2 Problem 2.1

2. We interleave recursion of our Cyclic Towers of Hanoi and Double Cyclic Towers of Hanoi algorithms as follows, where our inductive hypothesis becomes that both follow the recursive spec for for n. The recursive spec and algorithm for Double Cyclic Towers of Hanoi is given in part 3.

   $ToHc(n \in \mathbb{Z}^+, A, B, C)$: given posts A, B, C in cyclic order, that is, B is cyclically next from A, C is cyclically next from B, and A is cyclically next from C, moves the top n discs from post A to post B cyclically in the minimal number of moves while maintaining sorted order at all times.

Algorithm 4 $ToHc(n \in \mathbb{Z}^+, A, B, C)$:

procedure ToHc($n \in \mathbb{Z}^+, A, B, C$)
    if $n \leq 0$ **then**
        return
    end if
    ToHdc($n-1, A, B, C$)
    move top disc $A \to B$
    ToHdc($n-1, C, A, B$)
end procedure

3. $ToHdc(n \in \mathbb{Z}^+, A, B, C)$: given posts A, B, C in cyclic order, that is, B is cyclically next from A, C is cyclically next from B, and A is cyclically next from C, moves the top n discs from post A to post C cyclically in the minimal number of moves while maintaining sorted order at all times.

Algorithm 5 $ToHdc(n \in \mathbb{Z}^+, A, B, C)$:

procedure ToHdc($n \in \mathbb{Z}^+, A, B, C$)
    if $n \leq 0$ **then**
        return
    end if
    ToHdc($n-1, A, B, C$)
    move top disc $A \to B$
    ToHc($n-1, C, A, B$)
    move top disc $B \to C$
    ToHdc($n-1, A, B, C$)
end procedure

4. $ToHthick(n \in Z^+, A \in [A, B, C], B \in [A, B, C], C \in [A, B, C])$: move the top $3n$ rings, where there are 3 rings of each size n, from post A to post B, maintaining sorted order at all times, as fast as possible

Algorithm 6 $ToHthick(n \in Z^+, A \in [A, B, C], B \in [A, B, C], C \in [A, B, C])$:

```
procedure ToHthick(n ∈ Z+, A ∈ [A, B, C], B ∈ [A, B, C], C ∈ [A, B, C])
    if n > 0 then
        ToHthick(n − 1, A, C, B)
        move top 3 rings from A to B
        ToHthick(n − 1, C, B, A)
    end if
end procedure
```

5. $ToH3(n \in \mathbb{Z}^+, A, B, C)$, given that the smallest $3n$ discs are on post A, distributes the top $3n$ discs from post A, among A, B, C such that all posts have exactly one disc of each size $\leq n$, in the minimal number of moves while always maintaining sorted order of the discs.

Algorithm 7 $ToH3(n \in Z^+, A, B, C)$:

```
procedure ToH3(n ∈ Z+, A, B, C)
    if n ≤ 0 then
        return
    end if
    ToHthick(n − 1, A, C, B)
    move top ring A → B
    ToHthick(n − 1, C, B, A)
    move top ring A → C
    ToH3(n − 1, B, C, A)
end procedure
```

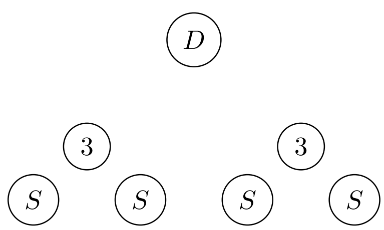
6. $murica(n \in Z^+, A \in [RED, WHITE, BLUE], B \in [RED, WHITE, BLUE], C \in [RED, WHITE, BLUE])$: maintaining sorted order at all times, sorts the top n rings in terms of color as fast as possible

Algorithm 8 $murica(n, A, B, C)$:

procedure MURICA(n, A, B, C)
    if $n > 0$ **then**
        let r be the bottomost ring of the top n rings
        if $r.color = A$ **then**
            $murica(n-1, A, B, C)$
        else
            let $X \leftarrow$ whatever both A and $r.color$ are not ($X \in [RED, WHITE, BLUE] - A - r.color$)
            $ToH(n-1, A, X, r.color)$
            move the top ring on A (which will be r) to $r.color$
            $murica(n-1, X, A, r.color)$
        end if
    end if
end procedure

8. *Bowling Pins of Hanoi.* Imagine a set of bowling pins with 3 pins taken out. The middle pin from the third row, and both pins from the row with two pins are also gone. Now transform the remaining pins into poles. All four of the pins in the bottom row (marked S) have n sorted rings, design an algorithm to move all the rings to the top pin (marked D), maintaining sorted order at all time, as fast as possible[1].

(*Hint:* A good subproblem to consider would be how to combine two poles with rings into one poll... But would doing 3 sets of 2 combines be faster than doing one set of combining 4?)



[1] Extra nuance can be added by introducing a cyclic(ish) order to ring movements like 2.1.2, rings at l_2 can only go to l_1, $l_1 \to l_0$, and $l_0 \to l_2$