1.1: 5, 15, 19, 28, 32, 36, 46
1.2: 6, 8, 17, 20, 25, 33

Math 2250
Josh Whitehead
U1069343

HW #1

1.1

5.) $y' = y + 2e^{-x}$; $y = e^{x} - e^{-x}$ $\qquad y' = e^{x} + e^{-x}$

$y' = (e^{x} - e^{-x}) + 2e^{-x} = e^{x} + e^{-x}$ ✓

15.) $y'' + y' - 2y = 0$; $\quad y = e^{rx}$, $y' = re^{rx}$, $y'' = r^2 e^{rx}$

$\rightarrow r^2 e^{rx} + re^{rx} - 2e^{rx} = 0 \rightarrow e^{rx}(r^2 + r - 2) = 0 \rightarrow r^2 + r - 2 = 0$

$(r+2)(r-1) = 0 \therefore \boxed{r = -2, 1}$

19.) $y' = y + 1$; $y(x) = Ce^{x} - 1$, $y(0) = 5$

$y'(x) = Ce^{x}$

$y' = Ce^{x} - 1 + 1 = Ce^{x}$ ✓ $\rightarrow y(0) = 5 = Ce^{0} - 1 = C - 1 \rightarrow \boxed{C = 6}$

28.) $m = \frac{dy}{dx}$, $\quad 0 = \frac{dy}{dx}\frac{x}{2} + b \rightarrow \cancel{b = \frac{dy}{dx}\frac{x}{2}}$ $\quad b = -\frac{x}{2}\frac{dy}{dx}$

$y = \frac{dy}{dx}x - \frac{x}{2}\frac{dy}{dx} = \frac{dy}{dx}\left(x - \frac{x}{2}\right) = \frac{dy}{dx}\left(\frac{x}{2}\right) \rightarrow \frac{2y}{x} = \boxed{\frac{dy}{dx} = \frac{2y}{x}}$

32.) $\boxed{\frac{dP}{dt} = k\sqrt{P}}$

36a.) $\boxed{\frac{dN}{dt} = kN(P-N)}$

46.) $\frac{dx}{dt} = kx^2$, $\quad \frac{dv}{dt} = kv^2$, $v(0) = 10\frac{m}{s}$, $a = -1\frac{m}{s^2}$ @ $v = 5\frac{m}{s}$

$\frac{dv}{dt} = a = kv^2 \rightarrow -1\frac{m}{s^2} = k(5\frac{m}{s})^2 \rightarrow k = \frac{-1}{25} \quad \therefore \frac{dv}{dt} = \frac{-1}{25}v^2$

$\frac{dv}{v^2} = -\frac{1}{25}dt \rightarrow \int \frac{1}{v^2}dv = -\frac{1}{v}$

$\rightarrow \int -\frac{1}{25}dt = \frac{-1}{25}t + C$

$\rightarrow -\frac{1}{v} = \frac{-1}{25}t + C \rightarrow \frac{-1}{10} = \frac{-1}{25}(0) + C$

$C = \frac{-1}{10}$

$\therefore \frac{-1}{v} = \frac{-1}{25}t - \frac{1}{10} \rightarrow \frac{50}{v} = 2t + 5 \rightarrow \boxed{v = \frac{50}{2t+5}}$

(a) $1 = \frac{50}{2t+5} \rightarrow 2t + 5 = 50 \rightarrow 2t = 45 \rightarrow \boxed{t = \frac{45}{2}\,s}$

(b) $\frac{1}{10} = \frac{50}{2t+5} \rightarrow \frac{2t+5}{10} = 50 \rightarrow 2t + 5 = 500 \rightarrow 2t = 495 \rightarrow \boxed{t = 495\,s}$

1.2

6.) $\frac{dy}{dx} = x\sqrt{x^2+9}$; $y(-4) = 0$

$\int dy = x\sqrt{x^2+9}\,dx$ $\quad u = x^2+9$, $du = 2x\,dx$

$y = \frac{1}{2}\int u^{1/2}\,du \Rightarrow y = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} \rightarrow y = \frac{1}{3}\sqrt{x^2+9}^{\,3} + C$

