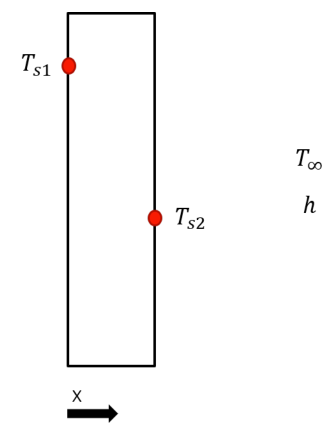
Heat Transfer Ch En 3453 | Homework 3 | Due Friday Sept. 17 at 5 PM

Scan as pdf and submit through Canvas

Note: Approximate solutions are the actual solution, but rounded to only 1 significant figure.

Problem 1

Solve the heat equation for the temperature profile, *T(x)*, with a constant surface temperature boundary condition on the left side and a convection boundary condition on the right side. Assume that h, T_{s2}, T_∞, k, and L (the wall thickness) are known and constant. Assume that the system is at steady state and that there is no generation.

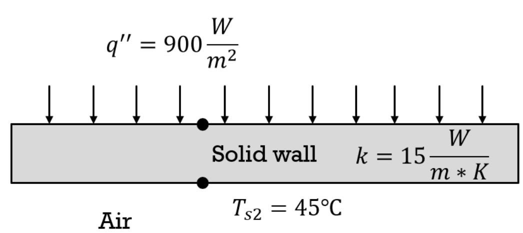


a. What is the temperature profile, *T(x)*, in terms of the variables: h, T_{s2}, T_∞, k, and L?
b. If h=95 W/m^{2}K, T_{s2} = 50°C, T_∞ = 20°C, k=25 W/mK, and L = 0.2 m, what is the left surface temperature T_{s1}?
c. What is the flux (q_x'') through the system?

Approximate solutions: (b) 70 °C, (c) 3000 W/m^2

Problem 2

Solve the heat equation for the temperature profile, *T(z)*, with a constant flux boundary condition on the top and a constant temperature boundary condition on the bottom. Assume that the system is at steady state and there is no generation within the solid.

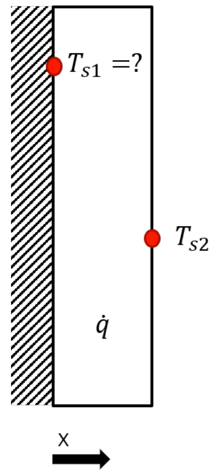


a. What is the temperature profile, *T(z)*, in terms of the variables provided (use L for the thickness)?
b. If L = 0.15 m, what is the surface temperature at the top, T_{s1} (i.e., $T(z$=0)).
c. What is the flux (q_z'') at z=0.10 m.

Approximate solutions: (b) 50 °C, (c) 1000 W/m^2.

Problem 3

Solve the heat equation for the temperature profile, *T(x)*, with a zero flux (i.e., well insulated) boundary condition on the left side and a constant surface temperature boundary condition on the right side when there is constant volumetric generation ($\dot{q}$) in the solid. Assume that $\dot{q}$, T_{s2}, k, and L (the wall thickness) are known and constant. Assume that the system is at steady state.

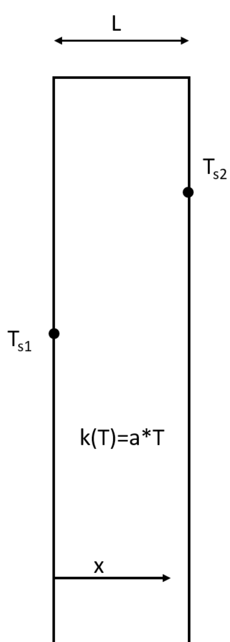


a. What is the temperature profile , *T(x)*, in terms of the variables provided?
b. If T_{s2} = 35°C, k=15 W/mK, $\dot{q}$ = 50,000W/m^3, and L = 0.15 m, what is the interface temperature T_{s1}?
c. What is the flux, $q_x''(x)$, through the system (as an equation)?
d. Calculate the flux exiting the system using two different methods: (1) Fourier's law and (2) an energy balance on the entire solid. How do these two values compare to each other?

Approximate solutions: (b) 70 °C, (d) 8000 W/m^2 and 8000 W/m^2

Problem 4

A plane wall of thickness L has a thermal conductivity that varies with temperature according to the equation k = a*T, where T is the temperature of the wall. Heat only flows in the x-direction. The system is at steady state.



(a) Starting with a differential energy balance (a.k.a., a "shell balance"), derive the 2nd order differential equation that can be solved for the temperature profile.

(b) With the plane wall, no generation, and steady-state assumptions, simplify the Heat Equation down to the 2nd order differential equation that can be solved for the temperature profile.

Compare the equations from (a) and (b). They should be the same.

(c) Derive an equation describing the steady-state wall temperature at any point (x), when given the wall surface temperatures T_{s1} and T_{s2}.

Compare the equations from (a) and (b). They should be the same.

