Exercise 12.10 Let $G = (V, E)$ be a directed graph with m edges and n vertices, where each vertex $v \in V$ is given an integer label $\ell(v) \in \mathbb{N}$. The goal is to find the length of the longest path[3] in G where the labels of the vertices are (strictly) increasing.

[3]Recall that a path is a walk that does not repeat vertices.

Exercise 12.10.1. Suppose G is a DAG. For this problem, either (a) design and analyze a polynomial time algorithm (the faster the better), or (b) prove that a polynomial time algorithm would imply a polynomial time algorithm for SAT.

Solution. □

Exercise 12.10.2. Consider now the problem for general graphs. For this problem, either (a) design and analyze a polynomial time algorithm (the faster the better), or (b) prove that a polynomial time algorithm would imply a polynomial time algorithm for SAT.

Solution. □

Exercise 12.10.3. Suppose instead we ask for the length of the longest path in G where G is a general graph and the labels of the vertices are weakly increasing.[4] For this problem, either (a) design and analyze a polynomial time algorithm (the faster the better), or (b) prove that a polynomial time algorithm would imply a polynomial time algorithm for SAT.

[4] A sequence $x_1, \ldots, x_k$ is weakly increasing if $x_1 \leq x_2 \leq \cdots \leq x_k$.

Solution. □