Additional Problems

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Problem 1

Show that the following hold.

(i) A hyperplane in $\mathbb{R}^n$ is a convex set.

(ii) A close half-space in $\mathbb{R}^n$ is a convex set.

(iii) An intersection of a finite collection of convex sets in $\mathbb{R}^n$ is convex.

(iv) Suppose $f : \mathbb{R}^a \to \mathbb{R}^b$ is a linear transformation. If $S \subseteq \mathbb{R}^a$ is a convex set, show that the following set is convex:

$$\{f(v) \mid v \in S\} \subseteq \mathbb{R}^b.$$

Problem 2

Consider the following LP

Maximize

$$z = 3x + 2y$$

subject to $x, y \geqslant 0$ and

$$\begin{aligned} 2x - y &\leqslant 6, \\ 2x + y &\leqslant 10. \end{aligned}$$

(i) Transform this problem to a problem n canonical form.

(ii) For each extreme point of the new problem, identify the basic variables.

(iii) Solve the given LP.

Problem 3

Consider the following simplex tableau that is missing its objective row and column.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	0	0	2	1	5/2	0	0	6/7
x_1	1	0	5	0	-3	0	-2	2/7
x_6	0	0	3	0	4	1	-4	5/7
x_2	0	1	0	0	3/2	0	0	1/7

Determine the departing variable if the entering variable is

(i) x_5;

(ii) x_3;

(iii) x_7.

Problem 4

Suppose $A \in \mathrm{Mat}_{m\times n}(\mathbb{R})$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$. Consider the following LP.

Maximize

$$z = c^\top x$$

subject to $x \geqslant 0$ and

$$Ax \leqslant b.$$

Show that if u and v are feasible solutions, then the following point is a feasible solution:

$$w = \frac{1}{3}u + \frac{2}{3}v.$$

Problem 5

Use one iteration of the Simplex Algorithm to obtain the next tableau from the following.

	x_1	x_2	x_3	x_4	x_5	
x_3	2/3					