- 1. (6 points) Consider the vector space  $V = \text{span}\{u(t), v(t)\}$  where u(t) and v(t) are linearly independent functions.
  - (a) Define two new functions p(t) = a(u(t) + v(t)) and q(t) = b(u(t) v(t)). Show that for any  $a, b \neq 0$  the set  $\{p(t), q(t)\}$  is also a basis for V.

Metars bel' del pare to 26 / 1/2 / 1

(b) Verify that  $u(t) = e^{it}$  and  $v(t) = e^{-it}$  are linearly independent solutions of x'' + x = 0.  $\sqrt{e^{it}} = e^{it} - e^{-it} - e^{-i$ 

(c) Use Euler's identity  $(e^{it} = \cos(t) + i\sin(t))$  along with part (a) to show that  $\{\cos(t), \sin(t)\}$  is a basis for the solution space of x'' + x = 0.

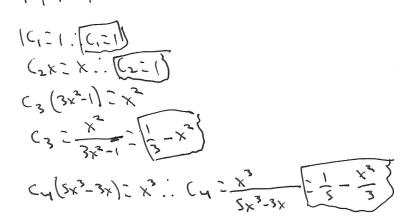
2. (7 points) Let P be the following set:

$$P = {\mathbf{p}_1(x), \ \mathbf{p}_2(x), \ \mathbf{p}_3(x), \ \mathbf{p}_4(x)},$$

where  $\mathbf{p}_1(x) = 1$ ,  $\mathbf{p}_2(x) = x$ ,  $\mathbf{p}_3(x) = 3x^2 - 1$ , and  $\mathbf{p}_4(x) = 5x^3 - 3x$ .

(a) Determine whether the set P is linearly independent or linearly dependent.

(b) Find a linear combination of elements of P that represents the polynomial  $y(x) = 1 + x + x^2 + x^3$ .  $(x_1 + x_2 + x_3) + (x_2 + x_3) + (x_3 + x_4) + (x_4 + x_3) + (x_4 + x_4) + (x_4 + x_4)$ 



(c) Does P span the vector space V of all polynomials of third-degree or less? Justify your answer.

Yes. Pis linearly independent and is 3" order