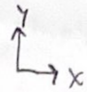
Name: Josh whitehead
Unid: u1069343



HW5 Answer template

Please use these for uploading to Gradescope. Put your final answer in the box provided.

Navier stokes

Problem 1

(a)

No slip ($v_f = v_{wall}$) top, bottom

(b)

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rightarrow \rho(0+0) = -\frac{\partial P}{\partial x} + 0 + \mu\left(0 + \frac{\partial^2 u}{\partial y^2}\right) \rightarrow$$

$$\mu\frac{d^2 u}{dy^2} = \frac{\partial P}{\partial x} \rightarrow u = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + C_1 y + C_2$$

$$y = h: \; u = 0 = \frac{1}{\mu}\frac{dP}{dx}\frac{h^2}{2} + C_1 h + C_2 \rightarrow C_1 = 0, \; C_2 = -\frac{dP}{dx}\frac{h^2}{2\mu}$$

$$\therefore u = -\frac{dP}{dx}\frac{h^2}{2\mu}\left(1 - \frac{y^2}{h^2}\right)$$

$$\boxed{u = -\frac{dP}{dx}\frac{h^2}{2\mu}\left(1 - \frac{y^2}{h^2}\right)}$$

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Problem 1 (continued)

(c)

$$\rho\left(u\cancel{\frac{\partial u}{\partial x}} + w\cancel{\frac{\partial u}{\partial z}}\right) = -\frac{\partial p}{\partial x} + \cancel{\rho g_x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\therefore +\frac{dp}{dx} = \mu\frac{\partial^2 u}{\partial z^2} \rightarrow u = \frac{1}{\mu}\frac{dp}{dx}\frac{z^2}{2} + C_1 z + C_2$$

$$y = \pm h: u = 0 \rightarrow C_1 = 0, \quad C_2 = -\frac{dp}{dx}\frac{h^2}{2\mu}$$

$$\therefore u = -\frac{dp}{dx}\frac{h^2}{2\mu}\left(1 - \frac{z^2}{h^2}\right)$$

$$\boxed{u = \frac{-dp}{dx}\frac{h^2}{2\mu}\left(1 - \frac{z^2}{h^2}\right)}$$

(d)

no final answer, show your work demonstrating answers in parts c and d are equivalent

$$u = \cancel{\frac{-dp}{dx}\frac{h^2}{2\mu}}\left(1 - \frac{y^2}{h^2}\right) = \cancel{\frac{-dp}{dx}\frac{h^2}{2\mu}}\left(1 - \frac{z^2}{h^2}\right)$$

$$\cancel{1} - \frac{y^2}{h^2} = \cancel{1} - \frac{z^2}{h^2} \rightarrow \sqrt{\frac{y^2}{\cancel{h^2}}} = \sqrt{\frac{z^2}{\cancel{h^2}}} \rightarrow y = z \checkmark$$

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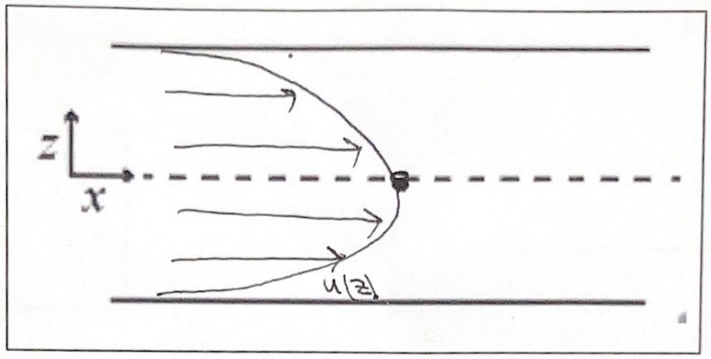
Problem 1 (continued)

(e)



(f)

Plane, steady, incompressible

Yes

(g) $u_{max} = \frac{-dP}{dx}\frac{h^2}{2\mu}$

$u = \frac{\partial \psi}{\partial z} = u_{max}\left(1 - \frac{z^2}{h^2}\right) \rightarrow \cancel{\psi = \int u_{max}\left(1-\frac{z^2}{h^2}\right)dz}$

$v = \frac{-\partial \psi}{\partial x} = 0$

$\psi = \int u_{max}\, dz - \int u_{max}\frac{z^2}{h^2}\, dz$

$\psi = u_{max}\left(z - \frac{z^3}{3h^2}\right)$

$$\boxed{\psi = \frac{-dP}{dx}\frac{h^2}{2\mu}\left(z - \frac{z^3}{3h^2}\right)}$$

Name: Josh Whitehead
Unid: u1063343

Problem 2

boundary 1

$\boxed{V_1\ ,\ Y_1}$

boundary 2

$\boxed{P_f = P_{atm}}$

boundary 3

$\boxed{\text{No slip}}$

boundary 4

$\boxed{V_2\ ,\ Y_2}$

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Problem 3

Does this satisfy continuity (yes/no)?

$\nabla \cdot v = 0$

$v = 3y\,i + 2x\,j$

$\nabla \cdot v = \frac{d\vec{u}}{dx} + \frac{d\vec{v}}{dy} = 0$

Yes

If so, find and sketch the stream function

$u = +\frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$

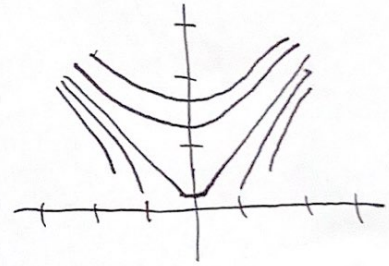
$u = 3y = \frac{\partial \psi}{\partial y} \rightarrow \int \partial\psi = \int 3y\,\partial y \rightarrow \underline{\psi = \frac{3}{2}y^2 + f(x)}$

$\frac{\partial \psi}{\partial x} = 0 + f'(x) = -2x$

$\therefore f(x) = -x^2$

$\therefore \psi = \frac{3}{2}y^2 - x^2$

Sketch:



Stream function:

$\boxed{\psi = \frac{3}{2}y^2 - x^2}$

$y = \sqrt{2/3\,(\psi + x^2}$