# CH EN 3353
# Homework #1

## Assigned 8/25/21. Due 9/1/21.

Note: “math_review.pdf” on Canvas will be helpful

**Problem 1. Derivatives**
*Differentiate. Show all work.*

(a) $\frac{d}{dx}(x^{-2.5})$

(b) $\frac{d}{dx}(x^2 \sin 2x + 2x \tan x)$

(c) $\frac{d}{dx}\left(\frac{6\sqrt{x}}{x^5-5}\right)$

(d) $\frac{d}{dx}(\ln(20-x))$

(e) $\frac{d}{dx}(\cos^2(x^4))$

**Problem 2. Integrals**
*Integrate. Show all work.*

(a) $\int \sin^2 x \ \ \cos x \ dx$

(b) $\int x^3 \cos(x^4+2)\, dx$

(c) $\int \ln(x) dx$

(d) $\int \frac{x^4+x}{x-1}$

**Problem 3. Partial derivatives**
*Show all work.*

(a) Given $f(x,y,z) = e\text{^}\{x+z\}]ln\{y\}$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

(b) Given $f(x,y) = \sin\left(\frac{x}{1+y}\right)$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

(c) Given $f(x,y) = 6x^3 + x^2y^2 - 7y^3$, find all 2$^{nd}$ partial derivatives

**Problem 4. Ordinary differential equations**
*Show all work.*

(a) $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$

(b) $\frac{dy}{dx} = x^3 y$

(c) $\frac{dy}{dx}(1 + \tan y) = x^2 + 1$

(d) $\frac{dy}{dx}(2y + e^{3y}) = x \cos x$, when $y(0) = 0$

**Problem 5. Linear algebra**
*Solve the following systems of simultaneous linear equations.*

(a) $x + 3y = 8, 2x - 9 = y$

(b) $2x + 3y - z = 5, 4x - y - z = -1, x + 4y + z = 12$

# HW1 Answer pages

Please use these for uploading to Gradescope. Put your final answer in the box provided.

Problem 1

(a)

$\frac{d}{dx}\left(x^{-2.5}\right) = -2.5x^{-3.5}$

$\boxed{-2.5x^{-3.5}}$

(b) $\frac{d}{dx}\left(x^2\sin(2x) + 2x\tan x\right)$

$= 2x\sin(2x) + 2x^2\cos(2x) + 2\tan(x) + 2x\sec^2(x)$

$= 2\left(x\sin(2x) + x^2\cos(2x) + \tan(x) + x\sec^2(x)\right)$

$\boxed{2(x\sin(2x) + x^2\cos(2x) + \tan(x) + x\sec^2(x)}$

(c) $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g(f') - fg'}{g^2}$ $\quad \frac{d}{dx}\left(\frac{6\sqrt{x}}{x^5-5}\right) =$

$= \frac{(x^5-5)(3x^{-1/2}) - (6x^{1/2})(5x^4)}{(x^5-5)^2}$

$\boxed{\frac{(x^5-5)(3x^{-1/2}) - (6x^{1/2})(5x^4)}{(x^5-5)^2}}$

(d) $\frac{d}{dx}\left(\ln(20-x)\right) = \frac{1}{20-x} \cdot -1 = \frac{-1}{20-x}$

$\boxed{\frac{1}{x-20}}$

(e) $\frac{d}{dx}\left(\cos^2(x^4)\right) \Rightarrow \left(\cos(x^4)\right)^2$

$= 2\cos(x^4) \cdot -\sin(x^4) \cdot 4x^3$

$= -8x^3\cos(x^4) \cdot \sin(x^4)$

$\boxed{-8x^3\cos(x^4)\sin(x^4)}$

Problem 2

(a) $\int vu' = vu - \int uv'$ $\quad v = \sin^2 x$, $v' = 2\sin x\cos x$ $\quad u = +\sin x$, $u' = \cos x$

$\int \sin^2 x\cos x = G = \sin^2 x \cdot + \sin x - \int \sin x \cdot 2\sin x\cos x$

$G = \sin^3 x - 2\underbrace{\int \sin^2 x \cos x}_{G}$ $\quad \therefore G = \sin^3 x - 2G$

