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4.1: 1, 12, 17, 19, 20, 31, 34
4.2: 2, 3, 9, 13

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Math HW 6

4.1

1.) $|a-b| = \begin{bmatrix}2\\5\\-4\end{bmatrix} - \begin{bmatrix}1\\-2\\-3\end{bmatrix} = \begin{bmatrix}1\\7\\-1\end{bmatrix} = \sqrt{1^2+7^2+1^2} = \boxed{\sqrt{51}}$

b) $2a+b = 2\begin{bmatrix}2\\5\\-4\end{bmatrix} + \begin{bmatrix}1\\-2\\-3\end{bmatrix} = \begin{bmatrix}4\\10\\-8\end{bmatrix} + \begin{bmatrix}1\\-2\\-3\end{bmatrix} = \boxed{\begin{bmatrix}5\\8\\-11\end{bmatrix}}$

c) $3a-4b = 3\begin{bmatrix}2\\5\\-4\end{bmatrix} - 4\begin{bmatrix}1\\-2\\-3\end{bmatrix} = \begin{bmatrix}6\\15\\-12\end{bmatrix} - \begin{bmatrix}4\\-8\\-12\end{bmatrix} = \boxed{\begin{bmatrix}2\\23\\0\end{bmatrix}}$

12.) $u = (4,1)$, $v = (-2,-1)$, $w = (2,-2)$

$au + bv = w \rightarrow \begin{bmatrix}4a\\a\end{bmatrix} + \begin{bmatrix}-2b\\-b\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

$\cancel{\rightarrow \begin{bmatrix}4 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix} \rightarrow \begin{bmatrix}1 & -\frac{1}{2}\\1 & -1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}}$

$\cancel{\begin{bmatrix}1 & -\frac{1}{2}\\0 & -\frac{3}{2}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix} \rightarrow -\frac{3}{2}b = -2}$

$b = \frac{4}{3}$

$\cancel{a - \frac{1}{2}\left(\frac{4}{3}\right) = 2 \rightarrow a - \frac{2}{3} = 2 \quad a = \frac{8}{3}}$

$\begin{bmatrix}4 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix} \qquad AA^{-1}x = A^{-1}b$

$A^{-1} = \frac{1}{4(-1)-1(-2)}\begin{bmatrix}-1 & 2\\-1 & 4\end{bmatrix} = \frac{1}{-2}\begin{bmatrix}-1 & 2\\-1 & 4\end{bmatrix} = \begin{bmatrix}\frac{1}{2} & -1\\\frac{1}{2} & -2\end{bmatrix}$

$\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}1/2 & -1\\1/2 & -2\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix} = \begin{bmatrix}\frac{1}{2}(2) + -1(-2)\\1/2(2) + -2(-2)\end{bmatrix} = \begin{bmatrix}1+2\\+1+4\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$

$\therefore \boxed{3u + 5b = w}$

4.1

17) $u = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 5 \\ 4 \\ -6 \end{bmatrix}$ $w = \begin{bmatrix} 8 \\ 3 \\ -4 \end{bmatrix}$

$$\det \begin{bmatrix} 3 & 5 & 8 \\ -1 & 4 & 3 \\ 2 & -6 & -4 \end{bmatrix} = 3\begin{vmatrix} 4 & 3 \\ -6 & -4 \end{vmatrix} + 1\begin{vmatrix} 5 & 8 \\ -6 & -4 \end{vmatrix} + 2\begin{vmatrix} 5 & 8 \\ 4 & 3 \end{vmatrix}$$

$$= 3(-16+18) + (-20+48) + 2(15-32) = 6 + 28 - 34 = 0$$

17) $u = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\det \begin{bmatrix} 1 & 3 & 1 \\ -1 & 0 & -2 \\ 2 & 1 & 2 \end{bmatrix} = -3\begin{vmatrix} -1 & -2 \\ 2 & 2 \end{vmatrix} + 0 - 1\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -3(-2+4) - (-2+1)$$

$$= -6 + 1 = -5 \neq 0 \quad \therefore \boxed{\text{linearly independant}}$$

19) $u = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}$

$$au + bv + cw = \begin{bmatrix} 2 & -3 & 0 \\ 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 1 & -2 \\ 0 & \frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$c = 1$, $1(b) = 2$, $a = 3$

$\therefore 3u + 2v + w = 0$ $\therefore$ $\boxed{\text{Linearly dependant}}$