Algorithms

- #DMFS

Augmented matrices

- Special type of matrix where we "augment" it by adding a column separator between one or more rows. #Matrices #Elementary Row Operators

Bases

- Let U be a subspace in R^n, a finite set of the subspace will be a base
- n vectors can at most span R^n
- All vectors in the base are independent and the base Span the space
- A subspace will always have infinitely many subspaces, unless U is 0
- A standard base will contain the elementary vectors where the n'th vector contains a 1 at the n'th position. Analogous to the Identity Matrix
- Given a Linear combination of Vectors , any given point in the Coordinates system can be represented by the coordinates in relation to the base, aka number of steps used of each Basis vector. If we have vector the basis vectors b_1 and b_2 and e, to find e we solve the matrix $[b_1, b_2|e]$
- $\mathcal{B}$ is a base for $\mathcal{U}$ if span of the Basis vectors are equal to the subspace
- Change of basis
- The Dimensionality of a basis is the number of vectors in said base

Basis vector

- A vector in a Bases

Change of basis

- Given two ordered Bases $\mathcal{B}, \mathcal{C}$, the shift matrix from C to B is defined as follows: $P_{\mathcal{C}\to\mathcal{B}} := ([c_1]_{\mathcal{B}}|...|[c_k]_{\mathcal{B}})$ It satisfies $[V]_{\mathcal{B}} = P_{\mathcal{C}\to\mathcal{B}}[V]_{\rfloor}\ \forall v \in \mathcal{U}$ where $\mathcal{U}$ is a mutual Subspaces
- Calculation of basis shift:
  - Solve for all the vectors of C in respect to a linear combination of the B vectors

Eksempel (Basisskift matrix) 3/3

Vi har

$$\mathbf{v} = \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\0\\1\end{pmatrix} - 3\begin{pmatrix}1\\1\\0\end{pmatrix} + 2\begin{pmatrix}1\\2\\0\end{pmatrix} = \mathbf{b}_1 - 3\mathbf{b}_2 + 2\mathbf{b}_3$$

$$\mathbf{v} = \begin{pmatrix}1\\1\\1\end{pmatrix} = 4\begin{pmatrix}1\\4\\1\end{pmatrix} - 3\begin{pmatrix}0\\2\\1\end{pmatrix} - 3\begin{pmatrix}1\\3\\0\end{pmatrix} = 4\mathbf{c}_1 - 3\mathbf{c}_2 - 3\mathbf{c}_3$$

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$$[\mathbf{v}]_{\mathcal{B}} = \begin{pmatrix}1\\-3\\2\end{pmatrix} \quad \text{og} \quad [\mathbf{v}]_{\mathcal{C}} = \begin{pmatrix}4\\-3\\-3\end{pmatrix}.$$

Og ganske rigtigt gælder

$$\mathbf{P}_{\mathcal{B}\leftarrow\mathcal{C}}[\mathbf{v}]_{\mathcal{C}} = \begin{pmatrix}1 & 1 & 0\\-6 & -6 & -1\\5 & 4 & 2\end{pmatrix}\begin{pmatrix}4\\-3\\-3\end{pmatrix} = \begin{pmatrix}1\\-3\\2\end{pmatrix} = [\mathbf{v}]_{\mathcal{B}}.$$

  -
- Mnemonic for finding $P_{B\leftarrow A}$ the matrix to solve is $[B|A]$
- The Inverse is a change of basis from the standard basis to a given basis $\mathcal{B}$.

Coefficient matrices

- The left part of an augmented matrix representing a System of linear equations #Matrices

Column space

- Column space is the Span of all the column vectors in a matrix
- The columns containing free variables are in the span of the other vectors in the set
- To find the column space of A, put the matrix in Reduced Row Echelon, all the columns containing a pivot should be picked from the original matrix, these columns make up the Span of the column space.

Dijkstras

- This algortihm operates on Directed Acyclic Graphs

Dimensionality

- Dimensionality is the number of free variables
- Dimensionality is the number of columns in A - rank of A
- $dim(row(A) = rank(A)$

Directed Acyclic Graphs

- This is a special category of Graphs, as the name implies, this type of graph does not contain any cycles, which allows it various algortihms to operate on it that would otherwise never terminate, such as Dijkstras.
- Trees are DAGs

DMFS

- TODO: ADD ALL THIS
  - RAM model
  - Stacks,queues, sets, sequences, sums
  - Logic
  - Proofs
  - Strong Induction <-> Induction
  - Combinatorics, probability theory
  - Order relatins, posets, trees
  - Lexical analysis, regex and finite automata
  - Syntax analysis, CFGs
  - Heaps and priority queues

Echelon Forms

- Row Echelon is a more general case of RRE where the following has to hold
  - All rows consisting of zeroes are at the bottom
  - Every column containing a leading entry has only zeroes below it
- Reduced Row Echelon can be computed using Gauss-Jordan elimination on a System of linear equations
  - In contrast to Row Echelon, this also requires every leading entry to be equal to 1
  - All values above a leading entry must also be 0

Elementary matrices

- A matrix which when premultiplied to a given matrix

  $$A$$

  results in a new matrix

  $$A'$$

  which is equivalent to applying the ERO that created the elementary matrix.