$3G = \sin^3 x$

$G = \frac{\sin^3 x}{3} + C$

$\boxed{\frac{\sin^3 x}{3} + C}$

(b) $\int x^3\cos(x^4+2)\,dx \longrightarrow u = x^4+2$, $du = 4x^3\,dx$

$\therefore = \frac{1}{4}\int \cos(u)\,du = \frac{1}{4}\sin(u) + C$

$= \frac{1}{4}\sin(x^4+2) + C$

$\boxed{\frac{1}{4}\sin(x^4+2) + C}$

(c) $\int \ln(x)\,dx = \int 1\cdot\ln(x)\,dx$ $\quad v = \ln x$, $u = x$; $v' = \frac{1}{x}$, $u' = 1$

$= x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int 1 = x\ln x - x$

$\boxed{x(\ln x - 1) + C}$

(d) $\int \frac{x^4+x}{x-1}$ $\quad u = x-1 \to du = 1dx$; $x^4 = (u+1)^4$, $x = u+1$

$\therefore = \int \frac{(u+1)^4 + u + 1}{u}\,du$

$(u+1)^4 = (u+1)(u+1)(u+1)(u+1) = (u^2+2u+1)(u^2+2u+1) = u^4 + 4u^3 + 6u^2 + 4u + 1$

$\to \int\left(\frac{u^4+4u^3+6u^2+4u+1+u+1}{u}\right)du = \int\left(u^3+4u^2+6u+5+\frac{2}{u}\right)du$

$= \frac{1}{4}u^4 + \frac{4}{3}u^3 + 3u^2 + 5u + 2\ln(u) + C$

$\boxed{\frac{1}{4}(x-1)^4 + \frac{4}{3}(x-1)^3 + 3(x-1)^2 + 5(x-1) + 2\ln(|x-1|) + C}$

Problem 3

(a) $f(x,y,z) = e^{x+z} \cdot \ln(y)$

$$\boxed{\begin{array}{l|l} \frac{\partial f}{\partial x} = \ln y \cdot e^{x+z} & \frac{\partial f}{\partial z} = \ln y \cdot e^{x+z} \\ \frac{\partial f}{\partial y} = \frac{e^{x+z}}{y} & \end{array}}$$

$\frac{\partial f}{\partial x} = \ln y \cdot e^{x+z}$ - treat y, z as constant

$$\frac{\partial f}{\partial y} = \frac{e^{x+z}}{y}$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial x}$$

(b) $f(x,y) = \sin\left(\frac{x}{1+y}\right)$

$$\boxed{\begin{array}{l} \frac{\partial f}{\partial x} = \frac{1}{1+y}\cos\left(\frac{x}{1+y}\right) \\ \frac{\partial f}{\partial y} = \frac{-x}{(1+y)^2}\cos\left(\frac{x}{1+y}\right) \end{array}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+y}\cos\left(\frac{x}{1+y}\right)$$

$$\frac{\partial f}{\partial y} = \frac{-x}{(1+y)^2}\cos\left(\frac{x}{1+y}\right)$$

(c) $f(x,y) = 6x^3 + x^2y^2 - 7y^3$

$$\boxed{\begin{array}{l} \frac{\partial^2 f}{\partial x^2} = 36x + 2y \\ \frac{\partial^2 f}{\partial y^2} = 2x^2 - 42y \end{array}}$$

$\frac{\partial}{\partial x} = 18x^2 + 2y^2x$ $\qquad \frac{\partial^2}{(\partial x)^2} = 36x + 2y$

$\frac{\partial}{\partial y} = 2x^2y - 21y^2$ $\qquad \frac{\partial^2}{(\partial y)^2} = 2x^2 - 42y$

Problem 4

(a)

$$\frac{dy}{dx} = \frac{6x^2}{2y+\cos y} \qquad \text{Seperable}$$

$$\boxed{y^2+\sin(y) = 2x^3+C}$$

$$\int (2y+\cos(y))\,dy = \int 6x^2\,dx$$

$$\rightarrow y^2+\sin(y) = 2x^3+C$$

(b)

$$\frac{dy}{dx} = x^3 y \qquad \text{Seperable}$$

$$\boxed{\ln|y| = \frac{1}{4}x^4+C}$$

$$\rightarrow \int \frac{1}{y}\,dy = \int x^3\,dx$$

$$\rightarrow \ln|y| = \frac{1}{4}x^4$$

(c)

