5.1: 6, 20, 27, 36, 40
5.2: 1, 8, 21, 27
5.3: 9, 12, 16, 20, 23, 25, 27

Math Hw8

5.1

6.) $y''+y'-6y=0$; $y_1=e^{2x}$, $y_2=e^{-3x}$, $y(0)=7$, $y'(0)=-1$

$y_1'=2e^{2x}$, $y_1''=4e^{2x}$: $4e^{2x}+2e^{2x}-6e^{2x}=0$ ✓
$y_2'=-3e^{-3x}$, $y_2''=9e^{-3x}$: $9e^{-3x}-3e^{-3x}-6e^{-3x}=0$ ✓

$r^2+r-6=0 \to (r+3)(r-2)=0 \therefore r=-3,2$

$\therefore y=C_1e^{-3x}+C_2e^{2x}$ $\qquad 7=C_1e^0+C_2e^0 \therefore C_1+C_2=7$
$y'=-3C_1e^{-3x}+2C_2e^{2x} \to -1=-3C_1e^{0}+2C_2e^0 \to -3C_1+2C_2=-1$
$C_2=\frac{3C_1-1}{2}$

$\therefore C_1+\frac{3C_1-1}{2}=7 \to$ $C_1+\frac{3C_1}{2}-\frac{1}{2}=7$
$C_1(1+\frac{3}{2})=\frac{15}{2} \to C_1=\frac{15}{2}\cdot\frac{2}{5}=3=C_1$, $C_2=4$

$\therefore \boxed{y=3e^{-3x}+4e^{2x}}$

20.) $f(x)=\pi$, $g(x)=\cos^2x+\sin^2x$ $\qquad C\pi=\cos^2x+\sin^2x \to C\pi=1$
$C=\frac{1}{\pi}$
Scalar multiples
$\boxed{\therefore \text{dependent}}$

27.) $y''+py'+qy=f(x)$ $\qquad \cancel{r^2+pr+q=0}$ $\qquad y=y_c+y_p$
$(y_c+y_p)''+p(y_c+y_p)'+q(y_c+y_p)=f(x)$

$y_c''+y_p''+py_c'+py_p'+qy_c+qy_p=f(x)$

$f(x)=y_p''+py_p'+qy_p$, $y_c''+py_c'+qy_c=0$

$\therefore y_p''+py_p'+qy_p+y_c''+py_c'+qy_c=f(x)+0$

$\to (y_p+y_c)''+p(y_p+y_c)'+q(y_p+y_c)=f(x)$ ✓

5.1

36.) $2y'' + 3y' = 0 \rightarrow 2r^2 + 3r = 0 \rightarrow r(2r+3)$ $\quad r = 0, \frac{-3}{2}$

$\therefore y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x} = C_1 e^{0x} + C_2 e^{-3/2 x}$ $= C_1 + C_2 e^{-3/2 x}$

40.) $9y'' - 12y' + 4y = 0 \rightarrow 9r^2 - 12r + 4 = 0$

$\frac{12 \pm \sqrt{144 - 4(9)(4)}}{2(9)} = 0.67, 0.67$

$\therefore y(x) = (C_1 + C_2 x) e^{r_1 x}$ $\boxed{= (C_1 + C_2 x) e^{0.67x}}$

5.2

1.) $f(x) = 2x,\ g(x) = 3x^2,\ h(x) = 5x - 8x^2$

$$W(2x, 3x^2, 5x - 8x^2) = \begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 2x & 3x^2 & 5x - 8x^2 \\ 2 & 6x & 5 - 16x \\ 0 & 6 & -16 \end{vmatrix}$$

~~$W = 0 - 6 \begin{vmatrix} 2x & 5x - 8x^2 \\ 2 & 5 - 16x \end{vmatrix} - 16 \begin{vmatrix} 2x & 3x^2 \\ 2 & 6x \end{vmatrix}$~~

~~$= -6(10x - 32x^2 - (10x - 16x^2)) - 16(12x^2 - 6x^2) = -6(-16x^2) - 16(-6x^2)$~~

$0 - 6 \begin{vmatrix} 2x & 5x - 8x^2 \\ 2 & 5 - 16x \end{vmatrix} - 16 \begin{vmatrix} 2x & 3x^2 \\ 2 & 6x \end{vmatrix}$

