1

5.6: 1, 11

10.1: 3, 9, 28, 31

Math HW 10

5.6

1.) $x'' + 9x = 10\cos(2t)$ $\quad x(0) = x'(0) = 0$

$r^2 + 9 = 0$

$r = \pm 3i$ $\quad x_c = c_1\cos(3t) + c_2\sin(3t)$

$f(t) = 10\cos(2t)$ $\quad \therefore y_p = A\cos(2t) + B\sin(2t)$

$f'(t) = -20\sin(2t)$ $\quad y_p' = -2A\sin(2t) + 2B\cos(2t)$

$y_p'' = -4A\cos(2t) - 4B\sin 2t$

$-4A\cos(2t) - 4B\sin 2t + 9(A)\cos(2t) + 9B\sin(2t) = 10\cos(2t)$

$\therefore (9A - 4A)\cos(2t) + (9B - 4B)\sin(2t) = 10\cos 2t + 0\sin(2t)$

$\therefore 5A = 10$, $\quad 5B = 0$ $\quad y_p = 2\cos(2t)$

$A = 2$ $\quad B = 0$

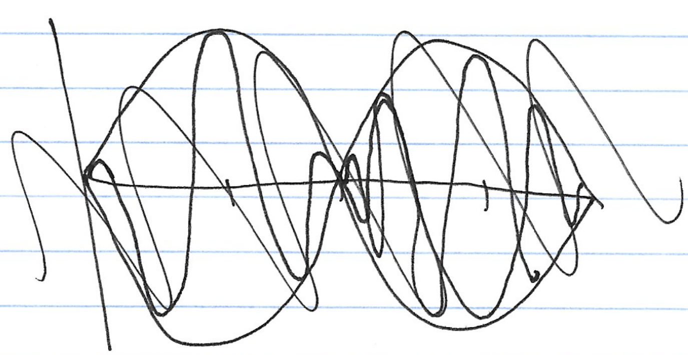
$x(t) = c_1\cos(3t) + c_2\sin(3t) + 2\cos(2t)$

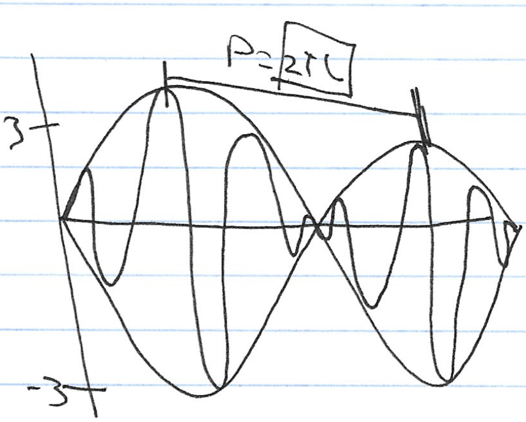
$x(0) = c_1 + 2 = 0$ $\quad c_1 = -2$

$x'(t) = -3c_1\sin(3t) + 3c_2\cos(3t) - 4\sin(2t)$

$x'(0) = 0 = 0 + 3c_2 + 0$ $\quad \therefore c_2 = 0$

$\therefore \boxed{x(t) = 2\cos(2t) - 2\cos(3t)}$





5.6

11) $x'' + 4x' + 5x = 10\cos(3t) \qquad x(0) = x'(0) = 0$

$m = 1 \quad c = 4 \quad k = 5 \qquad \therefore \omega_0 = \sqrt{5}$

$x_p = A\cos(3t) + B\sin(3t)$

$x_p' = -3A\sin 3t + 3B\cos(3t)$

$x_p'' = -9A\cos 3t - 9B\sin(3t)$

$\therefore -9A\cos(3t) \boxed{-9B\sin(3t)} + \boxed{-12A\sin 3t} + 12B\cos 3t + 5A\cos(3t) + \boxed{5B\sin(3t)}$

$= (12B - 4A)\cos 3t + (-4B - 12A)\sin(3t) = 10\cos 3t + 0$

$12B - 4A = 10 \qquad -36A - 4A = 10$

$-4B - 12A = 0 \qquad -40A = 10$

$-4B = 12A \qquad A = -\frac{1}{4}$

$B = -3A$

$B = \frac{3}{4} \qquad \therefore C = \sqrt{\frac{1}{16} + \frac{9}{16}} = \sqrt{\frac{10}{16}} = \frac{\sqrt{10}}{4}$

$\alpha = \pi + \tan^{-1}\left(\frac{B}{A}\right) = \pi + \tan^{-1}(-3) = 1.89$

$\therefore \boxed{x_{sp}(t) = \frac{\sqrt{10}}{4}\cos(3t - 1.89)}$

$x_c: r^2 + 4r + 5 = 0 \qquad \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$

$\therefore \cancel{x_c = e^{-2t}(c_1\cos(t) + c_2\sin(t))}$

$\cancel{x_c' = -2e^{-2t}(c_1\cos(t) + c_2\sin(t)) + e^{-2t}(-c_1\sin(t) + c_2\cos(t))}$