$$\frac{dy}{dx}(1+\tan(y)) = x^2+1 \qquad \text{Seperate}$$

$$\boxed{y-\ln|\cos y| = \frac{1}{3}x^3+x+C}$$

$$\int (1+\tan y)\,dy = \int (x^2+1)\,dx$$

$$\rightarrow y+\int \tan(y)\,dy = \frac{1}{3}x^3+x$$

$$\int \tan y\,dy = \int \frac{\sin y}{\cos y}\,dy \rightarrow \left.\begin{matrix} u=\cos y \\ du=-\sin y\,dy \end{matrix}\right\} \int -\frac{1}{u}\,du = -\ln|u| = -\ln|\cos y|$$

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(d)

$$\frac{dy}{dx}(2y+e^{3y}) = x\cos x \qquad y(0)=0$$

$$\boxed{y^2+\frac{1}{3}e^{3y} = x\sin x\cos x - \frac{2}{3}}$$

$$\rightarrow \int (2y+e^{3y})\,dy = \int x\cos x\,dx$$

$$\rightarrow y^2+\frac{1}{3}e^{3y} = \underbrace{\int x\cos x\,dx} \qquad \begin{matrix} v=x & u=\sin x \\ v'=1 & u'=\cos x \end{matrix}$$

$$= x\sin x - \int \sin x\,dx = x\sin x+\cos x$$

$$\rightarrow y^2+\frac{1}{3}e^{3y} = x\sin x+\cos x+C\Big|_{0,0} \rightarrow 0^2+\frac{1}{3}e^0 = 0+\cos(0)+C \rightarrow 0+\frac{1}{3} = 0+1+C$$

$$C = -\frac{2}{3}$$

Problem 5

(a) $x+3y=8$

$2x-9=y \rightarrow 2x-y=9$

$Ax=B$

$x=A^{-1}B$

$$\boxed{\begin{array}{l} x=5 \\ y=1 \end{array}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

$$A^{-1}=\frac{1}{1(-1)-3(2)}\begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}=\frac{1}{-7}\begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-7}\begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 9 \end{bmatrix}=\frac{1}{-7}\begin{bmatrix} -8-27 \\ -16+9 \end{bmatrix}=\frac{1}{-7}\begin{bmatrix} -35 \\ -7 \end{bmatrix}=\begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

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(b) $$\begin{bmatrix} 2 & 3 & -1 \\ 4 & -1 & -1 \\ 1 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ -1 \\ 12 \end{bmatrix}$$

$x=A^{-1}B$

$$\boxed{x=1,\ y=2,\ z=3}$$

$A^{-1}$: minor:
$$\begin{bmatrix} -1(1)-4(-1) & 4(1)-1(-1) & 4(4)-1(-1) \\ 3(1)-4(-1) & 2(1)-1(-1) & 2(4)-3(1) \\ 3(-1)+1(-1) & 2(-1)-4(-1) & 2(-1)-4(3) \end{bmatrix}=\begin{bmatrix} 3 & 5 & 17 \\ 7 & 3 & 5 \\ -4 & 2 & -14 \end{bmatrix} \rightarrow \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$\rightarrow \begin{bmatrix} 3 & -5 & 17 \\ -7 & 3 & -5 \\ -4 & -2 & -14 \end{bmatrix} \rightarrow T \rightarrow \begin{bmatrix} 3 & -7 & -4 \\ -5 & 3 & -2 \\ 17 & -5 & -14 \end{bmatrix}$$

$$\frac{-1}{26}\begin{bmatrix} -26 \\ -52 \\ -78 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

det(A): $2(-1+4)-3(4+1)-(16+1)=-26$

$$A^{-1}=\frac{-1}{26}\begin{bmatrix} 3 & -7 & -4 \\ -5 & 3 & -2 \\ 17 & -5 & -14 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{-1}{26}\begin{bmatrix} 3 & -7 & -4 \\ -5 & 3 & -2 \\ 17 & -5 & -14 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \\ 12 \end{bmatrix}=\frac{-1}{26}\begin{bmatrix} 15+7-48 \\ -25-3-24 \\ 17(5)+5-14(12) \end{bmatrix}$$