HW1_3

January 20, 2022

1 Problem 3

Problem 3.1 Find R_{top}

$$R_{top} = R + H\tan(90-\theta) \tag{1}$$

The supporting work for this problem is included with the pdf

Problem 3.2 Find V The differential volume is related to the differential height with the following relationship

$$dV = \pi r^2 dh \tag{2}$$

similar to finding R_top in (1), r can be found with

$$r = R + h\tan(90-\theta)$$

and then used in (2)

$$dV = \pi(R + h\tan(90-\theta))^2 dh = [\pi R^2 + 2\pi R\tan(90-\theta)h + \pi\tan(90-\theta)^2 h^2]dh$$

the next step is to integrate both sides

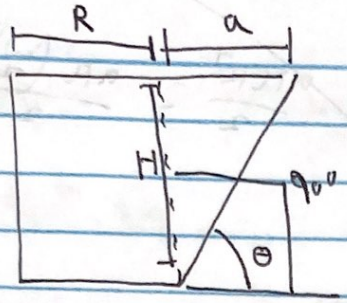
$$\int dV = \int_0^H \pi R^2 dh + \int_0^H 2\pi R\tan(90-\theta)hdh + \int_0^H \pi\tan(90-\theta)^2 h^2 dh$$

pull the constants out of the integral

$$V = \pi R^2 \int_0^H dh + 2\pi R\tan(90-\theta)\int_0^H hdh + \pi\tan(90-\theta)^2 \int_0^H h^2 dh$$

integrating gives

$$V = \pi R^2 H + \pi R\tan(90-\theta)H^2 + \frac{\pi}{3}\tan(90-\theta)^2 H^3$$



$$\tan(90-\theta) = \frac{a}{H}$$

$$\downarrow$$

$$a = H \tan(90-\theta)$$

$$\downarrow$$

$$R_{top} = R + H \tan(90-\theta)$$