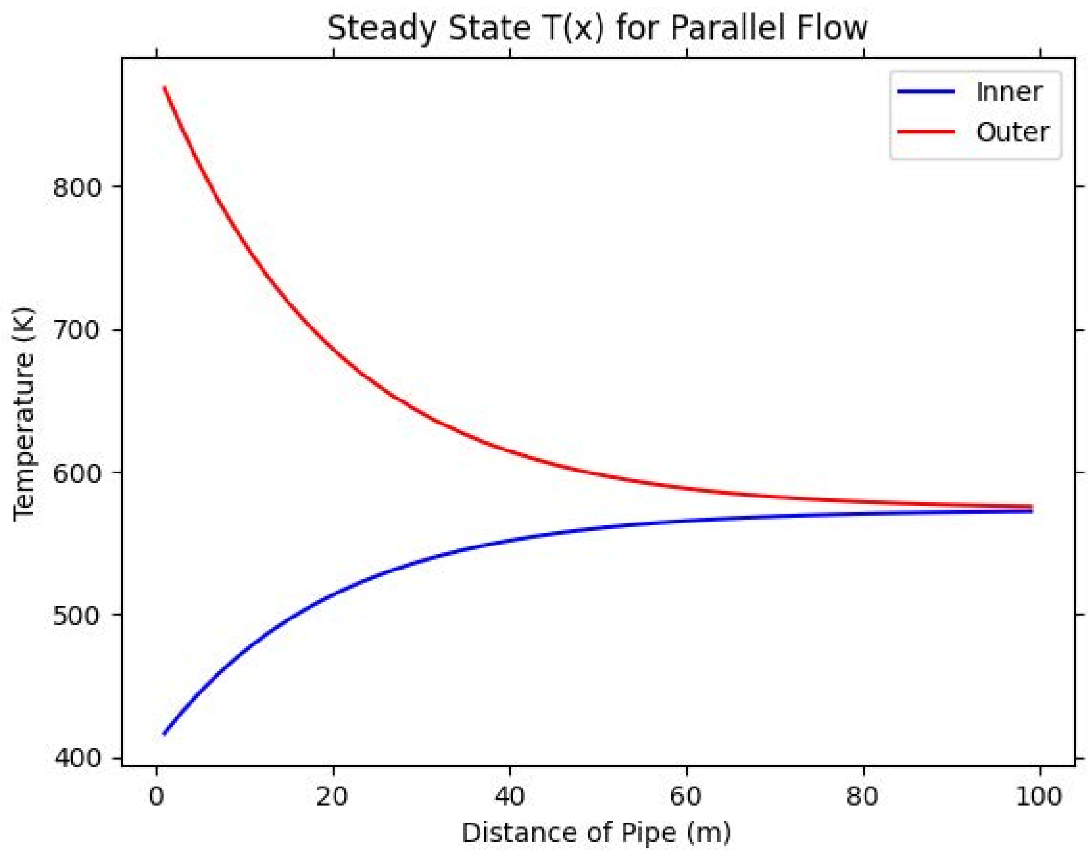
$T_2 \rightarrow$ plot backwards (see plot)

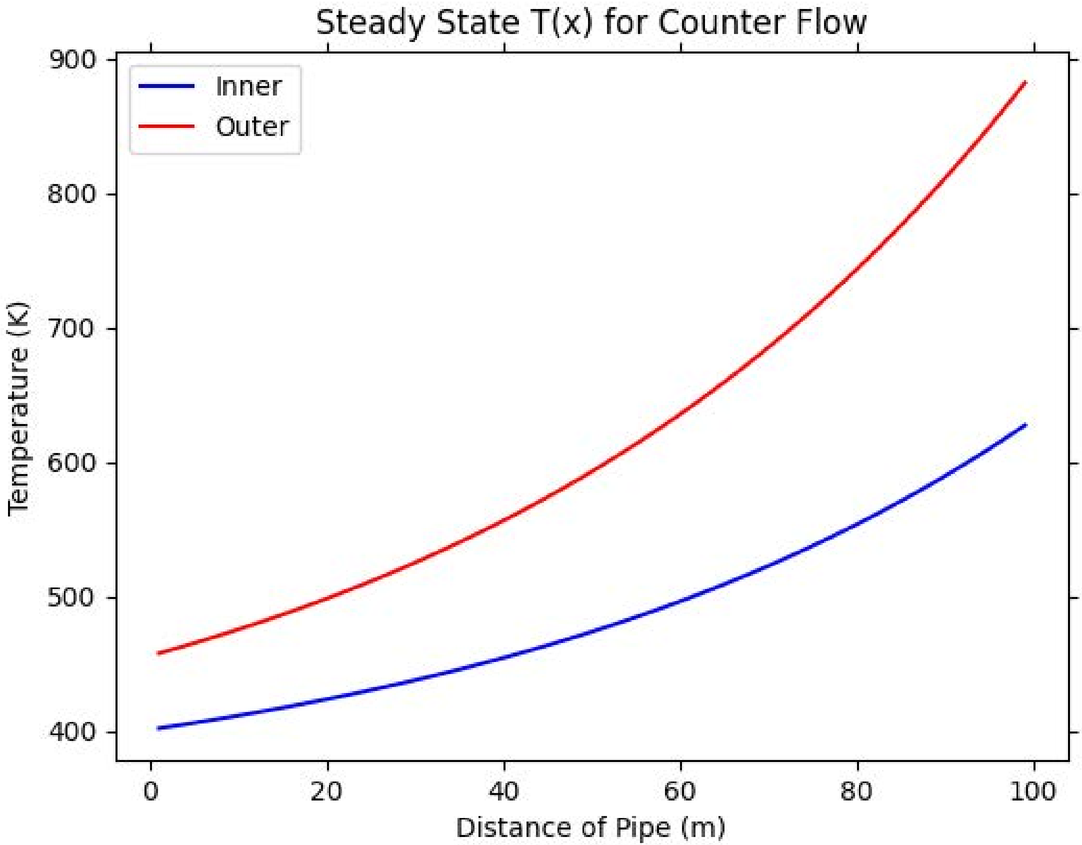
e) ~~$T_w = 62\cancel{7}.8\ K$~~ $\boxed{T_w = 627.5\ K}$

