Josh Park, Amy Kang, Diya Singh Prof. Kent Quanrud CS 390ATA Homework 7 (16.6)

 $\begin{array}{c} \mathbf{Spring} \ \mathbf{2025} \\ \mathbf{Page} \ \mathbf{1} \end{array}$

Exercise 16.6. Let G = (V, E) be an undirected graph with distinct nonnegative edge weights $w : E \to \mathbb{R}$. For a spanning tree T, we say that the *bottleneck weight of* T is the maximum weight edge in T, $\max_{e \in T} w(e)$.

Exercise 16.6.1. Prove that the MST is also a minimum bottleneck weight spanning tree of G.

Solution.

CS 390ATA Homework 7 (16.6)

Spring 2025 Page 2

Exercise 16.6.2. Design and analyze a O(m+n)-time algorithm for computing a minimum bottleneck weight spanning tree of G. (This is faster than any of our algorithms for MST.)⁴

⁴Here's step 1: compute the median edge weight in O(m) time.

 \Box