Elementary Row Operators

- EROs are used to solve System of linear equations with Gauss-Jordan elimination
- An elementary matrix can be created by applying an ERO to the Identity Matrix
- The following EROs exist:
  - Addition: $cr_j + r_i \to r_i$
  - Swap: $r_i \leftrightarrow r_j$
  - Scaling: $cr_i \to r_i$, where c is non-zero
- Likewise, the inverse EROs exist:
  - Subtraction: $-cr_j + r_i \to r_i$
  - Swap: $r_i \leftrightarrow r_j$
  - Scaling: $\frac{1}{c} r_i \to r_i$, where c is non-zero.
- For any matrix A it holds that applying the ERO followed by the inverse ERO will result in A

Gauss-Jordan elimination

- GJ elimination can be used to convert a matrix into Reduced Row Echelon

Graph Algorithms

- A graph algorithm is an algorithm that can operate on a graph
- Kruskals
- Dijkstras
- Breadth First Search also known as BFS
- Depth First Search also known as DFS, is a special case of Dijkstras where all edge weights are identical.

Graphs

- Graphs can operated upon by various Graph Algorithms such as Depth First Search, Dijkstras or Kruskals
  -
- Adjacency Matrix
  - To represent a graph as a matrix named A, you make an $n \times n$ matrix, if the graph has n nodes. Then for all edges going from $i \to j$ we set $A[i, j] := 1$ and otherwise.
- Strongly connected components
  - TODO
- #Trees
  -
-

Idempotent matrices

- A matrix A is idempotent $\iff A^2 = A$
- Idempotent matrices are always Square matrices

Identity Matrix

- The identity matrix is the zero matrix with ones on the diagonal.
- An

  $$n \times n$$

  identity matrix is denoted

  $$I_n$$

- This matrix has the interesting property of being the multiplicative identity of all square matrices. This means that if

  $$A$$

  is an

  $$m \times n$$

  matrix then the following holds:

  $$AI_m = A = I_n A$$

- The Rank of the identity matrix is equal to its size
- The ID matrix is idempotent, meaning all rows and columns are independent and that

  $$I_n^k = I_n \ \forall k \in \mathbb{N}$$

- The identity matrix is the only idempotent that is invertible Proof: Suppose we have an idempotent matrix $A = A^2$

  $$AA^{-1} = I$$

  $$A^2 A^{-1} = I$$

  $$A = I \ \blacksquare$$

Image

- The image of a linear transform is the Span of the Vectors of the transform, aka what can we reach with a given transform.

Inverse

- A Matrix A, a $n \times n$ matrix is said to be invertible if there exists a matrix A^{-1} so that $A^{-1}A = I_n = AA^{-1}$
- A Square matrices will always have both a left and right inverse
- Left and right inverses
  - A matrix does not necessarily have both inverses
  - If $A^{-1}A = I_n$ then then A has a left inverse
  - If $AA^{-1} = I_n$ then then A has a right inverse
- Solving a System of linear equations can be done using the inverse of matrix

$$Ax - b = 0$$

$$A^{-1}(Ax - b) = 0$$

$$x - A^{-1}b = 0$$

$$A^{-1}b = x$$

Kernel

- The kernel of a Linear transformations is all of the values that get sent to 0. If the transform is a matrix, it is analogous to finding the Null space.

Linear combination

- #TODO

Linear dependence

- Opposite of Linear independence
- For a space of dimensionality n, there can be at most n vectors in the space, otherwise they will be dependent. This is since the DoFs are 0

Linear independence

- #Linear dependence
- A set of vectors is linearly independent if a linear combination of the vectors only has one solution, 0.
- If all scalars of the linear combination are 0, then it is linearly dependent
- If Rank of the array of the System of linear equations is equal to the number of equations, then it is independent
- A square matrix is invertible iff the columns or rows are linearly independent.
- Theorem 3.3

Linear transformations

- Linear transforms can be composed
- A transform T is linear if the following holds: $T(u + v) = T(u) + T(v)$ $T(cu) = cT(u)$
- Examples of linear transforms might be: Rotation matrices Inverse Elementary Row Operators Matrices
- The Rank of a linear transform is the Dimensionality of the Image
- The nullity is the Dimensionality of the Kernel

Matrices

tags:: Linear Algebra, DMFS

- A matrix is a $m \times n$ (m by n) rectangular array with mn entries, where
  - $m =$ **number of rows**
  - $n =$ **number of columns**.
- Matrices are commonly denoted by boldfaced capital letters: **A, B, C, D,** etc.
- An $m \times 1$ matrix is called a **column vector** or **m-vector** (see, Vectors).
- Matrices can also represent Graphs in the form of an Adjacency Matrix
- Eigenvectors #TODO

