HW2_1

January 30, 2022

1 Problem 1

$$v_z = \frac{\Delta p R^2}{4\mu L}[1-\tilde{r}^2] = v_z^{max}[1-\tilde{r}^2] \tag{1}$$

$$c_A = c_{A_0}\tilde{r}^2 \tag{2}$$

where $\tilde{r} = \frac{r}{R}$ and $c_{A_0} = 0.10$c

Problem 1.1 Find and plot x_A and $\tilde{N}_A$ as functions of $\tilde{r}$ for $x_A(\tilde{r})$ the relationship $c_A = cx_A$ can be used:

$$x_A = \frac{c_A}{c}$$

then (2) can be substituted in for c_A to get

$$x_A = \frac{c_{A_0}\tilde{r}^2}{c} = \frac{0.10c\tilde{r}^2}{c} = 0.10\tilde{r}^2 \tag{3}$$

for $\tilde{N}_A(\tilde{r})$:

$$N_A = c_A v_z \tag{4}$$

given that $\tilde{N}_A = \frac{N_A}{c_{A_0}v_z^{max}}$ (4) can be substituted in for N_A to get

$$\tilde{N}_A = \frac{c_A v_z}{c_{A0}v_z^{max}} = \frac{cx_A v_z}{c_{A_0}v_z^{max}}$$

now (3) can be substituted in for x_A to get

$$\tilde{N}_A = \frac{\frac{c_{A_0}}{0.10}0.10\tilde{r}^2 v_z}{c_{A_0}v_z^{max}}$$

now substituting (1) for v_z

$$\tilde{N}_A = \frac{v_z^{max}(1-\tilde{r}^2)\tilde{r}^2}{v_z^{max}} = \tilde{r}^2(1-\tilde{r}^2) \tag{5}$$

[ ]:
```
"""import packages"""
import matplotlib.pyplot as plt
import numpy as np
```

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[ ]: """make functions that return x(r) and N(r)"""
def x(r):
    return 0.1*r**2

def N(r):
    return r**2*(1-r**2)

r = np.linspace(0,1,999)


plt.plot(r,x(r),label=r'$x_A$')
plt.plot(r,N(r),label=r'$\tilde{N}_A$')
plt.legend()
plt.title(''.join([r'$x_A$',' and ',r'$\tilde{N}_A$',' as functions of␣
 ↪',r'$\tilde{r}$']))
plt.grid()
plt.xlabel(r'$\tilde{r}$')



;
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[ ]: ''
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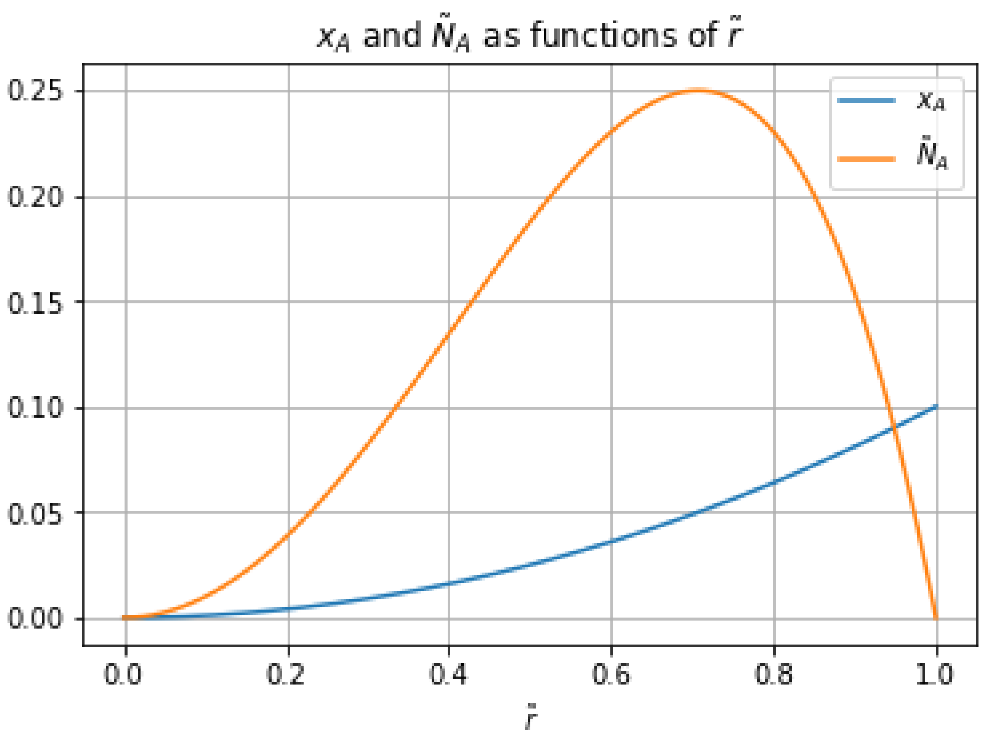

Problem 1.2 Find radial location ($\tilde{r}$) of $\tilde{N}_A^{max}$ For this problem, (5) and the above plot can be used to determine $\tilde{r}$ where $\tilde{N}_A$ is maximum

[ ]:
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N = N(r).tolist()
print(r[N.index(max(N))])
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0.7074148296593186
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$\tilde{r} = 0.707$ at $\tilde{N}_A^{max}$

Problem 1.3 Find molar flow rate of A ($\dot{n}_A$) and total volumetric flowrate ($\dot{V}$) in terms of v_z^{max}, c_{A_0}, and R

For $\dot{n}_A$:

$$\dot{n}_A = \int N_A dS = \int c_A v_z dS \tag{6}$$

substituting (1) and (2) in for v_z and c_A respectively and using the relationship $dS = 2\pi r dr$,

$$\dot{n}_A = \int_0^R c_{A_0}(\frac{r}{R})^2 v_z^{max}(1-(\frac{r}{R})^2)2\pi r dr = \frac{2\pi c_{A_0} v_z^{max}}{R^2}\int_0^R r^3 - \frac{r^5}{R^2} dr$$

integrating and evaluating from 0 to R gives

$$\dot{n}_A = \frac{2\pi c_{A_0} v_z^{max}}{R^2}[\frac{R^4}{4} - \frac{R^4}{6}] \tag{7}$$

for $\dot{V}$:

$$\dot{V} = \int v_z dS \tag{8}$$

substituting (1) for v_z and $dS = 2\pi r dr$ gives

$$\dot{V} = 2\pi v_z^{max}\int_0^R [1-(\frac{r}{R})^2]dr$$

integrating and evaluating from 0 to R gives

$$\dot{V} = 2\pi v_z^{max}(R - \frac{R}{3}) \tag{9}$$