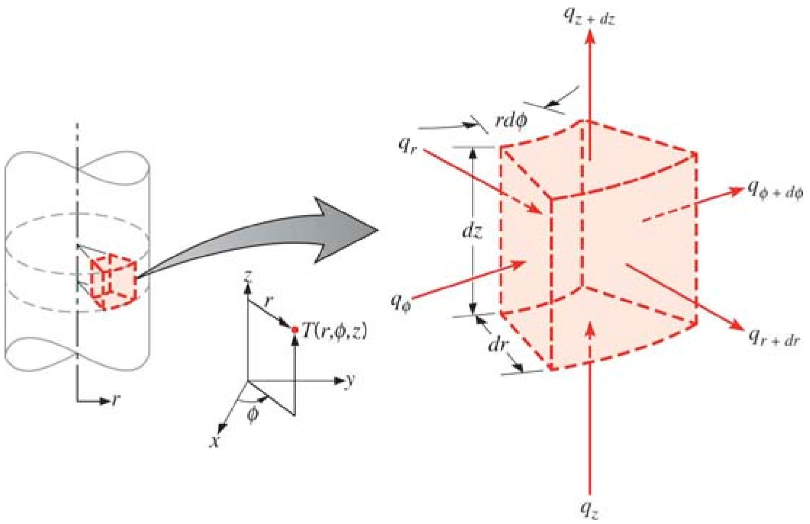
(d) If L = 10 cm, a = 0.01 W/m*K^2, T_{s1} = 300 K and T_{s2} = 600 K, what is the temperature at x = 3 cm (in K)?

Approximate answer for (d): 400 K

Problem 5

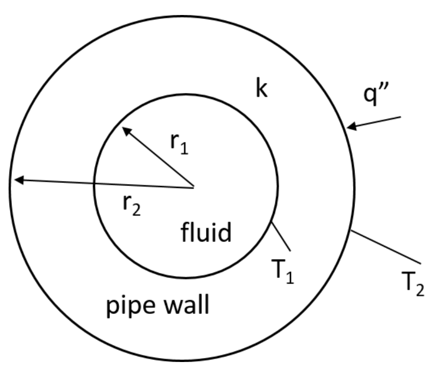
Derive the heat diffusion equation for cylindrical coordinates beginning with the differential control volume shown in the figure.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$



Problem 6

In order to prevent freezing of the fluid it carries, a buried pipe is heated from the outside with a constant heat flux of q” that enters the outside wall of the pipe (at r = r_2). The inner wall of the pipe (at r = r_1) is kept at a constant temperature T_1. The system is at steady state. A cross-section of the pipe is shown below.



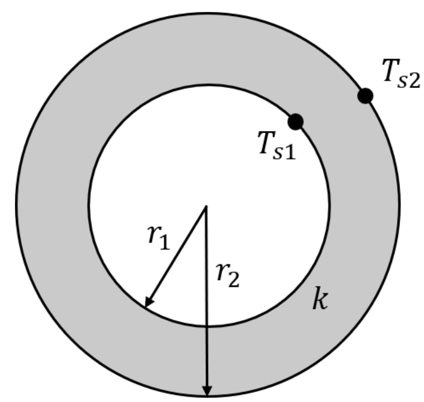
a. Derive an equation expressing the temperature profile between r_1 and r_2, T(r), in terms of the variables given above.

b. If r_1 = 5 cm, r_2 = 10 cm, k = 1 W/m*K, q” = 1200 W/m^2, and T_1 = 450 K, what is T_2 (in K)?

Approximate answer for (b): 500 K

Problem 7

A <u>spherical</u> wall contains a hot fluid, which creates an elevated temperature at its inner wall (T_{s1} located at $r = r_1$). The outer wall loses heat to the ambient, causing its surface temperature (T_{s2} located at $r = r_2$) to be cooler than that of the inner wall. Assuming temperature variation only in the r-direction and with the constant surface temperatures shown as boundary conditions, solve for the steady-state temperature profile, $T(r)$.



Problem 8

A nuclear reactor fuel element consists of a solid cylindrical pin of radius r_1 and thermal conductivity k_f. The fuel pin is in good contact with a cladding material of outer radius r_2 and thermal conductivity k_c. Consider steady-state conditions for which uniform heat generation occurs within the fuel at a volumetric rate $\dot{q}$ and the outer surface of the cladding is exposed to a coolant that is characterized by a temperature T_∞ and a convection coefficient h.

a. Obtain equations for the temperature distributions $T_f(r)$ and $T_c(r)$ in the fuel and cladding, respectively. Express your results exclusively in terms of the foregoing variables.
b. Consider a uranium oxide fuel pin for which k_f = 2 W/m*K and r_1 = 6 mm and cladding for which k_c = 25 W/m*K and r_2 = 9 mm. If $\dot{q} = 2 \times 10^8$ W/m^3, h = 2000 W/m^2*K, and T_∞= 300 K, what is the maximum temperature in the fuel element?

Approximate answer: (b) 1500 K