$\cancel{x(0) = c_1 = 0} \qquad x_c = e^{-2t}(c_1\cos(t) + c_2\sin(t))$

$\cancel{x'(0) =}$

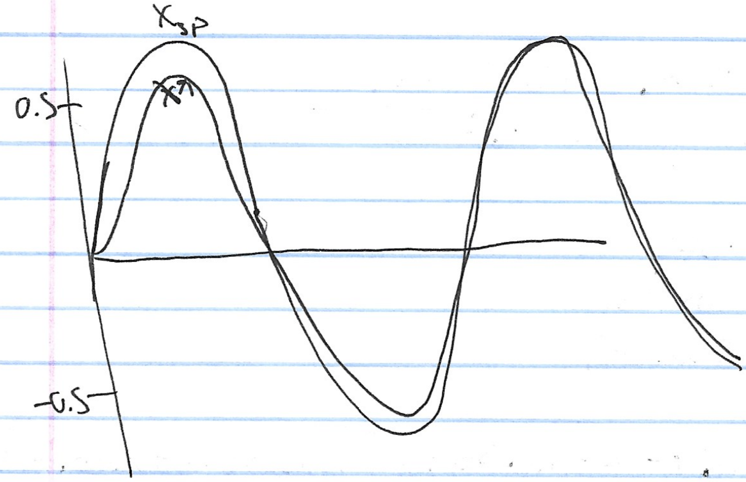
$x(t) = \frac{\sqrt{10}}{4}\cos(3t - 1.89) + e^{-2t}(c_1\cos(t) + c_2\sin(t))$

$x'(t) = \frac{-3\sqrt{10}}{4}\sin(3t - 1.89) - 2e^{-2t}(c_1\cos t + c_2\sin(t)) + e^{-2t}(-c_1\sin(t) + c_2\cos(t))$

$x(0) = \frac{\sqrt{10}}{4}\cos(-1.89) + c_1 \qquad \therefore c_1 = \frac{1}{4}$

$x'(0) = \frac{-3\sqrt{10}}{4}\sin(-1.89) - 2(c_1) + c_2 = 0 \qquad c_2 = -\frac{7}{4}$

$\therefore x(t) = \frac{\sqrt{10}}{4}(\cos(3t - 1.89)) + e^{-2t}(\cos(t)\frac{1}{4} - \frac{7}{4}\sin(t))$



x_{sp}
x
0.5
-0.5

4)

a.1

3.) $F(t) = \lim_{b\to\infty} \int_0^{b} e^{-st} \cdot e^{3t+1}\, dt = \lim_{b\to\infty} \int_0^{b} e^{-(s-3)t+1}\, dt$

$= \frac{-1}{s-3} \int_0^{b} e^{-(s-a)t+1} = -\frac{1}{s-3}\left[e^{-(s-3)t+1}\right]_0^{\infty} = -\frac{1}{s-3}\left(e^{-\infty} - e^{1}\right)$

$\boxed{\therefore F(t) = \frac{e}{s-3}}$

9.) $x(t) \begin{cases} 0 \le t \le 1 & x = t \\ t > 1 & x = 0 \end{cases}$

$u = t \quad v = \frac{-1}{s} e^{-st}$
$u' = 1 \quad v' = e^{-st}$

$F(t) = \int_0^1 e^{-st}\, t\, dt + \int_1^{\infty} 0 \cdot e^{-st}\, dt$

$\therefore F(t) = -\frac{1}{s} t e^{-st}\Big]_0^1 - \int_0^1 \frac{-1}{s} e^{-st} + \int_1^{\infty} 0\, dt$

$\therefore F(t) = \left(\frac{-1}{s} e^{-s}\right) + \frac{1}{s}\int_0^1 e^{-st} + \int_1^{\infty} 0\, dt$

$= \frac{-1}{s} e^{-s} + \left[\frac{1}{s} \cdot \frac{-1}{s} e^{-st}\right]_0^1 + \cancel{\int_1^{\infty} 0\, dt}$

$= -\frac{1}{s} e^{-s} + \left(\frac{-1}{s^2} e^{-s} + \frac{1}{s^2}\right) = -\frac{1}{s} e^{-s} + \frac{1}{s^2}\left(1 - e^{-s}\right)$

5)

10.1

28) $F(s) = \frac{3s+1}{s^2+4}$ $\mathcal{L}(\cos(kt)) = \frac{s}{s^2+4}$

$F(s) = \frac{3s}{s^2+4} + \frac{1}{s^2+4}$ $\therefore f(t) = 3\cos(2t) + \mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right) \rightarrow \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$

$\therefore \cancel{f(t) = 3\cos(2t) + \tfrac{1}{2}\cos(2t)}$

$\boxed{f(t) = 3\cos(2t) + \tfrac{1}{2}\sin(2t)}$

31) $F(s) = \frac{10s-3}{25-s^2} = -1\left(\frac{3-10s}{s^2-25}\right) = \cancel{-1\left(\frac{3}{s^2-25}\right)}$

$= -1\left(\frac{3}{s^2-25} - \frac{10s}{s^2-25}\right) = -1\left(\frac{3}{5}\cdot\frac{5}{s^2-25} - 10\frac{s}{s^2-25}\right)$

$\therefore f(t) = -1\cdot\frac{3}{5}\sin(5t) - 10\cos(5t) = \boxed{-\frac{3}{5}\sin(5t) - 10\cos(5t)}$