Review of Useful Math

CH EN 3853 Chemical Engineering Thermodynamics
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Geometric formulas

Circumference of circle: $2\pi r$

Area of circle: πr^2

Area of sphere: $4\pi r^2$

Volume of sphere: $\frac{4}{3}\pi r^3$

Area of triangle: $\frac{(base)(height)}{2}$

Exponents and logarithms

$a^{x+y} = a^x a^y$

$log_a(xy) = log_a x + log_a y$

$log_a \frac{x}{y} = log_a x - log_a y$

$\log_a\left(x^n\right) = n\log_a x$

$a^{\log_a y} = y$

Algebra

$\frac{x/a}{y/b} = \frac{bx}{ay}$

$x^{-n} = \frac{1}{x^n}$

$\sqrt[n]{x} = x^{1/n}$

Mistakes

$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$

$\frac{x+a}{y+a} \neq \frac{x}{y}$

Derivatives

$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$

$\frac{d}{dx}(f-g) = \frac{df}{dx} - \frac{dg}{dx}$

$\frac{d}{dx}(Cf) = C\frac{df}{dx}$ where C is a constant

$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g(df/dx) - f(dg/dx)}{g^2}$

$\frac{d\left(x^a\right)}{dx} = ax^{a-1}$

$\frac{d\left[e^{f(x)}\right]}{dx} = \frac{df(x)}{dx}e^{f(x)}$

$\frac{d\left[\sin f(x)\right]}{dx} = \frac{df(x)}{dx}\cos f(x)$

$$\frac{d\left[\cos f(x)\right]}{dx} = -\frac{df(x)}{dx}\sin f(x)$$

$$\frac{d\left[\ln f(x)\right]}{dx} = \frac{1}{f(x)}\frac{df(x)}{dx}$$

Maxima and minima

To find those values of x such that f(x) is a maximum or a minimum, solve $\frac{df}{dx} = 0$ for x. If $\frac{d^2f}{dx^2} > 0,$ then f is a minimum. If $\frac{d^2f}{dx^2} < 0,$ then f is a maximum.

Integrals

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$ if

$$\frac{dF(x)}{dx} = f(x)$$

$$\int x^a dx = \frac{1}{a+1}x^{a+1},\ a \neq -1$$

$$\int \frac{1}{x}dx = \ln x$$

Mean value theorem for integrals: there exists at least one number c, between a and b, such that

$$\int_a^b f(x)dx = f(c)\cdot(b-a)$$

Partial Derivatives

$$\frac{\partial}{\partial x}\left(x^2 y\right) = 2xy$$

$$\frac{\partial}{\partial t}\sin\left(zt^2 x\right) = 2tzx\cos\left(zt^2 x\right)$$

Partial derivatives of f(x(t),y(t)):

Chain rule

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

In differential form

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

Order of differentiation

$$\frac{\partial^2 f}{\partial x \partial y} = \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)_x\right]_y$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

Approximations:

$$f\left(x + \Delta x\right) \cong f\left(x\right) + \left.\frac{df}{dx}\right|_x \Delta x$$

$$f\left(x + \Delta x, y + \Delta y\right) \cong f\left(x, y\right) + \left.\frac{\partial f}{\partial x}\right|_x \Delta x + \left.\frac{\partial f}{\partial y}\right|_y \Delta y$$

Matrices and Inverses

Ax = b

Solution x = A^{-1}b if A^{-1}A = 1

Sums and Infinite Series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \text{ where } |x| < 1$$

$$\ln\left(1+x\right) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$f(x+\Delta x) = f(x) + \left.\frac{df}{dx}\right|_x \Delta x$$

$$+ \left.\frac{d^2 f}{dx^2}\right|_x \frac{(\Delta x)^2}{2!} + \cdots$$

Numerical Methods

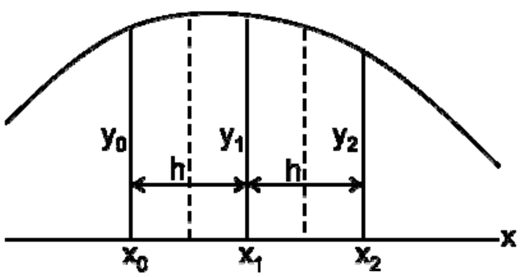
Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2D Newton's Method

$$g + g_x \Delta x + g_y \Delta y = 0$$
$$h + h_x \Delta x + h_y \Delta y = 0$$

Approximation of Derivatives



$$\left.\frac{dy}{dx}\right|_{\frac{1}{2}} \approx \frac{y_1 - y_0}{h}$$

$$\left.\frac{dy}{dx}\right|_{\frac{3}{2}} \approx \frac{y_2 - y_1}{h}$$

$$\left.\frac{d^2 y}{dx^2}\right|_1 \approx \frac{\left.\frac{dy}{dx}\right|_{3/2} - \left.\frac{dy}{dx}\right|_{1/2}}{h} = \frac{y_2 - 2y_1 + y_0}{h^2}$$

Iteration

$x_{n+1} = F(x_n)$ converges to fixed point

$x^* = F(x^*)$ if $|F'(x^*)| < 1$