HW2_2

January 30, 2022

1 Problem 2

Consider the funnel that we analyzed in homework 1, shown below, with a radius of the hole in the bottom of R and a height of H.

$$u = \sqrt{2gh} \tag{1}$$

Problem 2.1 Derive $\frac{dh}{dt}$ in terms of h, R, θ, $\dot{V}_{in}$, and $\dot{V}_{out}$ Combinging the relationships

$$\frac{dV}{dt} = \dot{V}_{in} - \dot{V}_{out} \tag{2}$$

and

$$dV = A_c dh \tag{3}$$

we get

$$\frac{dh}{dt} = \frac{\dot{V}_{in} - \dot{V}_{out}}{A_c} \tag{4}$$

substituting $A_c = \pi r^2$ and $r = R + h\tan(90-\theta)$ (solved in HW1) into (4) produces

$$\frac{dh}{dt} = \frac{\dot{V}_{in} - \dot{V}_{out}}{\pi(R + h\tan(90-\theta))^2} \tag{5}$$

Problem 2.2 Find steady state liquid level h at steady state, $\frac{dh}{dt} = 0 \therefore$ setting (5) equal to zero we get

$$0 = \dot{V}_{in} - \dot{V}_{out} = \dot{V}_{in} - uA_c$$

substitute (1) into u to get

$$\dot{V}_{in} = \dot{V}_{out} = \pi R^2 \sqrt{2gh} \tag{6}$$

rearange to get

$$h = \frac{(\frac{\dot{V}_{in}}{\pi R^2})^2}{2g} \tag{7}$$

Problem 2.3 Find R at steady state when funnel is half-full by volume The following code uses (6) and (7) along with an equation for the volume of the tank derived in HW1:

$$V_{ss} = \frac{\pi\tan(\theta)(R_{top}^3 - R^3)}{6} = \frac{\pi\tan(\theta)(R_{top}^3 - (r - h\tan(90-\theta))^3)}{6}$$

to solve for the three unknowns r, R, and h

```
[ ]: import numpy as np

Vin   = 50                # inlet flow rate, cm^3/s
Rtop  = 20                # radius of the top of the funnel, cm
theta = 50  * np.pi / 180 # angle - convert to radians
g     = 980               # cm/s^2

def resfun(x):
    r = x[0]  # unpack - ordering here must be consistent with the initial␣
 ↪guess.
    R = x[1]
    h = x[2]
    # calculate each residual and return them as an array
    return [
        (np.pi*np.tan(theta)*(Rtop**3-(R**3))-np.pi*np.
 ↪tan(theta)*(Rtop**3-(r-h*np.tan(np.pi/2-theta))**3))/6,
        h - (Vin/np.pi/R**2)**2/2/g,
        Vin-np.pi*R**2*np.sqrt(2*g*h)
    ]
```

```
[ ]: from scipy.optimize import fsolve

roots = fsolve(resfun, # the function to set to zero
               [1,1,5] # guesses for the solution values r, R, h
              )
# unpack the solution - the ordering consistent with the
# guess and how these are used in the residual function.
r = roots[0]
R = roots[1]
h = roots[2]

print('Hole in the bottom is {:.2f} cm'.format(R))
print('Liquid level is {:.2f} cm'.format(h))
print('Funnel height is {:.2f} cm'.format(np.tan(theta) * (Rtop - R)))

# If we got the right solution, the residual values should be very close to␣
 ↪zero.
# uncomment the next line to look at the residual values
print('residual values: ',resfun([r,R,h]))
```

```
Hole in the bottom is 0.83 cm
Liquid level is 0.28 cm
Funnel height is 22.85 cm
residual values:  [-7.882287415365378e-12, -1.2601031329495527e-14,
1.1226575225009583e-12]
```

Problem 2.4

Steady-State Height

[ ]:
```
R = 0.04   # cm
hss = 0.28
print('The steady-state height is {:.2f} cm'.format( hss ) )
```

```
The steady-state height is 0.28 cm
```

1.1 Plot of $h(t)$

The following code uses (5) to solve for h as a function of time

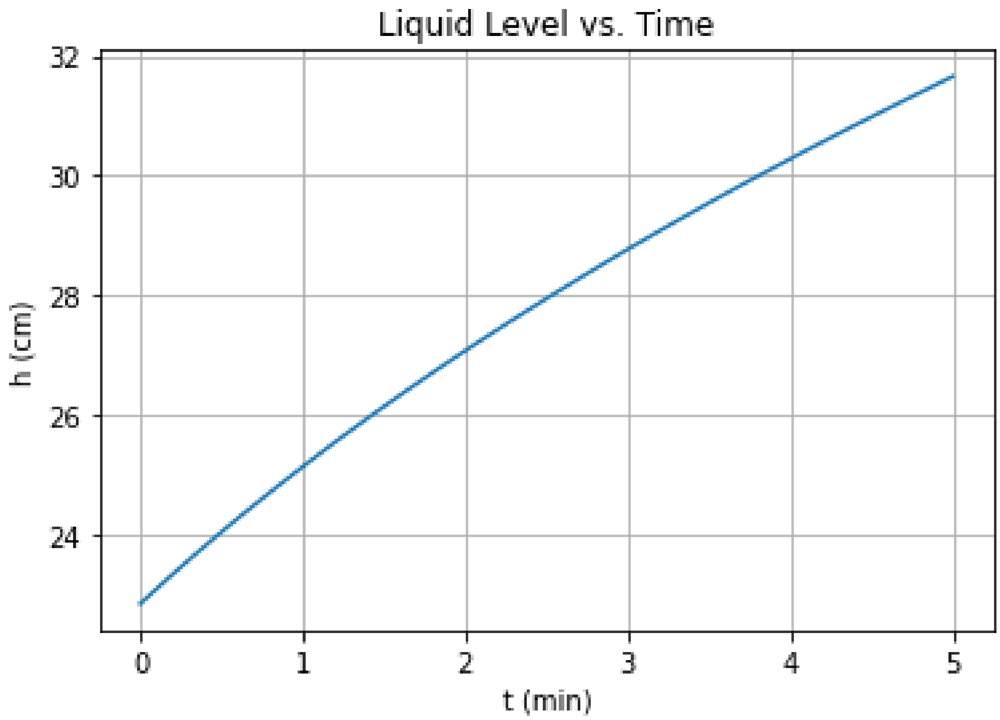
[ ]:
```
def dhdt(h,t):
    return (Vin-np.pi*R**2*np.sqrt(2*g*h))/(np.pi*(R+h*np.tan(0.5*np.
 ↪pi-theta))**2)
```

[ ]:
```
import matplotlib.pyplot as plt
from scipy.integrate import odeint

t  = np.linspace(0,5*60,400) # array of points in time we want to include in␣
 ↪the plot
H0 = 22.85                       # initial height

soln = odeint( dhdt, H0, t )
h = soln[:,0]

plt.plot(t/60,h)
plt.xlabel('t (min)')
plt.ylabel('h (cm)')
plt.title('Liquid Level vs. Time')
plt.grid()
plt.show()
```



1.1.1 Liquid level after 3 minutes

[ ]:
```
print('The liquid level at t={:.0f} minutes is {:0.2f} cm\n'.format( 5, h[-1] )
 ↪)
```

```
The liquid level at t=5 minutes is 31.66 cm
```