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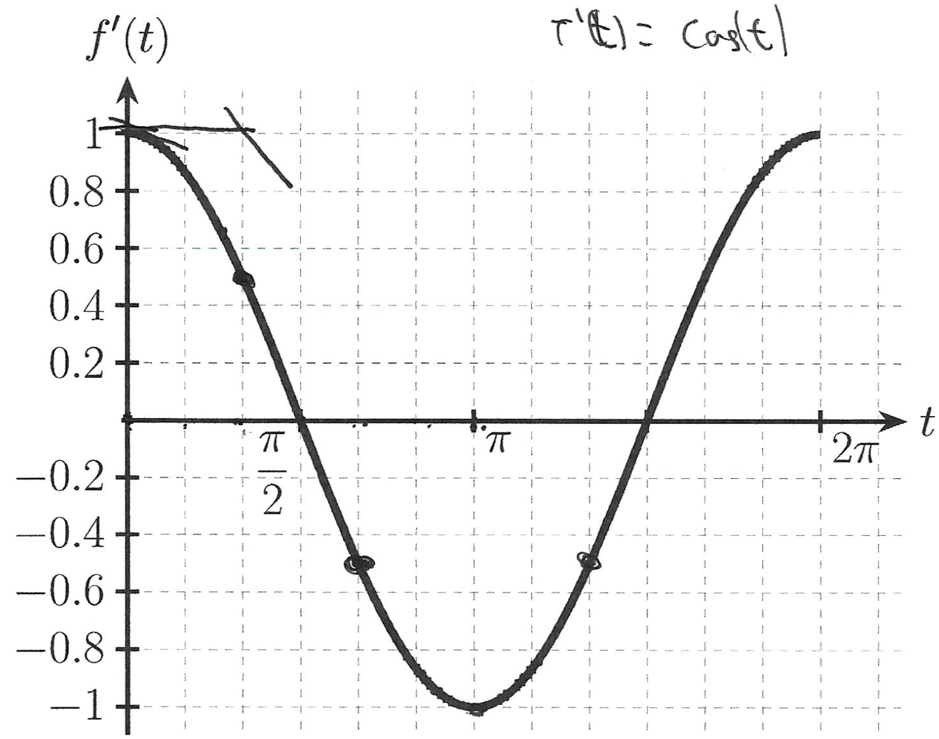
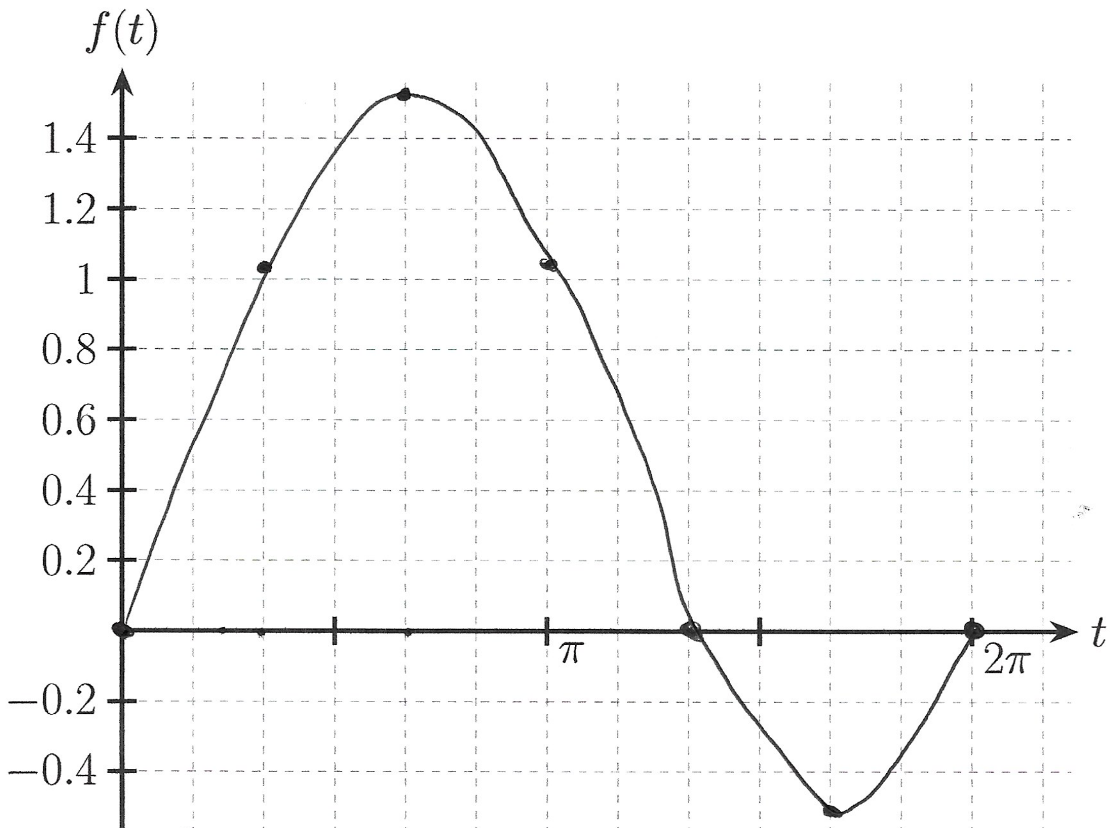
1. (6 points) **Visual Numerical Methods**