$= -6(10x - 32x^2 - (10x - 16x^2)) - 16(12x^2 - 6x^2)$

$= -6(-16x^2) - 16(6x^2) = 0$ $\therefore$ ~~dependent~~

$C_1 2x + C_2 3x^2 + C_3(5x - 8x^2) = 0$

$\rightarrow x^2(3C_2 - 8C_3) + x(2C_1 + 5C_3) = 0$

$3C_2 - 8C_3 = 0 \qquad 2C_1 + 5C_3 = 0$

$C_2 = \frac{8}{3} C_3 \qquad C_1 = -\frac{5}{2} C_3 \qquad C_3 = 6$

$C_2 = 16,\ C_1 = -15$ $\boxed{\therefore\ -15(2x) + 16(3x^2) + 6(5x - 8x^2) = 0}$

5.2

8.) $f(x) = e^x$, $g(x) = e^{2x}$, $h(x) = e^{3x}$

$$W(f,g,h) = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix}$$

$$W = e^x \begin{vmatrix} 2e^{2x} & 3e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} - e^x \begin{vmatrix} e^{2x} & e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} + e^x \begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix}$$

$$= e^x(18e^{5x} - 12e^{5x}) - e^x(9e^{5x} - 4e^{5x}) + e^x(3e^{5x} - 2e^{5x})$$

$$= e^x(6e^{5x} - 5e^{5x} + e^{5x}) = e^x(2e^{5x}) = 2e^{6x} \neq 0 \therefore \boxed{\text{independent}}$$

21.) $y'' + y = 3x$; $y(0) = 2$, $y'(0) = -2$

$y_c = C_1 \cos x + C_2 \sin x$; $y_p = 3x$

$y(x) = C_1 y_1(x) + C_2 y_2(x) + \dots + y_p(x)$

$y_c = 2 = C_1 \cos(0) + C_2 \cancel{\sin(0)}$ $\therefore$ $C_1 = 2$

$y_c' = -C_1 \sin x + C_2 \cos x \rightarrow y_c' = -2 = -C_1 \cancel{(0)} + C_2 \cos(0) \therefore C_2 = -2$

$\therefore \boxed{y(x) = 2\cos x - 2\sin x + 3x}$

5.2 27.) $C_1 + C_2x + C_3x^2 = 0$

$0 + C_2 + C_3x = 0$

$0 + 0 + C_3 = 0$

$\therefore 0+0+0=0 \quad \therefore \boxed{\text{independent}}$

$~~W \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix}~~$

$W = \begin{vmatrix} 1 & 2x \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$W = 2 \neq 0$ ✓

5.3 9.) $y'' + 8y' + 25y = 0 \rightarrow r^2 + 8r + 25 = 0$

$r = \dfrac{-8 \pm \sqrt{64 - 4(25)}}{2} = \dfrac{-8 \pm \sqrt{-36}}{2} = \dfrac{-8 \pm 6i}{2} = -4 \pm 3i$

$\therefore y = e^{ax}(C_1 \cos bx + C_2 \sin bx) = \boxed{e^{-4x}(C_1 \cos(3x) + C_2 \sin(3x))}$

12.) $y^{(4)} - 3y^{(3)} + 3y'' - y' = 0 \rightarrow r^4 - 3r^3 + 3r^2 - r = 0 \rightarrow r(r^3 - 3r^2 + 3r - 1) = 0$

$\therefore r = 0, 1 \qquad r^3 - 3r^2 + 3r - 1 = 0 \qquad r^3 = 1, \; 1, \; -3r^2 = 3r$

$\therefore$ ~~$r = 1$~~ ✓

$\therefore Y(x) = C_1e^{r_1x} + C_2e^{r_2x} = C_1e^0 + C_2e^x \boxed{\cancel{= C_1 + C_2e^x}}$

$\boxed{Y(x) = C_1 + C_2e^x + C_3xe^x + C_4x^2e^x}$

5.3

16) $y^{(4)} + 18y'' + 81y = 0 \rightarrow r^4 + 18r^2 + 81 = 0 \rightarrow (r^2+9)^2 = 0$