f) $$LMTD = \frac{(457.9 - 400) - (900 - 627.5)}{\ln(57.9/272.5)}$$

$q_{in} = q_{out} = U A_s LMTD = U \pi 2 \cdot r_i L \cdot LMTD$

$\boxed{= 5.35\ MW}$


Steady State T(x) for Parallel Flow
Inner
Outer
800
700
600
500
400
Temperature (K)
0
20
40
60
80
100
Distance of Pipe (m)


Steady State T(x) for Counter Flow
Inner
Outer
Temperature (K)
Distance of Pipe (m)
900
800
700
600
500
400
0
20
40
60
80
100

```python
###HW 8.8###
"""
29Oct2021
"""
import matplotlib.pyplot as plt
import numpy as np
## 1 is water in inner tube
## 2 is fluid in outer tube

L = 100
r1 = .15
r2 = .25
n = 50

m1 = 5
cp1 = 4180
ro1 = 1000

m2 = 5
cp2 = 2220
ro2 = 740

T0 = 300
T1in = 400
T2in = 900
U = 410

Ac1 = np.pi*r1**2
Ac2 = np.pi*r2**2-Ac1

t_fin = 300000
dt = 1

dx = L/n

x = np.linspace(dx/2,L-dx/2,n)
x2 = x[::-1]
T1 = np.ones(n)*T0
dT1dt = np.zeros(n)

T2 = np.ones(n)*T0
dT2dt = np.zeros(n)


t = np.arange(0,t_fin,dt)
```

```
# t2 = t[::-1]

for j in range(1,len(t)):
    dT1dt[1:n] = (m1*cp1*(T1[:n-1]-T1[1:n])+U*2*np.pi*r1*dx*(T2[1:n][::-1]-
T1[1:n]))/(ro1*cp1*dx*Ac1)
    dT1dt[0] = (m1*cp1*(T1in-T1[0])+U*2*np.pi*r1*dx*(T2[-1]-
T1[0]))/(ro1*cp1*dx*Ac1)

    dT2dt[1:n] = (m2*cp2*(T2[:n-1]-T2[1:n])-U*2*np.pi*r1*dx*(T2[1:n]-T1[1:n][::-
1]))/(ro2*cp2*dx*Ac2)
    dT2dt[0] = (m2*cp2*(T2in-T2[0])-U*2*np.pi*r1*dx*(T2[0]-T1[-
1]))/(ro2*cp2*dx*Ac2)

    T1 = T1 + dT1dt*dt
    T2 = T2 + dT2dt*dt
plt.text(10,600,str(j))
plt.cla()
plt.plot(x,T1,color='blue',label='Inner')
plt.plot(x,T2[::-1],color='red',label='Outer')
plt.legend()
plt.pause(0.01)
plt.xlabel('Distance of Pipe (m)')
plt.ylabel('Temperature (K)')
plt.title('Steady State T(x) for Counter Flow')
# plt.savefig('HW8_8d.png')

plt.show()

print(T1[-1],T2[-1])
As1 = 2*np.pi*r1*L


LMTD = ((57.85-272.5)/np.log(57.85/272.5))
print(LMTD)
print(U*2*np.pi*r1*L*(LMTD))
```