Consider the graph of $f'(t)$ below.

$f'(t) = \cos(t)$



(a) Use Euler's method to draw an approximation of the graph of $f(t)$. Use $h = \frac{\pi}{3}$, $n = 6$, and $f(0) = 0$. Estimate values from the graph of $f'(t)$. Show your work.



$Y_{n+1} = Y_n + h \cdot f'(x_n, Y_n)$

work on seperate paper

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$y_{n+1} = y_n + h \cdot f(x_n, y_n)$

1 a)

X-value	Approx - y value	exact y
0		0
$\pi/3$	$y_1 = 0 + \frac{\pi}{3} \cdot \cos(0) = \boxed{\frac{\pi}{3}}$	
$2\pi/3$	$y_2 = \frac{\pi}{3} + \frac{\pi}{3} \cdot \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{\pi}{2}}$	
π	$y_3 = \frac{\pi}{2} + \frac{\pi}{3} \cdot \cos\left(\frac{2\pi}{3}\right) = \boxed{\frac{\pi}{3}}$	
$4\pi/3$	$y_4 = \frac{\pi}{3} + \frac{\pi}{3} \cdot \cos(\pi) = \boxed{0}$	
$5\pi/3$	$y_5 = 0 + \frac{\pi}{3} \cdot \cos\left(\frac{4\pi}{3}\right) = \boxed{-\frac{\pi}{6}}$	
2π	$y_6 = -\frac{\pi}{6} + \frac{\pi}{3} \cdot \cos\left(\frac{5\pi}{6}\right) = 0$	

1 b)

$$y_{n+1} = y_n + h\left(\frac{f(x_n, \cancel{y_n}) + f(x_{n+1}, (\cancel{y_n} + hk_1))}{2}\right)$$