$y(-4) = 0 = \frac{1}{3}\sqrt{16+9}^{\,3} + C \Rightarrow C = -\frac{1}{3}\sqrt{25}^{\,3} = -\frac{1}{3}\cdot 125 = -\frac{125}{3}$

$\therefore \boxed{y(x) = \frac{1}{3}\left(\sqrt{x^2+9}^{\,3} - 125\right)}$

8.) $\frac{dy}{dx} = \cos(2x)$; $y(0) = 1$

$\int dy = \int \cos(2x)\,dx$ $\quad u = 2x$, $du = 2\,dx$

$y = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C \quad \frac{1}{2}\sin(2x) + C$

$y(0) = 1 = \frac{1}{2}\sin(0) + C \rightarrow 1 = 0 + C \rightarrow C = 1$

$\therefore \boxed{y(x) = \frac{1}{2}\sin(2x) + 1}$

17.) $a(t) = \frac{1}{(t+1)^3}$, $v_0 = 0$, $x_0 = 0$, $a(t) = \frac{dv}{dt}$, $v(t) = \frac{dx}{dt}$

$a(t) = (t+1)^{-3} = \frac{dv}{dt}$

$u = t+1 \quad du = dt$

$\int dv = \int (t+1)^{-3}\,dt \rightarrow v = -\frac{1}{2}(t+1)^{-2} + C$

$v(0) = 0 = \frac{-1}{2(1)^2} = -\frac{1}{2} \quad \therefore v(t) = -\frac{1}{2}(t+1)^{-2} - \frac{1}{2}$

$v(t) = \frac{dx}{dt} \rightarrow \int dx = \int -\frac{1}{2}\{(t+1)^{-2} - 1\}\,dt$

$x = -\frac{1}{2}\left[\int (t+1)^{-2}dt + \int 1\,dt\right] = -\frac{1}{2}\left[-1(t+1)^{-1} + t\right] + C$

$x(0) = 0 = \frac{1}{2}(1)^{-1} + 0 + C = \frac{1}{2} + C \rightarrow C = -\frac{1}{2}$

$\therefore \quad x(t) =$

1.2

17.) $a(t) = (t+1)^{-3}$, $v(0) = 0$, $x(0) = 0$

$a(t) = \frac{dv}{dt}$ $\quad v(t) = \frac{dx}{dt}$

$\int dv = \int (t+1)^{-3}\,dt \rightarrow v = \frac{-1}{2}(t+1)^{-2} + c$

$v(0) = \frac{-1}{2}(0+1)^{-2} + c \rightarrow c = \frac{1}{2}\left(\frac{1}{1}\right) = \frac{1}{2}$

$\therefore v(t) = \frac{-1}{2}(t+1)^{-2} + \frac{1}{2}$

$v(t) = \frac{dx}{dt}$

$\int dx = \frac{-1}{2}\int (t+1)^{-2}dt + \int \frac{1}{2}\,dt \rightarrow x = \frac{-1}{2}\left(-(t+1)^{-1}\right) + \frac{1}{2}t + c$

$x(0) = \frac{-1}{2}\left(-\left(\frac{1}{1}\right)\right) + 0 + c \rightarrow c = -\frac{1}{2}$

$\therefore \boxed{x(t) = \frac{1}{2}(t+1)^{-1} + \frac{1}{2}t - \frac{1}{2}}$

20.) ~~[0,5]~~: $\quad 0 \le x \le 5$: $\cancel{y} = \cancel{x} \rightarrow v(t) = t$ $\qquad v(t) = \frac{dx}{dt}$

$5 \le x \le 10$: $y = 5 \rightarrow v(t) = 5$

a) $\frac{dx}{dt} = t \rightarrow \int dx = \int t\,dt \rightarrow x = \frac{1}{2}t^2 + c \rightarrow x(0) = 0 = c + 0$

