# Q7

$3y'' + 7y' - 6y = 10e^{2x}$

$Y_c:\ 3r^2 + 7r - 6 = 0$

$$r = \frac{-7 \pm \sqrt{49 + 72}}{6} = \frac{-7 \pm \sqrt{121}}{6}$$

$$r = \frac{-7 \pm 11}{6} = \frac{2}{3},\ -3$$

$$Y_c = C_1 e^{2/3 x} + C_2 e^{-3x}$$

$Y_p:\ f(x) = 10e^{2x}$

$f'(x) = 20e^{2x}$

$\therefore Y_p = Ae^{2x}$

$Y_p' = 2Ae^{2x}$

$Y_p'' = 4Ae^{2x}$

$12A e^{2x} + 14Ae^{2x} - 6Ae^{2x} = 10e^{2x}$

$\therefore 20Ae^{2x} = 10e^{2x}$

$20A = 10$

$A = \frac{1}{2}$

$\therefore X(t) = C_1 e^{2/3 x} + C_2 e^{-3x} + \frac{e^{2x}}{2}$

$x'(t) = \frac{2}{3} C_1 e^{2/3x} - 3C_2 e^{-3x} + e^{2x}$

$X(0) = C_1 + C_2 = 0$

$x'(0) = \frac{2}{3}C_1 - 3C_2 = 0$

$-\frac{2}{3}C_2 = 3C_2$

$-\frac{11}{3}C_2 = 0 \quad \therefore C_1 = C_2 = 0$

$$\boxed{X(t) = \frac{e^{2x}}{2}}$$