TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q	
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M\tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Fin effectiveness: how much heat the fin dissipates relative to how much it would dissipate from base with no fin

Fin efficiency: how much heat the fin dissipates relative to how much it would dissipate if entire fin were at base temperature

$$\frac{\rho Vc}{hA_s}\ln\frac{\theta_i}{\theta} = t$$

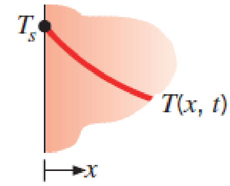
$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

$$\theta \equiv T - T_\infty \qquad \theta_i \equiv T_i - T_\infty \qquad Bi = \frac{hL_c}{k} < 0.1$$

Table 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

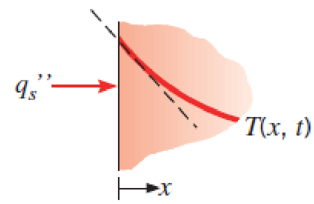
$$T(0, t) = T_s \tag{2.31}$$



2. Constant surface heat flux

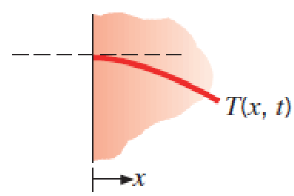
(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \tag{2.32}$$



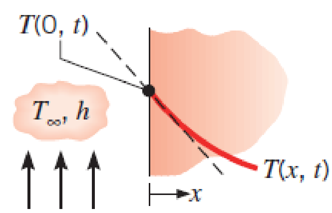
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \tag{2.33}$$



3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \tag{2.34}$$



Heat Equation in Cartesian, Cylindrical, and Spherical Coordinates

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$