The Mathematics of Decision Making I

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September 8, 2024

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1 Introduction

The mathematics of decision making is very closely tied to the field of mathematical optimization. One of the primary ways mathematics is used to help guide decisions is by maximizing (or minimizing) specific outcomes subject to a list of constraints. Mathematical optimization provides the formal tools to model and solve such problems.

There are many kinds of mathematical optimization. There are two basic types depending on whether the variables to optimize or discrete or continuous. A few types of optimization are[1]

- Linear Programming,
- Integer Programming,
- Stochastic programming,
- Combinatorial optimization,
- Dynamic programming.

Unsurprisingly there are many real-world applications; to list a few we have network optimization, pricing strategy, scheduling, supervised machine learning training, supply chain optimization, and transportation problems.

In this module, we will introduce the fundamentals of **linear programming**, also called *linear optimization* and *operations research*, such as the simplex method, polyhedral geometry, and the notion of duality. Depending on the time, we may also delve into **integer programming**.

[1] "Program" is not a computer program but comes from the United States military's use of the word for training and logistics schedules.

1.1 History

Mathematical optimization has quite an interesting history. In the 17th century, combinatorial optimization problems were solved using game theory, combinatorics, and ad hoc methods. In the 19th century, transportation problems involving post and rail were studied and solved. And in the 20th century with the two World Wars and rise of the assembly line, operations research took off developing the mathematics for all kinds of optimization problems.

One of the most influential figures in mathematical optimization, and linear programming in particular, is George Dantzig. He was the recipient of the President's National Medal of Science in 1975 [2] and was credited for

> *inventing linear programming and discovering methods that led to wide-scale scientific and technical applications to important problems in logistics, scheduling, and network optimization, and to the use of computers in making efficient use of the mathematical theory.*

The proof of the simplex method was classified and proved in 1960 by the Mathematics Division of the RAND Corporation [1].

1.2 Five examples

References

[1] G. B. Danzig. Inductive proof of the simplex method. https://apps.dtic.mil/sti/tr/pdf/AD0224306.pdf, 1960.

[2] National Science Foundation. The President's National Medal of Science: Recipient Details. https://www.nsf.gov/od/nms/recip_details.jsp?recip_id=95. Accessed: 2024-09-08.