

# Additional Problems

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## Problem 1

Show that the following hold.

- (i) A hyperplane in  $\mathbb{R}^n$  is a convex set.
- (ii) A close half-space in  $\mathbb{R}^n$  is a convex set.
- (iii) An intersection of a finite collection of convex sets in  $\mathbb{R}^n$  is convex.
- (iv) Suppose  $f : \mathbb{R}^a \rightarrow \mathbb{R}^b$  is a linear transformation. If  $S \subseteq \mathbb{R}^a$  is a convex set, show that the following set is convex:

$$\{f(v) \mid v \in S\} \subseteq \mathbb{R}^b.$$

## Problem 2

Consider the following LP

Maximize

$$z = 3x + 2y$$

subject to  $x, y \geq 0$  and

$$2x - y \leq 6,$$

$$2x + y \leq 10.$$

- (i) Transform this problem to a problem in canonical form.
- (ii) For each extreme point of the new problem, identify the basic variables.
- (iii) Solve the given LP.

### Problem 3

Consider the following simplex tableau that is missing its objective row and column.

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	
$x_4$	0	0	2	1	$5/2$	0	0	$6/7$
$x_1$	1	0	5	0	$-3$	0	$-2$	$2/7$
$x_6$	0	0	3	0	4	1	$-4$	$5/7$
$x_2$	0	1	0	0	$3/2$	0	0	$1/7$

Determine the departing variable if the entering variable is

- (i)  $x_5$ ;
- (ii)  $x_3$ ;
- (iii)  $x_7$ .

### Problem 4