X-value	Approx - y	
0		0
$\pi/3$	$y_1 = 0 + \frac{\pi}{3}\left(\frac{\cos(0) + \cos\left(\frac{\pi}{3}\right)}{2}\right) = \boxed{\frac{\pi}{4}}$	
$2\pi/3$	$y_2 = \frac{\pi}{4} + \frac{\pi}{3} \cdot \frac{1}{2}\left(\cos\frac{\pi}{3} + \cos\frac{2\pi}{3}\right) = \boxed{\frac{\pi}{4}}$	
π	$y_3 = \frac{\pi}{4} + \frac{\pi}{3} \cdot \frac{1}{2}\left(\cos\left(\frac{2\pi}{3}\right) + \cos(\pi)\right) = \boxed{0}$	
$4\pi/3$	$y_4 = 0 + \frac{\pi}{3} \cdot \frac{1}{2}\left(\cos(\pi) + \cos\left(\frac{4\pi}{3}\right)\right) = \boxed{-\frac{\pi}{4}}$	
$5\pi/3$	$y_5 = -\frac{\pi}{4} + \frac{\pi}{3} \cdot \frac{1}{2}\left(\cos\frac{4\pi}{3} + \cos\left(\frac{5\pi}{3}\right)\right) = \boxed{-\frac{\pi}{4}}$	
2π	$y_6 = -\frac{\pi}{4} + \frac{\pi}{3} \cdot \frac{1}{2}\left(\cos\left(\frac{5\pi}{3}\right) + \cos(2\pi)\right) = \boxed{0}$	

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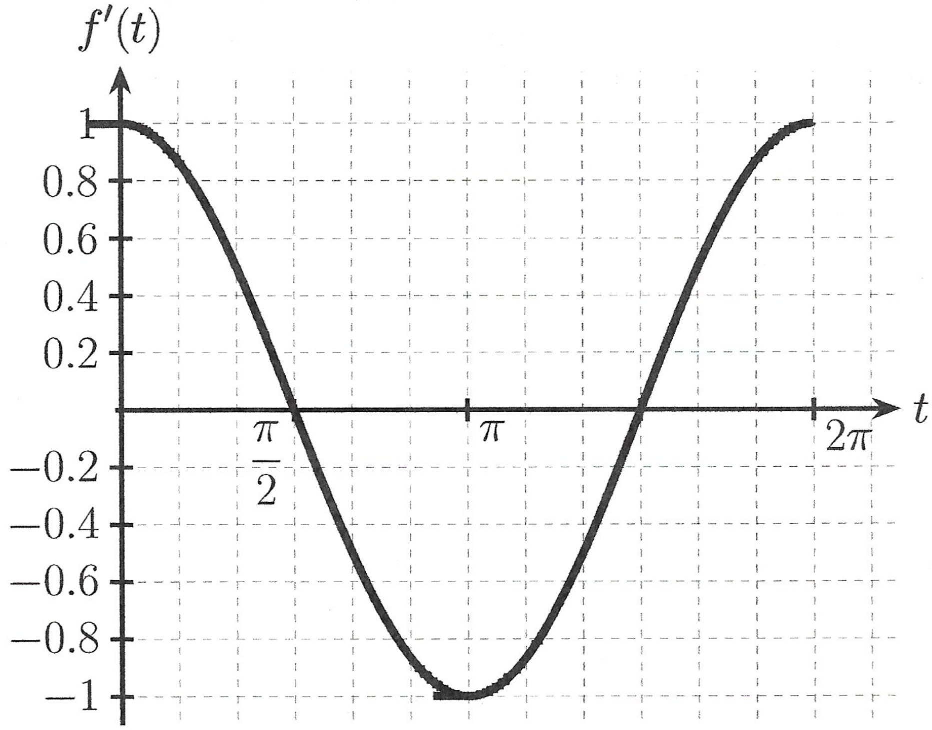
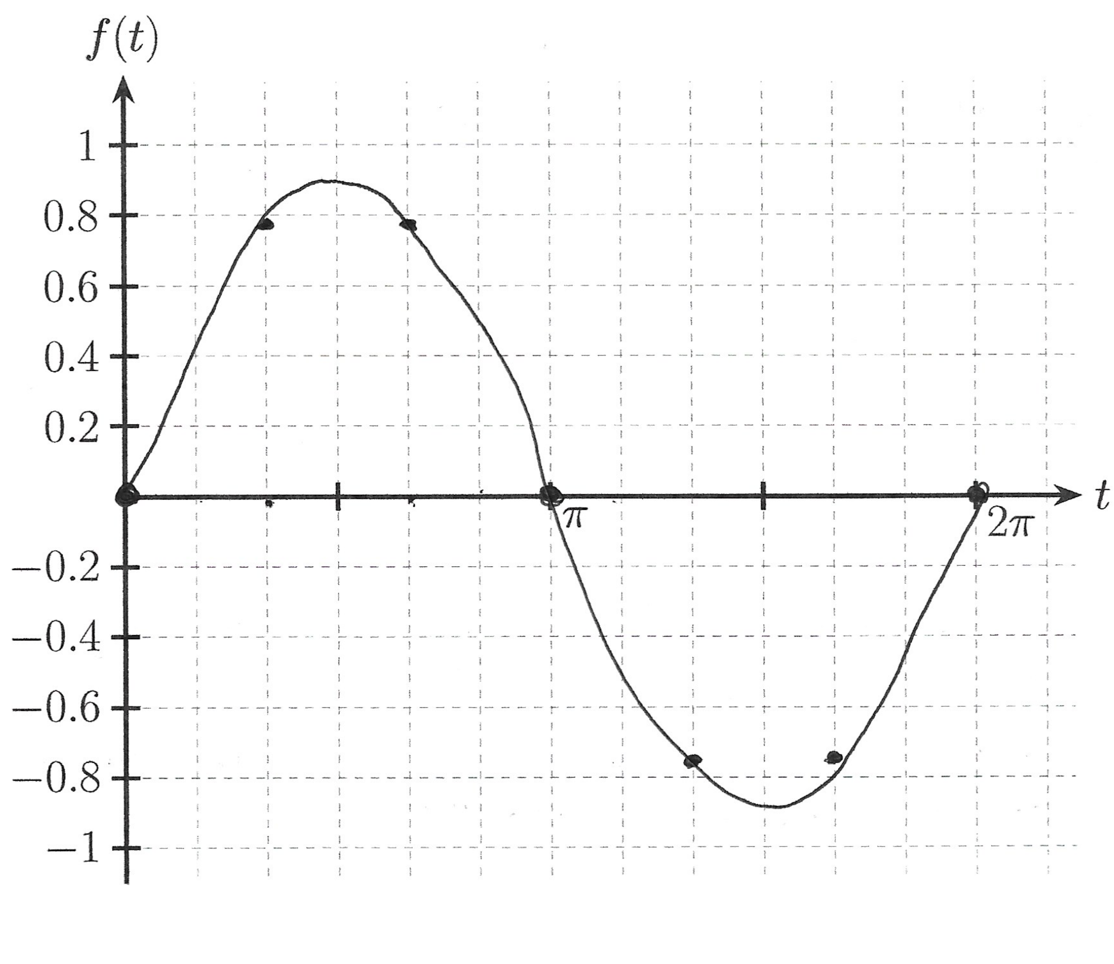
For reference here is the plot of $f'(t)$ again.