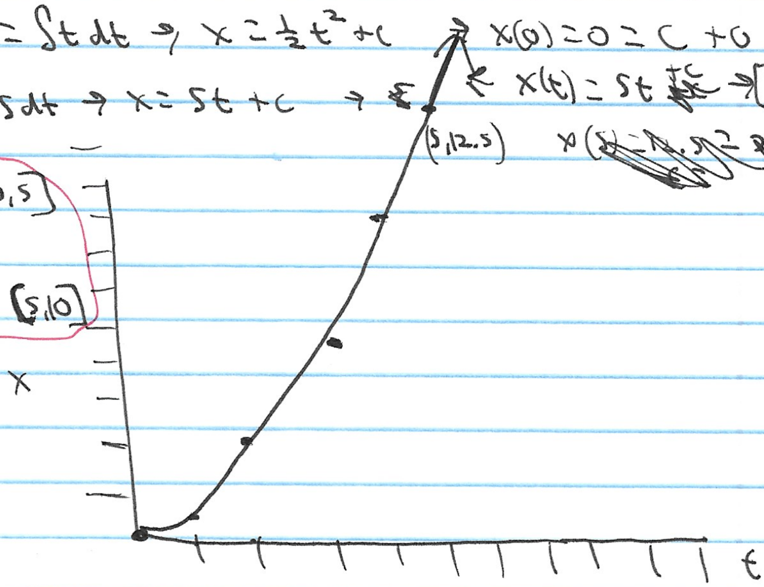
b) $\frac{dx}{dt} = 5 \rightarrow x = \int 5\,dt \rightarrow x = 5t + c \rightarrow$ $\quad x(t) = 5t \cancel{+c} \rightarrow \boxed{x(t) = 5t - \frac{25}{2}}$

(5, 12.5) $\quad x(5) = \cancel{12.5} = 5 \cdot 5 + c$

a.) $x(t) = \frac{1}{2}t^2$ [0,5]

~~b.) x(t) = ½ t~~

b.) $x(t) = 5t - \frac{25}{2}$ [5,10]



25.) $v_0 = 100 \frac{km}{hr}$ $\quad a(t) = -10 \frac{m}{s^2}$

$v_0 = 100 \frac{km}{hr} \times \frac{1000\,m}{1\,km} \times \frac{1\,hr}{60\,min} \times \frac{1\,min}{60\,sec} = 27.8 \frac{m}{s}$

$a(t) = \frac{dv}{dt} = -10 \rightarrow \int dv = -\int 10\,dt \rightarrow v(t) = -10t + c$

$v(0) = 27.8 \frac{m}{s} = 0 + c \rightarrow c = 27.8 \frac{m}{s}$

$\therefore v(t) = -10t + 27.8 \frac{m}{s}$ $\rightarrow v(t) = \frac{dx}{dt}$

$\int dx = \int -10t\,dt + \int 27.8 \frac{m}{s}\,dt \rightarrow x = -5t^2 + 27.8 \frac{m}{s} t + c$

~~find t when x=0~~

$\cancel{x(t) = 0 = -5t^2 + 27.8t = t(-5t + 27.8) = 0}$

$\cancel{-5t = -27.8}$

$\cancel{t = \frac{27.8}{5}}$

find C

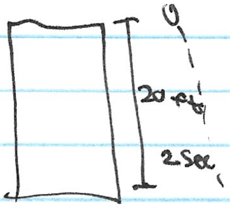
$x(0) = 0 = -5(0)^2 + 27.8 \frac{m}{s}(0) + c \rightarrow c = 0 \quad \therefore x(t) = -5t^2 + 27.8t$

find when v=0

$v(t) = 0 = -10t + 27.8 \rightarrow 27.8 = 10t \rightarrow t = 2.78$ sec

$x(2.78) = -5(2.78)^2 + 27.8(2.78) = \boxed{38.6\,m}$

1.2 33)



$\Delta Y = v_0 t + \frac{1}{2} a t^2$

$v_0 = 0 \therefore \Delta Y = 20 = 0 + \frac{1}{2} a \cdot 2^2$

$\rightarrow 20 = \frac{4}{2} a \rightarrow 20 = 2a \rightarrow a = 10 \frac{ft}{sec^2}$

$\Delta Y_2 = 200 = 0 + \frac{1}{2} \cdot 10 t^2 \rightarrow 200 = 5 t^2 \rightarrow t^2 = 40$

$\boxed{t = \sqrt{40} \ sec}$

$v_f = v_0 + at \rightarrow v_f = 10 \cdot \sqrt{40} = \boxed{10\sqrt{40} \ \frac{ft}{sec}}$