

# Introduction to Data Structures & Algorithms.

Every CS curriculum in the world covers DS & A. It is a fundamental subject and key to the pervasiveness of CS.

We interact with DS constantly:

- opening a file
- logging into a website
- searching the web

Without specialized algorithms to go with the improved data structures, we would not benefit nearly as much.

Analogy: swords / bows & arrows

~~One of the most basic algorithms.~~

What is a data structure? A collection of data, relationships between them, and functions that can be applied to the data.

Basic examples: strings, integers, lists.

What is an algorithm? A finite sequence of rigorous instructions.

Basic examples: we'll go through one soon, but one arises when solving a computational problem.

What is a computational problem? A task with a precise input and output that asks for a solution in terms of an algorithm.

Example: Searching through sorted lists of integers.

Data structure: sorted-list.

It's like a list, but it is always sorted in ascending order. The only entries are ints. Some functions associated to this might be:

- $\text{get}(i)$ : return the  $i^{\text{th}}$  entry.
- $\text{add}(x)$ : insert the integer  $x$ .
- $\text{len}()$ : return the length of the list.

Computational Problem:

Search

Input: sorted-list  $L$ , integer  $x$ .

Output:  $\begin{cases} \text{integer } i & \text{if } x \text{ in } L, \\ \text{null} & \text{if } x \text{ not in } L, \end{cases}$   
where  $\underbrace{L[i]}_{\text{"get(i)"}} == x$ .

Algorithm 1:

Simple Search (Python code)

1. for  $i$  in  $\text{range}(\text{len}(L))$ :
2. | if  $L[i] == x$ :
3. | | return  $i$
4. return None

Does Alg 1 solve the computational problem of Search?

(2)

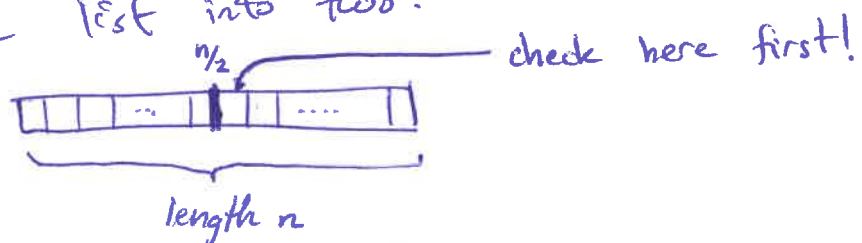
It does! This "check" requires a proof or a correctness proof as it's sometimes called. A correctness proof is a logical argument that convinces its readers that the algorithm solves (correctly!) the computational problem. This can be very challenging, which is why we like to try to keep algorithms ~~as~~ short — and hence computational problems simple and specific.

Is Alg1 a fast algorithm? What does this even mean?

- If  $L = [3]$  and  $x = 3$ , then it's essentially instantaneous.
- If  $L = [3, 4, \dots, 10^{100}]$  and  $x = 3$ , then same story.
- What about  $L = [1, 2, \dots, 10]$  and  $x = 10^{100} + 1$ ?
- $L = [1, 2, \dots, 10^{100}]$  and  $x = 10^{100} + 1$ ?
- What about  $L = [3, 4, \dots, 10^{100}]$  and  $x = 1$ ?

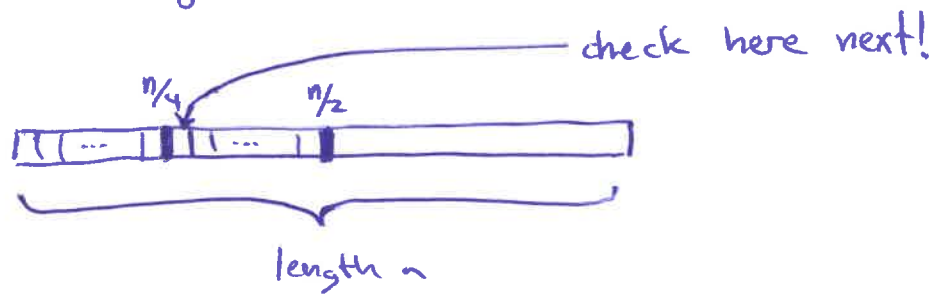
Notice in the last example we did not use the fact that we have a sorted-list. It should have been instantaneous, but it was not. Can we develop a better algorithm?

Let's divide the list into two:



③

Depending on the outcome, we shift down or up the list. For illustrating, let's assume our  $x$  is smaller.  
Next step:



Continue this process until we find a match or no such match exists.

(This is the algorithm. Code is not required to describe algorithms, but it can be very useful.)

### Algorithm 2:

#### Binary Search (Pseudo code)

1.  $low \leftarrow 0$
2.  $high \leftarrow \text{length}(L) - 1$
3. while  $low \leq high$
4.      $mid \leftarrow (low + high) / 2$
5.      $guess \leftarrow L[mid]$
6.     if  $x = guess$  then
7.         return  $mid$
8.     endif
9.     if  $x < guess$  then
10.          $high \leftarrow mid - 1$
11.     else
12.          $low \leftarrow mid + 1$
13.     endif
14. end while
15. return null

" $\leftarrow$ " means "assign".