$\rightarrow (r^2+9)(r^2+9) = 0 \qquad r^2 = -9 \qquad r = 3i \ (\times 4)$

$Y(x) = e^0 (C_1 \cos 3x + C_2 \sin 3x) + e^0 (C_3 \cos(3x) + C_4 \sin 3x)$

$\boxed{\therefore Y(x) = \cos(3x)(C_1 + C_3 x) + \sin(3x)(C_2 + C_4 x)}$

20) $y^{(4)} + 2y^{(3)} + 3y'' + 2y' + y = 0 \Rightarrow r^4 + 2r^3 + 3r^2 + 2r + 1 = 0$
$\rightarrow (r^2+r+1)^2 = 0 \rightarrow (r^2+r+1)(r^2+r+1) = 0$

$\dfrac{-1 \pm \sqrt{1-4(1)}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

$Y(x) = e^{-1/2x} \left(C_1 \cos\left(\tfrac{\sqrt{3}}{2}x\right) + C_2 \sin\left(\tfrac{\sqrt{3}}{2}x\right) + C_3 x \cos\left(\tfrac{\sqrt{3}}{2}x\right) + C_4 x \sin\left(\tfrac{\sqrt{3}}{2}x\right)\right)$

$\boxed{= e^{-x/2} \left(\cos\left(\tfrac{\sqrt{3}}{2}x\right)(C_1 + C_3 x) + \sin\left(\tfrac{\sqrt{3}}{2}x\right)(C_2 + C_4 x)\right)}$

23) $y'' - 6y' + 25y = 0 \qquad y(0) = 3, \quad y'(0) = 1$
$r^2 - 6r + 25 = 0$
$\dfrac{6 \pm \sqrt{36 - 4(25)}}{2} = \dfrac{6 \pm \sqrt{-64}}{2} = 3 \pm 4i$

$\therefore Y(x) = e^{3x} (C_1 \cos(4x) + C_2 \sin(4x))$
$y(0) = 3 = 1(C_1 \cos(0) + 0) \therefore C_1 = 3$
$y' = 3e^{3x} (3\cos(4x) + C_2 \sin(4x)) + e^{3x} (\cancel{-12\sin}(4x) + 4C_2 \cos(4x))$

$y'(0) = 1 = 3\cos(0) + 4C_2 \cos(0) \rightarrow 1 = 3 + 4C_2 \rightarrow C_2 = \frac{-2}{4} = \frac{-1}{2}$

$\therefore \boxed{Y(x) = e^{3x} \left(3\cos(4x) - \tfrac{1}{2}\sin(4x)\right)}$

6

§3 25.) $3y^{(3)} + 2y'' = 0 \qquad y(0) = -1 \qquad y'(0) = 0 \qquad y''\,0 = 1$

$3r^3 + 2r^2 = 0 \Rightarrow r^2(3r+2) = 0 \qquad \therefore\ r = 0, 0, \frac{-2}{3}$

$y(x) = (C_1 + C_2 x)e^{0x} + C_3 e^{-2/3x} = C_1 + C_2 x + C_3 e^{-2/3x}$

$y(0) = -1 = C_1 + C_3$

$y'(x) = C_2 - \frac{2}{3} C_3 e^{-2/3x} \rightarrow y'(0) = 0 = C_2 - \frac{2}{3} C_3$

$y''(x) = \frac{4}{9} C_3 e^{-2/3x} \rightarrow y''(0) = 1 = \frac{4}{9} C_3 \cancel{e^{-2/3}} \quad \therefore\ C_3 = \frac{9}{4}$

$C_2 = \frac{18}{12} = \frac{3}{2}$

$C_1 = \frac{-4}{4} - \frac{9}{4} = \frac{-13}{4}$

$\therefore\ \boxed{y(x) = \frac{-13}{4} + \frac{3}{2}x + \frac{9}{4} e^{-2/3x}}$

27.) $y^{(3)} + 3y'' - 4y = 0 \rightarrow r^3 + 3r^2 - 4 = 0 \rightarrow \cancel{(r+4)(r-1)} = 0 \quad \therefore\ \cancel{r = 1,\ r = -4}$

$\cancel{(r^2+4)(r-1)} \qquad r^3 + 3r^2 - 4 = 0$

$\cancel{r^3+3}.\quad 1 + 3 + 0 - 4 \;\underline{|a}$

$\quad a \quad 0 \quad 0$

$\quad 3+a \quad 0 \quad -4$

$3+a-4 = 0$

$a = 1$

$r = 1, -2$

$\boxed{y(x) = C_1 e^{x} + C_2 e^{-2x}}$