13.4

A polynomial time algorithm for this would imply a polynomial time algorithm for SAT. Duplicate G call this G_1, for $v \in G$, let v_1 represent the duplicated note in G_1. Now for each v_1 connect v_1 to $w \in G$, if $(v, w) \in E$, also connect v_1 to v. So what we have done is essentially duplicated each node and connected them all up.

I claim G' has an almost independent set of cardinality equal to double the cardinality of the maximal independent set of G, call the cardinality of the maximal independent of G as m, and the cardinality for the almost independent set of cardinality for G' as m'.

The $2m \leq m'$ is clear since we can take the independent set S for G and just select the duplicated nodes in G' to get an S' which is an almost independent set for G' of cardinality $2m$.

I now claim that for any maximal cardinality almost independent set for G',S', if say $v \in S'$ and $w_1 \in S'$, with $(v, w) \in E$, then there is an S'' which is the same except without w_1 and with v, this proves that the vertices in a maximal cardinality almost independent come in pairs.

Note that the subgraph of v, v_1, w, w_1 is a complete graph. So suppose that $v, w_1 \in S'$, then if we change this to $v, v_1 \in S''$, then S'' is still an almost independent set, this is clear since v has at most one neighbor in S' which is w_1, now v_1.

Additionally, if a $v \in S$ does not have a neighbor in S, then it isn't of maximal cardinality since I can add v' to S and still have it be an almost independent set.

Now will that done, the conversion from G to G' takes $O(n + m)$, the almost independent set algorithm is presumed to take polynomial time, and the decoding take $O(1)$. Hence, this would imply a polynomial runtime for maximal independent set, implying a polynomial runtime for SAT.