Null space

- Nullspace is a Subspaces of all values of x in $Ax = 0$
- Finding Bases for nullspace: Calculate Reduced Row Echelon read off the solutions of $Ax = 0$ The free variables are the basis vectors and are contained in the null space, the DOFs is equal to the Dimensionality

Polynomials

- Type of equation TODO

Principle Component Analysis

- Dimensionality reduction whic approximates a higher dimensionality space with a subspace

Proofs

- #DMFS #Algorithms Bla bla bla
  -

Rank

- Rank is the dimension of the Vector space spanned by its columns
  - This is the same as the maximum number of linearly independent columns of the matrix
-
- #System of linear equations
  - If the rank of the augmented matrix is greater than the coefficient matrix, then the system is inconsistent #Matrices
  - If the rank is equal, then it has at least one solution
-
- Def. 1 (p. 26)
  - Let $\mathbf{A}$ be an $m \times n$ matrix. The rank of $\mathbf{A}$, denoted by rank $\mathbf{A}$, is the number of nonzero rows in an echelon form for $\mathbf{A}$. We write rank $\mathbf{A} = r$. It also holds, for every matrix, that $r = n_p$ where n_p is the number of pivots. #definition
- Theorem 1 (p. 27)
  - Let $\mathbf{A}$ be an $m \times n$ matrix. The rank of $\mathbf{A}$, denoted by rank $\mathbf{A}$, holds rank $\mathbf{A} \leq \min\{m, n\}$.

Rank-nullity theorem

- The theorem states for Linear transformations that:
  - $T : V \rightarrow W$
  - $rank(T) + nullity(T) = dim(V)$
-
- For Matrices it states that:
  - $M \in \mathbb{R}^{m \times n}$
  - $rank(M) + nullity(M) = n$
  -

README

Public knowledge graph of various subjects on DIKU

How to use

- Install logseq https://logseq.com/downloads
- Import pages in the graph

Reduced Row Echelon

- A matrix is in reduced row echelon if it is
  - In Row Echelon
  - Leading entry is a one in each non-zero row
  - All columns containing a leading 1 has to be all zeroes otherwise
- Reduced row echelon, in contrast to Row Echelon, has only one unique matrix.

Row Echelon

tags:: Linear Algebra

-

Row space

- To find the row space of a matrix: Put the matrix into Reduced Row Echelon Read off the non-zero rows, these rows will make up the row space

Sorting algorithms

tags:: DMFS

- Sorting algorithms are used to sort structured data such as Matrices or Vectors.
  - Data must have a linear ordering (All elements in the data must have a unique ordering, Trees , Graphs and other non-linear data structures all violate this).
- #Quick Sort #Insertion Sort #Top Sort #Radix Sort

Span

- Let $\mathcal{S}$ be a set of Vectors in $\mathbb{R}^n$: Set span $\mathcal{S} = span\{v_1 ... v_k\} := \{x_1 v_1 + ... x_k v_k, x_k \in \mathbb{R}\}$
- Span is the set of all linear combinations of all vectors in the space.
- Span $\emptyset = \{0\}$
- You set up the span as a System of linear equations to figure out if the vector is in the span. If it has at least one solution, then the vector is in the span.
- $\mathcal{S}$ spans $\mathcal{U}$ if span S = U

Subspaces

- #Vector space #Bases #Span #Principle Component Analysis
- A subset $\mathcal{U} \subseteq \mathbb{R}^n$ is a subspace if the following holds:
  - $0 \in U$, origin is part of the subspace
  - $\forall u, v \in U$ holds that $u + v \in U$, subspaces are closed under addition
  - $\forall k \in \mathbb{R}, v \in U : kv \in U$, subspaces are closed under scalar multiplication
- A Span of vectors is always a subspace containing said vectors. (Theorem 3.2)
- The Dimensionality of a subspace is the number of vectors
- There are three categories of subspaces:
  - Null space
  - Row space
  - Column space

System of linear equations

tags:: Linear Algebra

- Is said to be in row echelon if its Augmented matrices is in Row Echelon
- A SoLE is said to be inconsistent if the canonical form contains one or more equations that can be reduced to 0=1
    - This also means it has zero solutions, and is equivalent to proving by contradiction that there are no solutions.
- A SoLE has infinite solutions if there are any free or independent variables
- A SoLE has one solution if there are no free variables and the system is consistent

test

test.md

- The kernel of a Linear transformations is all of the values that get sent to 0. If the transform is a matrix, it is analogous to finding the Null space.

Trees

- Minimum Spanning Trees
  - TODO

Vectors

tags:: Linear Algebra

- Vectors are a special case of Matrices

Vector space

- #Vectors
- Vector multiplication has vector addition (and the inverse), as well as scalar multiplication, analogous to Elementary Row Operators
    - Vector space (V) is represented as $\mathbb{R}^n$ and has to hold the following properties from Def 7.1:
        - Addition: $V \times V \to V$
        - Scalar multiplication $R \times V \to V$
- Matrices are a vector space since matrices are a generalised vector.
- Polynomial space is an abstract vector space
- Functions are also within a abstract vector space if they are defined across a set
- Theorem 3.1 contains various operations and identities for vector space