(b) Use Improved Euler's method to draw an approximation of the graph of $f(t)$. Use $h = \frac{\pi}{3}$, $n = 6$, and $f(0) = 0$. Estimate values from the graph $f'(t)$. Show your work.



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2. (7 points) **Numerical Approximation Sometimes Fails**

When we use numerical approximation techniques we are mainly concerned with how accurate a given numerical algorithm is, i.e. how small the error is for a given step size h. In some cases though, a numerical method might result in a solution that is completely wrong. To see this, consider the IVP:

$$y' = -8y, \qquad y(0) = 1,$$

$\frac{dy}{dt} = -8y \rightarrow \frac{dy}{y} = -8dt$

where y is a function of time, t, with domain $0 \leq t \leq 2$.

(a) Determine the true solution $y(t)$ to the IVP.

$\ln|y| = -8t + C$

$y = Be^{-8t}$

$1 = B \therefore \boxed{y = e^{-8t}}$

(b) On the provided slope field:

- sketch the approximations to the IVP using Euler's method with step size $h = 0.5$,
- sketch the approximations to the IVP using Euler's method with step size $h = 0.25$,
- and plot the true solution.

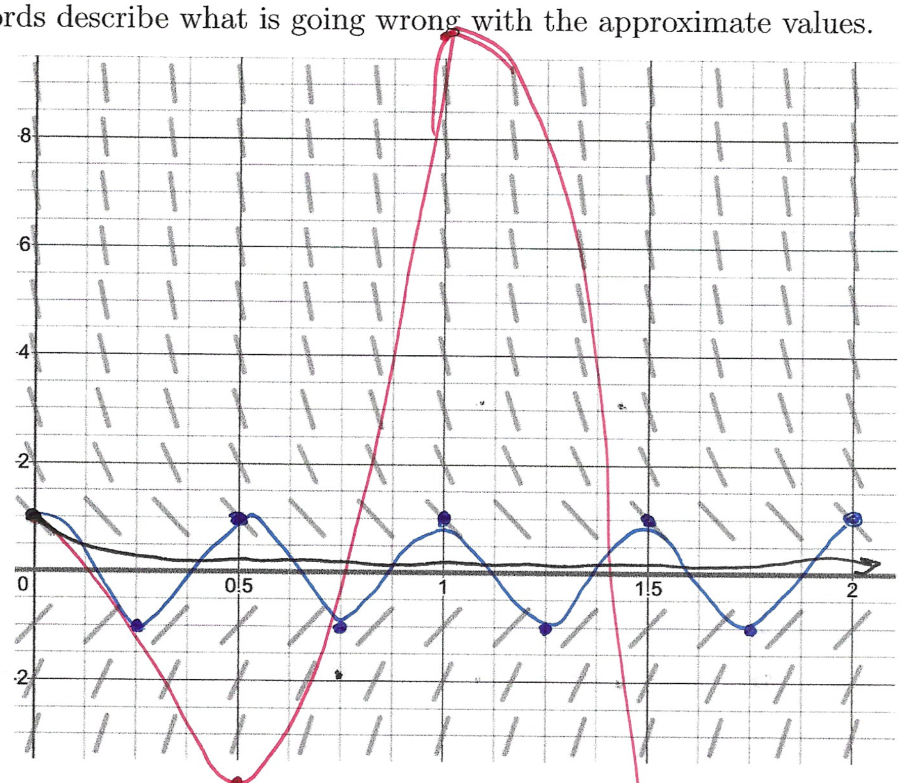
In your own words describe what is going wrong with the approximate values.

— true

— $h = 0.5$

— $h = 0.25$

There is a horizontal assymptote @ $y=0$ so approx keep oscillating



(c) Describe the qualitative properties of the true solution. For what range of step sizes will the approximate solution reflect these qualitative properties? That is, determine T such that $0 < h < T$ will provide a valid approximation to the IVP.

assymptote @ $y=0$. To not cross assymptote, $h(-8y) < 1$

since $y_0 = 1$, $h(-8) < 1 \therefore \boxed{h < 0.125}$

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2.b)

X-Val	Y-Val	exact
0	—	1
0.5	$Y_1 = 1 + 0.5\left(-8(1)\right) = -3$	
1	$Y_2 = -3 + \frac{1}{2}\left(-8(-3)\right) = 9$	
1.5	$Y_3 = 9 + \frac{1}{2}\left(-8(9)\right) = -27$	
2	$Y_4 = -27 + \frac{1}{2}\left(-8(-27)\right) = \sim$	
0	—	1
$\frac{1}{4}$	$Y_1 = 1 + \frac{1}{4}\left(-8(1)\right) = -1$	
$\frac{1}{2}$	$Y_2 = -1 + \frac{1}{4}\left(-8(-1)\right) = 1$	
$3/4$	$Y_3 = 1 + \frac{1}{4}\left(-8(1)\right) = -1$	
1	1 ⋮	
$5/4$	-1 ⋮	
$3/2$	1	
$7/4$	-1	
2	1	